Final Exam, Math 20F Signature ____________________

July 31, 1998

Section # __________ Name ____________________

(Print, with non-eraseable pen)

This exam is closed book, closed notes and closed calculator. Please work alone, and show and explain your work and justify your answers–give clear explanation in plain language combined with suitable equations. Each problem is worth 55 points for a total of $10 \times 55 = 550$ points.

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1. Let
$$\begin{bmatrix} 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & d \end{bmatrix}$$
be the echelon form of the augmented matrix of the system $Ax = b$, with A a 4×5 matrix.

   Given the general solution in the 3 cases: $c \neq 0 \neq d$; $c \neq 0 = d$; and $c = 0 \neq d$.

2. For $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ give a basis of Null(A) and a basis of Col(A).

3. Find the general solution of the system
$$\begin{aligned} 2x_1 + 3x_2 + 11x_3 + 6x_4 &= 11 \\ x_2 + 6x_3 + 5x_4 &= 6 \\ x_1 + x_2 + 2x_3 \quad &= 1. \end{aligned}$$

4. Find the inverse A^{-1} of $A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 7 \\ -3 & 6 & -2 \end{bmatrix}$.

5. Find a basis of the vector space spanned by the 4 vectors.
$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ -1 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} -4 \\ 3 \\ -3 \\ 5 \end{bmatrix}.$$

6. Compute the determinant of
$$A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 3 & 1 & -1 \\ -1 & 1 & 2 & 3 \\ -1 & 1 & 1 & -1 \end{bmatrix}.$$

7. Find an orthogonal basis of the vector space V spanned by the 3 vectors.

$$\begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}.$$

8. The matrix

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

has the eigenvalue $\lambda = 4$. Find a basis of the corresponding eigenspace.

9. The symmetric matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

has the eigenvectors

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Find the eigenvalues and write A in the form $A = UDU^T$, with D a diagonal matrix and U an orthogonal matrix.

10. Find the eigenvalues and eigenvectors for the matrix exponential e^A where A is the 3×3 matrix of problem 9. **Hint**. Use a suitable diagonalization for A to study e^A.

Remember to explain your answer in every case.