Math 20E Midterm 1, Fall 98, Lindblad.

1. Given three points $P = (-1, 2, 0)$, $Q = (1, 2, 2)$ and $R = (0, 3, 2)$
(a) Find the equation of the plane containing the three points.
(b) Find the area of the triangle with vertices at the three points.

2. Consider the curve C given in parametric form by
$\mathbf{R}(\mathbf{t}) = \mathbf{i}(1+t)^{3/2} + \mathbf{j}(1-t)^{3/2} + \mathbf{k}t$, $0 \le t \le 1$.
(a) Find the arc length of the curve C.
(b) Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$, where $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + x\mathbf{k}$.

3. Let $\mathbf{F} = y\mathbf{i} + x\mathbf{j}$.
(a) Find the flow line C for $\mathbf{F}$ that passed through $(1, 2, 2)$.
(b) Is there a scalar field ϕ such that $\nabla\phi = \mathbf{F}$?
If there is such a ϕ find it otherwise explain why there is none.
(c) Let C be a line segment from $(0, 0, 0)$ to $(1, 2, 2)$.
Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$.

4. Let $\mathbf{F} = \dfrac{-y\mathbf{i}}{\sqrt{x^2+y^2}} + \dfrac{x\mathbf{j}}{\sqrt{x^2+y^2}}$.
(a) Find $\nabla \times \mathbf{F}$.
(b) Find $\nabla \cdot \mathbf{F}$.
(c) Is there a scalar field ϕ such that $\nabla\phi = \mathbf{F}$?
If there is such a ϕ find it otherwise explain why there is none.