

Light travels more slowly in matter

Velocity slower

Wavelength slower

Frequency

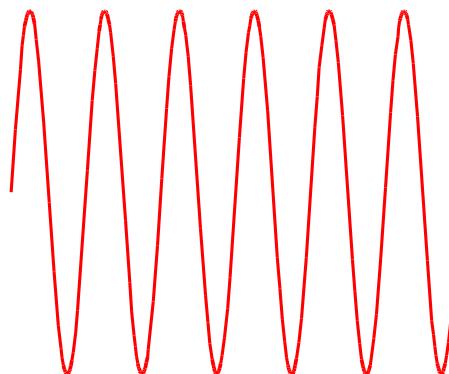
unchanged

Energy unchanged

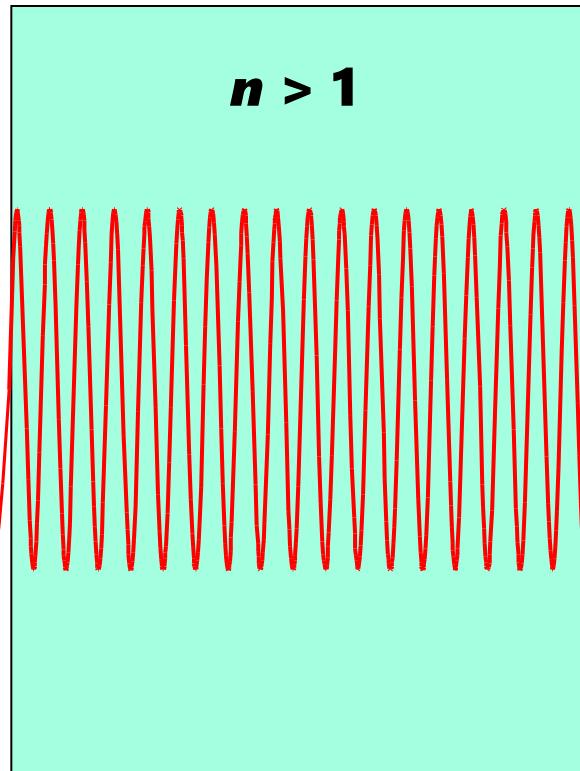
$$E = h\nu$$

$$\nu = c/\lambda$$

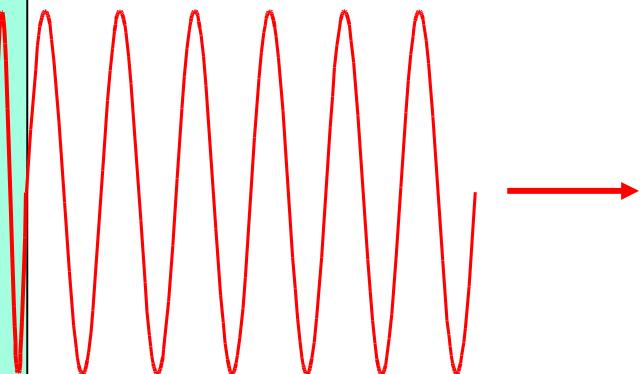
$$n = 1$$



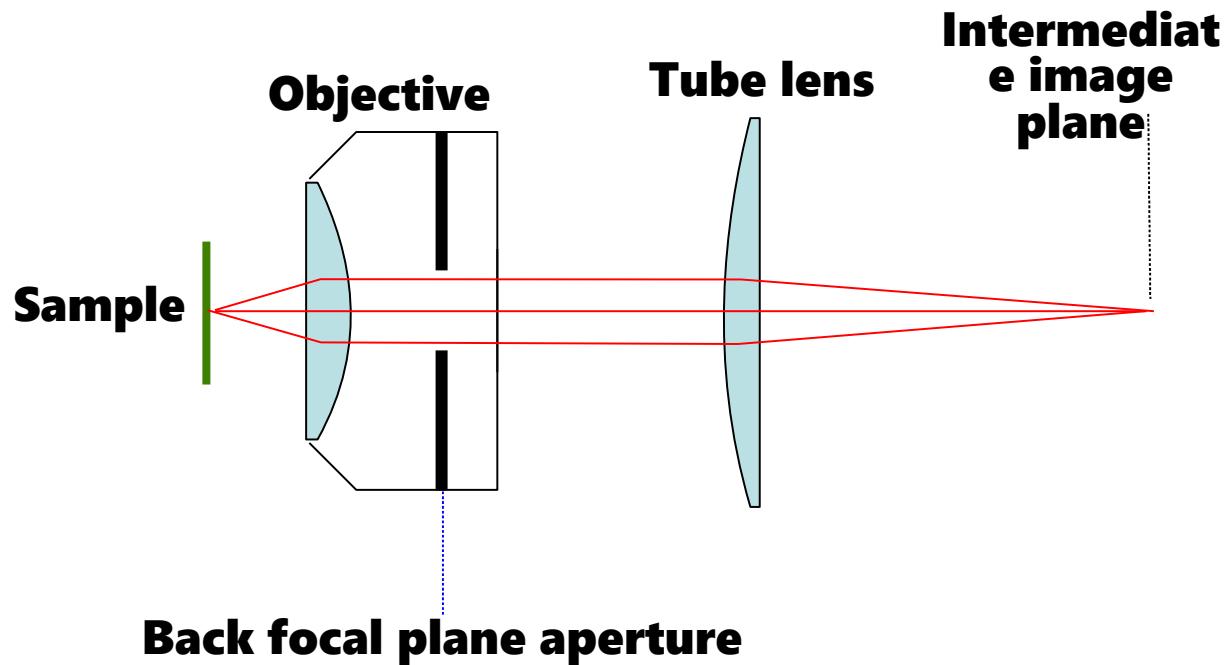
$$n > 1$$



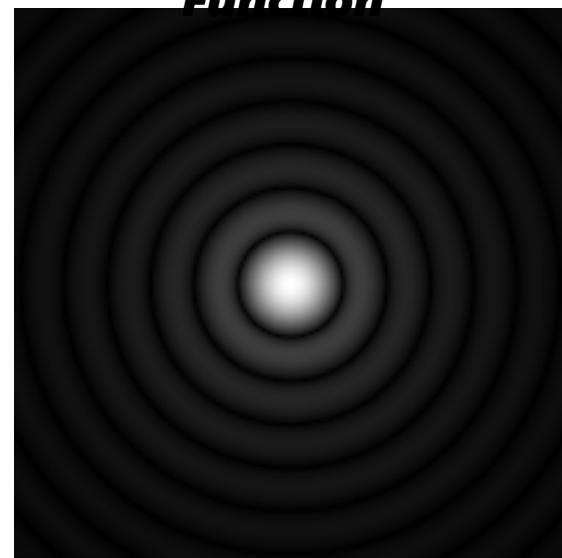
$$n = 1$$



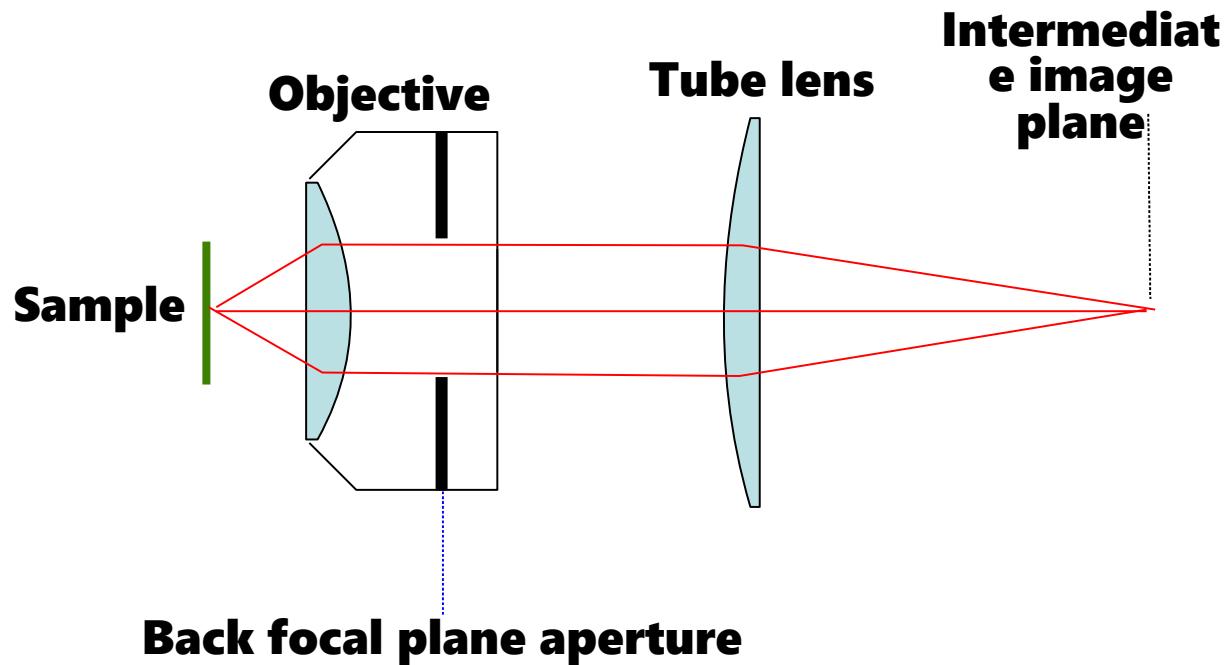
Aperture and Resolution



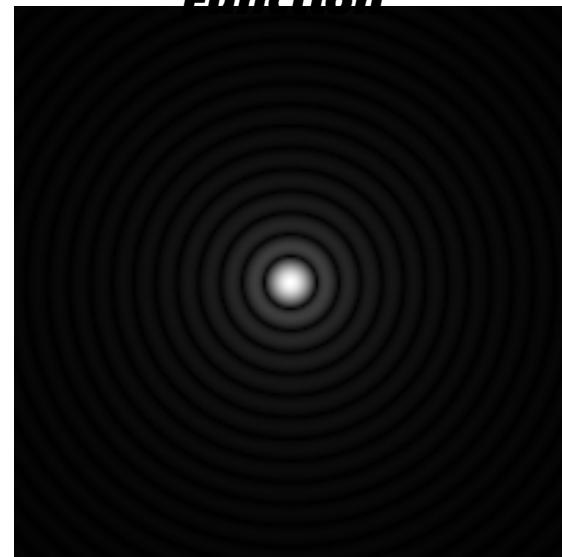
**Diffraction spot
on image plane
= *Point Spread
Function***



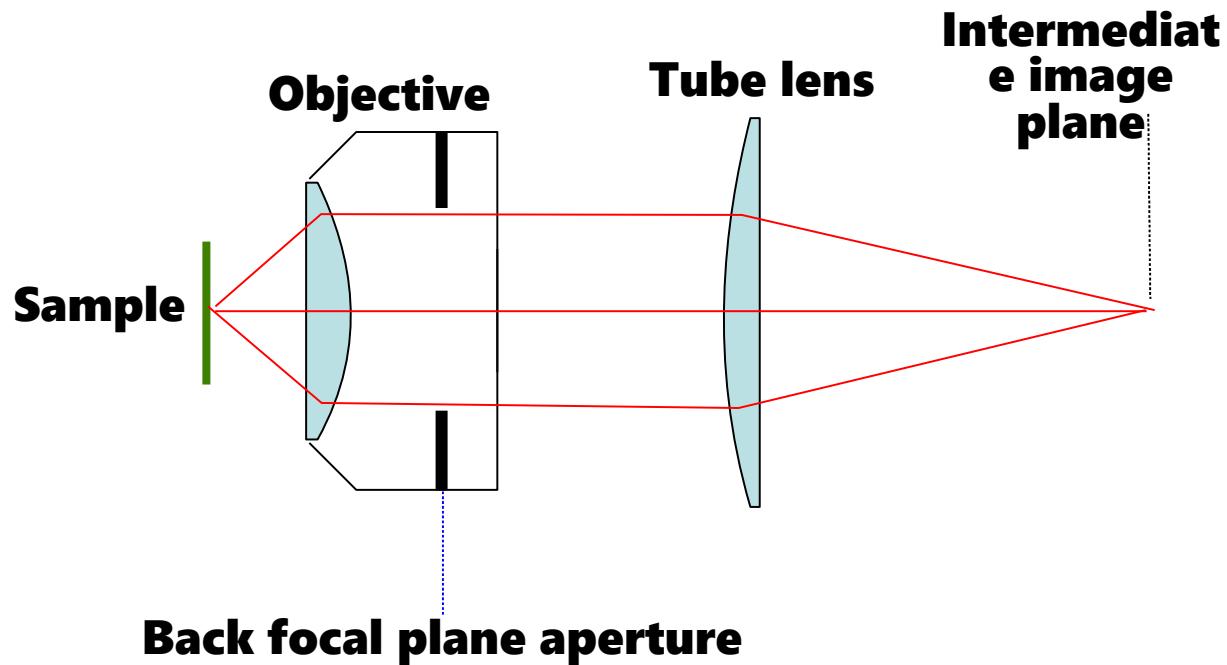
Aperture and Resolution



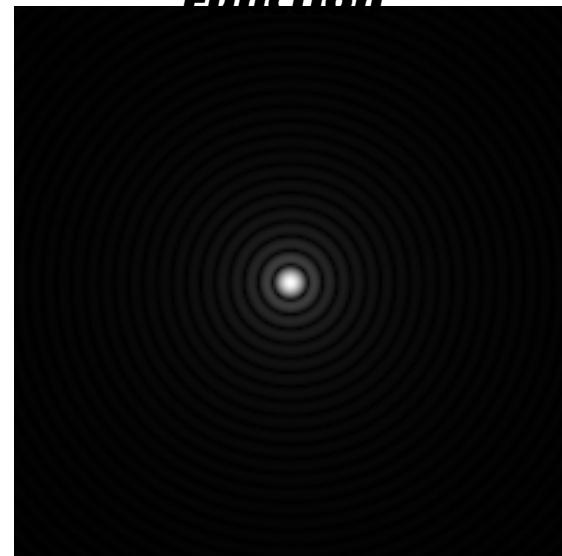
**Diffraction spot
on image plane
= *Point Spread
Function***



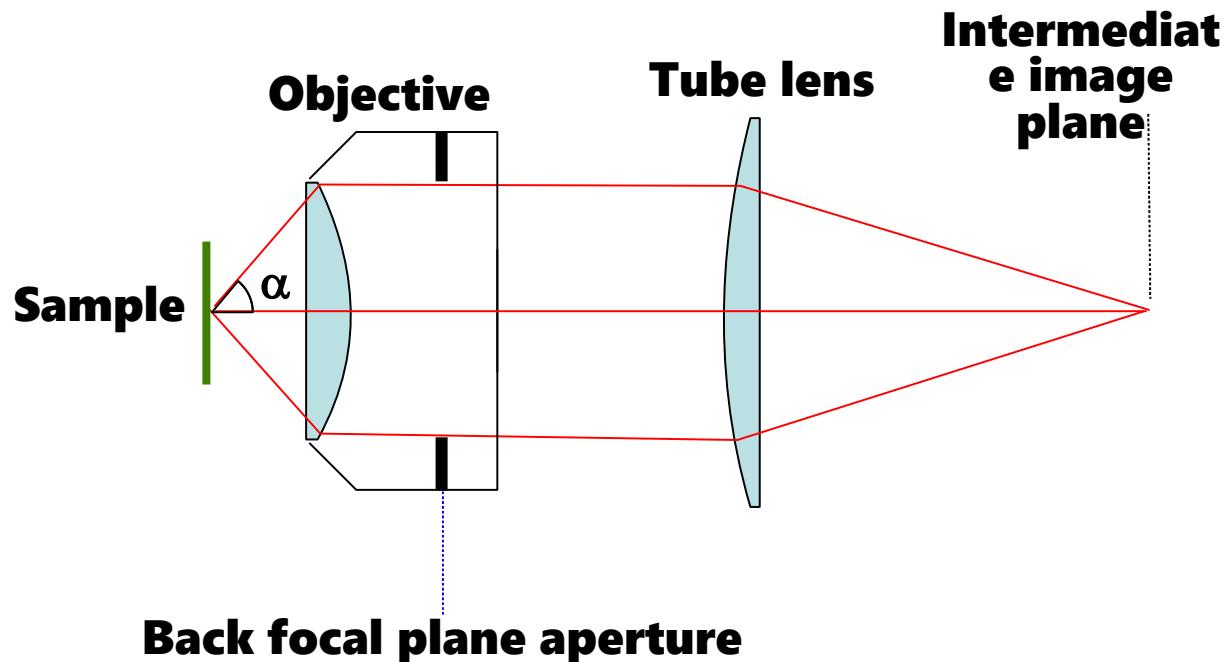
Aperture and Resolution



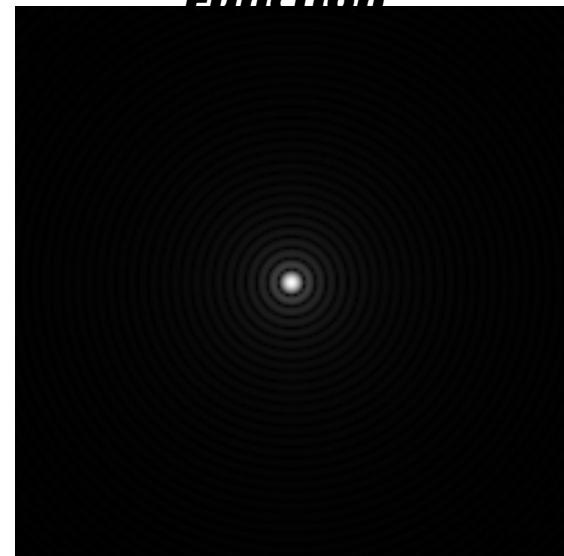
**Diffraction spot
on image plane
= *Point Spread
Function***



Aperture and Resolution



**Diffraction spot
on image plane
= Point Spread
Function**



- **Image resolution improves with Numerical Aperture (NA)**

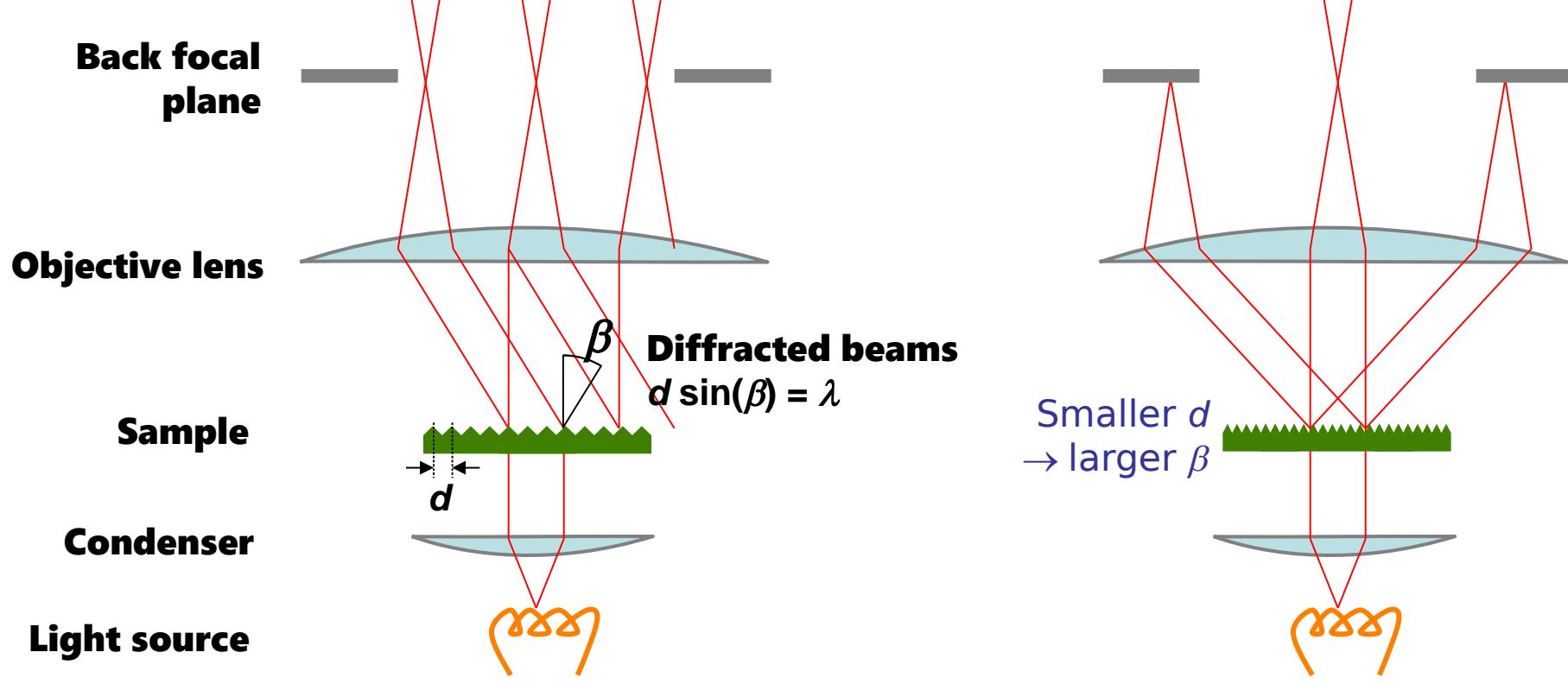
$$NA = n \sin(\alpha)$$

where: α = light gathering angle
 n = refractive index of sample

Resolution

Ernst Abbe's argument (1873)

Consider a striped sample \approx a diffraction grating

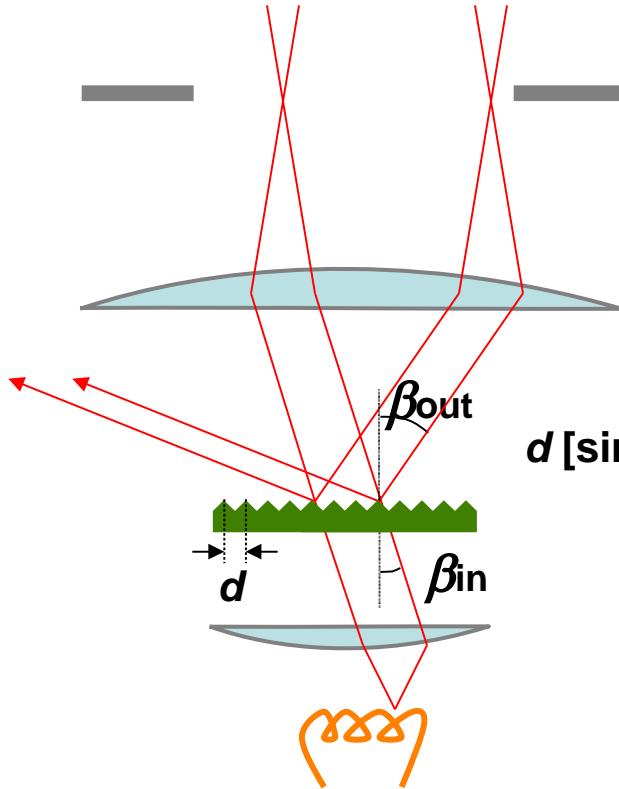


If $\beta > \alpha$, only one spot makes it through
 \Rightarrow no interference \Rightarrow no image formed

Resolution (smallest resolvable d):
 $d_{\min} = \lambda_{\text{sample}} / \sin(\alpha) = \lambda / n \sin(\alpha) = \frac{\lambda}{NA}$

(Abbe's argument, continued)

**Now consider oblique illumination
(an off-axis source point):**



**One spot hopelessly lost,
but two spots get through
→ interference → image formed!**

$$d [\sin(\beta_{in}) + \sin(\beta_{out})] = \lambda$$

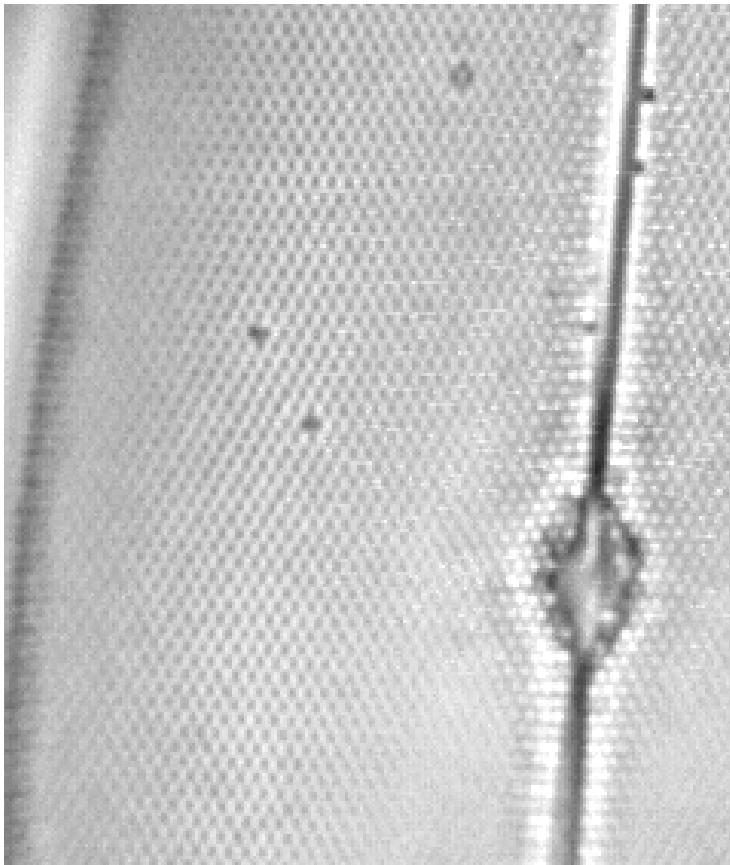
**Two spots get through if
 $\beta_{out} < \alpha$ and $\beta_{in} < \alpha$.**

**Resolution (smallest resolvable d)
with incoherent illumination (all possible illumination directions):**

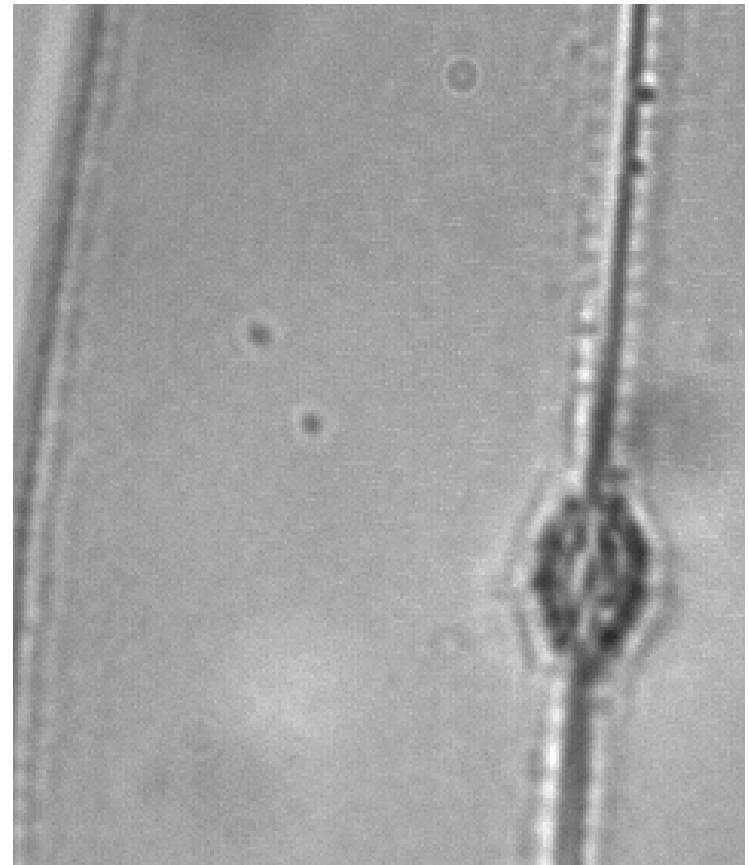
$$d_{min} = \lambda / (NA_{obj} + NA_{condenser})$$

→ $\lambda/2 \text{ NA}$ if $NA_{condenser} \geq NA_{obj}$ ("Filling the back focal plane")

NA and Resolution



High NA Objective

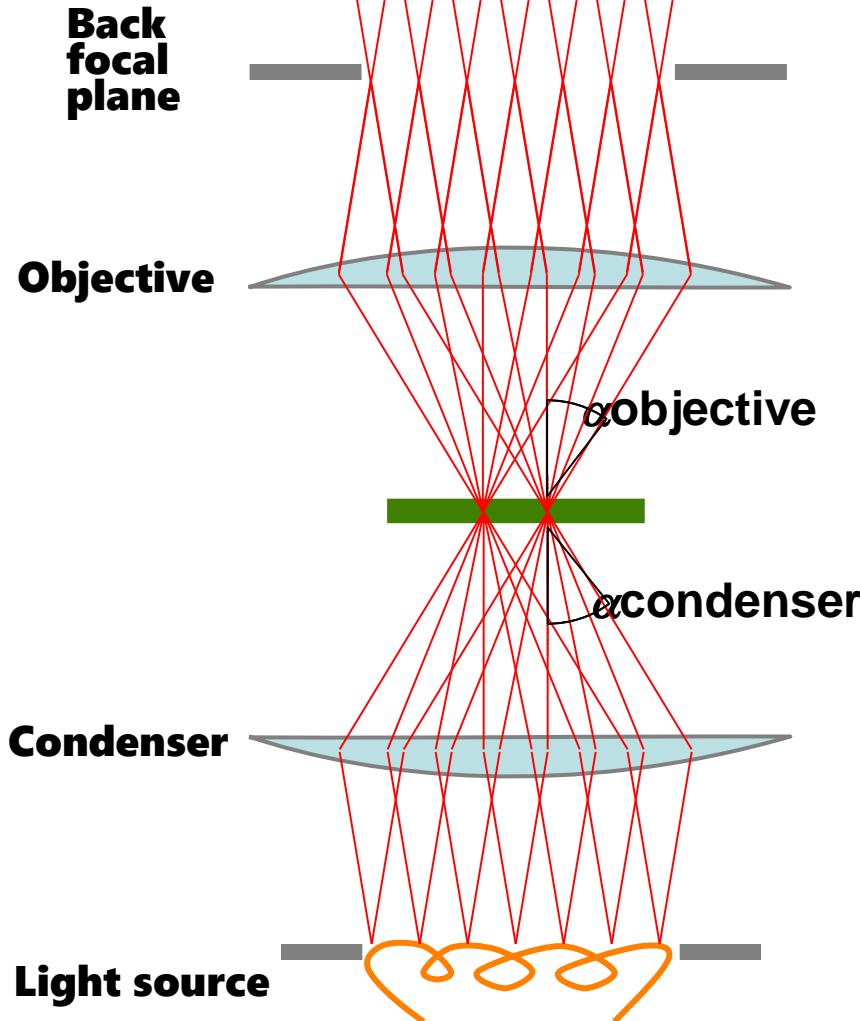


Low NA Objective

Filling the back focal plane

In trans-illumination microscopy, to get maximum resolution,

the illumination must “fill the back focal plane”



For the highest resolution, we need to have

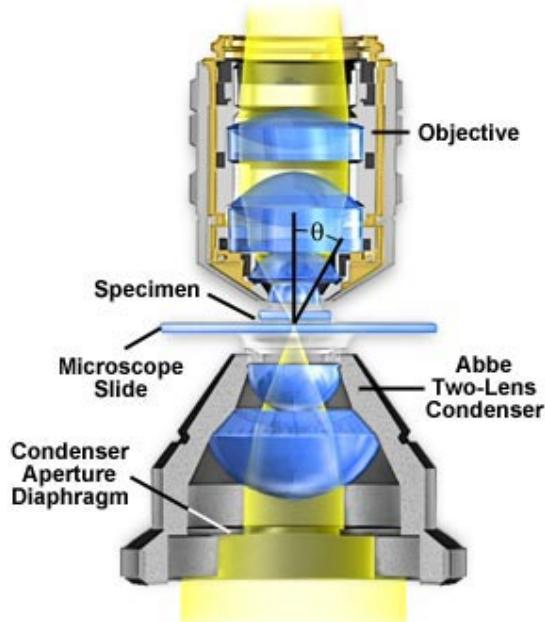
$$\alpha_{\text{condenser}} \geq \alpha_{\text{objective}}$$

$$N\!A_{\text{condenser}} \geq N\!A_{\text{objective}}$$

⇒ with oil immersion objectives, we need an *oil immersion condenser!*

The Condenser

Abbe Condenser Optical Pathway



Tasks:

- Illuminate at all angles $< \alpha_{\text{objective}}$
- Concentrate light on the field of view for *all objectives to be used*

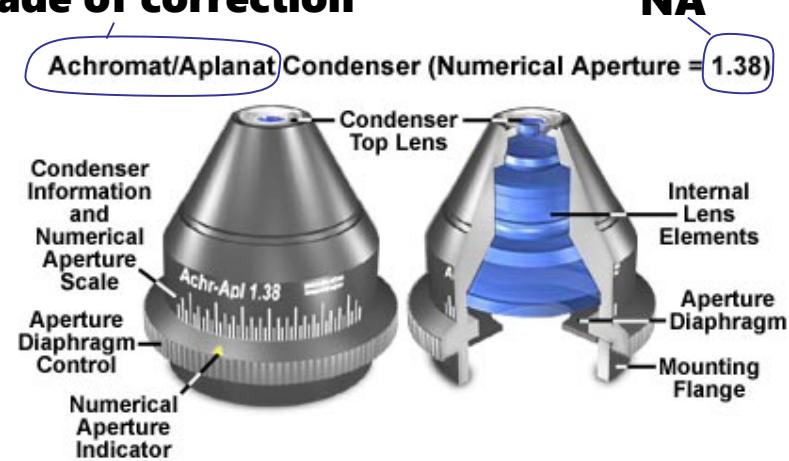
Problem:

- Low mag objectives have large FOV,
- High mag objectives have large α
(With 2X and 100x objectives we need $(100/2)^2 = 2500$ times more light than any objective uses!)

Solutions:

- Compromise
- Exchangable condensers, swing-out front lenses,...

Grade of correction



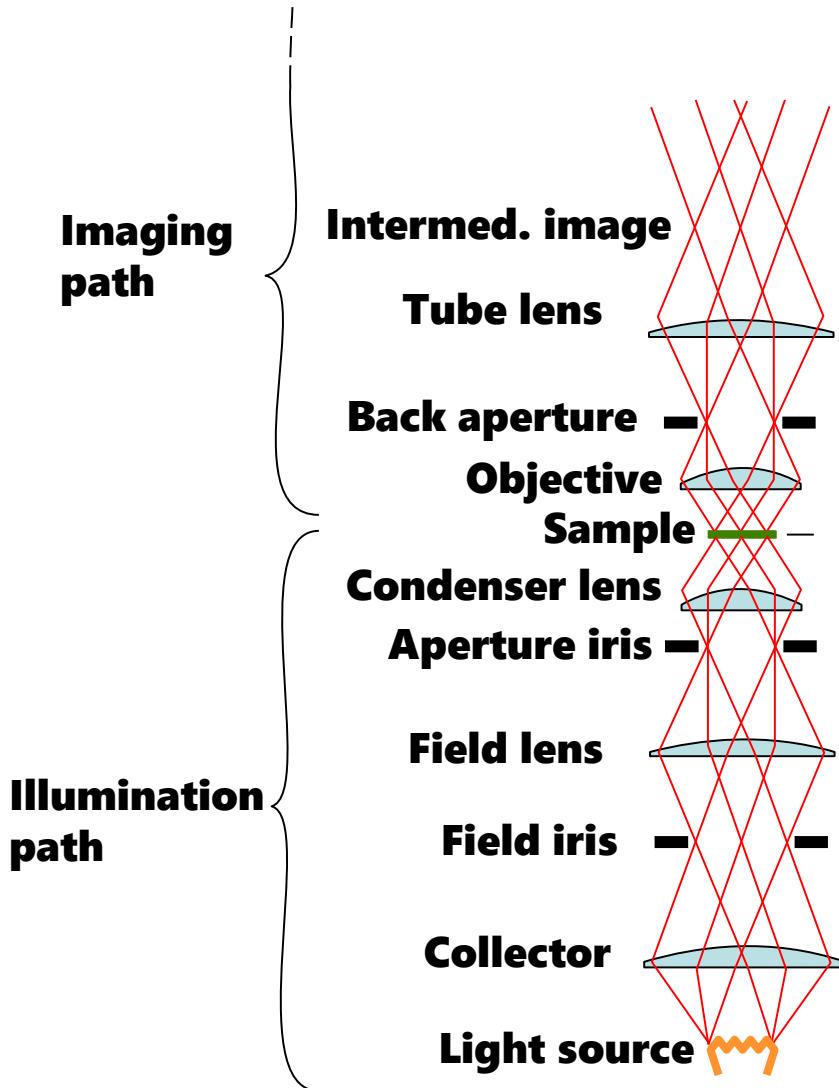
NA

Swing-Out Top Lens Condenser (Numerical Aperture = 1.35)



Aperture, Resolution & Contrast

Can adjust the condenser NA with the aperture iris

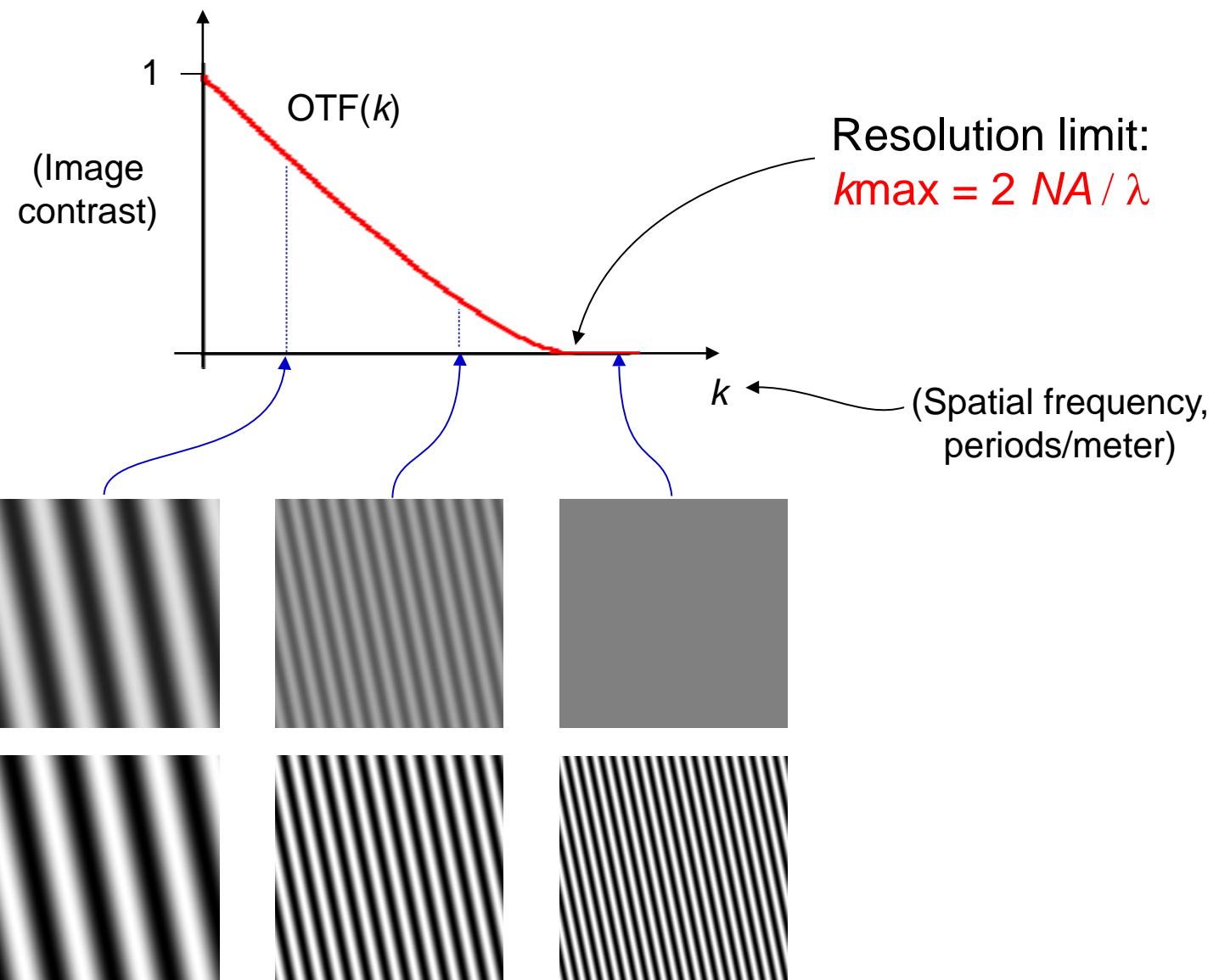


Q: Don't we always want it full open??

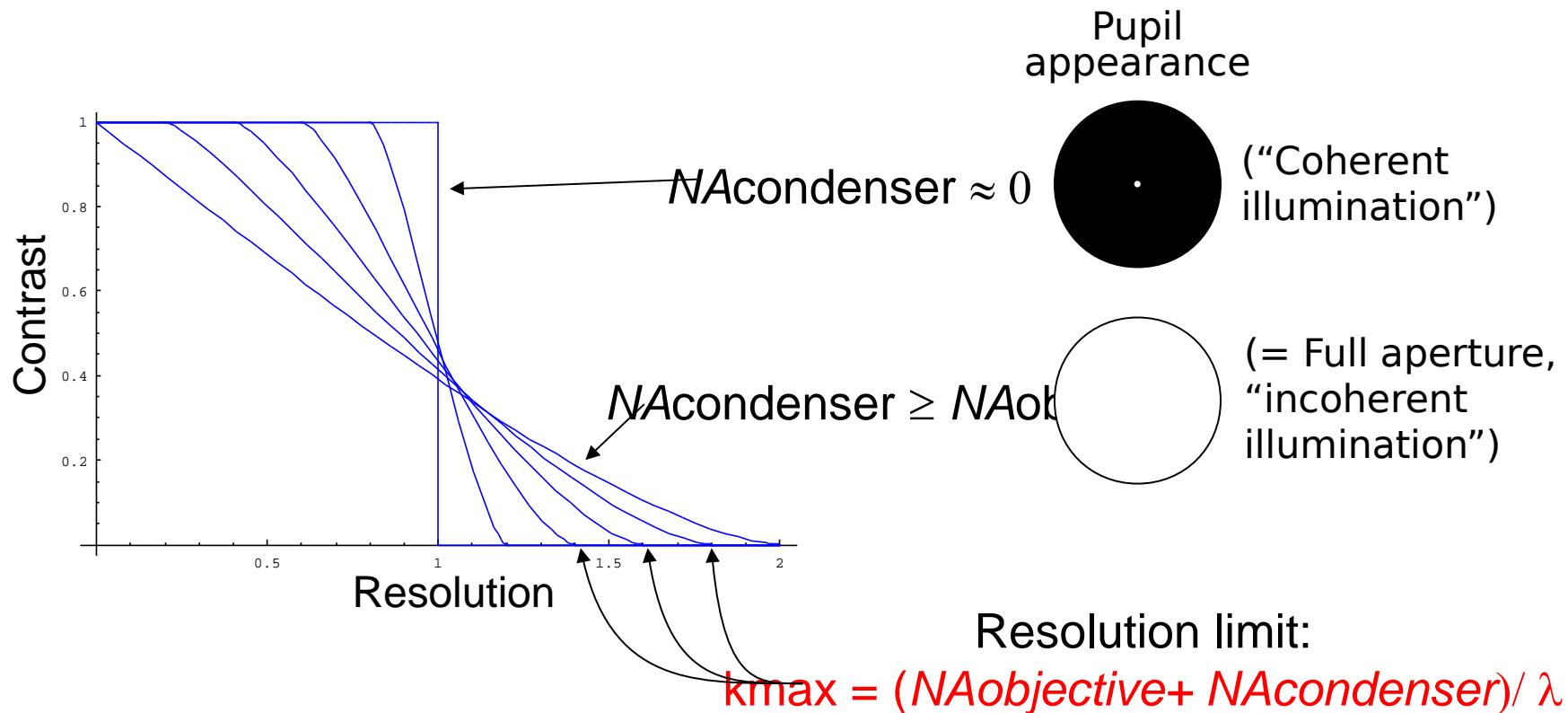
A: No

Why? Tradeoff:
resolution vs. **contrast**

Spatial frequencies & the Optical Transfer Function (OTF)



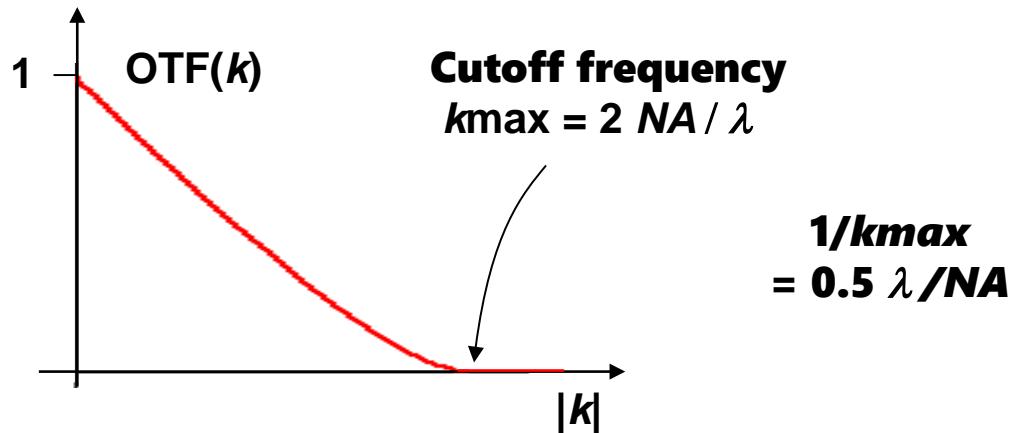
Resolution & Contrast vs. Illumination aperture



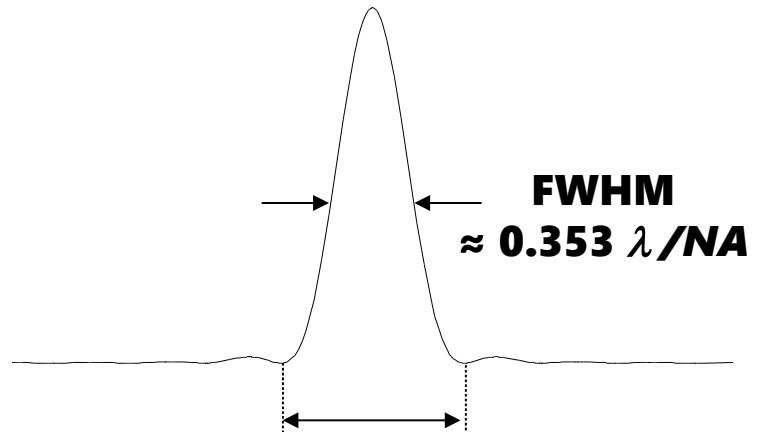
Increasing the illumination aperture increases resolution but decreases contrast

Definitions of Resolution

As the OTF cutoff frequency



As the Full Width at Half Max (FWHM) of the PSF



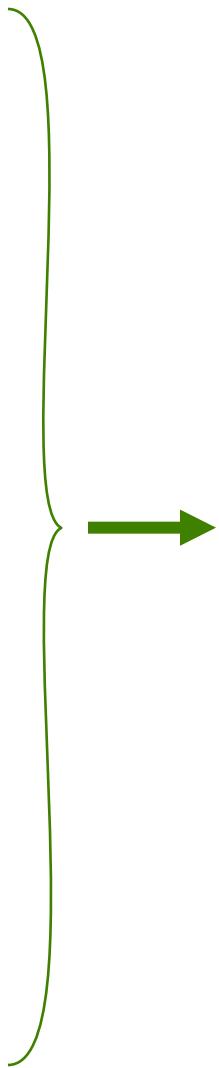
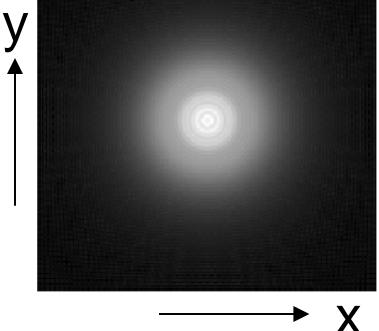
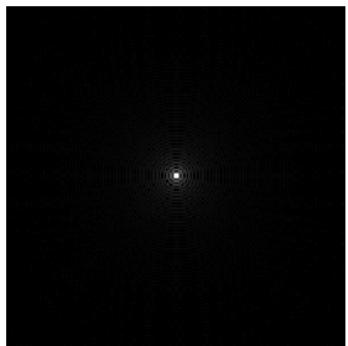
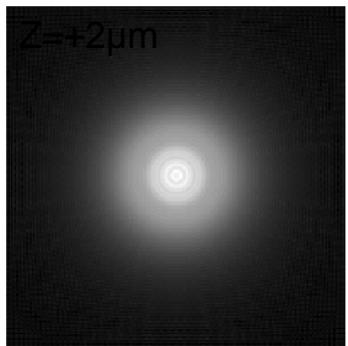
As the diameter of the Airy disk (first dark ring of the PSF)
= "Rayleigh criterion"

Airy disk diameter
 $\approx 0.61 \lambda / \text{NA}$

The 3D Point Spread Function (PSF)

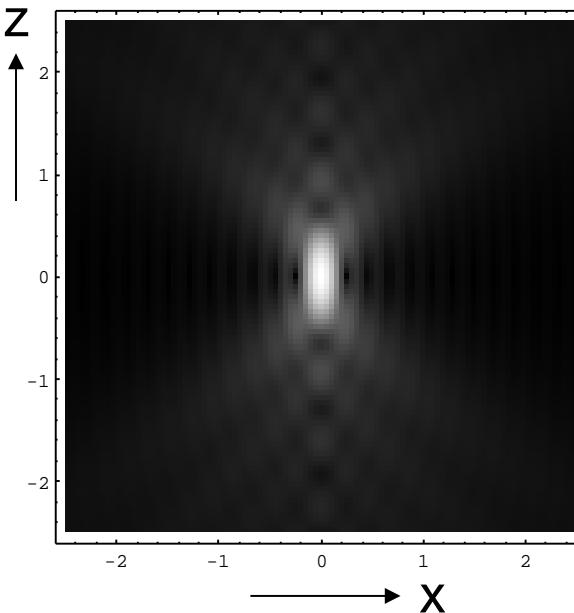
The image of a point object

**2D PSF
for different defocus**

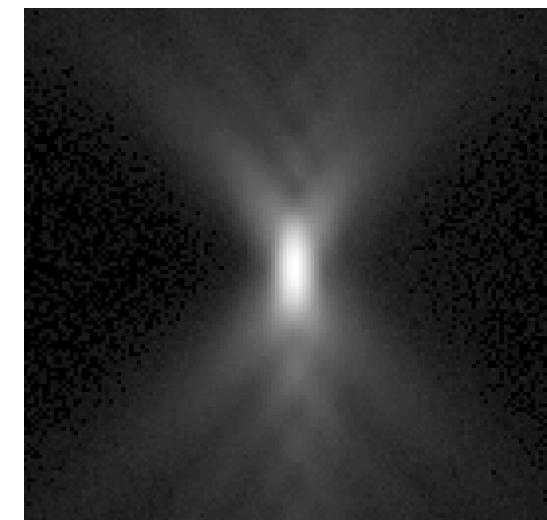


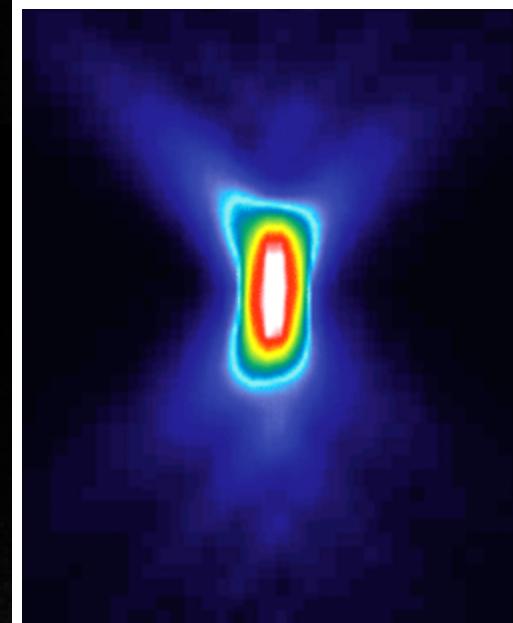
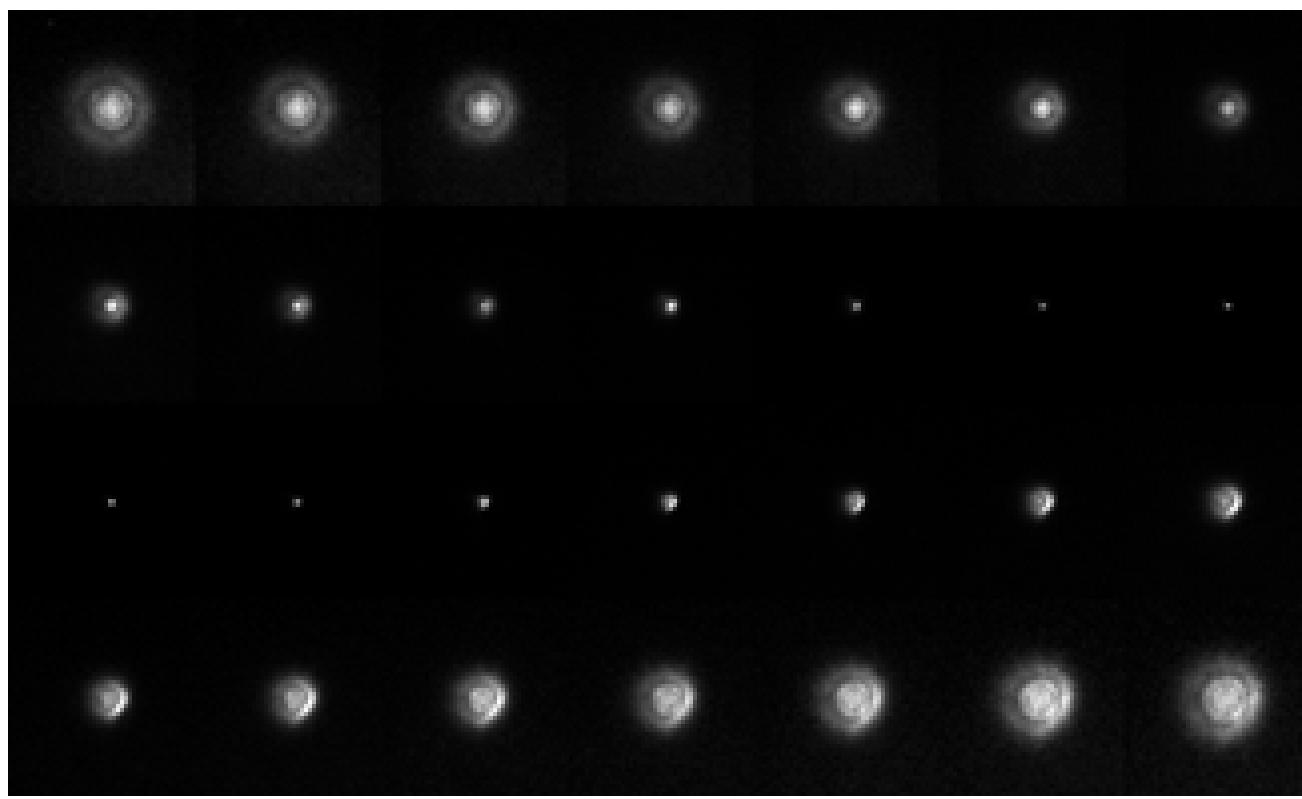
3D PSF

Calculated



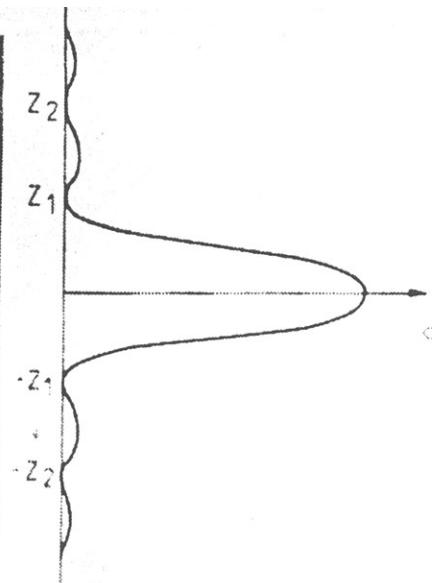
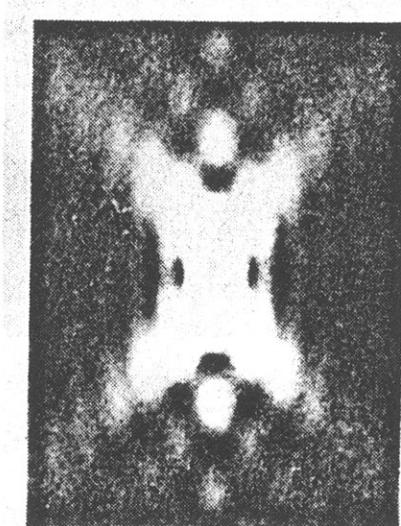
Measured





X - Z

Z-resolution, a.k.a. depth of field, for widefield microscopy



Z-resolution:
 $2\lambda n / NA^2$

NA	Resolution (nm; X-Y)	depth of field (μm)
0.3	1017	11.1
0.75	407	1.77
0.95	321	1.11
1.4	218	0.773

Summary: Numerical Aperture and Resolution

- Numerical aperture, ***not magnification***, sets the smallest details you can resolve in an image
- Increasing NA also increases the amount of light collected by the lens, thereby increasing the brightness of the image – this scales as NA^2

Specifications for some common objectives

Magnification	NA	Resolution (nm)	Depth of Field (nm)	Light gathering (arb. units)
10	0.3	1017	16830	0.09
20	0.75	407	2690	0.56
40	0.95	321	1680	0.90
40	1.3	235	896	1.69
60	1.2	254	926	1.44
60	1.4	218	773	1.96
100	1.4	218	773	1.96

Aberrations

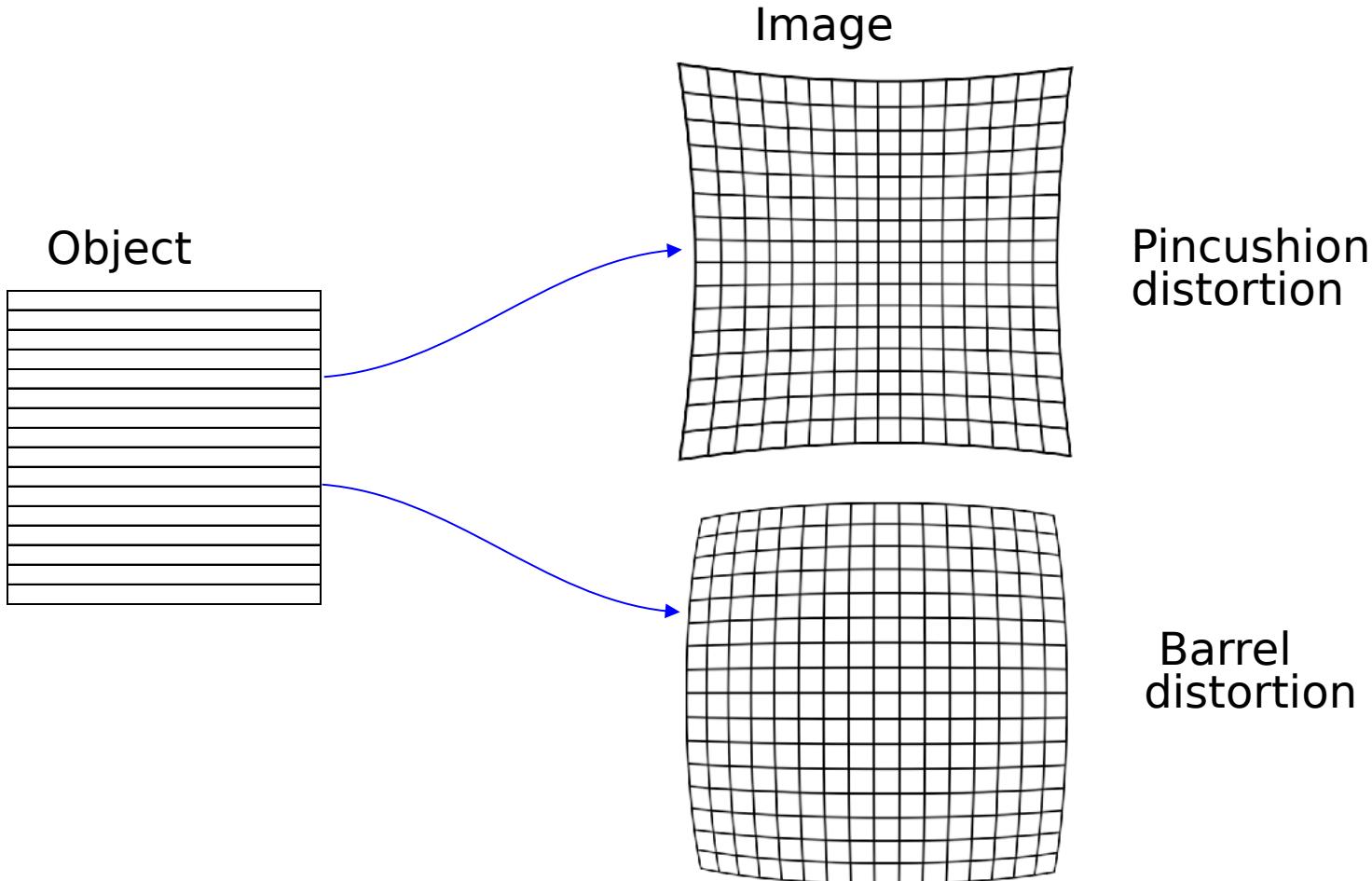
They are the enemy

Aberrations

- Chromatic aberrations
 - Longitudinal chr. Ab.
 - Lateral chr. Ab.
- Wavefront aberrations
 - Spherical aberration
 - Astigmatism
 - Coma
 - ...
- Curvature of field
- Distortion

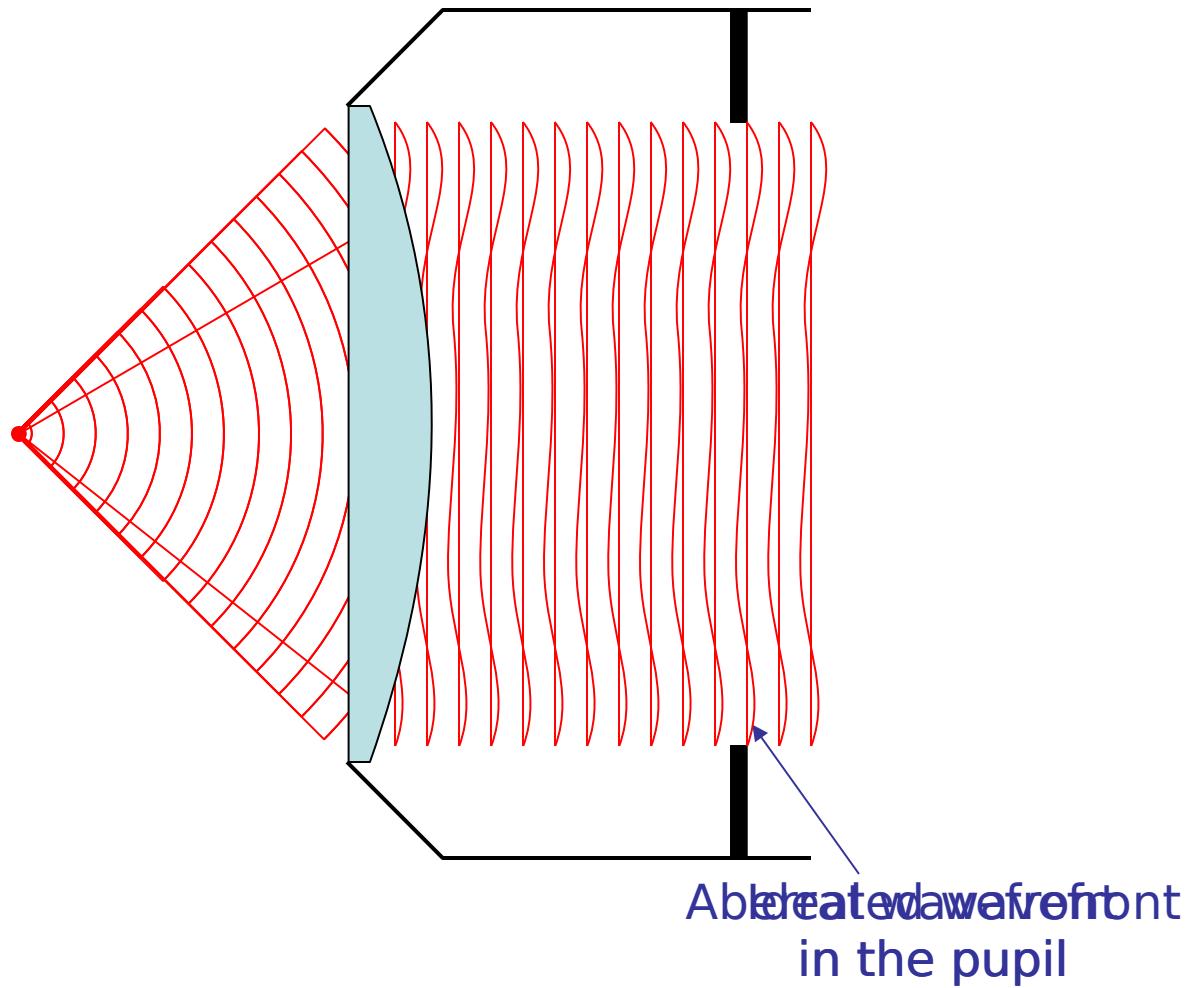
Geometric Distortion

= Radially varying magnification

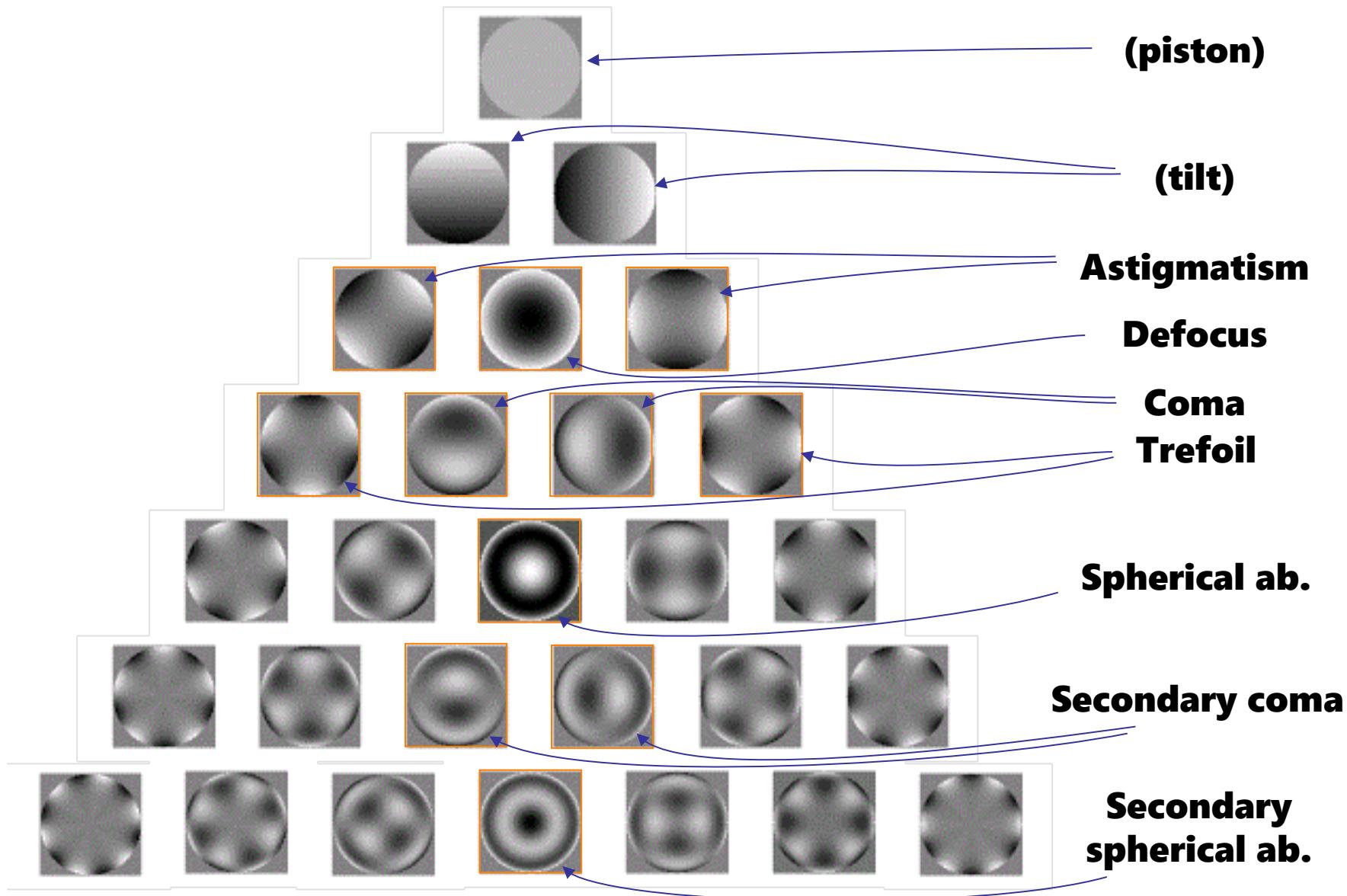


May be introduced by the projection eyepiece

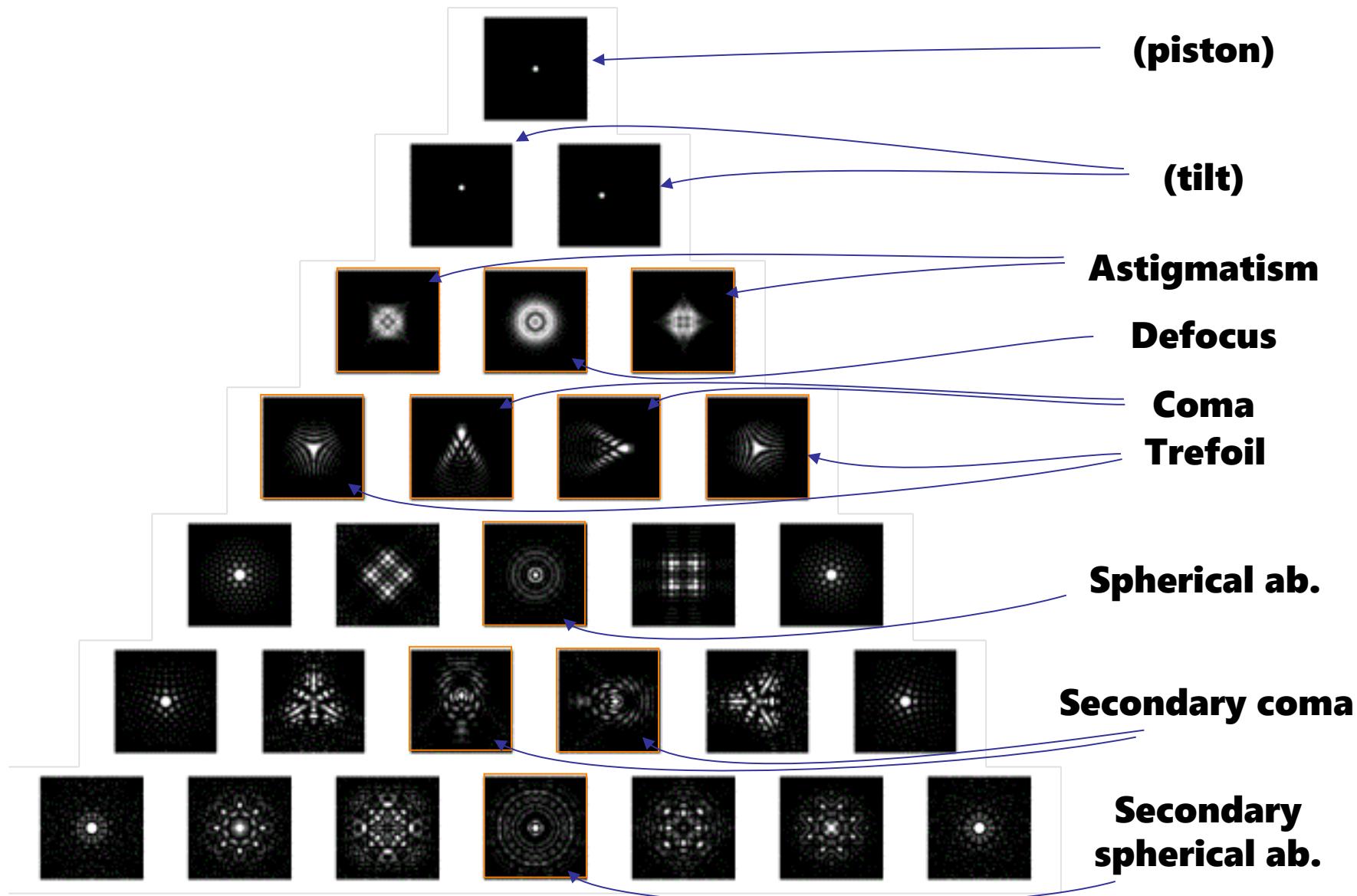
Wavefront Aberrations



Wavefront Aberrations

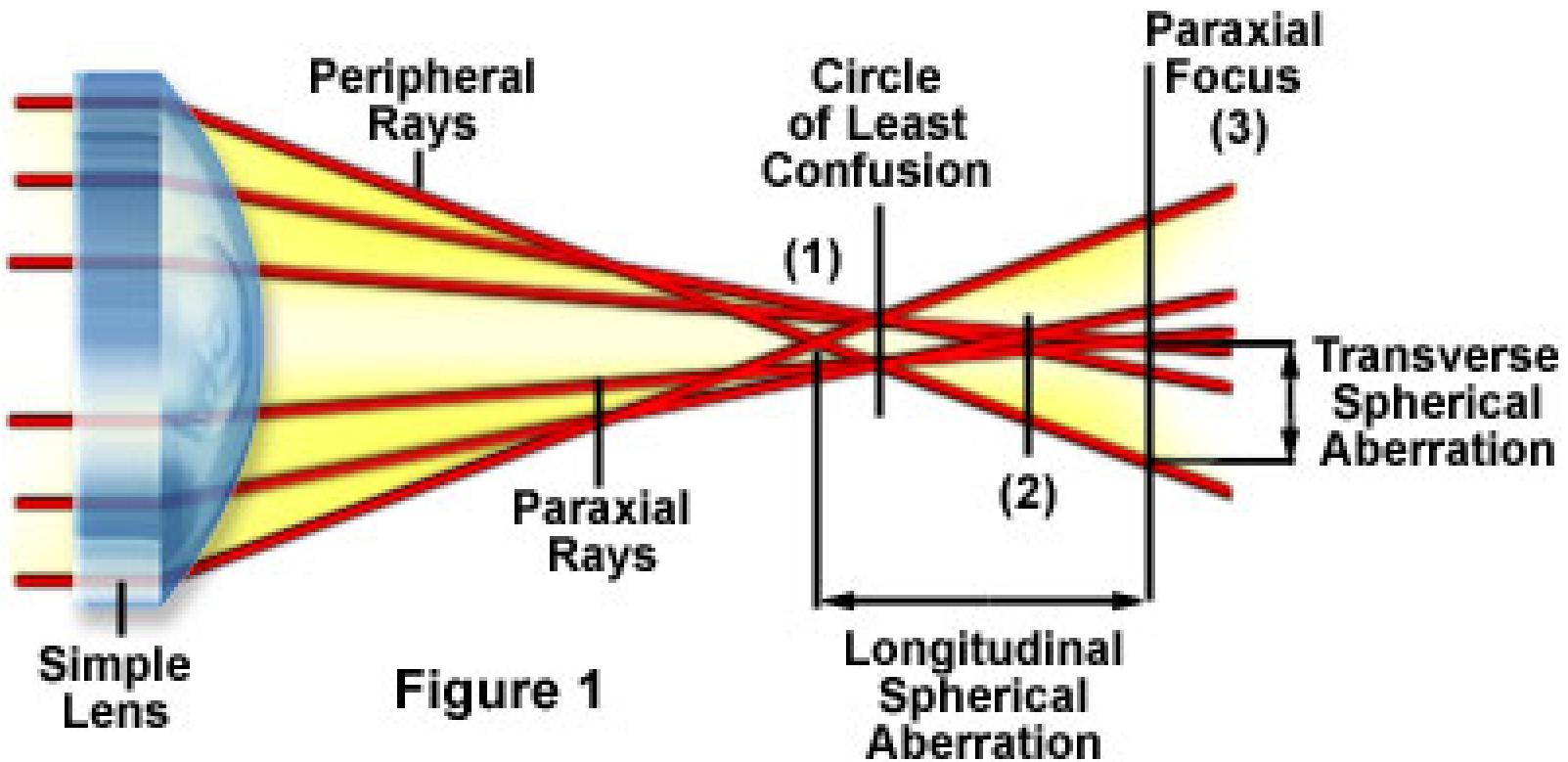


PSF Aberrations



Spherical Aberration

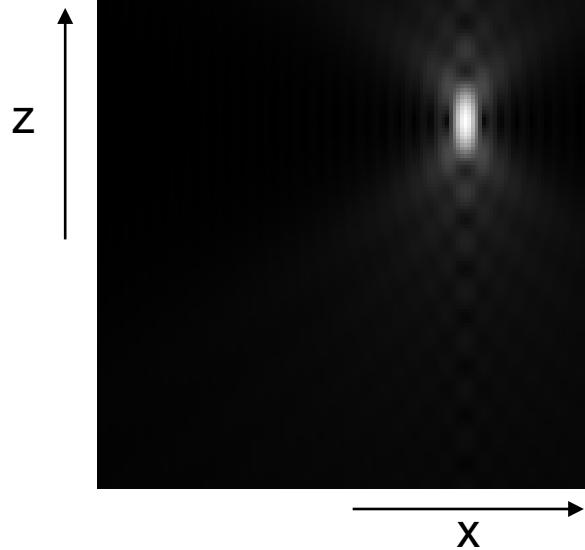
Longitudinal and Transverse Spherical Aberration



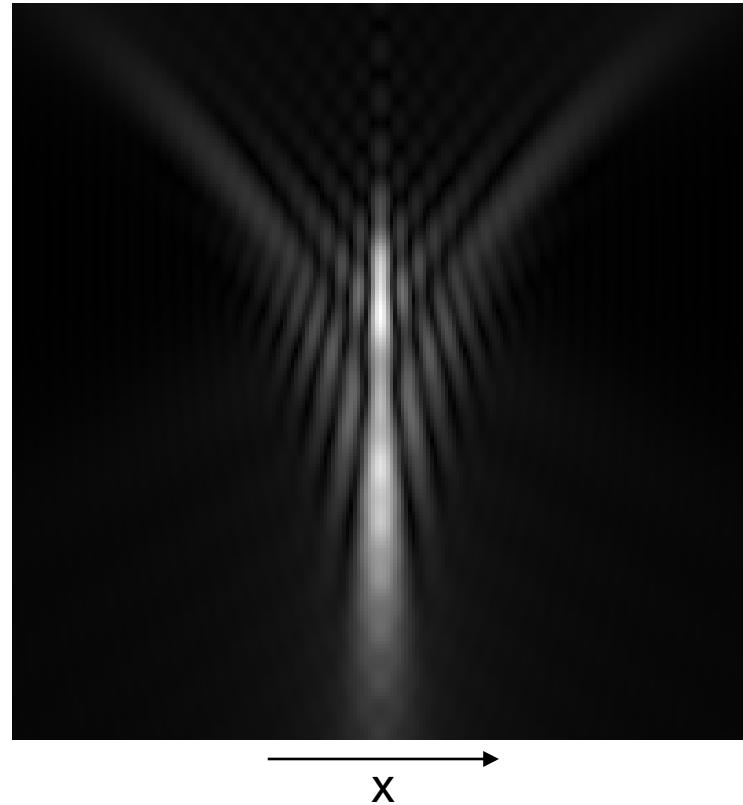
Spherical Aberration

Point spread functions

Ideal



1 wave of spherical ab



Causes of spherical aberration

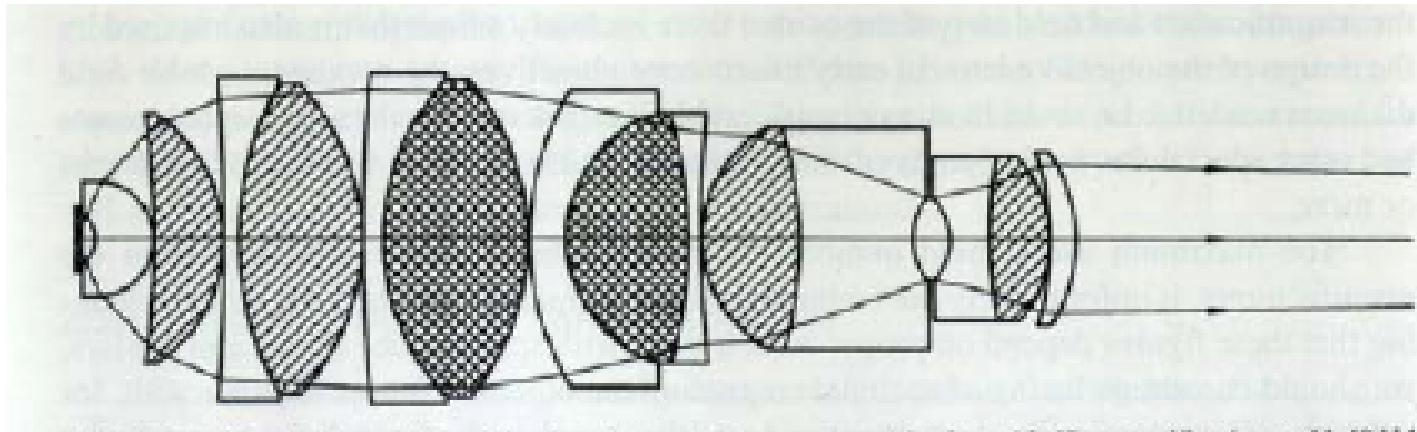


FIGURE 2-36. Design of Nikon Plan Apochromat oil immersion objective with 60 \times magnification and 1.40 NA.

- Modern objectives are complicated!
- The optical design requires specifying the optical path length between the sample and the back focal plane of the lens
- $OPL = l_1n_1 + l_2n_2 + \dots + l_{nn}$

Sources of Spherical Aberration

Design compromises

Manufacturing tolerances

Immersion fluid index error

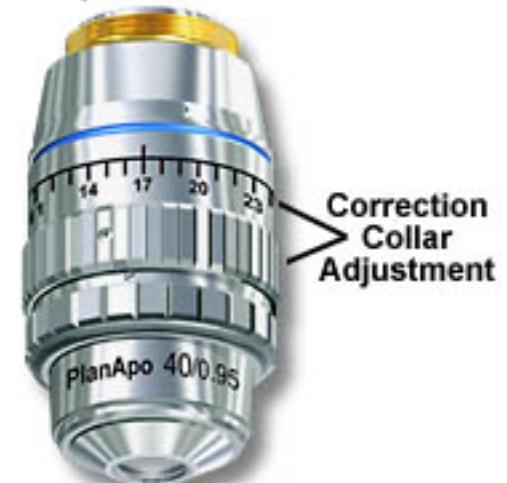
Temperature variation

Cover slip thickness

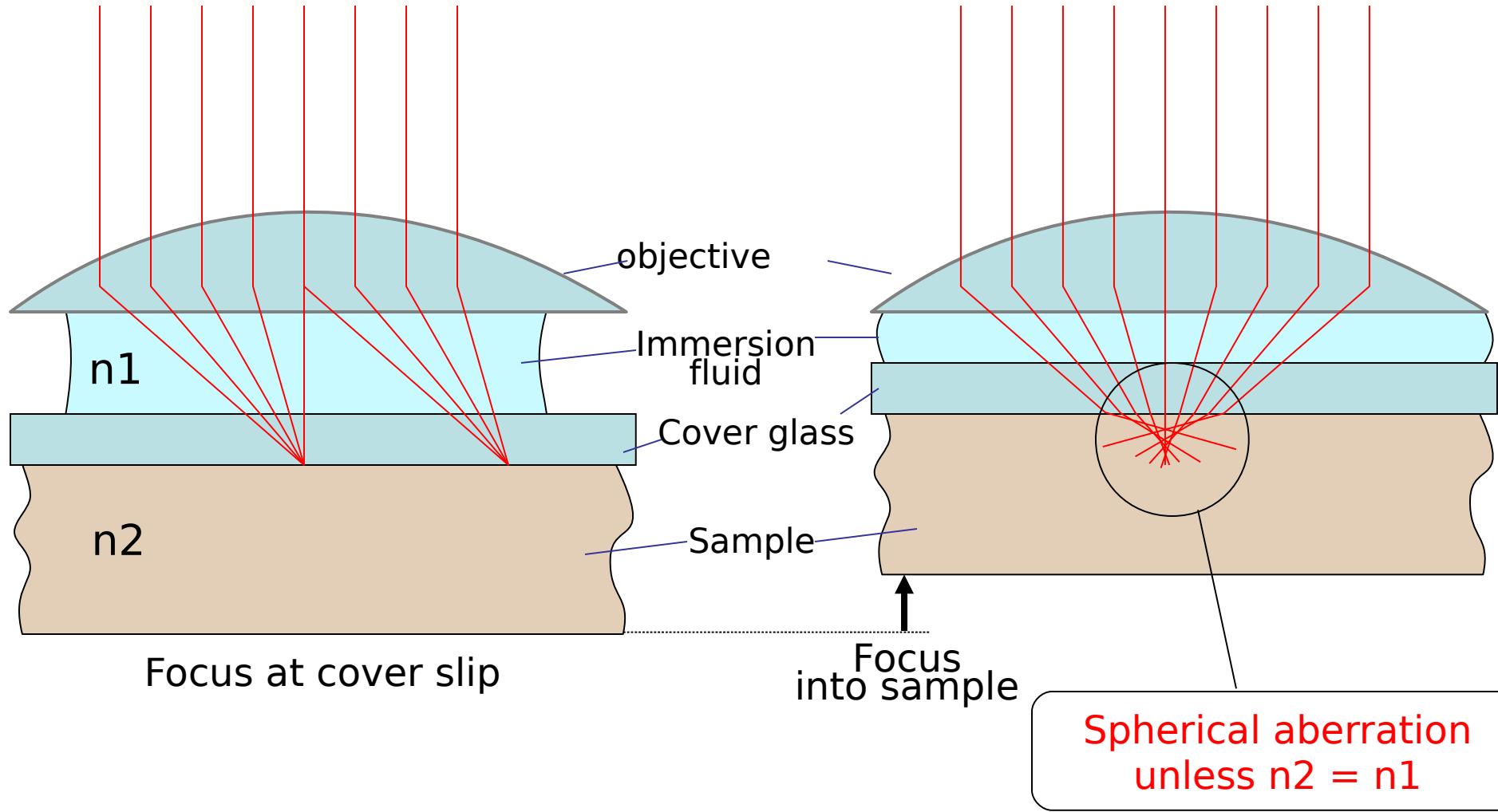
(high-NA objectives except oil immersion)

Correction collar setting

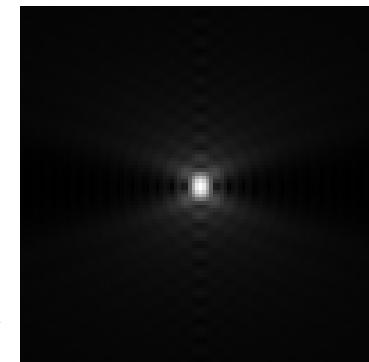
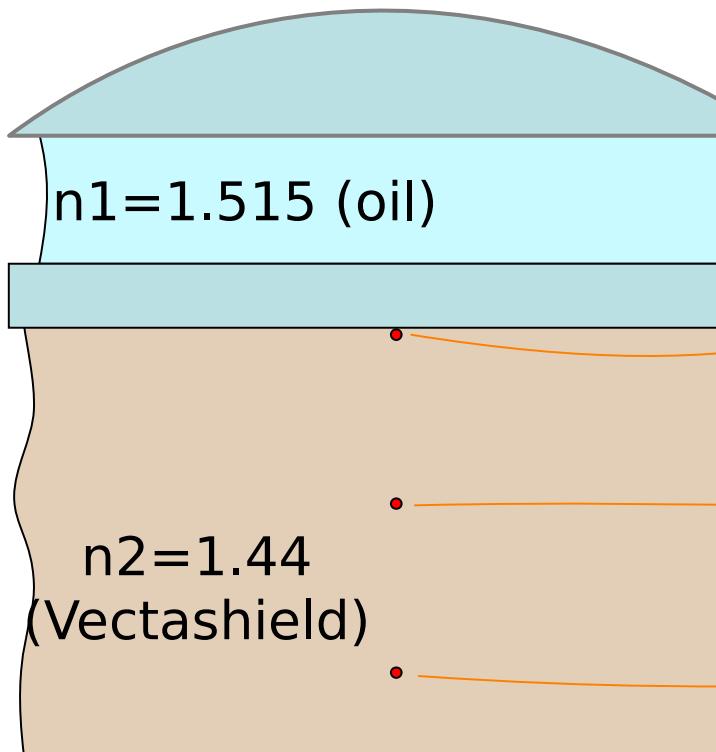
Sample refractive index mismatch



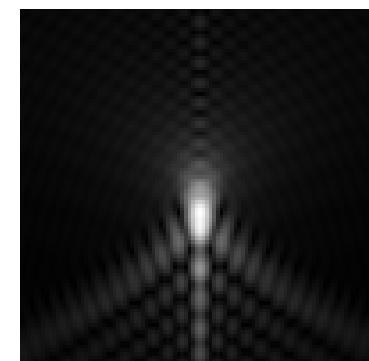
Index Mismatch & Spherical Aberration



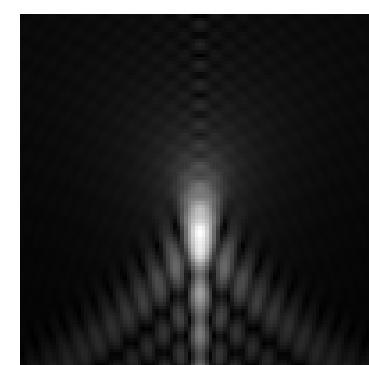
Index Mismatch & Spherical Aberration



$z = 0 \mu\text{m}$



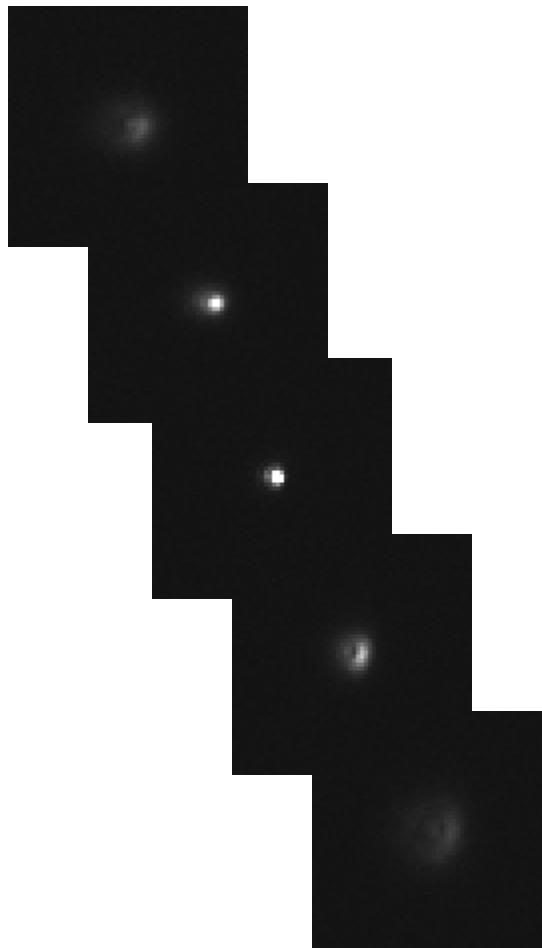
$z = 25 \mu\text{m}$



$z = 50 \mu\text{m}$

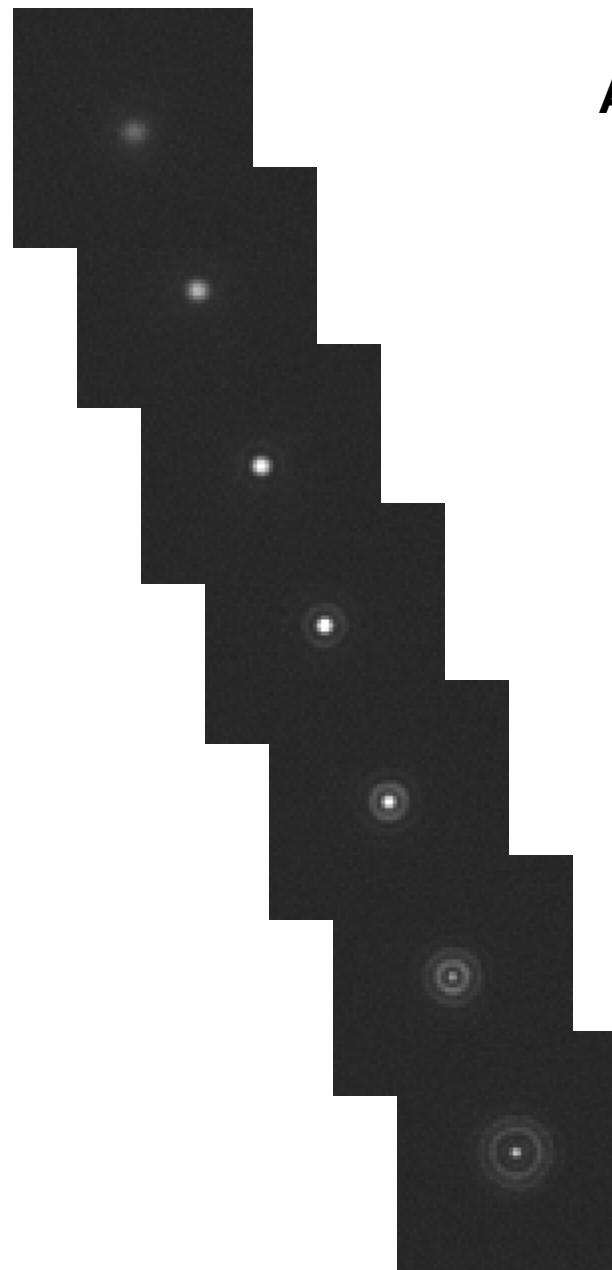
How to recognize spherical aberration

Unaberrated



0.5 μm steps

Aberrated

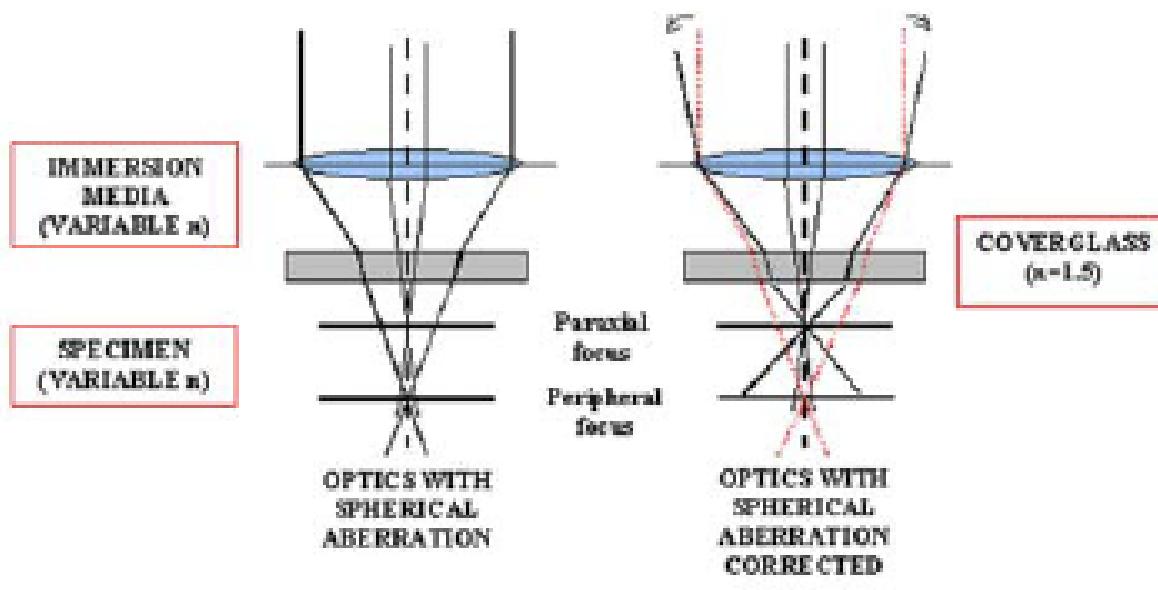


1 μm steps

What can you do about spherical aberration?

- Use 0.17 mm coverslips (~ #1.5)
- Work close to the coverslip
- Match lenses to the refractive index of your samples, and vice versa
 - For aqueous samples, use water immersion / water dipping lenses
 - For fixed samples and oil immersion lenses, mount your sample in a medium with $n = 1.515$
- Adjust objective correction collar when available
- Use lower NA lenses

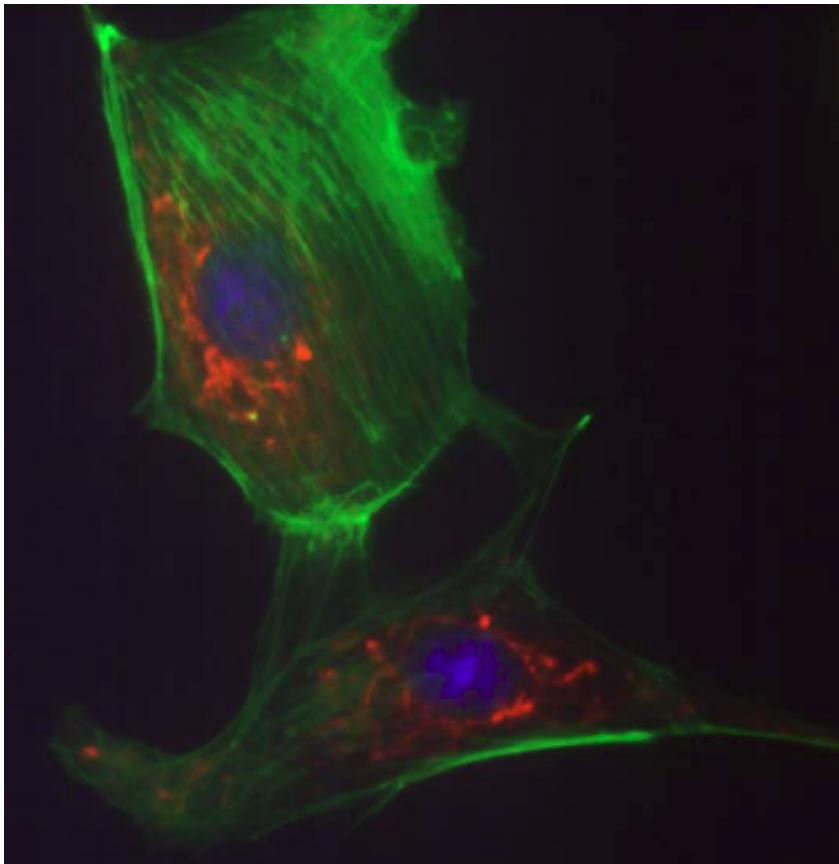
Correction collars



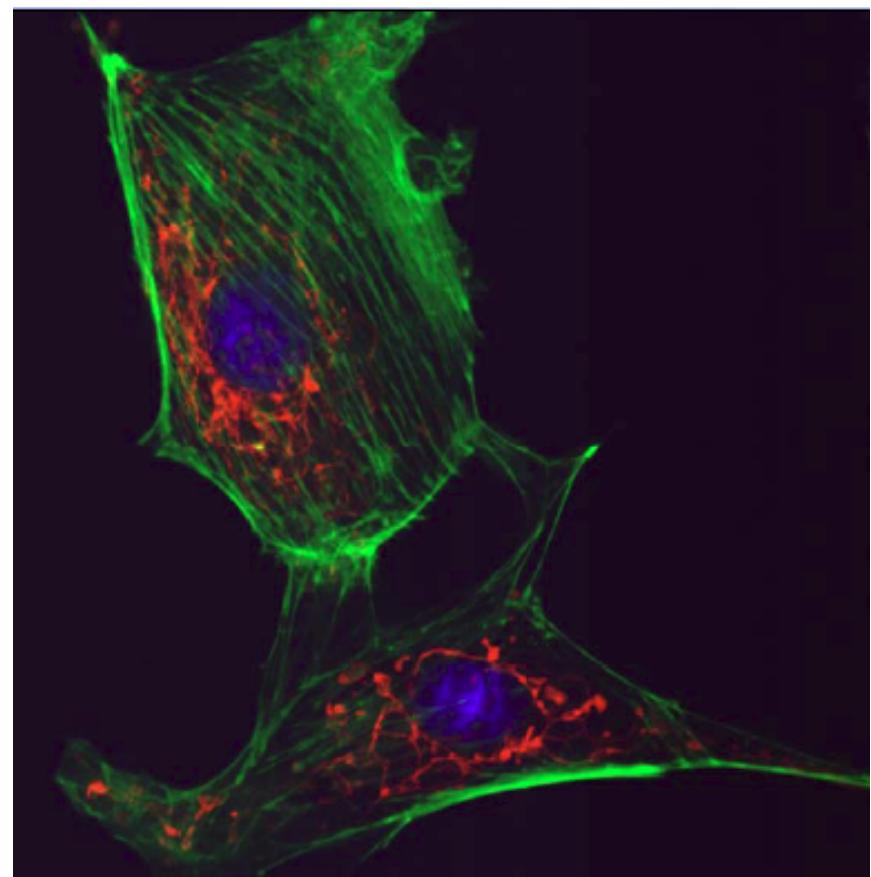
A correction collar can only eliminate spherical aberration at a single focal plane

Example

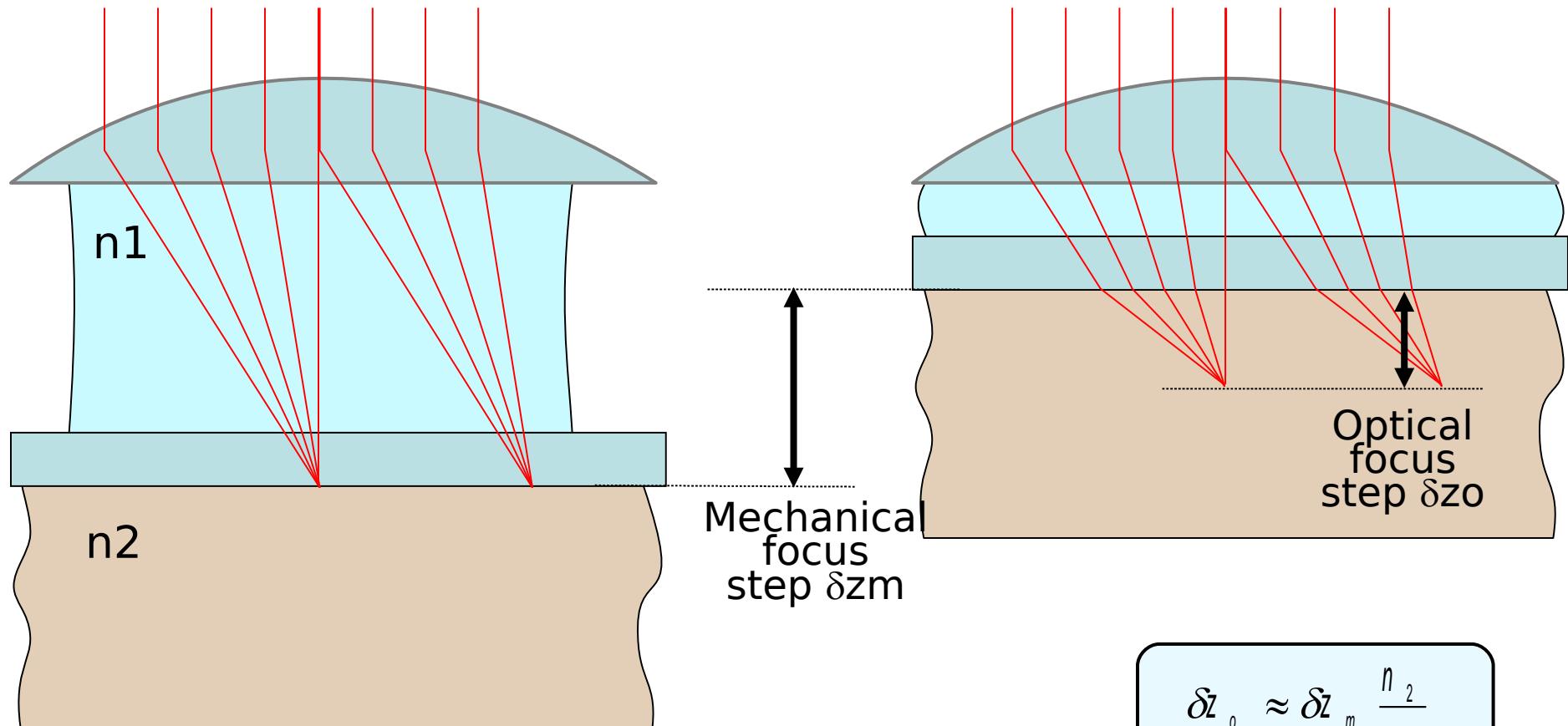
Aberrated



Unaberrated



Index Mismatch & Axial Scaling



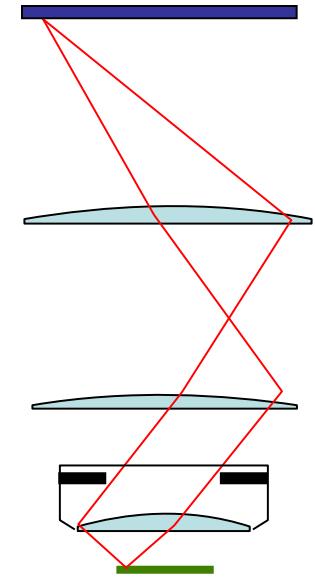
$$\delta z_o \approx \delta z_m \frac{n_2}{n_1}$$

If there is index mismatch,
your z pixel size is not what you think

Sources of Astigmatism & Coma

Off-axis (edges of field of view)

- All objectives have some
- Present in the design
- You get what you pay for

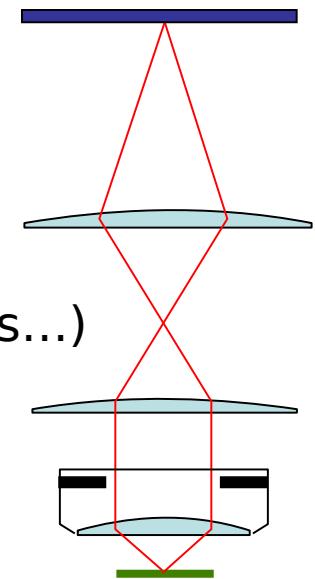


On-axis (center of field of view)

Should be none, by symmetry.

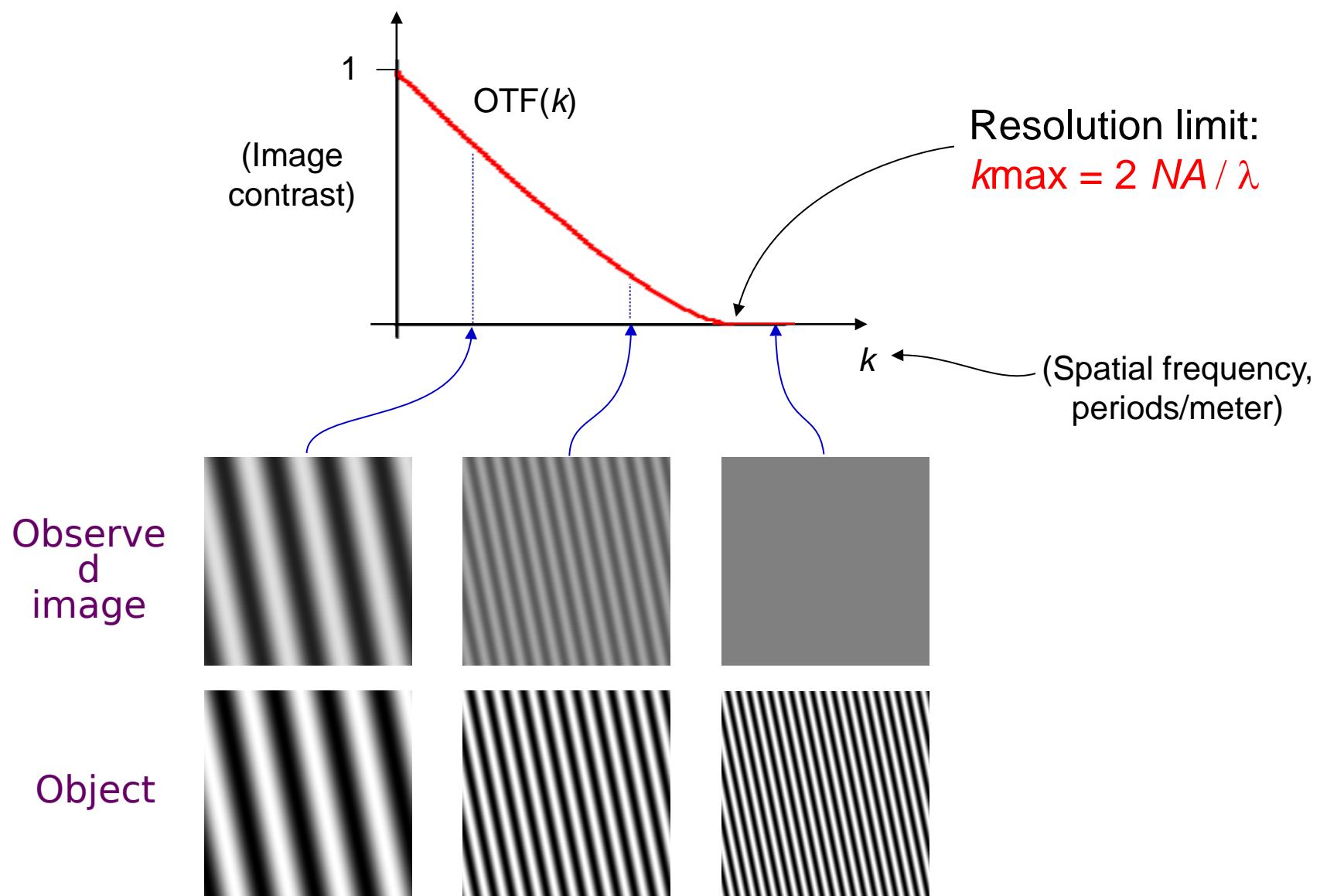
If they are there, they could be from:

- manufacturing or assembly tolerances
- dirt or abuse
- Misalignment (tilt, off-axis shift of something)
- **bad downstream components** (mirrors, dichroics, filters...)
- **Air bubble** in the immersion fluid or sample
- **Tilted cover slip**
(dry and water-immersion high-NA lenses)



More about Spatial frequencies & the Optical Transfer Function (OTF)

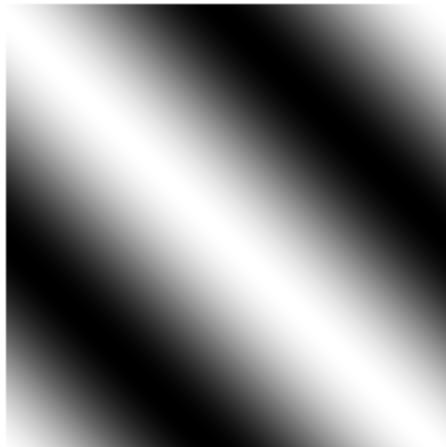
The response to pure waves is well-defined by the Optical Transfer Function (OTF)



Think of Images as Sums of Waves

... or “spatial frequency components”

one wave



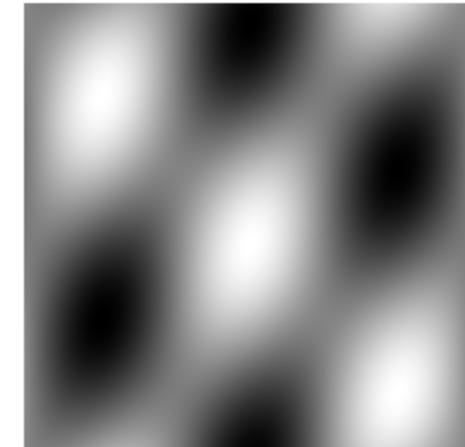
+

another wave

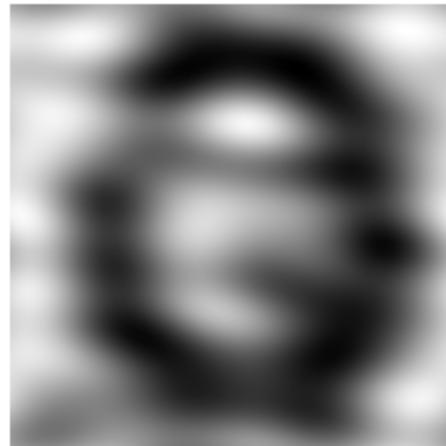


=

(2 waves)



(25 waves)



+ (...) =

+ (...) =

(10000 waves)



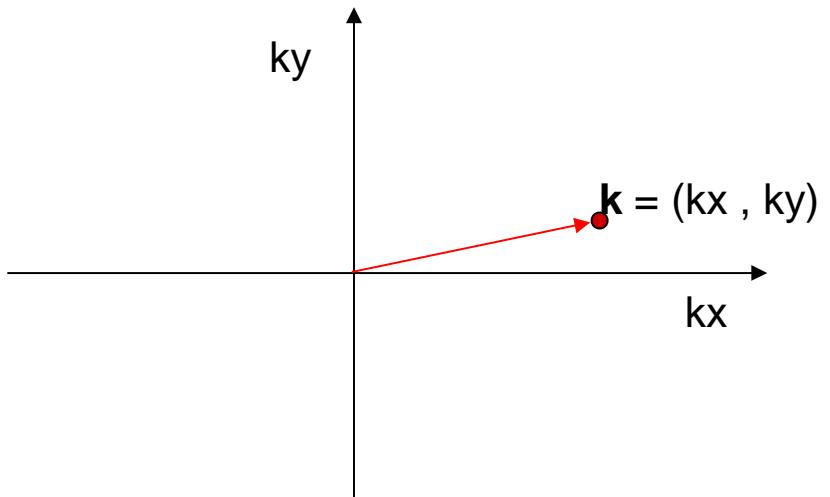
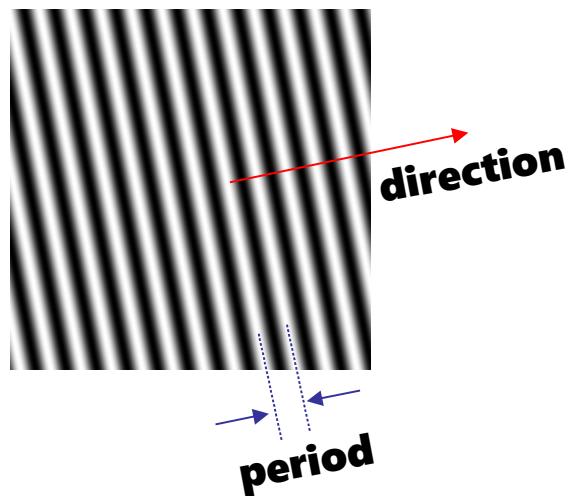
Frequency Space

To describe a wave,
we need to specify its:

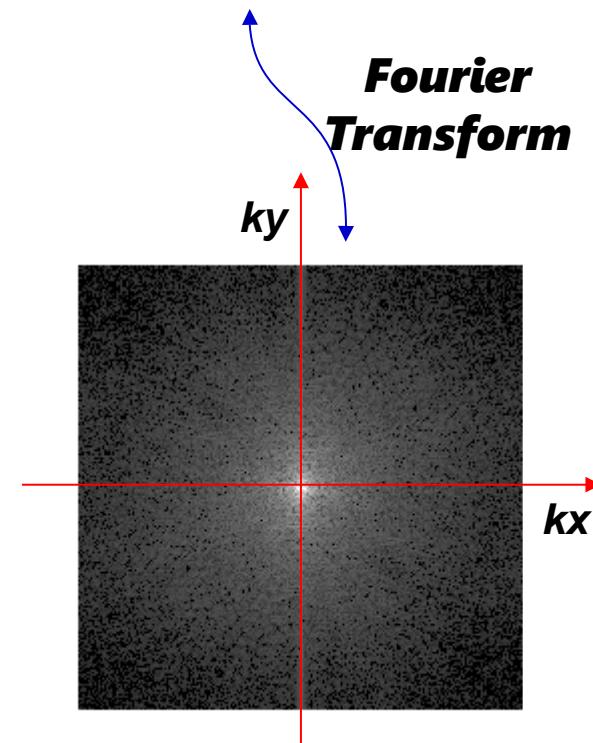
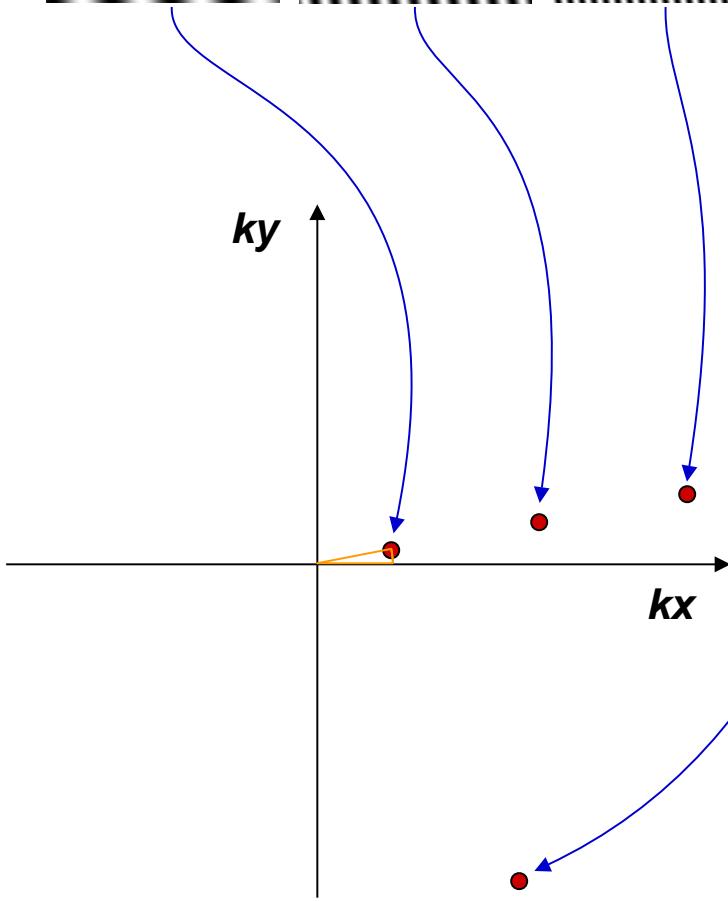
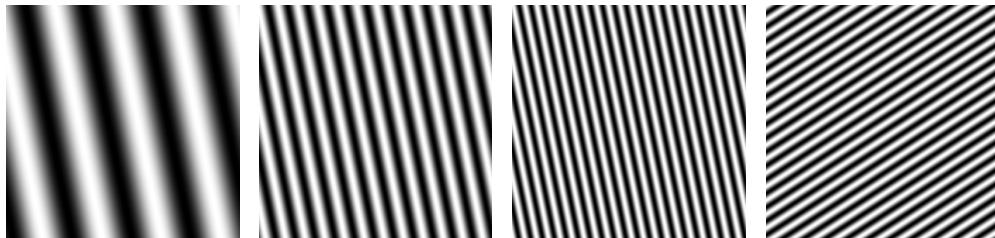
- Frequency (how many periods/meter?) → Distance from origin
- Direction → Direction from origin
- Amplitude (how strong is it?) → Magnitude of value
- Phase (where are the peaks & troughs?) → Phase of value

Can describe it by
a *value* at a *point*

complex



Frequency Space and the *Fourier Transform*



Properties of the Fourier Transform

$$F(k) = \int f(r) e^{-2\pi i k \cdot r} dr$$

Completeness:

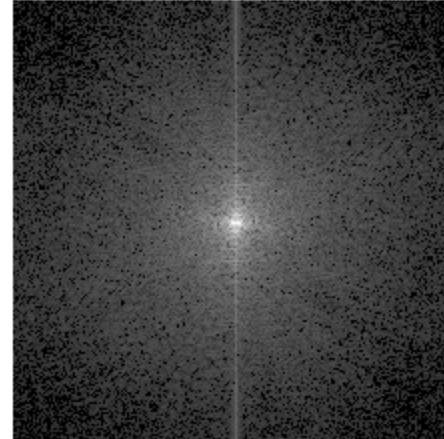
The Fourier Transform contains all the information of the original image

Symmetry:

The Fourier Transform of the Fourier Transform is the original image



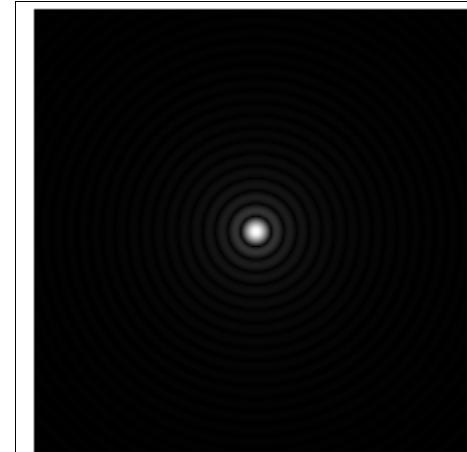
Fourier transform



The OTF and Imaging



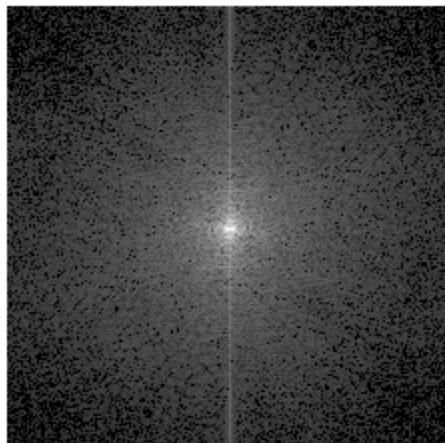
convolution



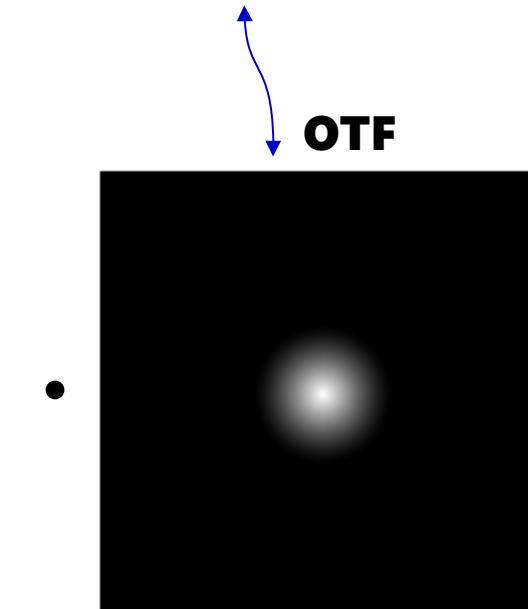
Observed Image



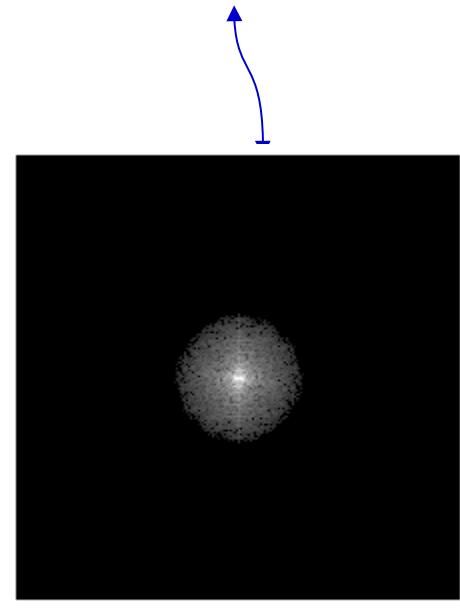
Fourier Transform



OTF



=



Convolutions

$$(f \otimes g)(r) = \int f(a) g(r-a) da$$

Why do we care?

- **They are everywhere...**
- **The convolution theorem:**

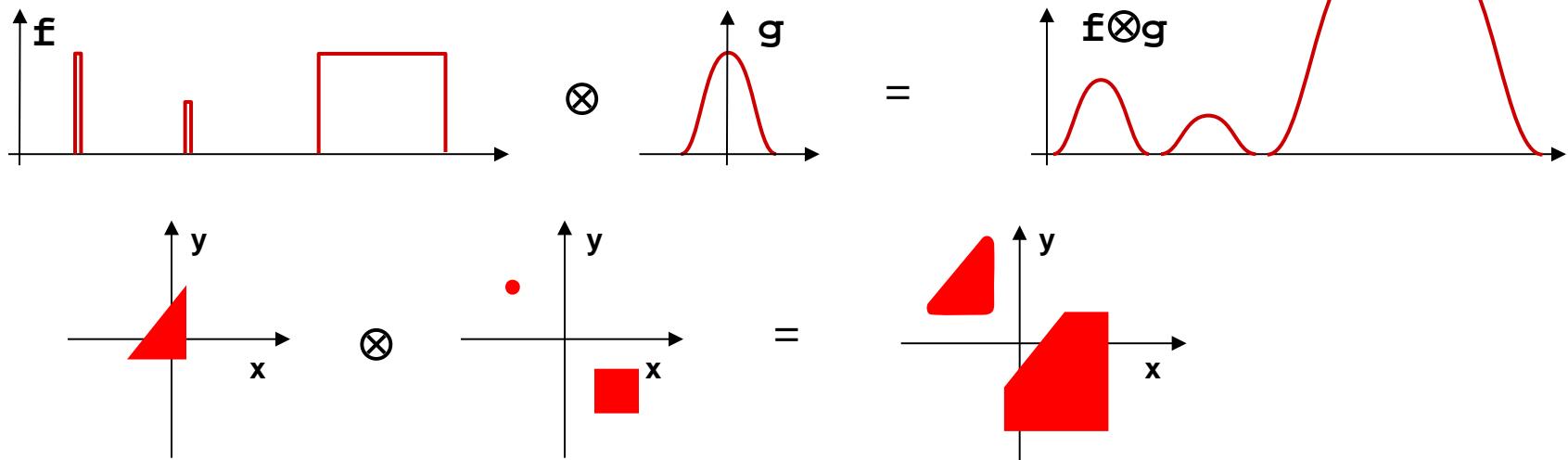
If $\tilde{h}(r) = \tilde{f} \otimes \tilde{g}(r)$,
then $\tilde{h}(k) = \tilde{f}(k) \tilde{g}(k)$

A convolution in real space becomes
a product in frequency space & vice versa

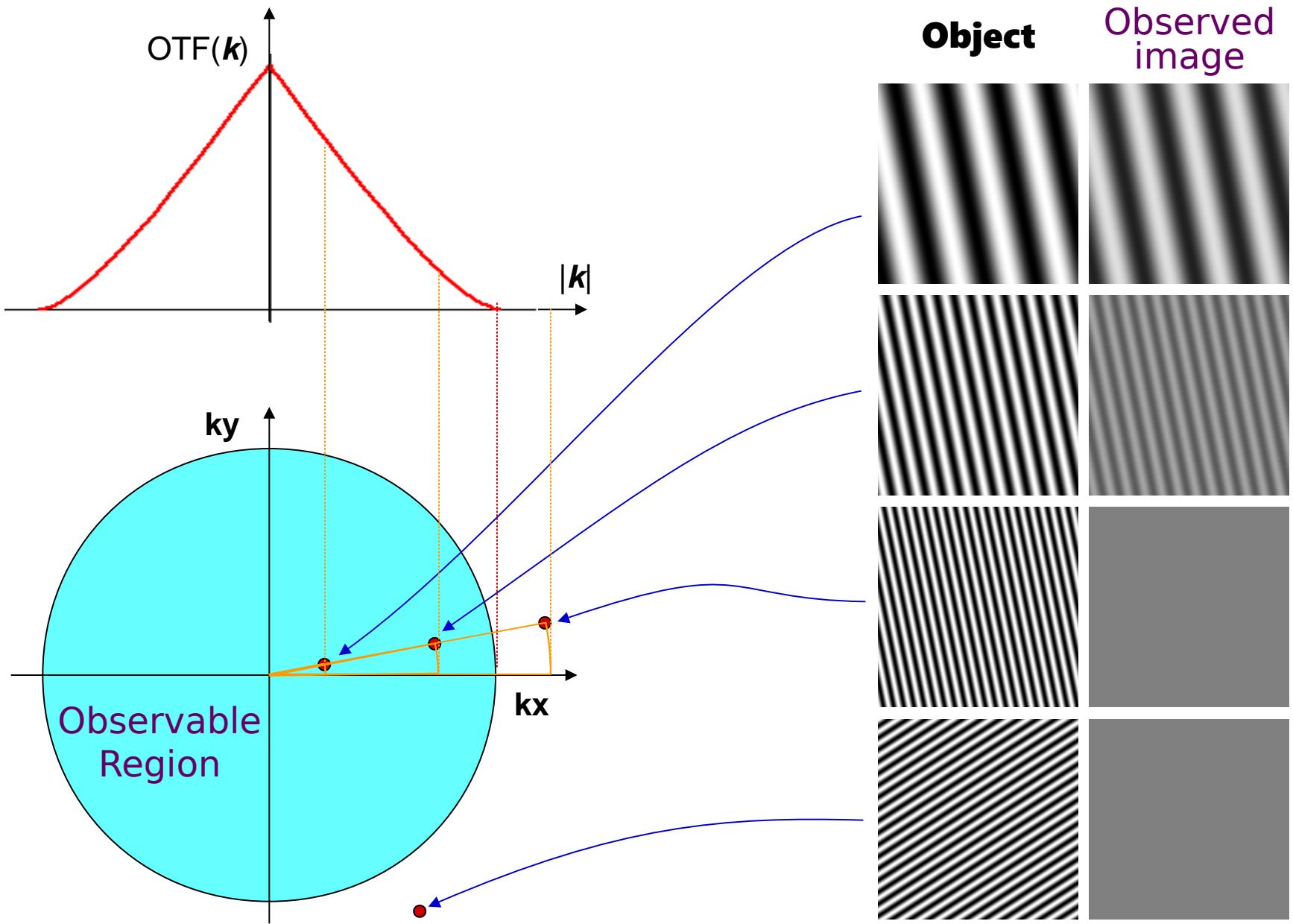
Symmetry: $g \otimes f = f \otimes g$

So what is a convolution, intuitively?

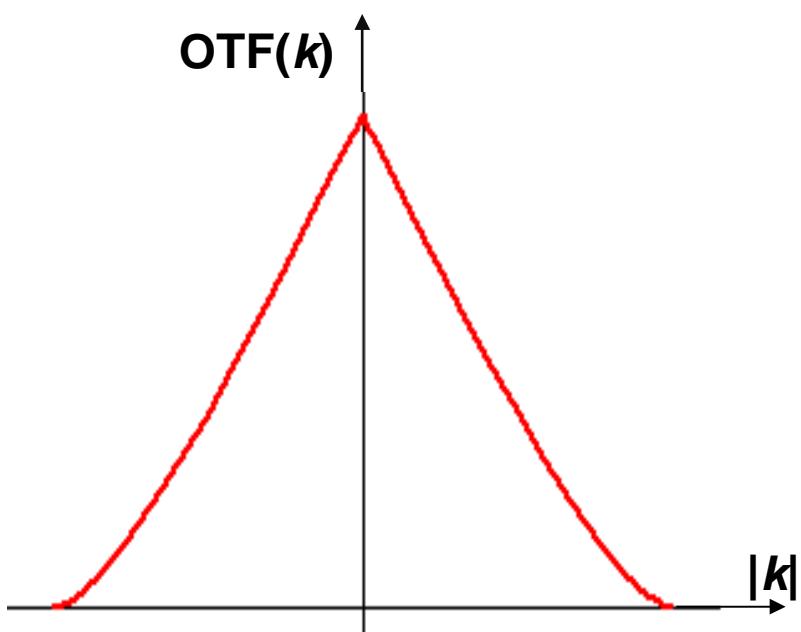
- “Blurring”
- “Drag and stamp”



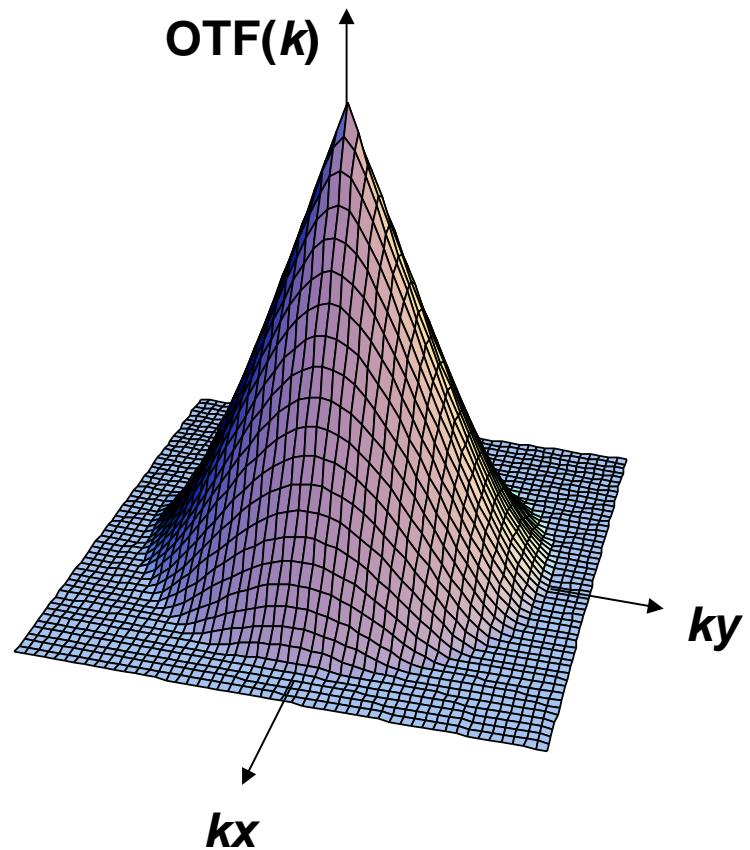
The Transfer Function Lives in Frequency Space



The 2D In-focus Optical Transfer Function (OTF)

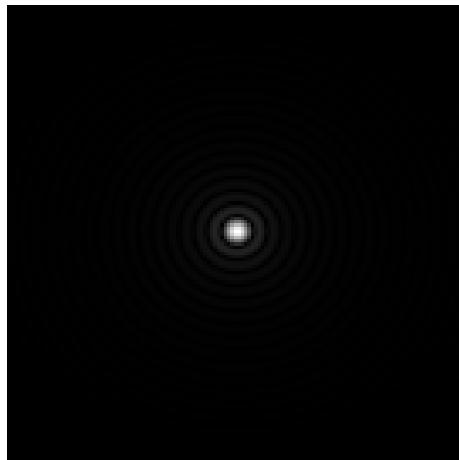


(Idealized
calculations)



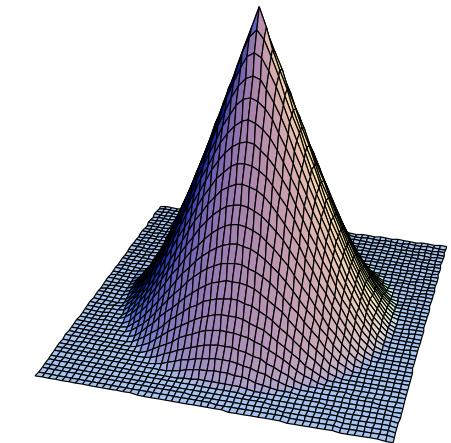
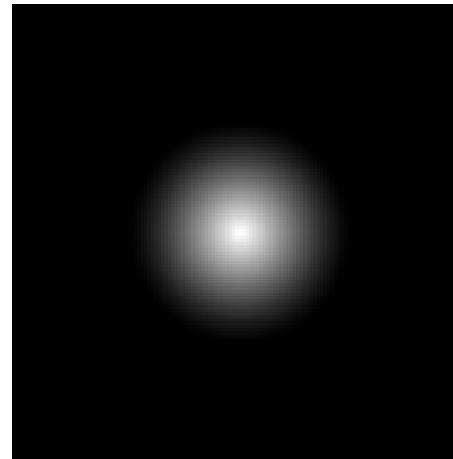
The 3D OTF

2D PSF

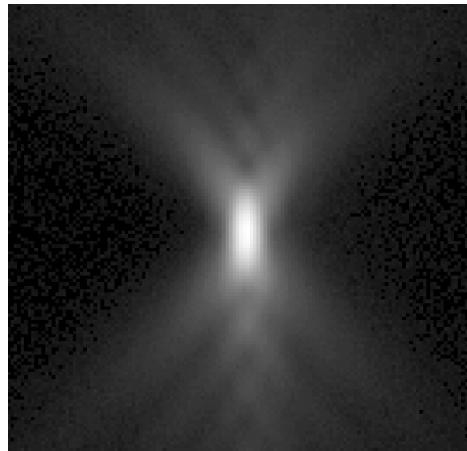


↔
**2D
F.T.**↔

2D OTF

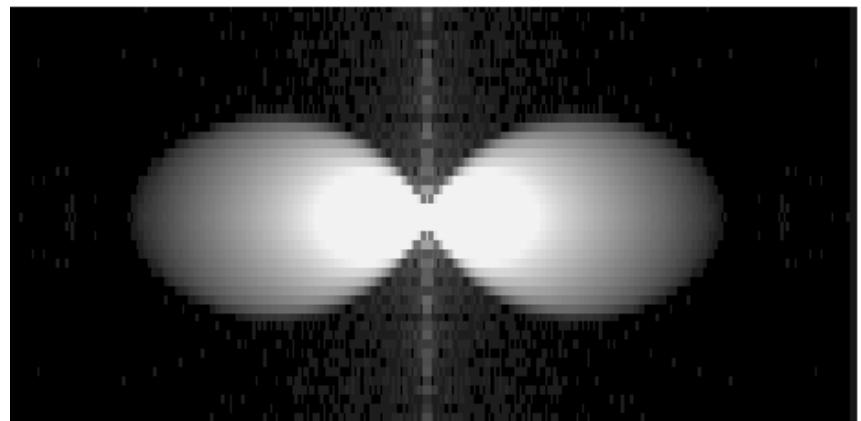


3D PSF

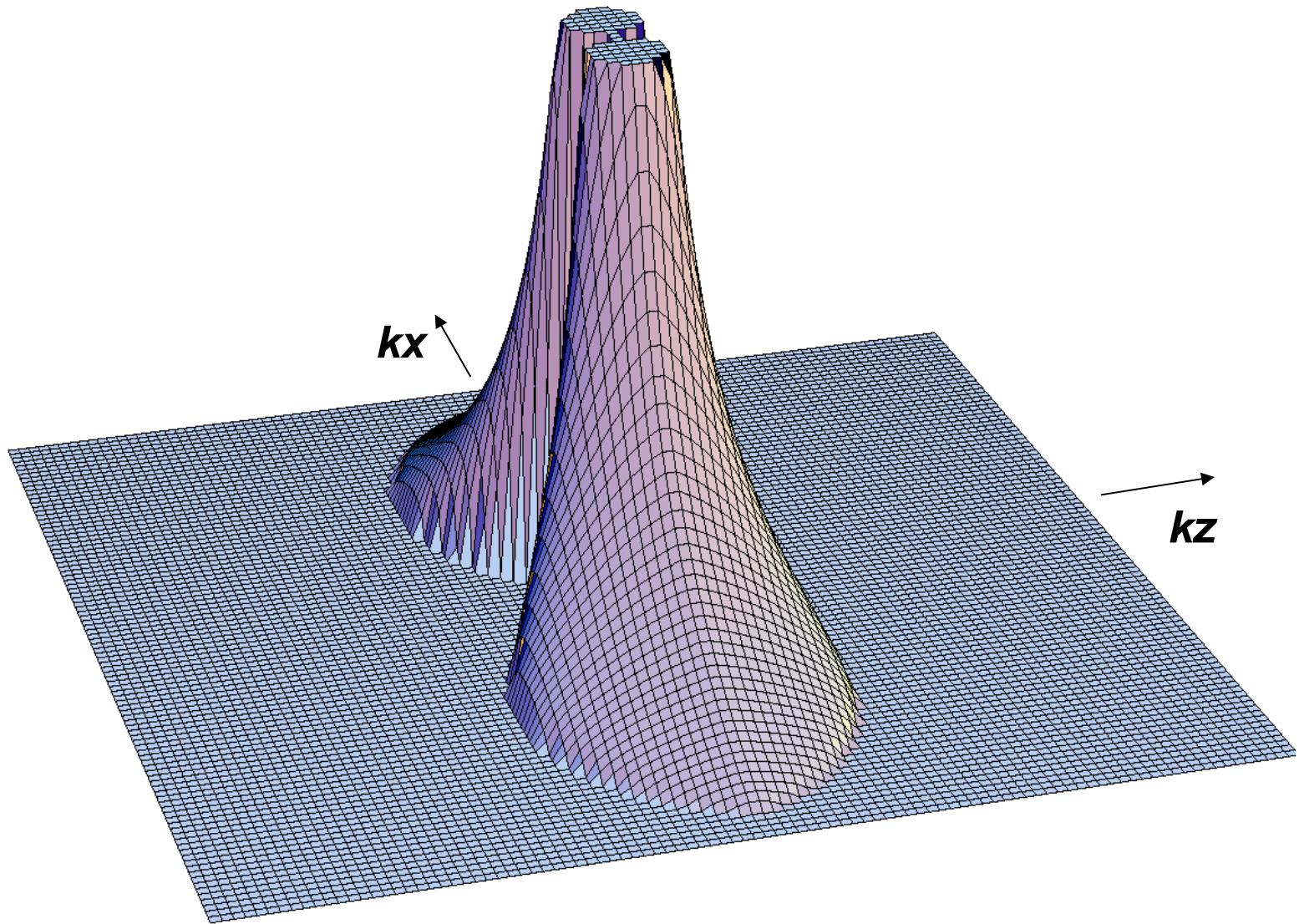


↔
3D F.T.↔

3D OTF



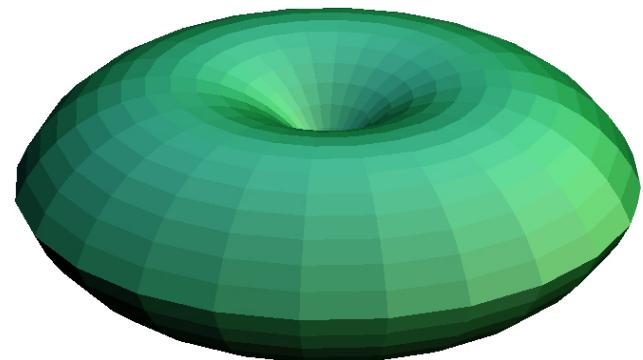
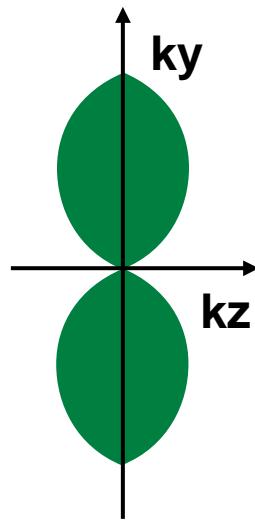
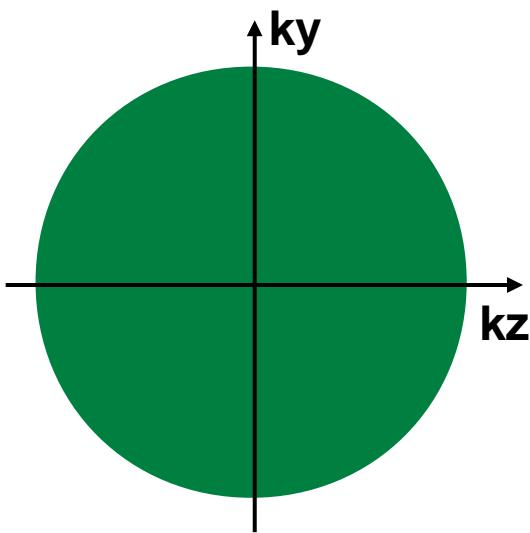
Values of the 3D OTF



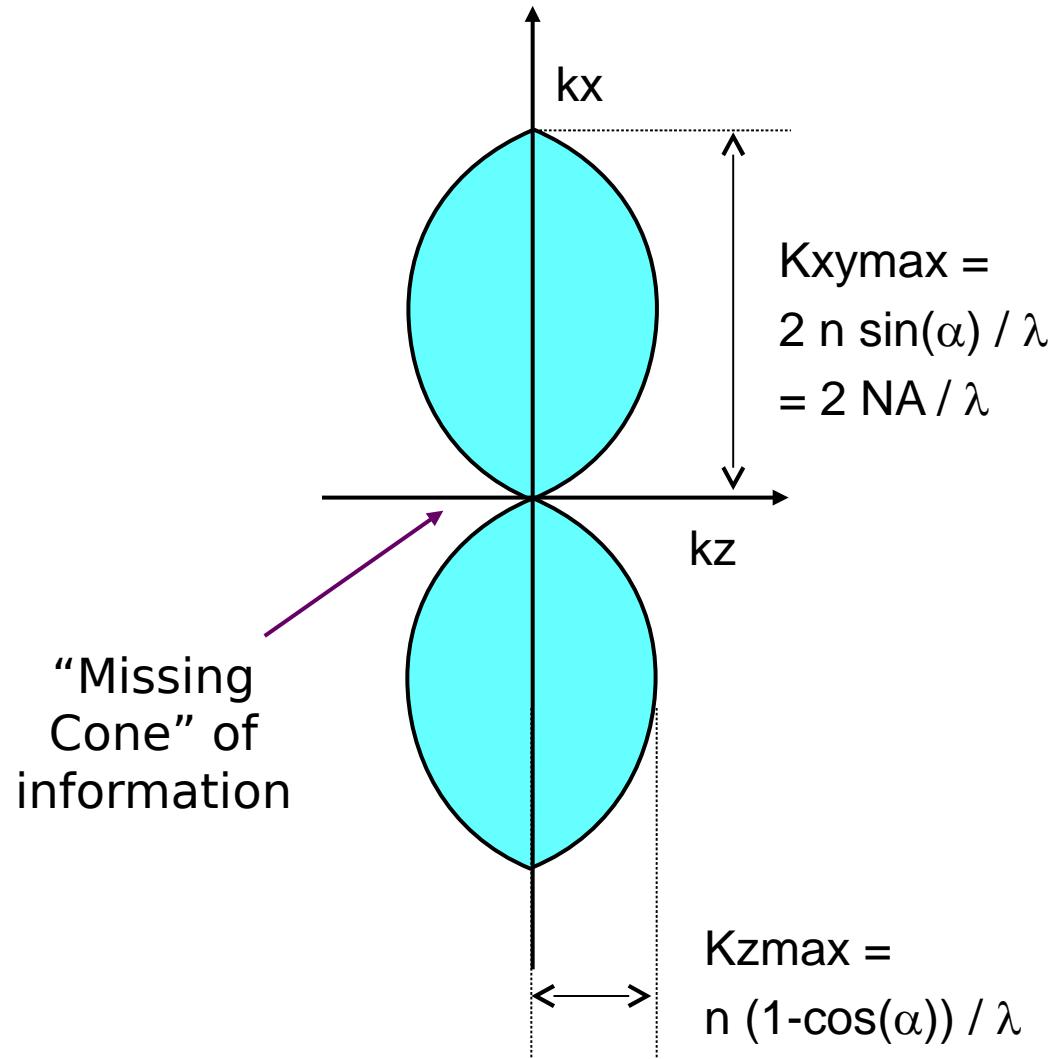
3D Observable Region

= OTF support

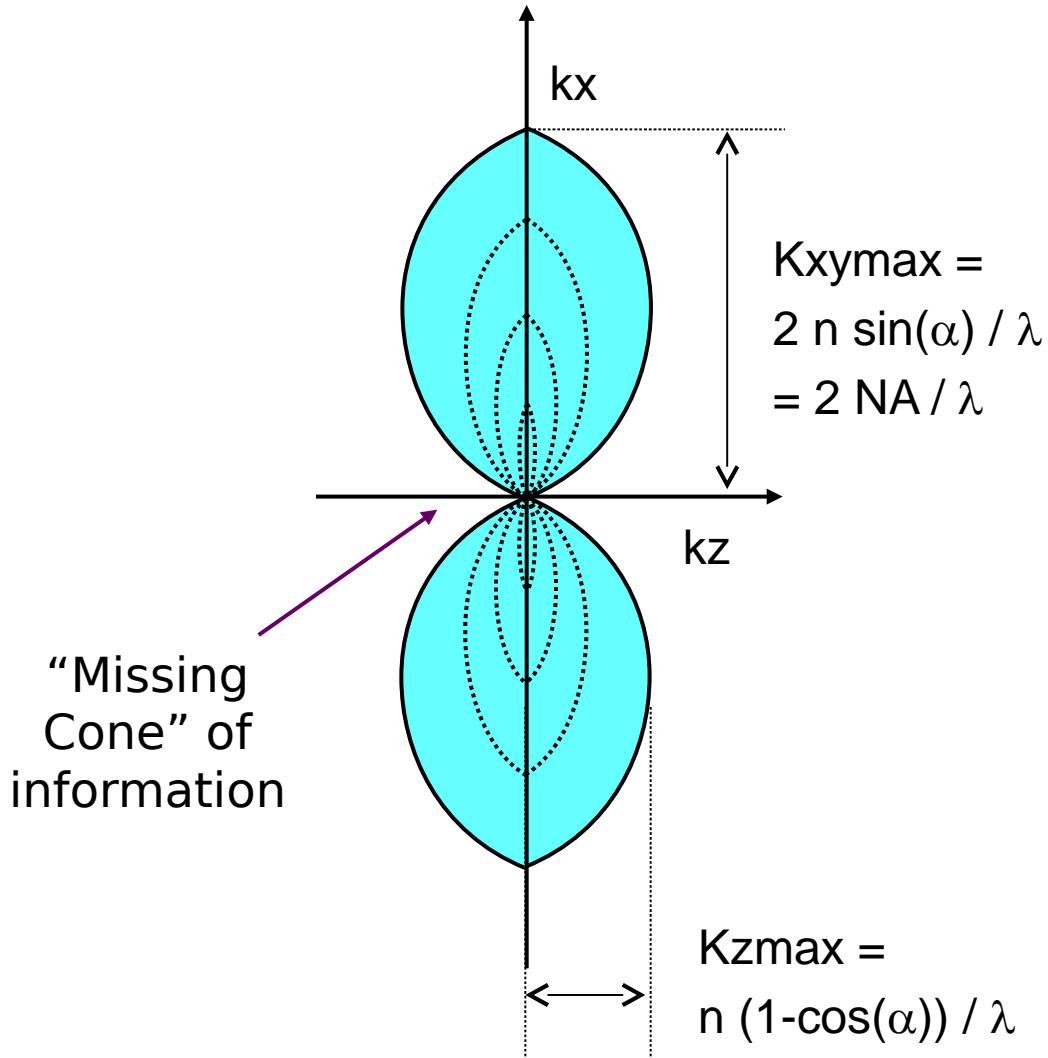
= Region where the OTF is non-zero



So what is the resolution?



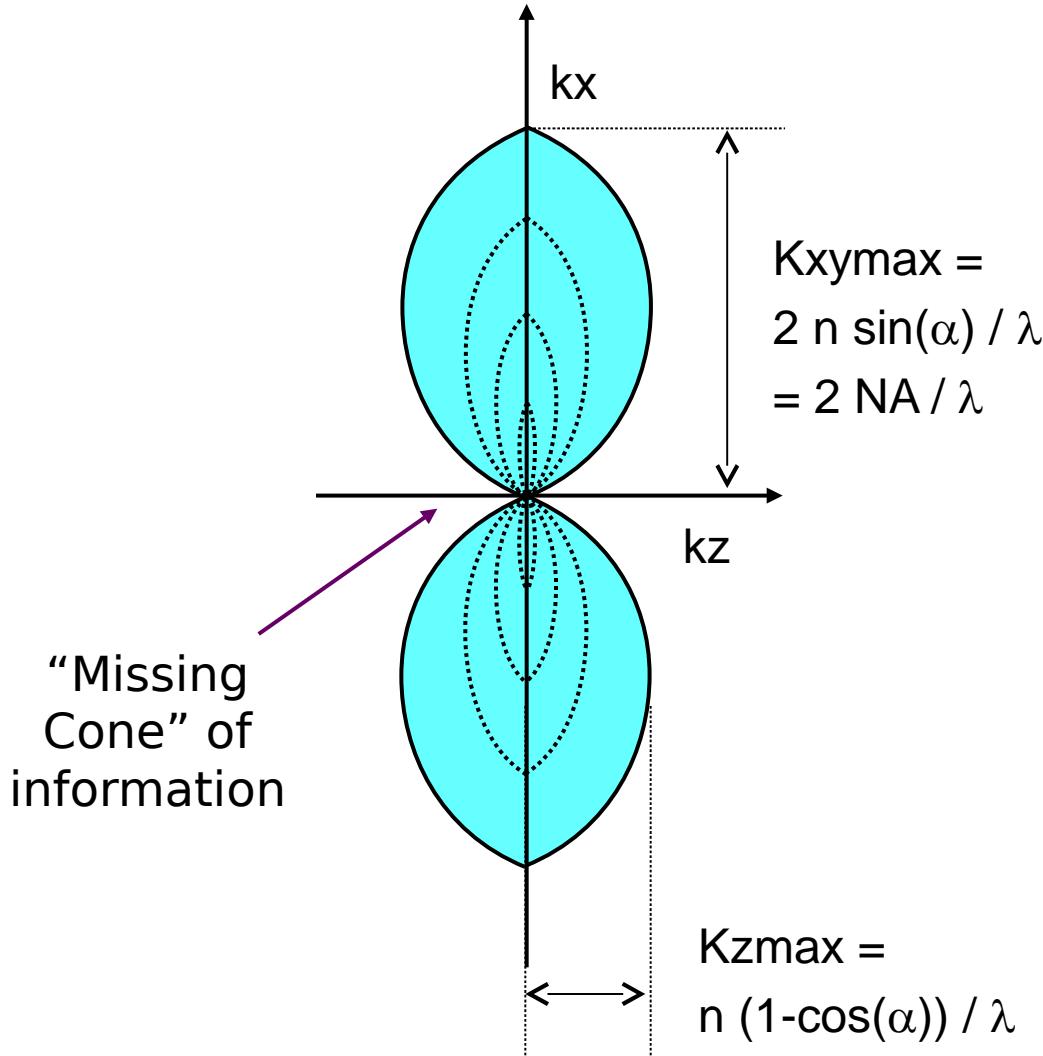
So what is the resolution?



Lowering the NA
Degrades the
axial resolution
faster than the
lateral resolution

But low axial resolution
= long *depth of field*
This is *good*,
if 2D is enough

So what is the resolution?



Example:
a high-end objective

$$\text{NA} = 1.4$$

$$n = 1.515$$

$$\rightarrow \alpha = 67.5^\circ$$

$$\lambda = 600 \text{ nm}$$



Lateral (XY) resolution:

$$1/K_{x\text{max}} = 0.21 \mu\text{m}$$

Axial (Z) resolution:
 $1/K_{z\text{max}} = 0.64 \mu\text{m}$

Nomenclature

- Optical Transfer Function, OTF
Complex value with amplitude and phase
- Contrast Transfer Function, CTF
- Modulation Transfer Function, MTF
Same thing without the phase information

Resources

Slides available at: <http://nic.ucsf.edu/edu.html>

<http://www.microscopyu.com>

<http://micro.magnet.fsu.edu>

Douglas B. Murphy "Fundamentals of Light Microscopy and Electronic Imaging"

James Pawley, Ed. "Handbook of Biological Confocal Microscopy, 3rd ed."

Acknowledgements

Steve Ross, Mats Gustafsson