Math 166A Exam Spring Quarter June 1991 Buss

(1) Let $L_1 = \{w : w \text{ has 3 or 4 } a\text{'s}\}$. $\Sigma = \{a, b\}$. Give a DFA for L_1. Give a regular expression α_1 for L_1.

(2) Let $L_2 = \{w : w$ has 3 a's **or** w has b's$\}$. $(\Sigma\{a, b\})$. Given and NFA M_2 s.t. $L(M_2) = L_2$. Give a regular expression α_2 s.t. $L(\alpha_2) = L_2$.

(3) Prove or disprove: $\{a^i b^{i^2} : i \geq 0\}$ is context-free. (Either give a PDA or a CFG; or prove its not context-free.

(4) Prove or disprove: $\{a^{i^2} : i \geq 0\}$ is context-free.

(5) Prove or disprove: $\{a^i b^j : i \leq j$ or i is a perfect square$\}$ is not regular.

(6) Prove or disprove: $\{a^i b^j : 100 \leq i < j \leq 1000\}$ is context-free.

(7) Prove or disprove: $\{a^i b^j : 100 \leq i < j \leq 1000\}$ is regular.

(8) Let L be a CFL. Let k, K be as in the strong pumping lemma for L. Prove that if $L \neq \emptyset$ then L has a string of length $\leq K$.

(9) Construct a PDA which accept $\{a^i b^j c^k d^\ell : i = k \text{ or } j = \ell\}$.

(10) Give a CFG that generates this language $\{a^i b^j c^k d^\ell : i = k \text{ or } j = \ell\}$.

*(11) Show $\{a^m b^n : m > n$ **or** $(m$ is prime and $n \leq m)\}$ is not context-free. **Hint**: for any $k > 0$, there are arbitrarily large primes p such that $p \pm i$ is composite for $1 \leq i \leq k$.