Name ______________________
PID ______________________

Practice Midterm Exam - Math 102

70 minutes

- The test covers sections 1.1 through 3.4 of your textbook, excluding section 1.7.
- The actual midterm is on Thursday during class - **please bring a blue book**.
- The format of the actual midterm will be the same - seven questions each worth an equal amount.
- I recommend doing this practice midterm in a test-like environment after you have done a good amount of studying. Give yourself 70 minutes, and don't look at any notes.
- This is the only practice midterm I will make, but if you want more practice, there are lots of good problems in the chapter reviews of your textbook.
- Remember to show all of your work.

1. True or False?

   (a) The vector $\frac{1}{3}u$ is a linear combination of u and v.

   (b) A set of five vectors must span $\mathbb{R}^4$.

   (c) If v_1, v_2, and v_3 are vectors in $\mathbb{R}^3$ and v_3 is not a linear combination of v_1 and v_2, then $\{v_1, v_2, v_3\}$ is linearly independent.

   (d) If a square matrix A satisfies the property that $Ax = b$ has at least one solution for every b, then $Ax = b$ has exactly one solution for every b.

   (e) If v is orthogonal to each vector in a basis for W, then v must be orthogonal to any vector in W.

2. Find the LDU factorization of

$$A = \begin{bmatrix} 4 & 2 & 4 \\ 2 & -8 & 3 \\ -8 & 13 & 2 \end{bmatrix}$$

   .

3. Show that for any A and b, only one of the following systems has a solution.

   i $Ax = b$

   ii $A^T y = 0$ and $y^T b \neq 0$.

   That is, either $b \in Col(A)$ or there is a $y \in Null(A^T)$ such that $y^T b \neq 0$ but not both.

4. (a) On the vector space $\mathbb{P}_3$ of polynomials of degree less than or equal to three, what matrix A represents taking the second derivative?

   (b) Find a basis for $Null(A)$ and interpret what this means in terms of polynomials.

   (c) Find a basis for $Col(A)$ and interpret what this means in terms of polynomials.

5. Show that for any matrix A, $Null(A^T A) = Null(A)$.

6. Show that the left nullspace of A and the column space of A are orthogonal complements in $\mathbb{R}^m$.

7. Find the least-squares solution of $Ax = b$ for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.