Give explanations which tell us clearly every step of what you are doing.

Let $F(x, y, z) = (x^2 + y^2 x^2) + x + 300$. Suppose the graph of F represents a shiny smooth hill.

1. What are the partial derivatives of F with respect to x and y.

2. What is the gradient?

Suppose you are standing over the point $x = 1, y = 2$ on the hill.

3. If you drop a marble, in which direction will it start to roll? You only need to specify the direction in the x, y plane which it will take.

4. What is the slope of the hill in the direction of the vector $(2, 6)$?

5. Suppose you lay a ruler (1 foot) down on the hill pointing in the northeast direction with one tip at your feet. Approximately how much higher is the other tip? Here we take the positive y-axis to be north and the positive x-axis to be east.

6. You hold a road map of the region and walk along a road which on the map is given by the equation $xy = 2$. What is the highest point on this road?

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7. **(Counts triple.)** Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ subject to the constraint $x^2 + y^2 z^2 = 40$.

8. What is the closest point on the curve parameterized by

$$x(r) = 2 + r \qquad y(r) = 0 \qquad z(r) = r + 50$$

To the circle $x^2 + y^2 = 9$, $z = 0$. Hint: It might be fastest to visualize this and give an argument which is mostly geometric.