

Place all books, papers and calculators under your desk. Write your name and section number on your blue book . In a column number one to four. Box your answers and show all your work. Good Luck!

1. (a) 5 pts. Find 3 complex numbers that satisfy  $z^3 = i$ .  
  
(b) 5 pts. Find 2 unit vectors that make an angle of  $\pi/6$  with  $\vec{j}$ .  
  
(c) 5 pts. Find the equation of a plane perpendicular to  $2\vec{i} + \vec{j} - \vec{k}$  passing through  $(1, 2, 3)$ .  
  
(d) 5 pts.  $P = (2, -1, 0)$ ,  $Q = (-1, 0, 3)$ ,  $R = (2, 1, 1)$  are vertices of a parallelogram. If  $S$  is the 4th vertex opposite  $P$ , find  $S$ .
2. (a) 10 pts. Let  $f(x, y) = x^2 e^y + \cos(xy)$ . Let  $x(t) = \sqrt{t}$ ,  $y = t^2$ . Compute  $f_x$ ,  $f_y$ ,  $f_{xy}$ . Use the chain rule to find  $\frac{df}{dt}$ .  
  
(b) 10 pts. A function  $g(x, y)$  has directional derivative  $2\sqrt{2}$  at  $(1, 2)$  in the direction of  $\vec{i} + \vec{j}$  and directional derivative 2 in the direction of  $\vec{i} + 2\vec{j}$ . What is  $\vec{\nabla}g(1, 2)$ ? Check your answer.
3. 20 pts. Find all critical points, local maxima, minima and saddle points of  $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$ .
4. (a) 10 pts. Use Lagrange multipliers to find the maxima and minima of the function  $f(x, y) = xy$  subject to the constraint  $x^2 + y^2 = 1$ . Find the points where maximum and minimum occur and also the maximum and minimum values of  $f$ .  
  
(b) 10 pts. Sketch level curves of  $f$  and the graph of  $x^2 + y^2 = 1$ .