Calculus

1. Limits

- Keywords
    - Converge/Diverge
    - Monotonic
- Theorems/Properties
    - Every convergent sequence is bounded.
    - If a sequence is monotonic and bounded, it's convergent.
    - Sums/Products/Quotients(with nonzero denominator) of convergent sequences converge.
    - For k>0, 1/n^k goes to 0. |k|>1, 1/k^k goes to 0.
    - Squeeze Theorem
- Limits of functions
    - Right hand/left hand limit
    - Epsilson/delta definition
    - Limits at infinity
    - Continuity
        - Can't divide by zero, take square root of negative, can't take log of 0 or negative
    - Theorems of Continuous Functions
        - Extreme Values
        - Intermediate value theorem

2. Derivatives

- What is derivative
    - Limit definition
    - Slope of Tangent line
    - When is there not a derivative (limit doesn't exist)
    - If not continuous then not differentiable (If differentiable, then continuous).
- Rules for finding derivatives
    - Sum/Product/Chain
    - Inverse Function
    - Remember Derivatives of trig functions, log, exp etc.
- Find equation of tangent line
    - Tangent line gives an approximation for the function
- Implicit Differentiation
- Higher Order Derivatives
- Curve Sketching
    - First Derivative
        - Increasing/Decreasing
        - Critical Points - Local Min/max
    - Second Derivative
        - Concavity

        - Inflection Points
        - Second derivative Test - classifying CPs
    - Can use curve sketch to find number of zeroes of a function
- Therorems of Differentiablity
    - Mean Value Theorem
        - Special Case - Rolle's Thm
- Min/Max Problems
    - Global Max/Min can occur at critical points and along boundary
    - On closed bounded domain, always have global min/max
- Related Rates
    - Write an equation relating your variables. Take the derivative. Solve for unknown.

3. Integrals

- Techniques
    - U-substitution
    - Intergration by parts
    - Trig Sub
        - sqrt(a^2-u^2) u=a sin x
        - sqrt(a^2+u^2) u=a tan x
        - sqrt(u^2 -a^2) u=a sec x
    - Partial Fractions
        - Degree of numerator one less than degree of denominator
        - If you have a term to a power (say n) include it it the first, second, ... to the nth
- Definite Integration
    - area under a curve
    - Riemann Sums
    - Limit Definition
- Fundamental Theorem of Calculus
    - Integral of Derivative (1st Fund Thm)
    - Derivative of Integral (2nd Fundamental Thm)
        - Chain rule
- Average Value of a function
- Area between curves
- Polar Coordinates
    - x=r*cos(t) y=r*sin(t)
    - r= sqrt(x^2+y^2) tan (t)=y/x
    - Area of equation or r in terms theta A= $\int_{\alpha}^{\beta} 1/2r^2 \, d\theta$
- Volumes of Solids of Revolution
    - V= $\int_{a}^{b} \pi[f[x]^2] dx$

- Arc Length
- Exponentials and Logs
    - e=$\lim_{n \to \infty}$(1+1/n)^n
    - log x = $\int_1^x 1/t$ *dt*
    - We can use logarithmic differentiation to solve for derivatives of functions like $f(x)^{g(x)}$. Write this as $e^{g(x) log\, f(x)}$. Then use implicit differentiation.
- L'Hopital's Rule
    - Need 0/0 or ∞ / ∞
    - $\lim_{x \to a} f(x)/g(x) = \lim_{x \to a} f'(x)/g'(x)$
- Improper Integrals
    - If either bound is $\pm\infty$or the function is undefined at the endpoint then the integral is improper
    - Let the bad bound be r. Take the limit as r goes to whatever it needs to.
    - Make sure to check that the function is defined for the entire interval. You may need to split it into two improper integrals.
- Infinite Series
    - Adding an infinite Sequence
    - Geometric Series
        - Common ratio
        - $\sum_{n=0}^{\infty} r^n = 1/(1-r)$
    - Divergence Test
        - if the terms don't go to 0, the sum can't
        - Converse is False e.g. Harmonic Series
    - p-series
    - Comparison Test
    - Ratio Test
        - Let $\lim_{n \to \infty} a_{n+1}/a_n$=L
        - If L<1 $\sum a_n$ converges, if L>1 $\sum a_n$ diverges
        - L=1 inconclusive
    - Root Test
        - Let $\lim_{n \to \infty} \sqrt[n]{a_n} = L$
        - If L<1 $\sum a_n$ converges, if L>1 $\sum a_n$ diverges
        - L=1 inconclusive
    - Integral Test

        - Let f(x) be positive monotonically decreasing function for $x \geq 1$ such that $f(n) = a_n$
        - $\sum_{n=1}^{\infty} a_n$ converges $\Leftrightarrow$ $\int_1^{\infty} f(x)dx$ converges
    - Alternating Series Test
        - $\sum_{N=1}^{\infty} (-1)^{n+1} a_n$ converges if a_n is a positive decreasing sequence that converges to 0
        - A series that converges but whose absolute value does not converge is called a **conditionally convergent** series
        - If a series and its absolute value converge is called **absolutely convergent**
- Power Series
- Taylor Series