1. Find the first 3 terms of the Taylor expansion of xe^x about

   (a) $x = 0$ (b) $x = 2$

2. (a) Write $3e^{-i5\pi/2}$ in the form $a + bi$.

   (b) Write $\frac{1+i}{1-i}$ in the form $a + bi$ and in the form $re^{i\theta}$. Here $r \geq 0,\ 0 \leq \theta < 2\pi$.

3. Find an equation for the plane through the points $(2, 4, 5), (1, 5, 7), (-1, 6, 8)$.

4. Find an equation for the plane through the origin and containing the line $x = 1 + 2t,\ y = -1 + 3t,\ z = 4t$.

5. Reduce the equation $4x^2 + y^2 - z^2 - 2y + 2z = 0$ to a standard form and classify it.

6. If $z = f(x, y),\ x = r + t$ and $y = r - t$, then show that

$$\frac{\partial z}{\partial r}\frac{\partial z}{\partial t} = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 .$$

7. Let $f(x, y) = e^{xy} - e^y$.

   (a) Find the differential of f.

   (b) Find the directional derivative of f at the point $(1, 2)$ in the direction of the vector $(2, 4)$.

   (c) Find the maximum rate of change of f at the point $(1, 2)$ and the direction in which it occurs.

8. The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant?

9. (Counts double) Find three real numbers whose sum is 9 and the sum of whose squres is as small as possible.

10. Find the volume of the solid under the surface $z = x+4$ and above the region bounded by $y = 4 - x^2$ and $y = 3x$.

11. Compute the integral

$$\iint_D x^2 + 2y \; dxdy$$

over the domain

$$D = \{(x, y) : 0 \leq x \leq 1, y \geq x^2\}$$

12. Solve the following differential equation.

$$x^2 \frac{dy}{dx} = xy - y^2$$

13. (a) Find all real and complex valued solutions of

$$y'' - 4y' + 4y = 0$$

(b) Find all solutions which satisfy $y(0) = 3$. Does this condition determine the solution uniquely?

14. Find all real valued solutions of

$$y'' - 4y' + 4y = \cos 3t$$

15. Use polar coordinates to evaluate $\displaystyle\int_0^{\sqrt{z}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy.$

16. (Counts double) A rectangular box with corners at $(\pm x, \pm y, \pm z)$ is inscribed in the ellipsoid $x^2 = 1 - x^2 - 4y^2$. What choices of x, y and z would maximize the total length of its twelve edges?

17. Level curves of the difference of July and January mean temperatures in the Eastern Hemisphere are shown in figure below. What are the approximate values of this function at its absolute maximum? At local maxima in Africa?