

## 1 Rules and Guidelines

1. Participants must correctly evaluate indefinite or definite, proper, single real variable integrals in the time allotted.
2. In each round, both participants will approach the board and an integral will be shown. The timer will then begin a 10 second countdown at their discretion, during which neither participant may write on the board. When the timer announces the end of the countdown, participants may begin writing their work and solutions on the board.
3. At any time during the round, a participant may circle or box their final solution. When the circle/box is completed, both participants must stop writing. The clock will be stopped, while the judge(s) consider the solution.
4. If at any point a boxed or circled solution is deemed to be correct, the round is over and the corresponding participant advanced to the next round.
5. If a boxed or circled solution is found to be incorrect, the timer will provide a 5 second countdown, after which point the clock will be restarted and the contestants may resume writing.
6. A competitor may present no more than two solutions per round. Two incorrect solutions does not disqualify a competitor, however - if their opponent does not arrive at a correct solution within the allotted time, both competitors may participate in an additional round.
7. Rounds will last at most 3 minutes. If no correct answer is arrived at by either contestant within this time, the same participants will start a new round with a new integral.
8. In your answers, it is not necessary to include the arbitrary constant  $C$  in an indefinite integral, nor the absolute value sign around the argument of a logarithm.

## 2 Prizes

1. Top 4:  
\$10 Gift Card to Jamba Juice
2. Top 2:  
Above, plus a math book
3. 1st Place:  
All of the above, plus The Grand Integrator's Hat and a box of Hagoromo Fulltouch Chalk

## 3 Misc.

1. Signup sheets located at <http://tinyurl.com/2017integrationbee>
2. General seeding order:

$$140/142 > 120 > 180 > 130 > 110 > 31 > 109 > 20d/e > 20c > 20b > 20a$$

3. Bracket generator located at [http://challonge.com/tournament/bracket\\_generator](http://challonge.com/tournament/bracket_generator)

## 4 Warmups

**Techniques:** Power rule, exponentials,  $u$ -substitution, single applications of integration by parts.

$$\int \sin x + \cos x + \csc x + \sec x \, dx = -\cos(x) + \sin(x) + \ln\left(\tan\left(\frac{x}{2}\right)\right) + \ln(\tan(x) + \sec(x)) \quad (1)$$

$$\int \frac{(9^3 + 10^3)x^{1728}}{\sin^2 x + \cos^2 x} \, dx = x^{1729} \quad (2)$$

$$\int \frac{x+1}{x^2+2x+3} \, dx = \frac{1}{2} \ln(x^2+2x+3) \quad (3)$$

$$\int \frac{x^{2017} \, dx}{(x^{2018} + \pi^{2018})} = \frac{1}{2018} \ln(x^{2018} + \pi^{2018}) \quad (4)$$

$$\int e^{e^x} e^x \, dx = e^{e^x} \quad (5)$$

$$\int \sin(\sin x) \cos x \, dx = -\cos(\sin x) \quad (6)$$

$$\int \sin(x) \cos(x) \cot(x) \tan(x) \, dx = \frac{\sin^2(x)}{2} \quad (7)$$

$$\int 5x\sqrt{49-4x^2} \, dx = -\frac{5(49-4x^2)^{3/2}}{12} \quad (8)$$

$$\int \cos^3(x) \sin(x) \, dx = -\frac{\cos^4(x)}{4} \quad (9)$$

$$\int \sin^2(\sin(x)) \cos(x) \, dx = \frac{1}{2} \left( \sin(x) - \frac{1}{2} \sin(2 \sin(x)) \right) \quad (10)$$

$$\int \frac{4x+6}{2x^2+5x-3} \, dx = \frac{2}{7} (3 \ln(4x+12) + 4 \ln(4x-2)) \quad (11)$$

$$\begin{aligned} \text{(alternatively)} \quad &= \frac{8}{7} \ln(1-2x) + \frac{6}{7} \ln(3+x) \end{aligned} \quad (12)$$

$$\int \frac{\cos(\ln(x))}{x} \, dx = \sin(\ln(x)) \quad (13)$$

$$\int \sin^2(x) \, dx = \frac{x}{2} - \frac{1}{4} \sin(2x) \quad (14)$$

$$\begin{aligned} \text{(alternatively)} \quad &= \frac{1}{2} (x - \cos(x) \sin(x)) \end{aligned}$$

$$\int x e^x \, dx = x e^x - e^x \quad (15)$$

$$\int \frac{2x^2 + 3}{x - 2} \, dx = x(4 + x) + 11 \ln(-2 + x) \quad (16)$$

$$\int x \ln(x) \, dx = \frac{1}{4} x^2 (-1 + 2 \ln(x)) \quad (17)$$

$$\int \frac{\ln(x) \, dx}{x^2} = -\frac{\ln(x)}{x} - \frac{1}{x} \quad (18)$$

$$\int_{-\pi}^{\pi} x \sin(x) \, dx = 2\pi \quad (19)$$

$$\int \frac{\cos(x) \, dx}{\sqrt{1 + 16 \sin^2(x)}} = \frac{1}{4} \ln \left( 4 \sin(x) + \sqrt{1 + 16 \sin^2(x)} \right) \quad (20)$$

$$\int \frac{x}{\sqrt{1 + x^2}} \, dx = \sqrt{1 + x^2} \quad (21)$$

## 5 Intermediate

**Techniques:** Non-obvious  $u$ -substitution, integration by parts with multiple steps, partial fraction decomposition, trigonometric identities

$$\int \frac{2x+6}{x^2+3x+2} dx = 4 \ln(x+1) - 2 \ln(x+2) \quad (22)$$

$$\int e^{12x} \sqrt{e^{12x} - \pi} dx = \frac{1}{18} \left( e^{12x} - \pi \right)^{3/2} \quad (23)$$

$$\int \frac{\ln(\ln(x))}{x} dx = \ln(x) [-1 + \ln(\ln(x))] \quad (24)$$

$$\int x^3 \cos(2x) dx = \frac{3}{8}(-1+2x^2) \cos(2x) + \frac{1}{4}x(-3+2x^2) \sin(2x) \quad (25)$$

$$\int \frac{14-7x}{2x^2+5x-3} dx = \frac{3}{2} \ln(2x-1) - 5 \ln(x+3) \quad (26)$$

$$\int \sin(\sqrt{x}) = -2\sqrt{x} \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \quad (27)$$

$$\int \tanh x dx = \ln(\cosh x) \quad (28)$$

$$\int \sec^8 x \tan x dx = \frac{\sec^8 x}{8} \quad (29)$$

$$\int x\sqrt{x+1} dx = \frac{2}{5}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{3/2} \quad (30)$$

$$\int \sin^2(x) \cos^2(x) dx = \frac{1}{8} \left( x - \frac{1}{4} \sin(4x) \right) \quad (31)$$

$$\int \pi^x dx = \frac{\pi^x}{\ln \pi} \quad (32)$$

$$\int x^2 \ln(x) dx = \frac{x^3}{3} \ln(x) - \frac{x^3}{9} \quad (33)$$

## 6 Hard

**Techniques:** "Clever"  $u$ -substitutions, Trigonometric substitutions, Hyperbolic trigonometric identities, adding "odd forms of zero", multiplying by "odd forms of one", integrals of powers of trigonometric functions, completing the square

$$\int \cos x \sqrt{\sin^2 x + 1} \, dx = \frac{1}{2} \sin(x) \sqrt{1 + \sin^2(x)} + \frac{1}{2} \ln \left( \sin(x) + \sqrt{1 + \sin^2(x)} \right) \quad (34)$$

$$\int \cosh^{-1}(x) \, dx = x \cosh^{-1}(x) - \sqrt{(x-1)(x+1)} \quad (35)$$

$$\int \frac{x^2}{1+x^2} = -\tan^{-1}(x) + x \quad (36)$$

$$\int \frac{1 + \sin x}{1 + \cos x} = -2 \ln \left[ \cos \left( \frac{x}{2} \right) \right] + \ln \left( \frac{x}{2} \right) \quad (37)$$

$$\text{also} \quad = \ln \left[ \tan^2 \left( \frac{x}{2} \right) + 1 \right] + \tan \left( \frac{x}{2} \right) \quad (38)$$

$$\int \frac{1}{1-x+x^2} = \frac{2}{\sqrt{3}} \arctan \left( \frac{2}{\sqrt{3}} \left( x - \frac{1}{2} \right) \right) \quad (39)$$

$$\int_{-2017}^{2017} \sin(\sqrt[3]{x}) \, dx = 0 \quad (40)$$

$$\int \frac{1}{1+e^x} \, dx = x - \ln(1+e^x) \quad (41)$$

$$\begin{aligned} \int (1+2x^2)e^{x^2} \, dx &= xe^{x^2} \\ \int \frac{e^{ix}}{x^2+1} \, dx &= \frac{\pi}{e} \end{aligned} \quad (42)$$

$$\int x(1-x)^{2017} = -\frac{1}{2}(1-x)^2 + \frac{1}{2019}(1-x)^{2019} \quad (43)$$

$$\int_{-\pi}^{\pi} \frac{x^3 - 2x}{\sqrt{x^4 + 1}} = 0 \quad (44)$$

$$\int \frac{1}{x(x^5+1)} \, dx = \frac{1}{5} \ln \left( \frac{x^5}{x^5+1} \right) \quad (45)$$

$$\begin{aligned} \text{also} \quad &= \frac{1}{5} [\ln(x^5) - \ln(x^5-1)] \end{aligned} \quad (46)$$

## 7 Challenging

$$\int \sinh x \sin x \, dx = \frac{1}{2}(\cosh x \sin x - \sinh x \cos x) \quad (47)$$

$$\int_{-\infty}^{\infty} \frac{1}{x^4 + 4} \, dx = \frac{\pi}{4} \quad (48)$$

$$\int \frac{\ln x \cos x - \frac{1}{x} \sin x}{\ln^2 x} \, dx = \frac{\sin x}{\ln x} \quad (49)$$

$$\int_{-\infty}^{\infty} \frac{1}{1 + x^2} \, dx = \pi \quad (50)$$

$$\int_0^{\infty} \frac{3\sqrt{3}}{1 + x^3} \, dx = 2\pi \quad (51)$$