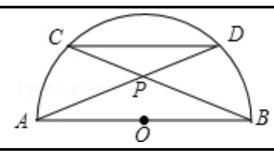
#### **Input Image**



• Question: Given a semicircular figure with the diameter represented by AB and intersecting chords AD and BC at point P, where angle DPB measures 60.0 degrees, calculate the value of tan(angle ADC) under the condition that D denotes the midpoint of the arc BC. Choices: A:  $\frac{1}{2}$ , B:

2, C: 
$$\frac{\sqrt{3}}{3}$$
, D:  $\sqrt{3}$ 

• Original Answer: C

## VL-Thinking

exact arcs involved.

<think>



Okay, so I'm trying to solve this geometry problem. Let me see if I can visualize it correctly. There's a semicircle with diameter AB. Points C and D are on the curved edge, and chords AC and BD intersect at point P. Below AB, there's the center O. The angle DPB is 60 degrees, and D is the midpoint of arc BC. I need to find the value of  $tan(\angle ADC)$ .

is the center, so AO = OB. Points C and D are on the semicircle, with D being the midpoint of arc BC. That means arc BD is equal to arc DC. Since it's a semicircle, the total arc from B to C is 180 degrees, so each arc BD and DC would be 90 degrees each. Wait, no, because D is the midpoint of arc BC, which is part of the semicircle. So if BC is some arc, D is halfway along it. But I'm not sure about the exact measure yet.

First, I should probably draw this out to get a better sense. I imagine a semicircle with AB as the diameter. O

Chords AC and BD intersect at P, and angle DPB is 60 degrees. I remember that when two chords intersect, the products of the segments are equal. So  $AP \cdot PC = BP \cdot PD$ . But I'm not sure if that's directly useful here.

Since D is the midpoint of arc BC, it might have some special properties. In a semicircle, the midpoint of an

arc would be equidistant from B and C. Also, angles subtended by the same arc are equal. Maybe I can use some properties of cyclic quadrilaterals here, but since it's a semicircle, all points lie on the circumference, so ABCD is a cyclic quadrilateral. Wait, but ABCD is a quadrilateral inscribed in a semicircle, which makes it a cyclic quadrilateral. In such

with a semicircle, not a full circle. Let me think about the angles. Angle DPB is 60 degrees. Since P is the intersection of chords AC and BD, maybe I can use the property that the angle between two chords is equal to half the sum of the measures of

the intercepted arcs. So angle DPB would be half the sum of arcs DB and AC. But I'm not sure about the

cases, opposite angles sum to 180 degrees. But I'm not sure if that's applicable here because we're dealing

Alternatively, maybe I can use triangle properties. Since D is the midpoint of arc BC, triangle DBC might be isosceles. Wait, no, because D is on the semicircle, so triangle DBC is actually a right triangle because any triangle inscribed in a semicircle is a right triangle. So angle BDC is 90 degrees.

Wait, that's an important point. Since D is on the semicircle, triangle BDC is right-angled at D. So angle BDC is 90 degrees. That might help in finding angle ADC.

I need to find  $\tan(\angle ADC)$ . Let's denote angle ADC as  $\theta$ . So  $\tan(\theta)$  = opposite/adjacent. To find this, I might need to find the lengths of the sides of triangle ADC or use some trigonometric identities.

Let me consider the coordinates approach. Let's place the semicircle on a coordinate system with center O at (0,0), and AB as the diameter along the x-axis from (-r,0) to (r,0), where r is the radius. Then point D, being the midpoint of arc BC, would be somewhere on the semicircle. But without knowing the exact position of C, this might be tricky.

of point P with respect to the circle is equal for both chords. So  $PA \cdot PC = PB \cdot PD$ . But without specific lengths, I'm not sure how to proceed. Wait, maybe I can assign some variables. Let's let the radius of the semicircle be r. Then AB = 2r. Let me

Alternatively, maybe I can use power of a point. Since P is the intersection of chords AC and BD, the power

assume r = 1 for simplicity, so AB = 2. Then O is at (0,0), A is at (-1,0), B is at (1,0). Let me place point C somewhere on the semicircle, say at an angle  $\theta$  from the x-axis. Then point D, being the midpoint of arc BC, would be at an angle  $\frac{(\theta + 180)}{2}$  from the x-axis, but I'm not sure. Alternatively, since D is the midpoint of arc BC, the arc BD equals arc DC. So if I consider the central

angles, angle BOD equals angle DOC. Since the total arc BC is some measure, say  $2\alpha$ , then each arc BD and DC would be  $\alpha$ . But I'm not sure about the exact measure. Wait, maybe I can use the fact that angle DPB is 60 degrees. Using the intersecting chords theorem, angle

DPB is equal to half the sum of the measures of arcs DB and AC. So angle DPB =  $\frac{1}{2}$  (arc DB + arc AC) = 60

degrees. Therefore, arc  $DB + \operatorname{arc} AC = 120$  degrees. But I'm not sure about the measure of arc AC. Alternatively, maybe I can express arc AC in terms of other arcs. Since the total semicircle is 180 degrees, arc AB is 180 degrees. Arc AC is part of that, so arc

Alternatively, since D is the midpoint of arc BC, arc BD = arc DC. Let's denote arc  $BD = \operatorname{arc} DC = x$ . Then arcBC = 2x. Since the total semicircle is 180 degrees, arc AB = 180 degrees, so arc  $AC = 180 - \operatorname{arc}BC = 180 - 2x$ .

Now, from the intersecting chords theorem, angle DPB =  $\frac{1}{2}(\operatorname{arc}DB + \operatorname{arc}AC) = 60$  degrees. So  $\frac{1}{2}(x + (180 - 2x)) = 60$ . Simplifying,  $\frac{1}{2}(180 - x) = 60$ . Multiply both sides by 2: 180 - x = 120. So arc BD = arc DC = 60 degrees, and arc BC = 120 degrees. Then arc AC = 180 - 120 = 60 degrees. Now, I need to find tan(angle ADC). Let's consider triangle ADC. Since D is on the semicircle, angle ADC is an inscribed angle. The measure of angle ADC is half the measure of arc AC. Since arc AC is 60 degrees, angle

ADC = 30 degrees. Therefore,  $tan(angle ADC) = tan(30 degrees) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ . Wait, but let me double-check. If angle ADC is 30 degrees, then tan(30) is indeed  $\frac{\sqrt{3}}{3}$ . So the answer should

</think> The value of tan(angle ADC) is  $\frac{\sqrt{3}}{3}$ .

### 1. Understanding the Problem:

**Step-by-Step Explanation:** 

be option C:  $\frac{\sqrt{3}}{3}$ .

AC = 180 - arcCB. But I'm not sure.

- We have a semicircle with diameter AB and center O. - Points C and D are on the semicircle, with D being the midpoint of arc BC.

- We need to find  $tan(\angle ADC)$ .

2. Key Properties and Theorems:

- Angle DPB is formed by chords BD and AC, so:

- Intersecting Chords Theorem: The measure of an angle formed by two intersecting chords is half the sum of the measures of the intercepted arcs.

- Chords AC and BD intersect at point P, forming an angle DPB of 60 degrees.

3. Applying the Intersecting Chords Theorem:

- Midpoint of Arc: Since D is the midpoint of arc BC, arcs BD and DC are equal.

- Inscribed Angle Theorem: An angle inscribed in a circle is half the measure of its intercepted arc.

# $\angle DPB = \frac{1}{2}(\operatorname{arc}DB + \operatorname{arc}AC)$ - Given $\angle DPB = 60^\circ$ , we have: $60^\circ = \frac{1}{2} (\text{arc } DB + \text{arc } AC)$

$$arc DB + arc AC = 120^{\circ}$$

- Substituting into the earlier equation:  

$$\operatorname{arc} AC = 180^{\circ} - \operatorname{arc} BC = 180^{\circ} - 2x$$

 $arc AC = 180^{\circ} - arc BC = 180^{\circ} - 2x$ 

### 5. Finding Angle ADC:

- Angle ADC is an inscribed angle intercepting arc AC. - By the Inscribed Angle Theorem:
- $\angle ADC = \frac{1}{2} \times \operatorname{arc} AC = \frac{1}{2} \times 60^{\circ} = 30^{\circ}$

$$-\tan(30^{\circ}) = \frac{\sqrt{3}}{3}.$$

**Answer**:  $\frac{\sqrt{3}}{3}$  (Option C)