Name ______________________
PID ______________________

Practice Final Exam - Math 102

3 hours

- The test is cumulative, though more emphasis will be placed on later material.
- The final is on Friday from 11:30 to 2:30 - **please bring a blue book**.
- I recommend doing this practice midterm in a test-like environment after you have done a good amount of studying. Give yourself 3 hours, and don't look at any notes.
- There are 18 questions, which gives 10 minutes per question.
- This is the only practice final I will make, but if you want more practice, there are lots of good problems in the chapter reviews of your textbook.
- Remember to show all of your work.

1. (a) Show that if a real symmetric matrix A has an LDU factorization then $U = L^T$.

   (b) Find the LDU factorization of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

2. True or False?

   (a) The vectors b that are not in column space of A form a subspace.

   (b) If the column space of A contains only the zero vector, then A is the zero matrix.

   (c) The column space of $2A$ equals the column space of A.

   (d) The column space of $A - I$ equals the column space of A.

3. Is $T : \mathbb{C} \to \mathbb{C}$ where $T(a + bi) = a - bi$ a linear transformation? In other words, is complex conjugation linear?

4. In this problem, you will show that every straight line remains a straight line after a linear transformation is applied. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ such that $T(v_1) = w_1$ and $T(v_2) = w_2$. Let v be a point halfway along the line between v_1 and v_2. Show $T(v)$ is halfway along the line between w_1 and w_2.

5. Find the projection matrix P onto the plane spanned by $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$. Project $b = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ onto the plane.

6. The vectors $v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$ form a basis for $\mathbb{R}^2$. Write $x = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ as a linear combination of v_1 and v_2.

7. Find three vectors that are orthogonal to $Null(A)$, where $A = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix}$.

8. Find an orthogonal basis for $V = span(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix})$.

9. Show that the product Q_1Q_2 of two orthogonal matrices, Q_1 and Q_2, is also an orthogonal matrix.

10. (a) Let $A = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$. Find $det(A)$ in any way.

    (b) Let $B = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 3 & 1 \\ 3 & 7 & 5 \end{bmatrix}$. Find $det(B)$ using row exchanges.

11. Diagonalize the following matrix, or explain why it is impossible.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

12. If $H_0 = 4$, $H_1 = 2$, and $H_{k+2} = 2H_{k+1} + 3H_k$, find a general formula for the k^{th} term of the sequence, H_k.

13. Suppose a Markov matrix A has the following eigenvalues and eigenvectors.

$$\lambda_1 = 1, x_1 = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}, \lambda_2 = 0.73, x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \lambda_3 = 0.41, x_3 = \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$$

    If $u_k = A^k u_0$, find the steady state vector u_∞, which will not depend on the initial distribution u_0.

14. Show that if A is similar to B and B is similar to C, then A is similar to C. Similarity is transitive.

15. Suppose $||Ax|| = ||A^H x||$ for any vector $x \in \mathbb{C}^n$. By choosing x appropriately, show that the i^{th} column of A has the same length as the i^{th} row.

16. Find the spectral decomposition of the following matrix, or explain why it is impossible.

$$A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

17. For each part, give an example of a 3×3 matrix that meets the description.

    (a) unitarily triangularizable

    (b) normal

    (c) defective

    (d) Hermitian

    (e) positive definite

18. Find the singular value decomposition of the following matrix, or explain why it is impossible.

$$A = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$$

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