Math 110A: Introduction to Partial Differential Equations
Fall 2016

Midterm 1 Review

1. Concepts of partial differential equations (PDE). What is the order of a PDE? How to determine if a PDE is linear or not? What is a homogeneous equation?

2. Find the general solution to $u_{xx} = 0$ for $u = u(x, y)$.

3. Solution to the first-order linear equation: $au_x + bu_y = 0$. What are characteristic lines for this equation? Solution to the first-order linear equation with variable coefficients. What are characteristic curves. Examples 2 and 3 on page 9. Additional exercise problems 3 and 5 on page 10.

4. Examples of Transport equations, wave equations, and diffusion equations in one-space dimension. Example 4: derivation of diffusion equation. Laplace's equation: $\Delta u = 0$. (Omit the derivation in Examples 5–7 on pages 16–18. But the final equations (10) and (11) are included.)

5. Initial conditions. Three types of boundary conditions. Initial-value problems, boundary-value problems, and initial-boundary-value problems. Well posed problems.

6. Classification of second-order linear equations. Theorem 1 on page 28. Examples 1 on page 29 and Example 2 on page 31.

7. The general solution to the wave equation $u_{tt} = c^2 u_{xx}$ for one-space dimension. The solution to the initial-value problem of this equation: Eq. (8) on page 36. Concept: domain of influence and domain of dependence. Exclude Example 2 on pages 36 and 37. Proof of conservation of energy: page 40.

8. The maximum principle for diffusion equation on a finite interval. Uniqueness and stability for initial-boundary-value problem for diffusion equation, using both the maximum principle and the energy method. See also Exercise 15 on page 53.

9. The solution to the initial-value problem of diffusion equation $u_t - ku_{xx} = 0$ $(-\infty < x < \infty, t > 0)$ with $u(x, 0) = \phi(x)$ $(-\infty < x < \infty)$ is given by

$$u(x,t) = \int_{-\infty}^{\infty} S(x-y,t)\phi(y)\,dy \quad (-\infty < x < \infty, t > 0).$$

   How to define the initial condition $u(x, 0) = \phi(x)$ with this formula? What is the formula of $S(x, t)$? $S(x, t) = \partial Q(x, t)/\partial x$ where $Q(x, t) = g(x/\sqrt{4kt})$ solves the same diffusion equation $Q_t - kQ_{xx} = 0$ with the special initial condition: $Q(x, 0) = 0$ if $x < 0$ and $Q(x, 0) = 1$ if $x > 0$. No need to memorize the formula of the error function in Eq. (10) in Section 2.4.

10. Given a function $\phi = \phi(x)$ defined on $x > 0$. How to extend it to be an odd (or even) function on the entire line $-\infty < x < \infty$?

11. Consider the diffusion equation $u_t - ku_{xx} = 0$ on the half line $x > 0$ with the initial condition $u(x, 0) = \phi(x)$ $(x > 0)$ and some boundary condition at $x = 0$. Find the solution if the boundary condition is: (1) $u(0, t) = 0$ $(t > 0)$; or (2) $u_x(0, t) = 0$ $(t > 0)$.