Math 142a **Name**:

October 31, 1997

Midterm

1. (9 pts.) Give an example of a function $f(x)$ defined on $\{0 < x < 1\}$ which is continuous, but which is not bounded from below. (Hint: it's not hard.)

2. (12 pts.) Use the Mean Value Theorem to prove

$$\log x < x - 1 \quad \text{for } x > 1.$$

3. (15 pts) Prove directly from the definition of the limit of a sequence that if $\lim a_n = 1/2$ and $a_n > 1/5$ for all n, then $\lim \frac{1}{a_n} = 2$.

4. (15 pts) Find $\limsup a_n$ and $\liminf a_n$ for the sequence

$$a_n = (-1)^n \frac{\sin \frac{1}{n}}{\frac{2}{n}} + \frac{\sin 2n}{n}$$

(Show work. You may use the identity $\lim_{x \to 0} \frac{\sin x}{x} = 1$ without proving it.)

5. (15 pts) Let

$$a_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}}.$$

(i) Show that $a_n \leq 1$ for each n.

(ii) Show that $a_n \geq \frac{1}{\sqrt{1+\frac{1}{n}}}$

(iii) Find $\lim a_n$.

6. (15 pts) Find $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x$.

7. (9 pts.) Suppose that f is a continuous function on $I = \{a \leq x \leq b\}$ such that $f(a) = 1$ and $f(x) \neq 0$ for all $x \in I$. Show that $f(x) > 0$ for all $x \in I$. (Hint: cite a theorem.)

8. (9 pts.) If $f(x) = |x|$ for $-1 \leq x \leq 1$, then $f(-1) = f(1)$, but there is no x_0, $-1 < x_0 < 1$ for which $f'(x_0) = 0$. Explain why the Mean Value Theorem does not apply to this example.