# Homework 2

*Collaboration in the sense of discussion is allowed, however, the assignment is **individual** and the work you turn in should be entirely your own. See the collaboration and academic integrity statement here:* https://natanaso.github.io/ece276b. *Books may be consulted but not copied from. Please acknowledge **in writing** people you discuss the problems with and provide references for any books or papers you use.*

## Submission

Upload your solutions on **Gradescope** by the deadline shown at the top right corner. You may use latex, scanned handwritten notes (write legibly!), or any other method to prepare a pdf file. Do not just write the final result. Present your work in detail, explaining your approach at every step.

## Problems

In square brackets are the points assigned to each part.

1. Consider the deterministic shortest path (DSP) problem in Fig. 1 with start node $S$ and goal node $T$.

   (a) [5 pts] Define the vertex set $\mathcal{V}$ of the graph. How many elements are in $\mathcal{V}$? What is the planning horizon of the equivalent deterministic optimal control problem?

   (b) [10 pts] Apply the (backward) dynamic programming algorithm to compute the minimum cost to travel from each node to $T$. State the shortest path and its cost for each node.

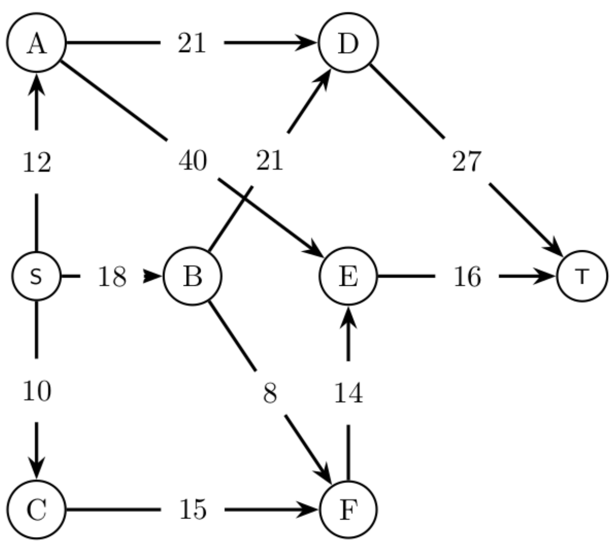


Figure 1: A deterministic shortest path problem.

2. [25 pts] Consider a robot with position $(x, y) \in [-5, 5]^2$ and orientation $\theta \in [-\pi, \pi)$. Let the body of the robot be a line segment of length 1 such that the body frame of the robot is at the center of the line segment and its $x$ axis is in the direction of the line segment. In other words, if the robot's position is $(0, 0)$ with orientation $0°$, the front tip of the robot is located at $(0.5, 0)$, while its back tip is located at $(-0.5, 0)$. Similarly, if the robot's position is $(0, 0)$ with orientation $30°$, the front tip of the robot is located at $(0.433, 0.25)$, while its back tip is located at $(-0.433, -0.25)$. Now, suppose that there is a circular obstacle at $(0, 0)$ with radius 1. Plot the configuration space of the robot, including the configuration space obstacle in Python.

3. A heuristic function $h$ is **admissible** if $h(\mathbf{x}_i) \leq \mathbf{dist}(\mathbf{x}_i, \mathbf{x}_\tau)$ for every node $i$ with coordinates $\mathbf{x}_i$, where $\mathbf{dist}(\mathbf{x}_i, \mathbf{x}_\tau)$ is the shortest distance from $\mathbf{x}_i$ to the goal $\mathbf{x}_\tau$. A heuristic function $h$ is **consistent** if:

   - $h(\mathbf{x}_\tau) = 0$ for the goal node $\tau$ with coordinates $\mathbf{x}_\tau$,
   - $h(\mathbf{x}_i) \leq c(\mathbf{x}_i, \mathbf{x}_j) + h(\mathbf{x}_j)$ for every node $i$ with coordinates $\mathbf{x}_i$ and its children $j$ with coordinates $\mathbf{x}_j$.

   Finally, a heuristic $h$ is **$\epsilon$-consistent** with $\epsilon \geq 1$ if:

   - $h(\mathbf{x}_\tau) = 0$ for the goal node $\tau$ with coordinates $\mathbf{x}_\tau$,
   - $h(\mathbf{x}_i) \leq \epsilon c(\mathbf{x}_i, \mathbf{x}_j) + h(\mathbf{x}_j)$ for every node $i$ with coordinates $\mathbf{x}_i$ and its children $j$ with coordinates $\mathbf{x}_j$.

   (a) [5 pts] Prove that if $h^{(1)}$ and $h^{(2)}$ are consistent heuristics, then $h := \max\{h^{(1)}, h^{(2)}\}$ is also consistent.

   (b) [5 pts] Prove that if $h^{(1)}$ and $h^{(2)}$ are consistent heuristics, then $h := h^{(1)} + h^{(2)}$ is $\epsilon$-consistent.

4. [30 pts] Consider the graph in Fig. 2 with nodes $\mathcal{V} = \{1, \ldots, 10\}$ and edge costs $c_{ij} = c_{ji} = 1$. Apply the Real-time Adaptive A* (RTAA*) algorithm to find a path from node 1 to node 10 using the heuristic function provided in Fig. 3 and expanding $N = 4$ nodes per RTAA* iteration. If two nodes $i, j \in \mathcal{V}$ have the same f-values, $f_i = f_j$, then expand the node with the smaller index, e.g., $i < j$, first. For each RTAA* iteration, show the CLOSED list, the heuristic function for all nodes, the OPEN list, and the f-values for all nodes in the OPEN list. After an RTAA* iteration, move $N = 4$ steps before starting the next RTAA* iteration. Report the final path from 1 to 10 obtained by RTAA*.

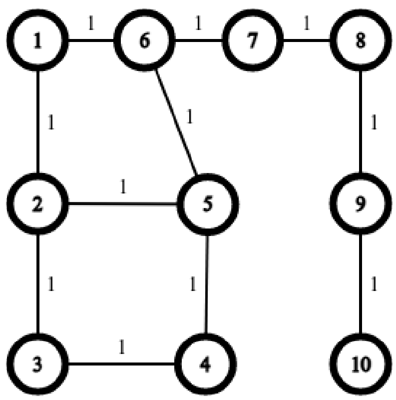


Figure 2: A weighted undirected graph.

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $h_i$ | 5 | 4 | 3 | 2 | 3 | 4 | 3 | 2 | 1 | 0 |

Figure 3: Heuristic function

5. Consider the deterministic shortest path problem in Fig. 4 with initial node 2 and goal node 4.

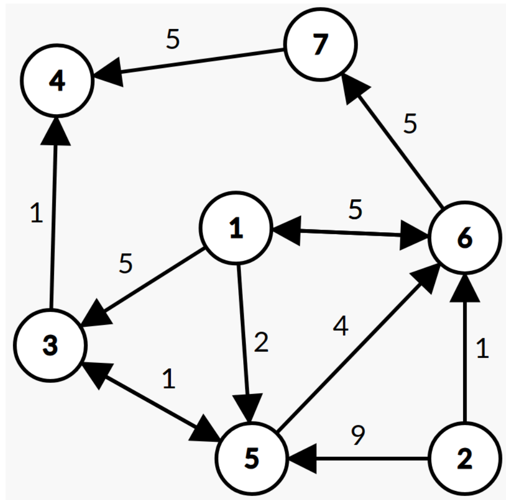


Figure 4: Deterministic shortest path problem on a graph with seven nodes and directed edges labeled by transition costs.

(a) [10 pts] Perform 4 iterations of the weighted A* algorithm on Slide 21 of Lecture 8 with weight $\epsilon = 2$ using the heuristic function in Table 1. Report the $g$ values computed by A* by filling out Table 2.

(b) [10 pts] Which of the 7 nodes are inconsistent after the 4 iterations of A*?

Table 1: Heuristic function for the deterministic shortest path problem in Fig. 4.

| $h_1$ | $h_2$ | $h_3$ | $h_4$ | $h_5$ | $h_6$ | $h_7$ |
|---|---|---|---|---|---|---|
| 1 | 10 | 3 | 0 | 2 | 7 | 5 |

Table 2: Weighted A* Algorithm

| Iteration | Node exiting OPEN | OPEN | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | – | $\{2\}$ | $\infty$ | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| 1 | 2 | | | | | | | | |