Math 20E Midterm 1, Winter 2005, Lindblad.

1. Let ϕ be a function such that $\dfrac{\partial\phi}{\partial x}(0,0,0)=2$, $\dfrac{\partial\phi}{\partial y}(0,0,0)=3$ and $\dfrac{\partial\phi}{\partial z}(0,0,0)=4$.

(a) Let $w(t)=\phi\big(\mathbf{c}(t)\big)$, where $\mathbf{c}(t)=t\,\mathbf{i}+t^2\mathbf{j}+3t\mathbf{k}$ is a curve. Find $\dfrac{dw}{dt}(0)$!

(b) In which direction is the rate of increase of ϕ largest at the point $(0,0,0)$?

(c) Let $\mathbf{F}=\mathbf{grad}\ \phi$. Find $\mathbf{curl}\,\mathbf{F}$.

2. Let $f(x,y)=x\cos(x+y)$

(a) Calculate the second order Taylor polynomial of f about the point $(1,-1)$.

(b) Use your answer to (a) to write down an estimate for $f(1.1,-0.8)$.

(c) Use the linear approximation to find an estimate for $f(1.1,-0.8)$.

3. Let $\mathbf{G}=-y\mathbf{i}+x\mathbf{j}$ be a vector field.

(a) Show that the curves $\mathbf{c}(t)=r\cos t\,\mathbf{i}+r\sin t\,\mathbf{j}$, where r is a constant, are flow lines for $\mathbf{G}$.

(b) Sketch the vector field $\mathbf{G}$ at the points $(1,0)$, $(0,1)$, $(-1,0)$ and $(0,-1)$ and sketch the flow line passing through $(1,0)$.

4. Let $\mathbf{F}(x,y,z)=(y^2+x)\,\mathbf{i}-(x^2-y)\,\mathbf{j}+z\mathbf{k}$.

(a) Find $\mathbf{curl}\,\mathbf{F}$.

(b) Find $\operatorname{div}\mathbf{F}$.

(c) Find the derivative matrix $\mathbf{DF}$ (i.e. the matrix of partial derivatives).