Putnam Workshop: Polynomials

Led by: Devanshi Jain

October 27th 2022

2005 : A1 Show that every positive integer is a sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$. Also note that the representations need not be unique: for instance, $11 = 2+9 = 3+8$.)

This problem is originally due to the Hungarian mathematician, Paul Erdos.

2019 : B5 Let F_m be the mth Fibonacci number, defined by $F_1 = F_2 = 1$ and $Fm = Fm-1 + Fm-2$ for all $m \geq 3$. Let $p(x)$ be the polynomial of degree 1008 such that $p(2n+1) = F2n+1$ for $n = 0, 1, 2, \ldots, 1008$. Find integers j and k such that $p(2019) = F_j - F_k$.