

Name: _____

Section:

Final

Math 20B

VERSION A

Part I

1. Which of the following identities are true?

(a) $\int (\sin x + x \cos x) dx = x \sin x + 5 + C$

(b) $\int 2e^4 x e^{x^2} dx = e^{(x^2+4)} + C$

(c) $\int \sin x \cdot e^{2x} dx = e^{2x} \cos x + 2e^{2x} \sin x + C$

(d) $\int \frac{x+1}{x^2} dx = -\frac{1}{x} + \ln(x) + 5 + C$

Please justify your answers.

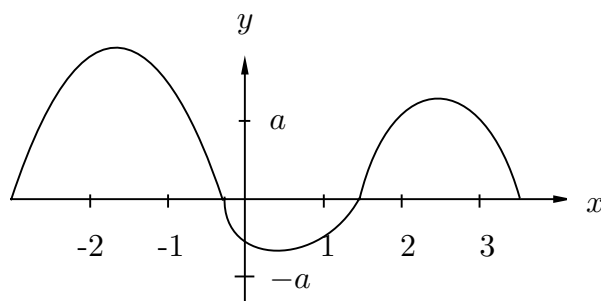
2. Find the integrals:

(a) $\int x \sin x^2 \, dx$

(b) $\int \frac{5x + 3}{x^2 + 4} dx$

3. The function x^3 is rotated around the x -axis and this surface bounds a solid. What is the volume of the solid between $x = 0$ and $x = 2$? Set up the integral clearly. Then integrate it by hand (not using the numerical integration program).

4. The graph of a function $f(x)$ is shown below:



What are the signs (+ or -) of the following definite integrals:

(a) $\int_{-2}^0 f(x) \, dx :$

(b) $\int_{-2}^1 f(x) \, dx$

(c) $\int_0^1 f(x) \, dx :$

(d) $\int_{-2}^2 f(x) \, dx.$

(Note: You will receive no credit for this problem if any of your answers is incorrect.)

5. Evaluate the following definite integral (without calculators):

$$\int_0^\pi \frac{d}{dx} \left(x^2 e^{-x^2} \right) dx$$

Part II

6. A population grows in proportion to its size.
- (a) What differential equation does this give?
 - (b) Initially, the population is 5,000,000 and it is changing at the rate of 1 500 persons per day. What is the population after 5 years?
7. Compute the area of the ellipse (without calculators):

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

8. After Mt. St. Helens erupted in 1980, it was found that ash was spread in decreasing density as a function of distance r from the center of the crater. Say that the density δ of ash at a distance r (meters) from the center of the crater is given as follows:

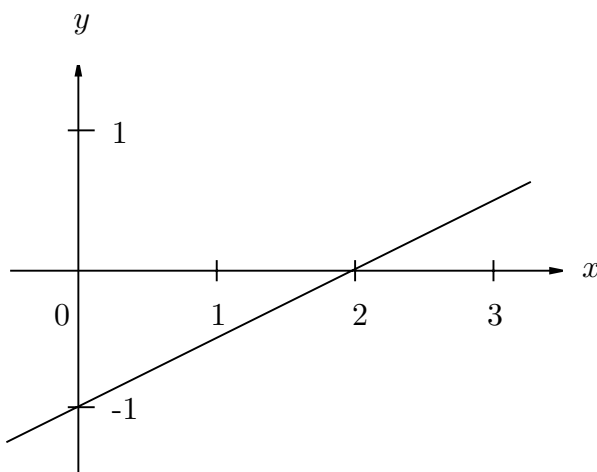
$$\delta(r) = \frac{2000}{1 + r^2} \text{ kg/m}^2$$

- (a) Write a Riemann sum that approximates the total mass of ash deposited within a 1000-meter radius of the center of the crater.
 - (b) Turn your Riemann sum from part (a) into a definite integral and evaluate that integral to find the exact value of the total mass of ash with 1000 meters of the center of the crater.
9. Consider the following Equations:

$$\begin{cases} \frac{dy}{dx} = 2x + y, 0 \leq x < \infty, \\ y(0) = 3. \end{cases}$$

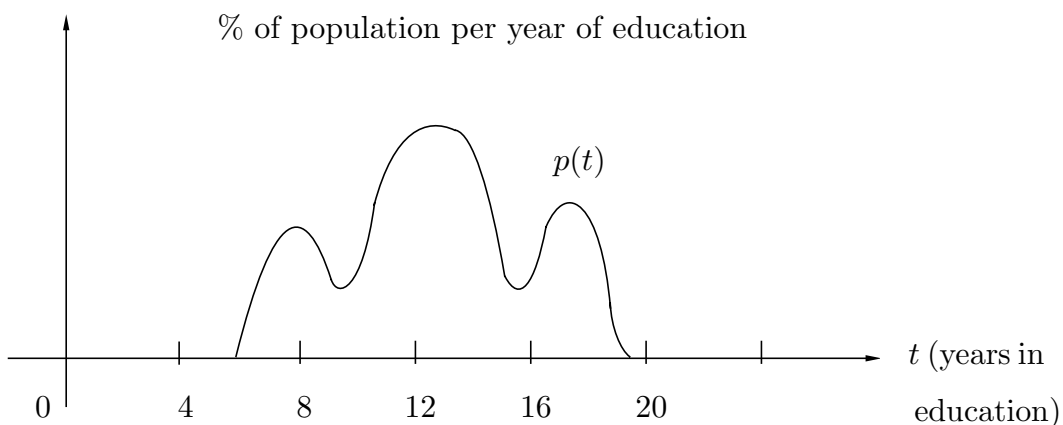
Start from the point $(x_0, y_0) = (0, 3)$ and take $\Delta x = 0.2$. Use Euler's method to find the approximate value of $y(0.6)$. (Calculators allowed only for the purpose of additions, subtraction, multiplication and division. Please show the idea of Euler's method.)

10. The graph of a function in $y = f(x)$ is the line shown below:



Consider the integral $\int_0^3 f(x)dx$. What do LEFT(4), RIGHT(4), MID(4) mean? Compute their values by hand. (Show all your steps, no calculators allowed for this problem.)

11. The graph below shows the distribution of the number of years of education completed by adults in a population.



- What is your best estimate of the percentage of adults who has completed between 14 and 16 years of education?
- What is your best estimate of the percentage of adults who has completed between 8 and 12 years of education?