MATH 20E   FINAL EXAM Spring 2000, Lindblad.

1. The temperature of space is given by $\phi(x, y, z) = xy + xz$. A fly is flying in space and at each point (x, y, z) of its journey it flies in the direction $\mathbf{F}(x, y, z)$ in which the rate of increase of temperature is maximum.

(a). Calculate $\mathbf{F}(x, y, z)$.

(b). Find the curve along which the fly travels if it starts at the point $(1, 0, 1)$.

2. Let T be the triangle in the $xy-$plane consisting of all (x, y, z) such that $z = 0$ and $x \geq 0$, $y \geq 0$ and $2x+y \leq 2\}$. Let C be the boundary curve of T going in mathematically positive direction in the $xy-$plane, i.e. C consists of the 3 directed line segments $(0, 0, 0)$ to $(1, 0, 0)$, from $(1, 0, 0)$ to $(0, 2, 0)$ and from $(0, 2, 0)$ to $(0, 0, 0)$.
Let $\mathbf{F} = y\mathbf{i} + x\mathbf{k}$. Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$.

3. Let S be the surface given as the graph of $z = x^2 + y$ over the region $D = \{(x, y);\ x \geq 0,\ y \geq 0,\ 2x + y \leq 2\}$ in the xy-plane. Let $\mathbf{G} = x\mathbf{k}$. Find the flux of G out from S: $\iint_S \mathbf{G} \cdot \mathbf{n}\, dS$, where $\mathbf{n}$ is the unit normal, satisfying $\mathbf{n} \cdot \mathbf{k} > 0$.

4. a) For $\mathbf{G} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$, determine whether or not $\mathbf{G}$ is conservative in space.

b) For $\mathbf{F} = \dfrac{-y}{x^2 z}\mathbf{i} + \dfrac{1 - xyz}{xz}\mathbf{j} - \dfrac{y}{xz^2}\mathbf{k}$, find a potential ϕ with $\nabla\phi = \mathbf{F}$.

c) Let C be the curve $y = e^{-x}$, $z = x$ for $1 \leq x < \infty$.
For the vector field $\mathbf{F}$ in part (b), find $\int_C \mathbf{F} \cdot d\mathbf{R}$.

5. a) Sketch the region R defined by $1 \leq x^2 - y^2 \leq 4, \quad 0 < x/4 \leq y \leq x/2$.

b) Calculate $\iint_R \dfrac{1}{x^2}\, dxdy$, by making the change of variables $u = x^2 - y^2$, $v = y/x$.

6. (a) State Greens theorem for a domain D in the plane bounded by a curve C. Explain how one can use Greens theorem to find the area of the domain D.

(b) Find a parametrization for the ellipse C given by $x^2/a^2 + y^2/b^2 = 1$.

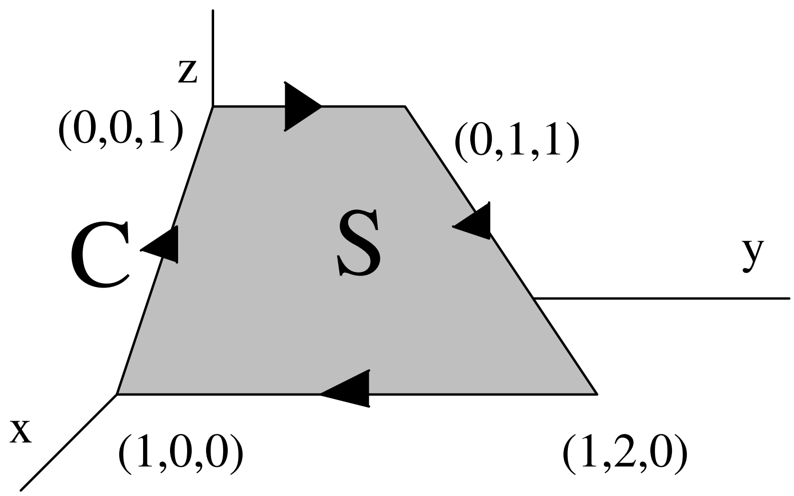
(c) Compute the area D bounded by the ellipse C in (b).

7. (a) Let S be the surface of the ellipsoid given by $x^2 + y^2 + z^2/4 = 1$.
Find $\iint_S \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F} = 2z\mathbf{k}$.

(b) Let $E = \{(x, y, z);\ x^2 + y^2 + z^2/4 \leq 1\}$ be the solid ellipsoid. Find $\iiint_E \nabla \cdot \mathbf{F}\, dV$.

(c) Find the volume of E.

8. Let $\mathcal{S}$ be the quaderalateral with vertices $(1, 0, 0)$, $(0, 0, 1)$, $(0, 1, 1)$, $(1, 2, 0)$ and let C is the boundary of $\mathcal{S}$ directed as shown. Let $\mathbf{F} = y\mathbf{k}$.



a) Let $\mathbf{n}$ be the unit normal on $\mathcal{S}$ compatible with the direction of C. Calculate $\mathbf{n}$.

b) Calculate $\iint_{\mathcal{S}} \nabla \times \mathbf{F} \cdot \mathbf{n} dS$. (Do not use Stoke's theorem.)

c) Calculate $\int_C \mathbf{F} \cdot d\mathbf{R}$. (Don't use Stoke's theorem.)

9. Let S be the sphere of radius 2; $x^2 + y^2 + z^2 = 4$. Find $\iint_S y^2\, dS$.