GRE Differential Equation

Summary:

- Separation method
- Technique for homogeneous equation
- Solving an exact equation by variation of parameters
- Solving a non-exact equation by finding an integrating factor or variation of parameter
- Solving a homogeneous higher-order ODE

Since there is no discussion of variation of parameter in the GRE textbook which turns out a very powerful method, let me discuss it here. Using variation of parameter, we can solve an inhomogeneous 1st order ODE

$$\frac{dy}{dx} + P(x)y = Q(x)$$

For example

$$\frac{dy}{dx} = 5x - \frac{3y}{x} ------(1)$$

First, we consider an homogeneous one

$$\frac{dy}{dx} = \frac{3y}{x} ------(2)$$

We have $y = C\frac{1}{x^3}$ to be a solution of (2)

We assume $y = C(x)\frac{1}{x^3}$ to be a solution of (1) where C(x) is a differentiable function. Put the solution to the equation (1). We have

$$C'(x)\frac{1}{x^3} - \frac{3y}{x} = 5x - \frac{3y}{x} \Rightarrow C'(x) = 5x^4 \Rightarrow C(x) = x^5 + C$$

Therefore, a solution of equation (1) is

$$y = \frac{x^5 + C}{x^3}$$

Q37(96)

Which of the following is the general solution of the differential equation?

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

(A) $C_1e^t + C_2te^t + C_3t^2e^t$ (B) $C_1e^{-t} + C_2te^{-t} + C_3t^2e{-}t$ (C) $C_1e^t - C_2e^{-t} + C_3te^{t^2}$
(D) $C_1e^t + C_2e^{2t} + C_3e^{3t}$ (E) $C_1e^{2t} + C_2te^{-2t}$

Q5 (87)

All functions f defined on the xy-plane such that

$$\frac{\partial f}{\partial x} = 2x + y \quad \text{and} \quad \frac{\partial f}{\partial y} = x + 2y$$

are given by $f(x, y)$=
(A) $x^2 + xy + y^2 + C$ (B) $x^2 - xy + y^2 + C$ (C) $x^2 - xy - y^2 + C$ (D) $x^2 + 2xy + y^2 + C$ (E) $x^2 - 2xy + y^2 + C$

Q40(87)

Let $y = f(x)$ be a solution of the differential equation

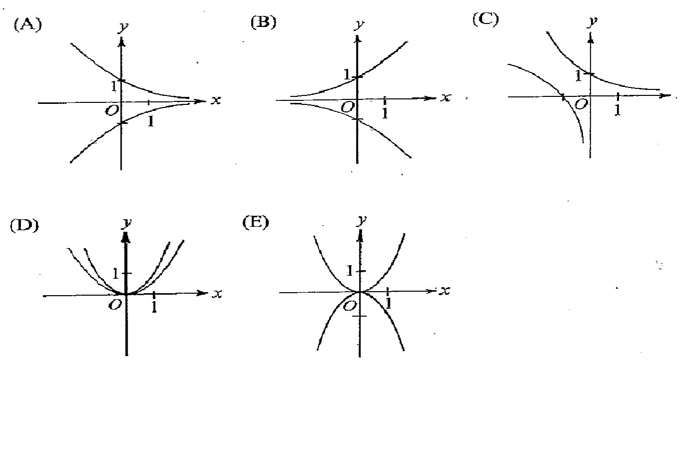
$$xdy + (y - xe^x)dx = 0$$

such that $y = 0$ when $x = 1$. What is the value of $f(2)$?
(A) $1/(2e)$ (B) $1/e$ (C) $e^2/2$ (D) $2e$ (E) $2e^2$

Q40(87)

Which of the following indicates the graphs of two functions that satisfy the differential equation

$$(\frac{dy}{dx})^2 + 2y(\frac{dy}{dx}) + y^2 = 0$$



Contact me if you have any question: p1tong@ucsd.edu