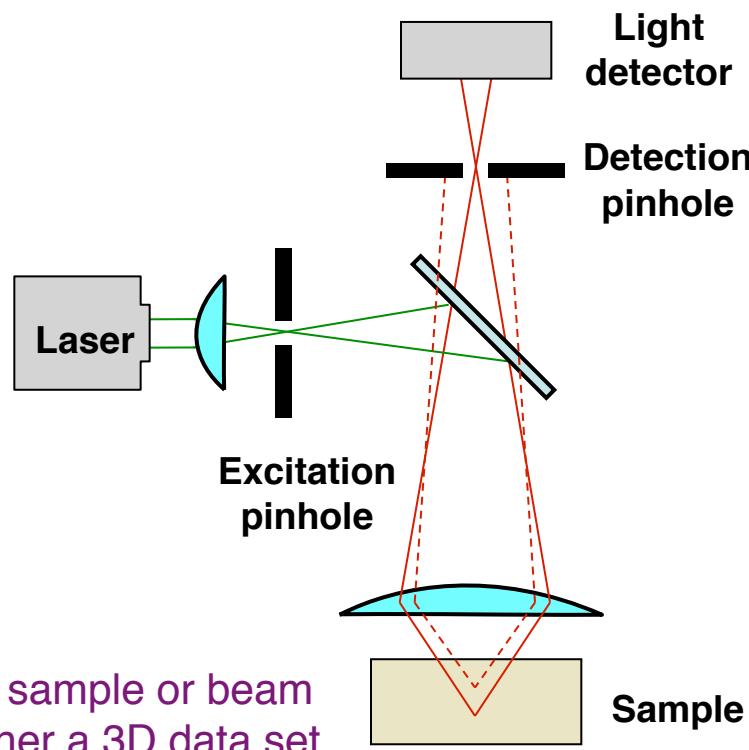


# 3D fluorescence microscopy

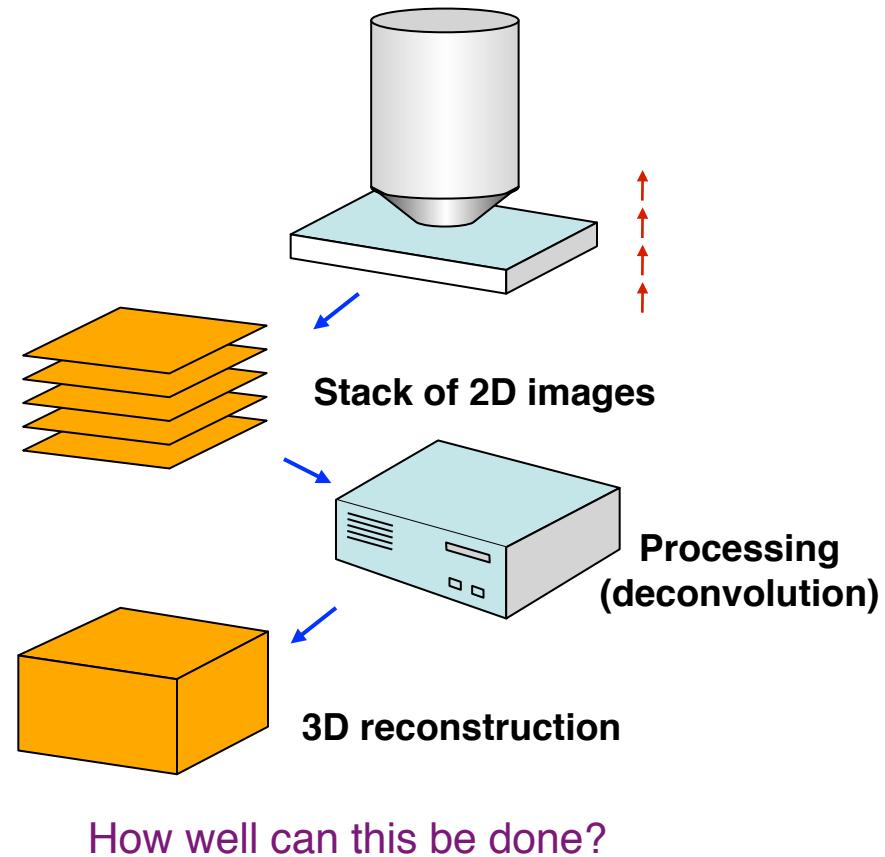
For 3D, acquire a “focal series” (stack) of images:  
Take an image, refocus the sample, take another image, refocus, etc.

Problem: Each image contains out-of-focus blur from other focal planes

**Approach 1:**  
**Physically exclude the blur by**  
**confocal microscopy**



**Approach 2:**  
**Remove the blur computationally**

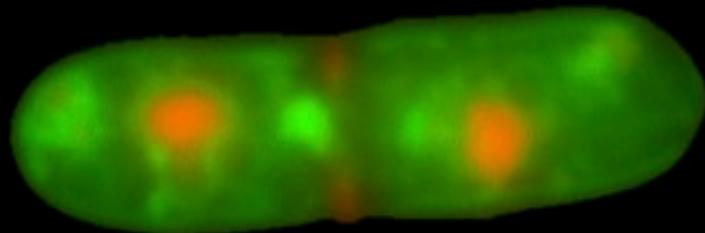


**Scan sample or beam**  
to gather a 3D data set

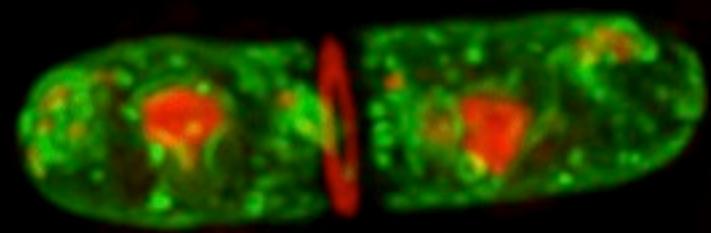
# Deconvolution of 3D data

(Dividing fission yeast cell)

Raw data



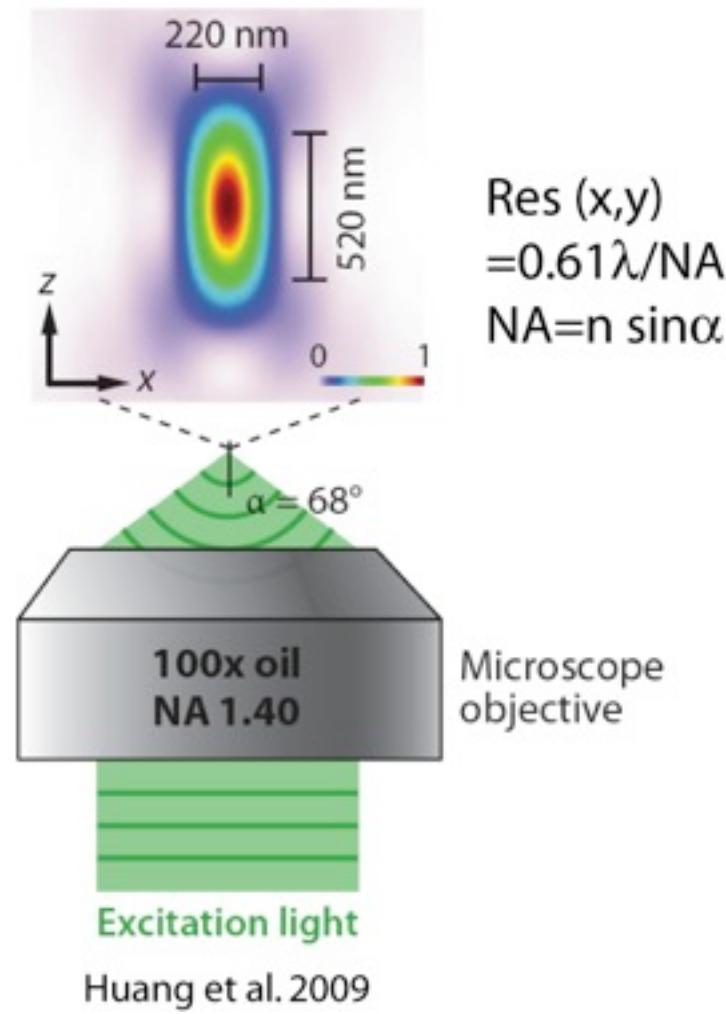
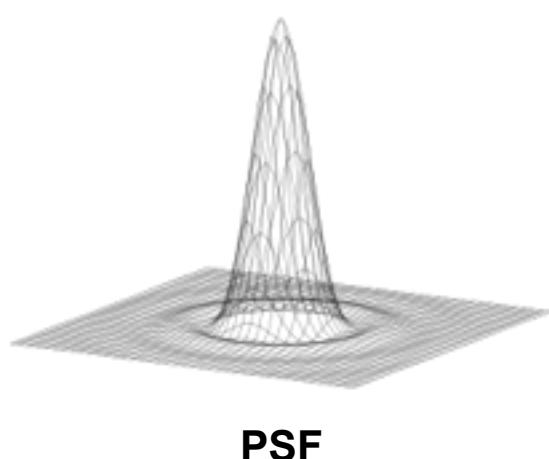
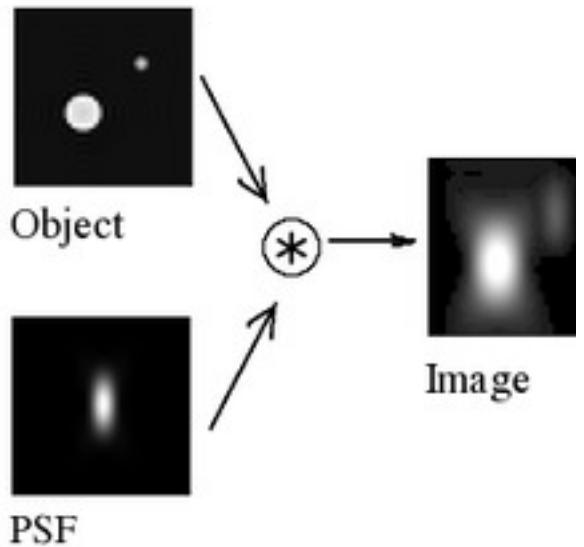
Deconvolved



4  $\mu\text{m}$

4  $\mu\text{m}$

# Light microscopy imaging is defined by the Point Spread Function



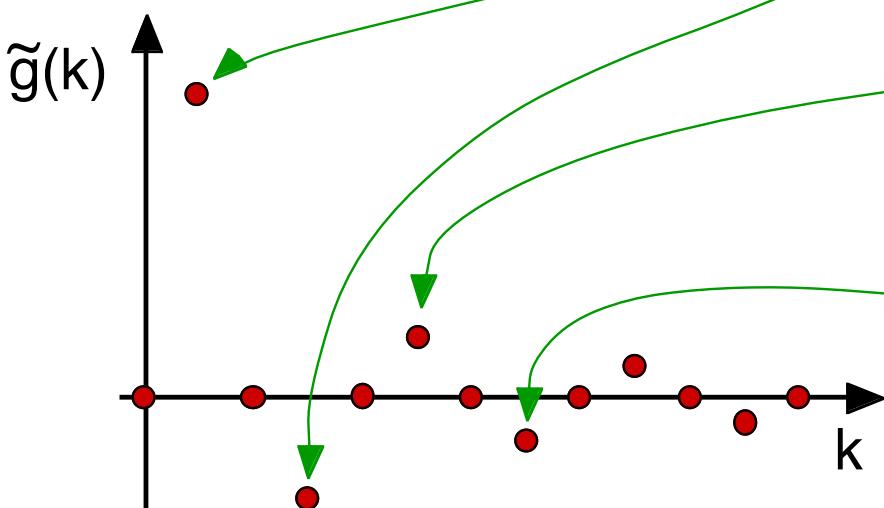
Huang et al. 2009

# The Fourier Transform

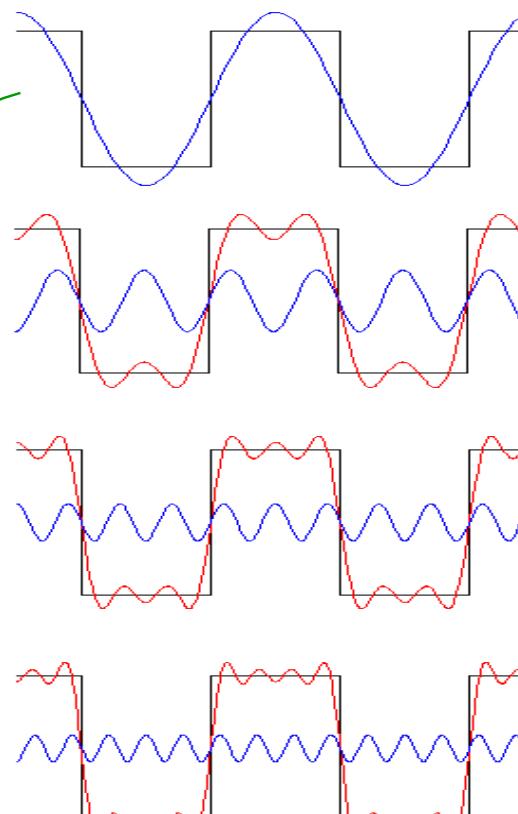
Any (nice) function  $g(x)$  can be *equally well* described as a sum of waves.

The “Fourier transform”  $\tilde{g}(k)$  specifies the amplitude  $A$  and the phase  $\phi$  for the component wave of wavelength  $L = 1/k$

Long wavelength (low resolution)  
info is close to the origin

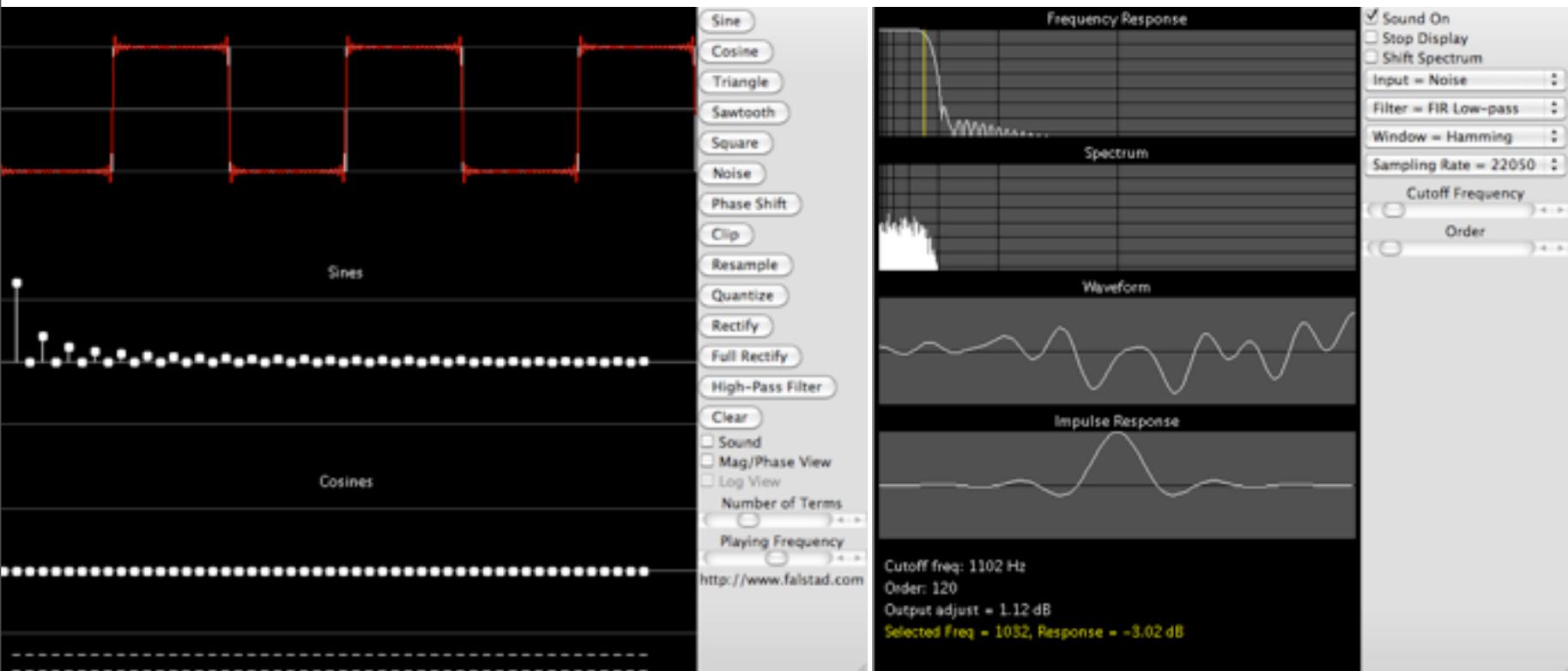


$$\tilde{g}(k) = \int g(x) e^{-2\pi i k x} dx$$



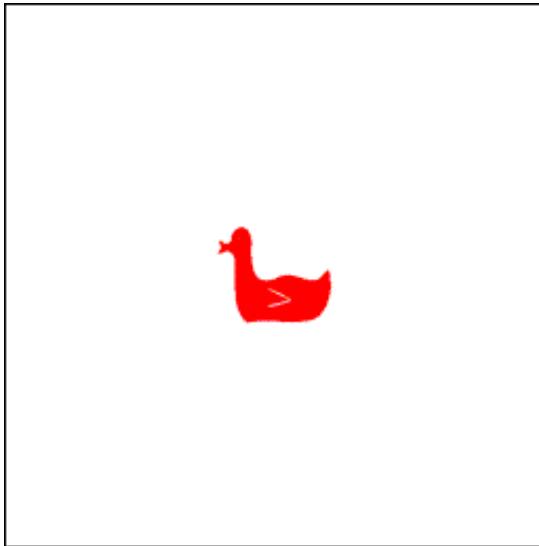
# Fourier Transform and Digital Filter Applets

<http://www.falstad.com/mathphysics.html>

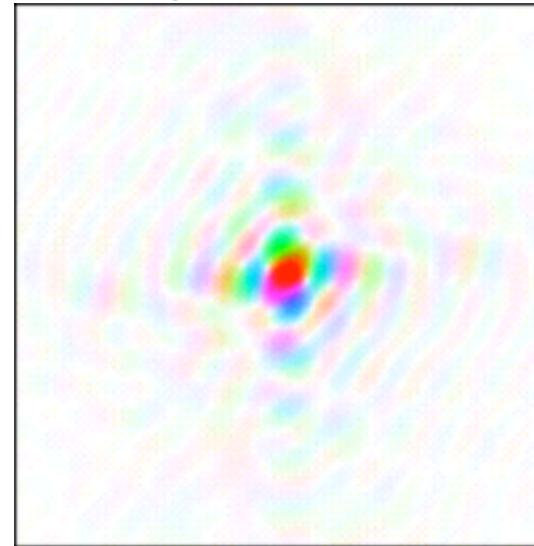


# Low Resolution Duck

A Duck

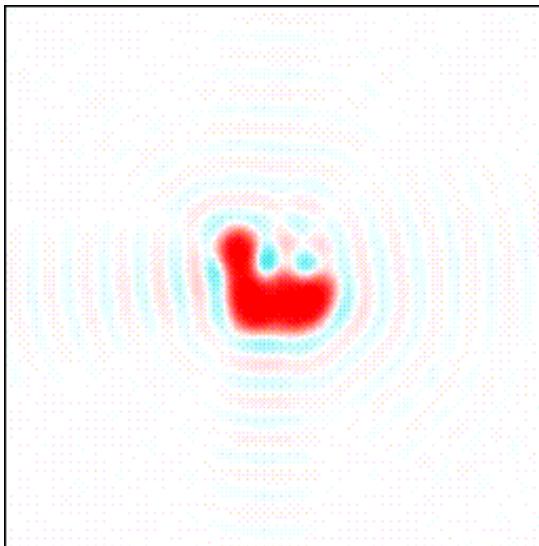


and its scattering function (Fourier Transform):

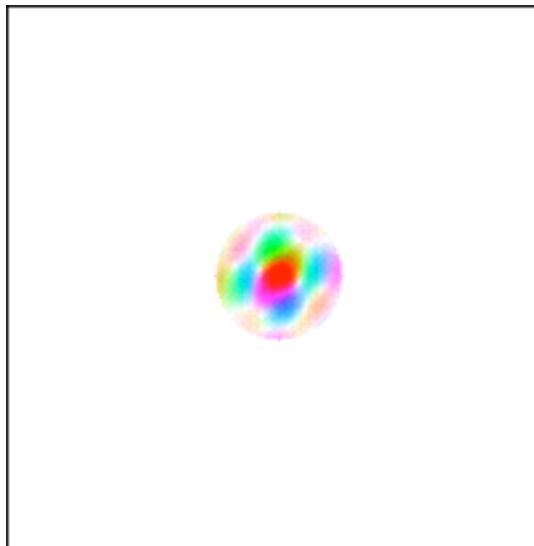


a blurred duck

<=



Using only low resolution data



# Convolutions

$$(f \otimes g)(r) = \int f(a) g(r-a) da$$

Why do we care?

- They are everywhere...
- The convolution theorem:

If  $h(r) = (f \otimes g)(r)$ ,

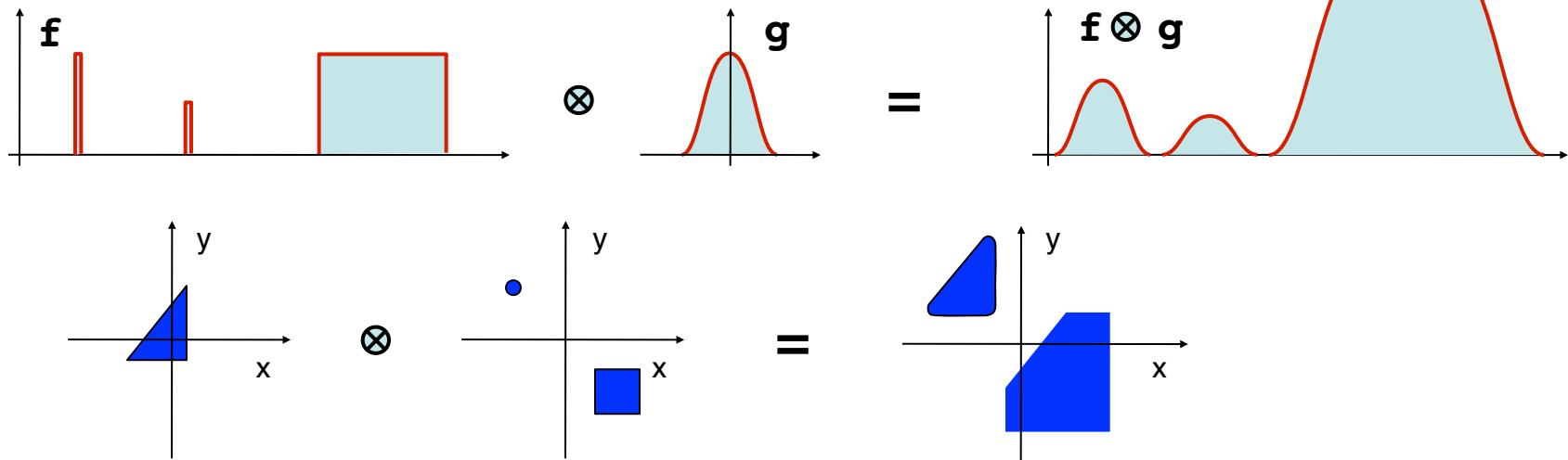
then  $\tilde{h}(k) = \tilde{f}(k) \tilde{g}(k)$

A convolution in real space becomes a product in reciprocal space & vice versa

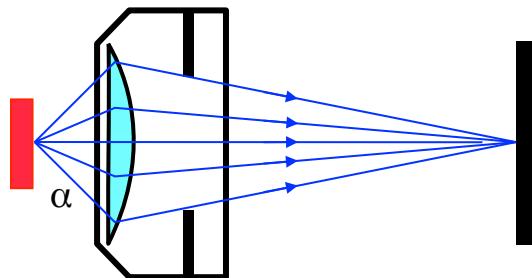
Symmetry:  $g \otimes f = f \otimes g$

So what is a convolution, intuitively?

- “Blurring”
- “Drag and stamp”



# 2D PSF, OTF of an in-focus lens viewing a point



$\alpha$  = maximum angle

$$\text{N.A.} = \eta \sin \alpha,$$

the highest spatial frequency is then  $f_c = (2\eta \sin \alpha)/\lambda$   
 $f_c = 0.178 \mu\text{m}$  for  $\lambda = 500\text{nm}$ , 1.4NA oil immersion lens

using Raleigh's criterion, the smallest separation  
 between two points that can be resolved is  $1/(1.22 \cdot f_c)$   
 $d_{min} = 0.146 \mu\text{m}$

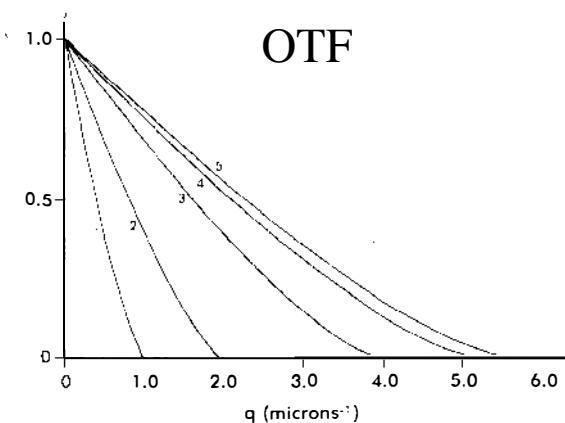


Figure 3 The in-focus contrast-transfer functions for several Zeiss lenses are compared. The curves are (1) 10 × 0.25 N.A., (2) 25 × 0.5 N.A., (3) 50 × 1.0 N.A. oil, (4) 100 × 1.25 N.A. oil, (5) 63 × 1.4 N.A. oil.

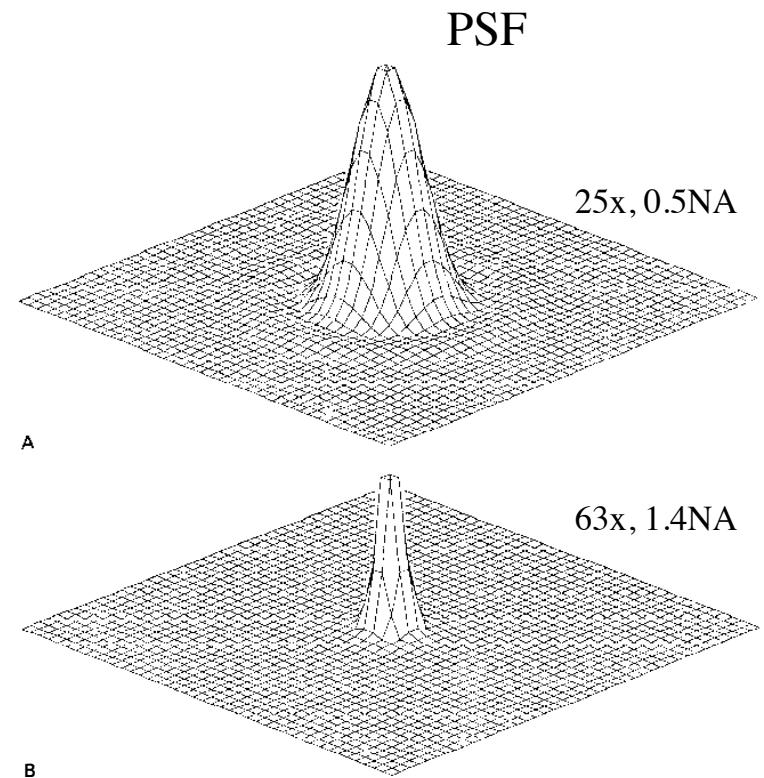
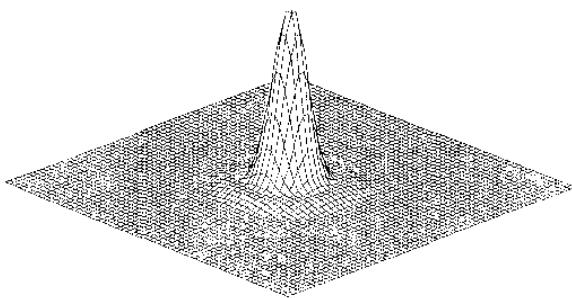
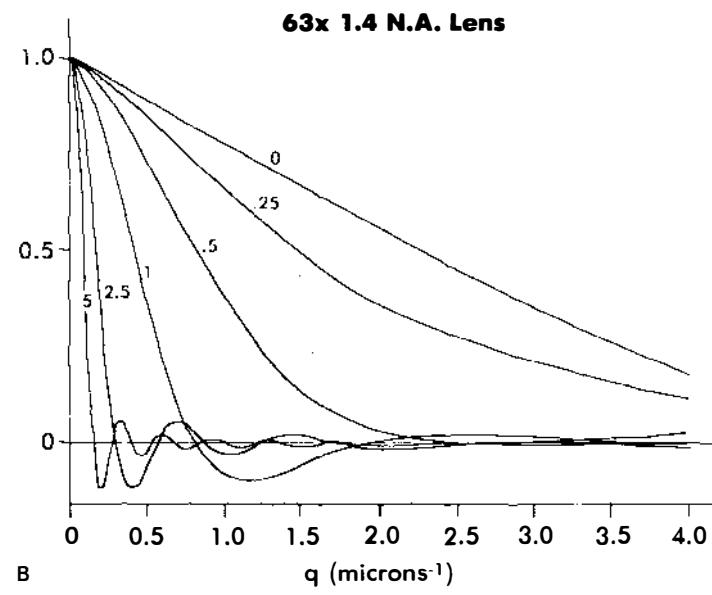
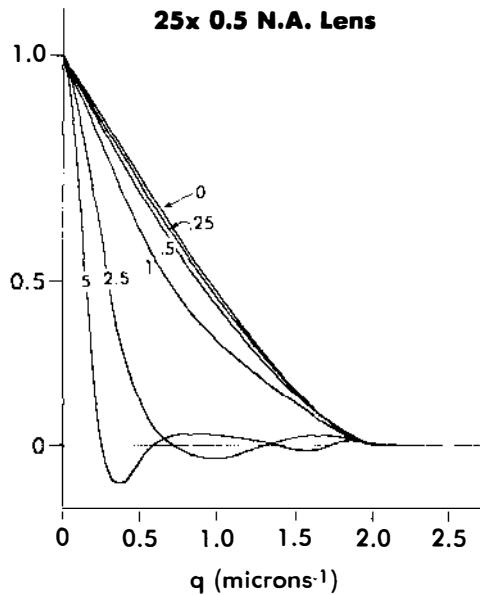
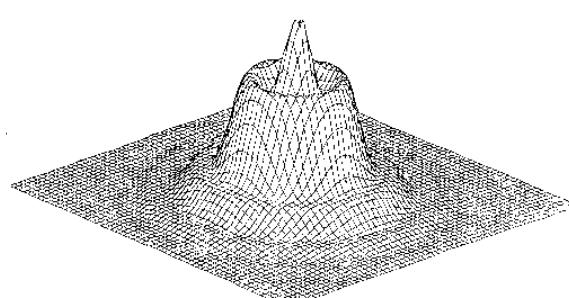


Figure 4 The point-spread functions calculated by Fourier transformation of the CTF's of Figure 3. (A) 25 × 0.5 N.A. lens (#2); (B) 63 × 1.4 N.A. lens (#5). Each grid square is  $\lambda/6$  or 833 Å. Note how much sharper the 1.4 N.A. lens is than the 0.5 N.A. lens.

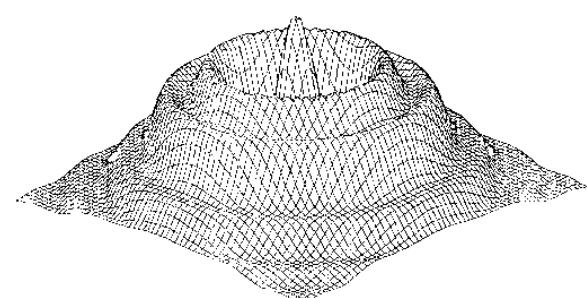
# OTF, PSF of a defocused lens



25x lens 0  $\mu\text{m}$  defocus



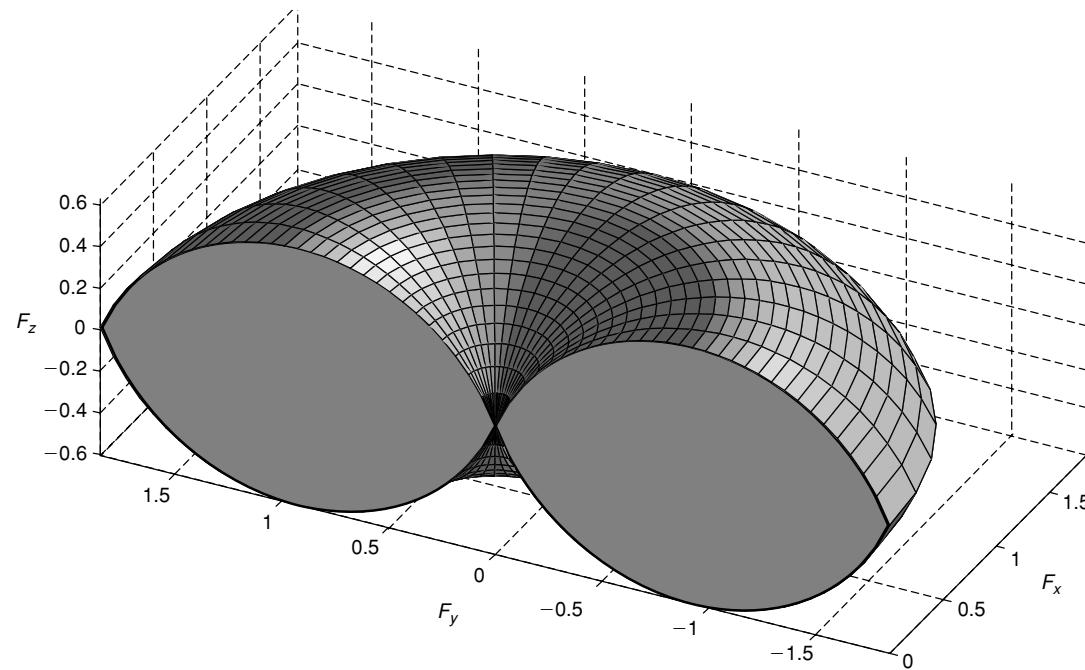
25x lens 2.5  $\mu\text{m}$  defocus



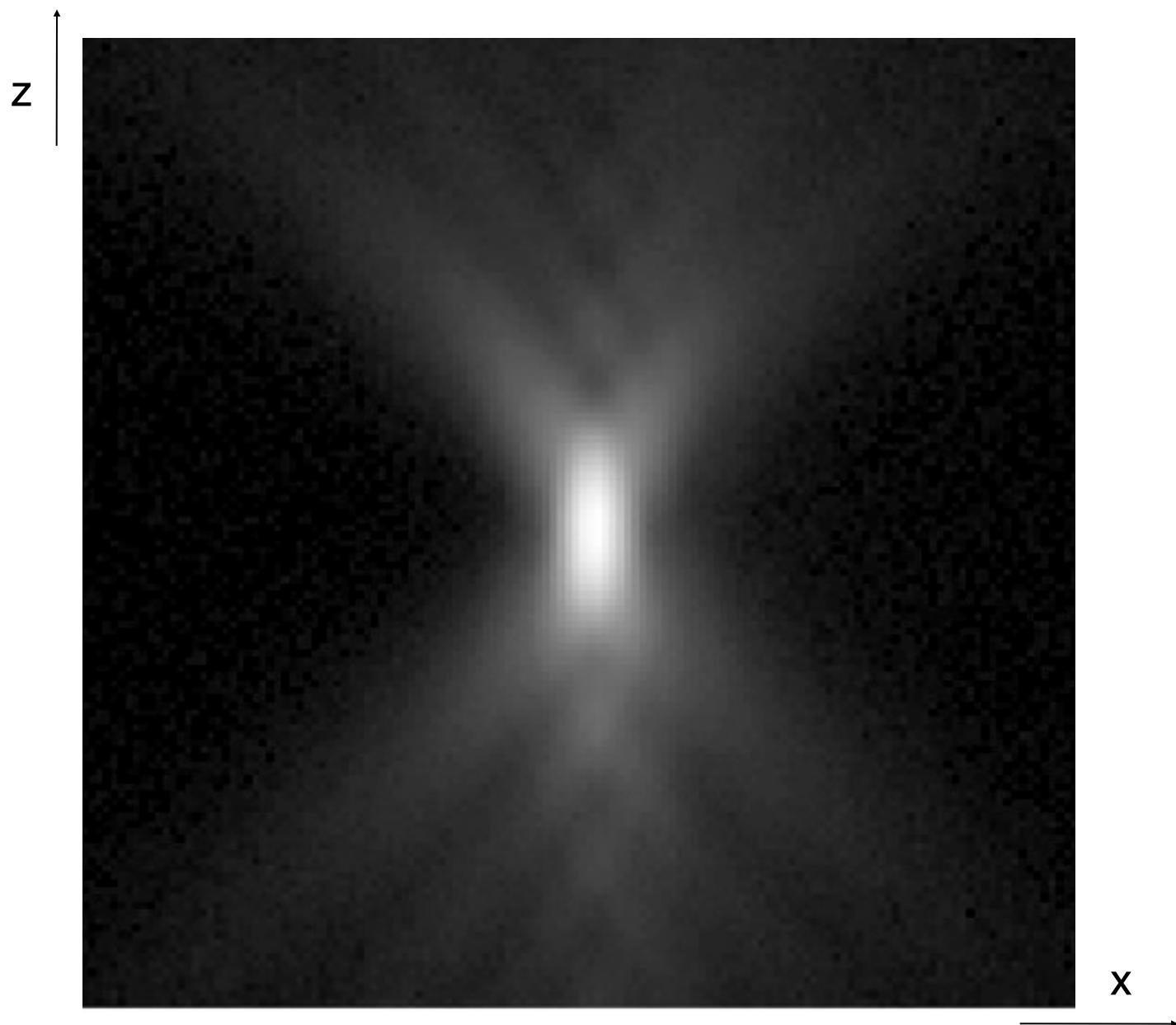
25x lens 5  $\mu\text{m}$  defocus

# 3D OTF

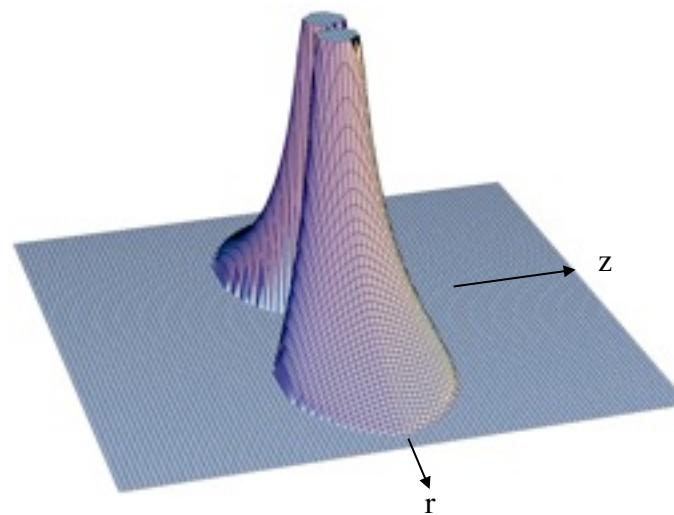
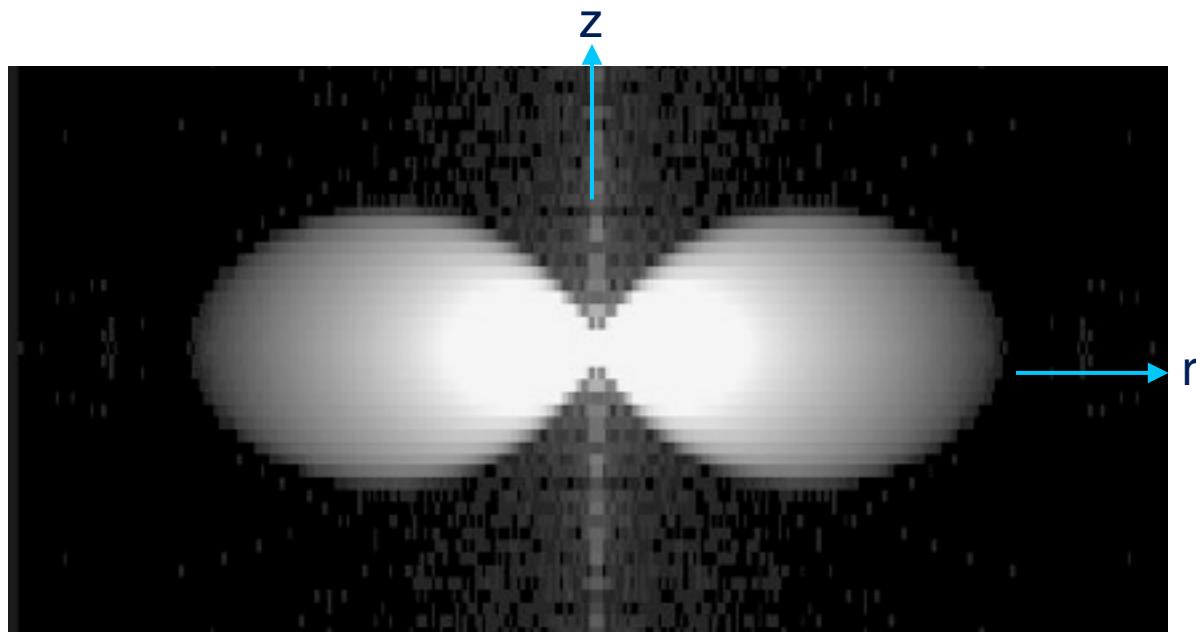
Combining different defocus OTFs from a stack of optical sections gives a donut shaped observable region (support) in Fourier Space



## Experimentally measured PSF



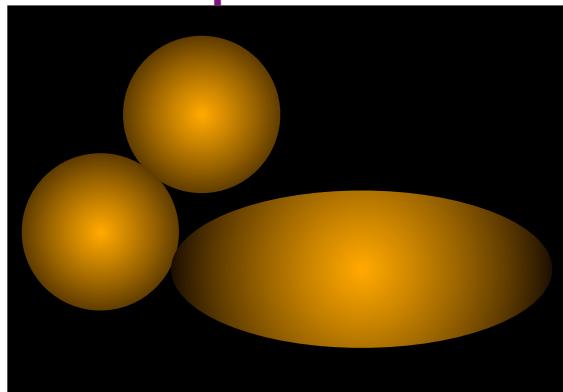
# Experimentally measured OTF



- Intensity is very peaked at origin
- data along Z direction is missing

# PSF, OTF & deconvolution

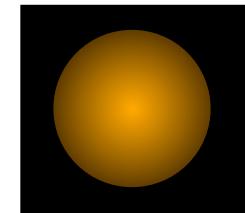
In real space:



Observed Image( $r$ )



True Object( $r$ )



Point Spread  
Function, PSF( $r$ )

In reciprocal space the convolution becomes a product:

$$\text{Image} = \text{Object} \cdot \text{PSF}$$

This suggests:

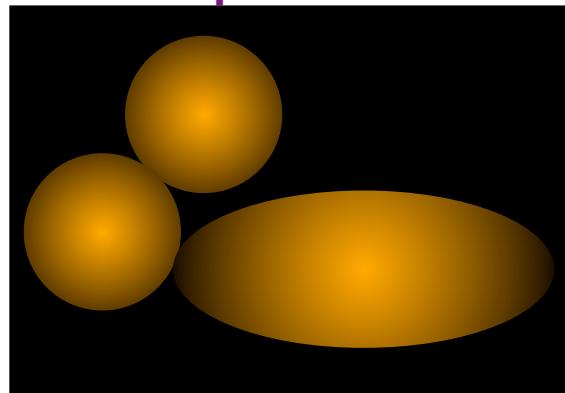
$$\text{Object} = \frac{\text{Image}}{\text{OTF}}$$

??? ("Deconvolution")

PSF is called the  
"Optical Transfer Function",  
OTF

# PSF, OTF & deconvolution

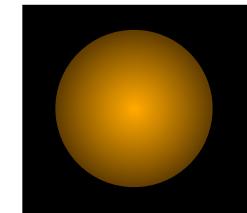
In real space:



Observed Image( $r$ )



True Object( $r$ )



Point Spread  
Function, PSF( $r$ )

In reciprocal space the convolution becomes a product:

$$\text{Image} = \text{Object} \cdot \text{PSF}$$

This suggests:

$$\text{Object} = \frac{\text{Image}}{\text{OTF}}$$

PSF is called the  
“Optical Transfer Function”,  
OTF

??? (“Deconvolution”)

What's the catch??

A: We can't divide by OTF( $k$ ) if it is zero (or small because of noise)

# Deconvolution strategies

Nearest neighbor: simplest method, only takes into account adjacent sections  
ie subtract out blurred version of adjacent sections from central section

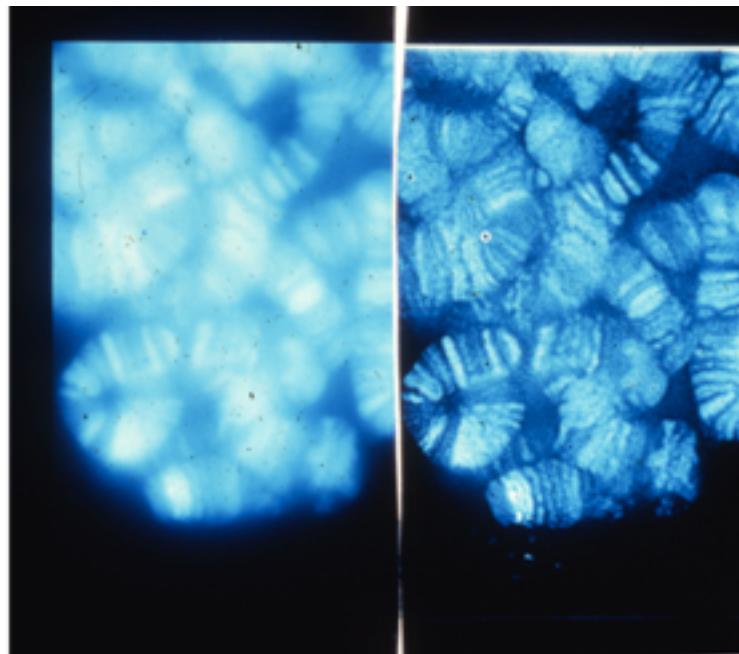
$$I_j \approx c_1 [ O_j - c_2 (\text{PSF}_{\Delta z} \otimes O_{j+1} + \text{PSF}_{-\Delta z} \otimes O_{j-1}) ]$$

for speed do the convolutions as Fast Fourier Transforms (FFT), multiplication, FFT<sup>-1</sup>

$$\text{PSF}_{\Delta z} \otimes O_{j+1} = \text{FFT}^{-1} [ \text{FFT}(O_{j+1}) \cdot \text{OTF}_{\Delta z} ]$$

optical section from a DAPI  
stained polytene nucleus

before and after nearest neighbor



# Deconvolution strategies

much better to consider the contributions of all the sections to one another = 3D

Weiner filter: simplest 3D method, linear processing, takes care of zeros

$$\text{Object} = \frac{\text{Image}}{\text{OTF} + \gamma}$$

$\gamma$  is related to the signal to noise, sets maximum amplification

or if OTF is complex:

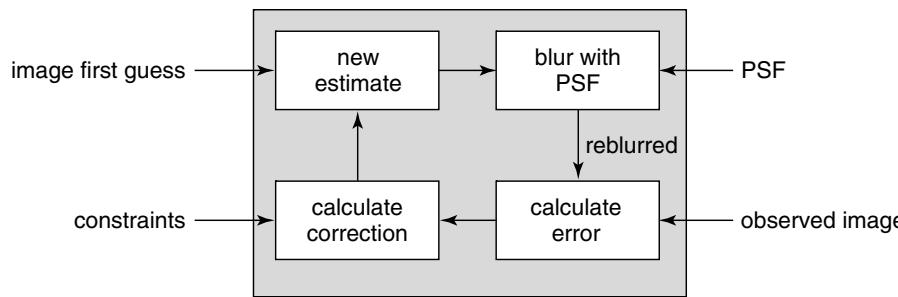
$$\frac{\text{Image} \cdot \text{OTF}^*}{\text{OTF} \cdot \text{OTF}^* + \gamma}$$

# Deconvolution strategies

even better to include *a priori* knowledge about solution such as positivity: Object  $\geq 0$

family of iterative constrained methods

work by calculating convolution, followed by update



arithmetic update (vanCittert's method)

start with  $I^0 = O$

$$I^{k+1} = I^k + (O - \text{PSF} \otimes I^k) \quad \text{with positivity constraint: if } I^{k+1} < 0 \text{ then set } I^{k+1} = 0$$

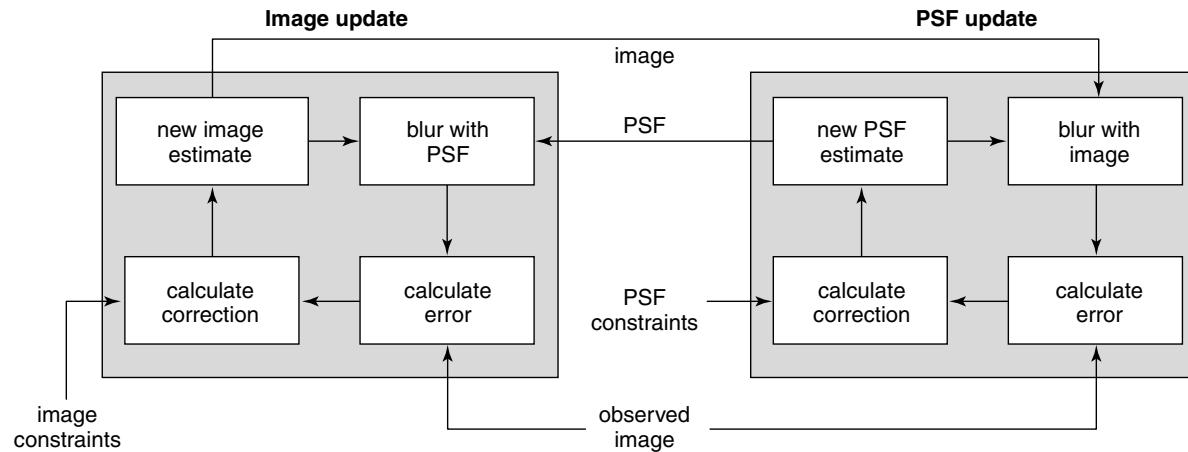
or use multiplicative update (Gold's method)

$$I^{k+1} = I^k \cdot \frac{O}{\text{PSF} \otimes I^k}$$

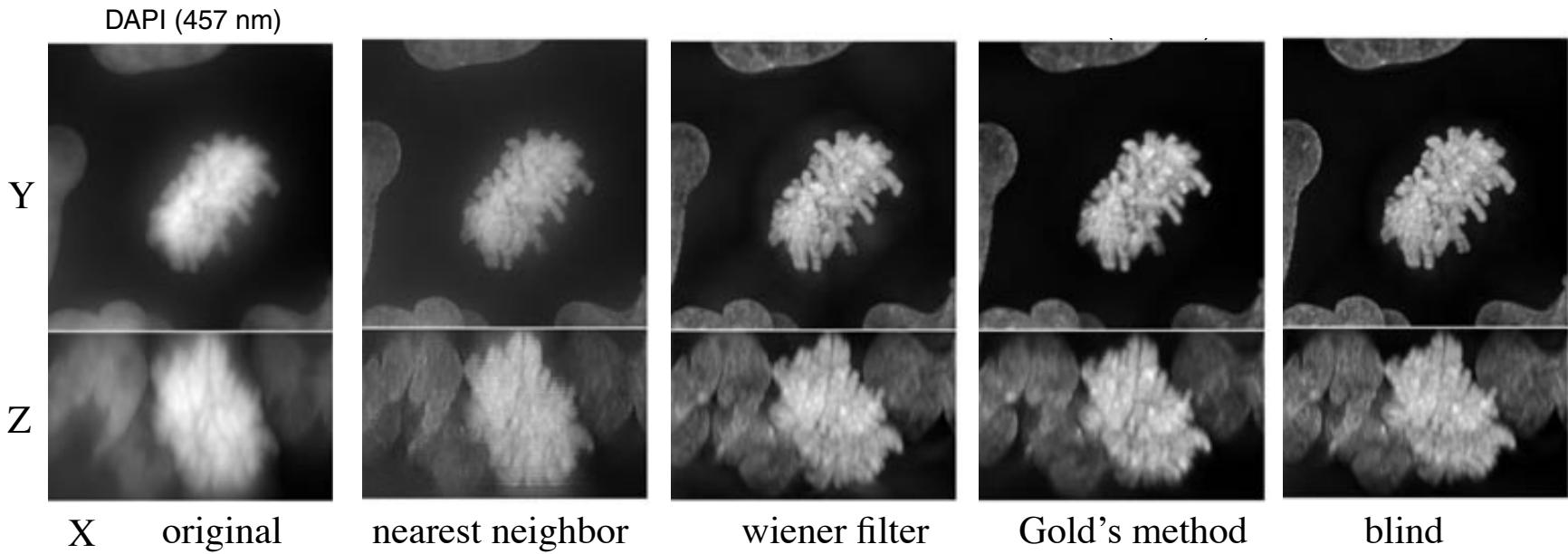
# Deconvolution strategies

what if your PSF is not accurate?

blind deconvolution seeks to estimate both Image and PSF



## Some comparisons: Hela cells, DAPI stained

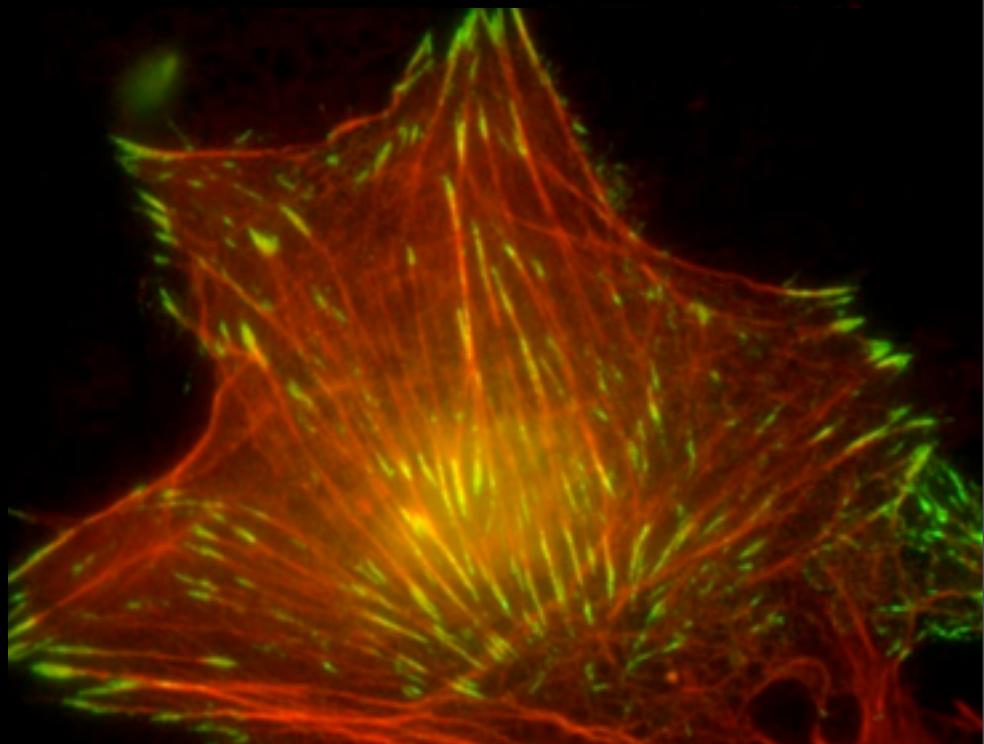
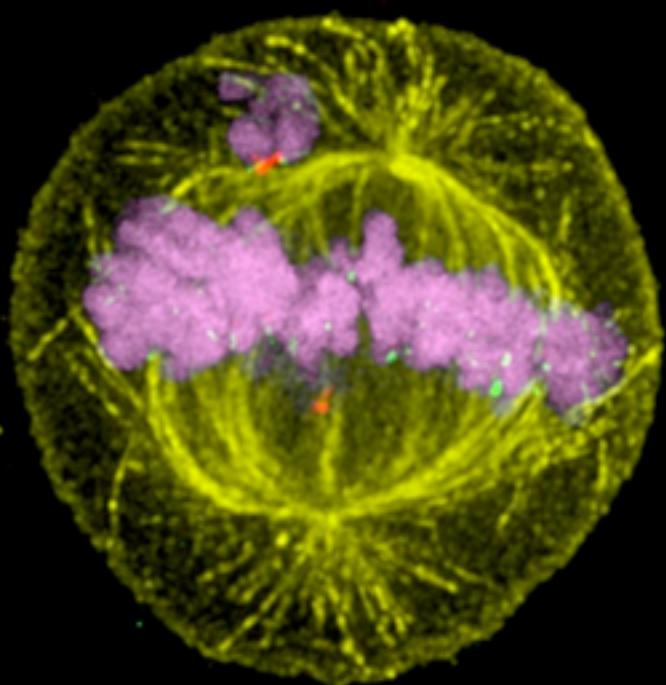
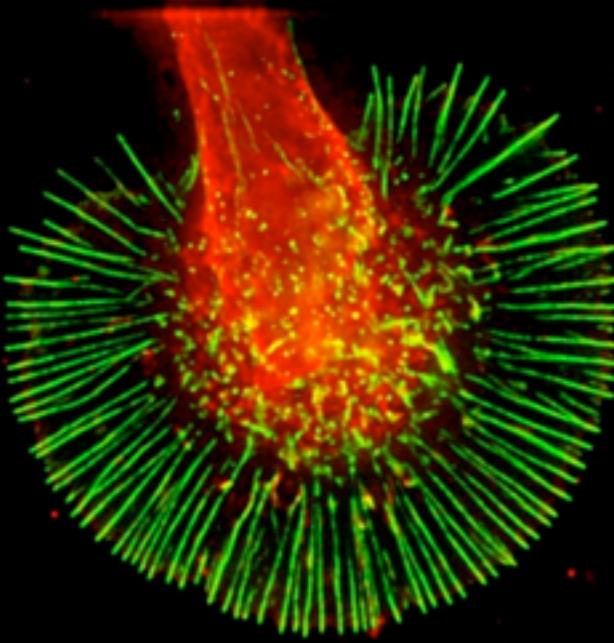
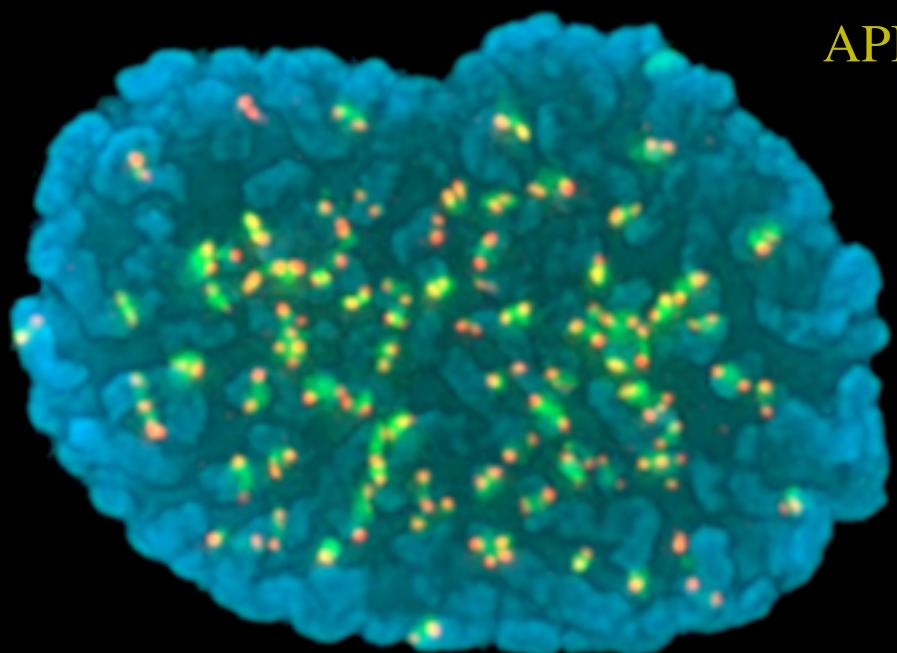


now many related variations

practical issues are signal to noise, accuracy of PSF

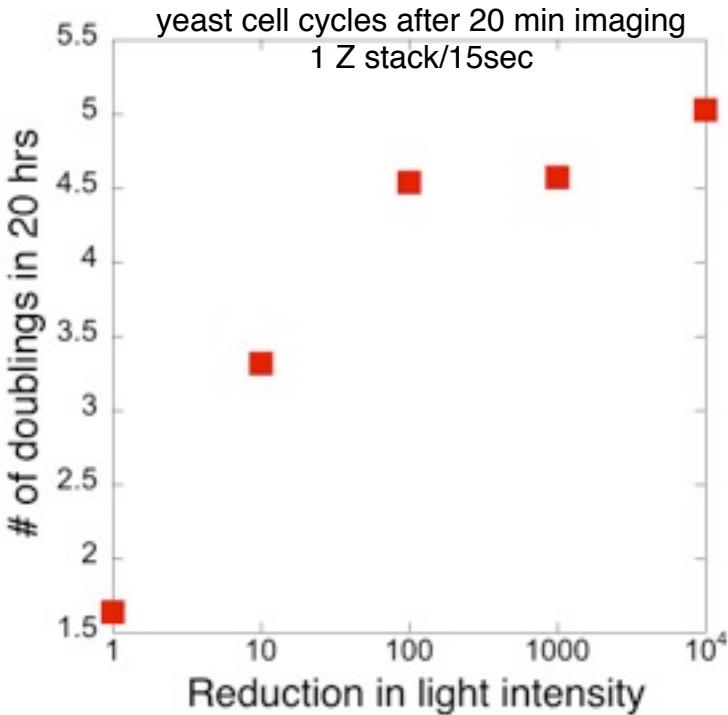
if sample is very thick then OSF can vary throughout sample  
(minimize by matching index of refraction)

API web site

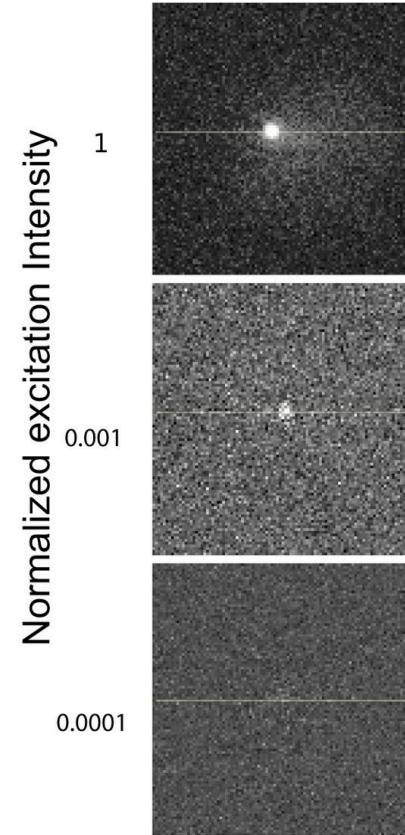


# *The in vivo imaging problem: phototoxicity*

*the challenge of live imaging:*



**maximum allowable photon flux that allows unperturbed viability is  $4.8 \text{ nW}/\mu\text{m}^2$**



**“With this dose level, and with the current state-of-the-art deconvolution, the signal was insufficient for any quantitative measurement, even tracking”**

Pete Carlton et al, PNAS, 2010

## *Formulation of the deconvolution problem*

$$g_{opt}(r) = \arg \min_g \sum_r (h(r) * g(r) - f(r))^2 + \lambda R(g)$$



*Regularization functional*  
defines how noise is handled

$$R(g) = \sum_r \left[ \log \left( 1 + \alpha \left( g^2(r) + \sum_{i=1}^6 (L_i(r) * g(r))^2 \right) \right) + \sum_r N(g(r)) \right]$$

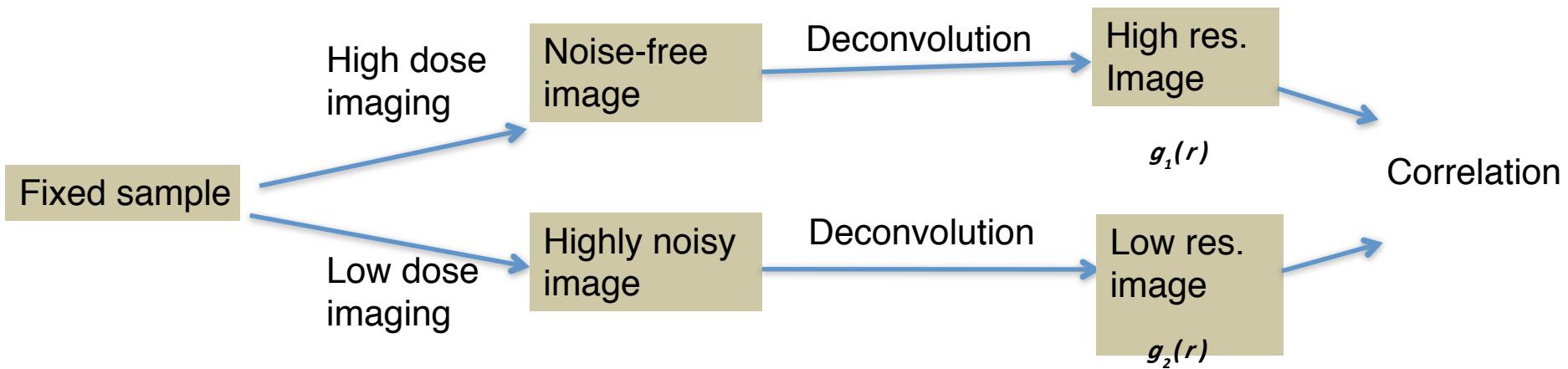
use entropy based optimization (ER-Decon)

$\{L_1, L_2, \dots, L_6\}$ : Second derivative filters

$\alpha$ : Nonlinearity factor

$$N(s) = \begin{cases} s^2 & \text{for } s < 0 \\ 0 & \text{for } s \geq 0 \end{cases}$$

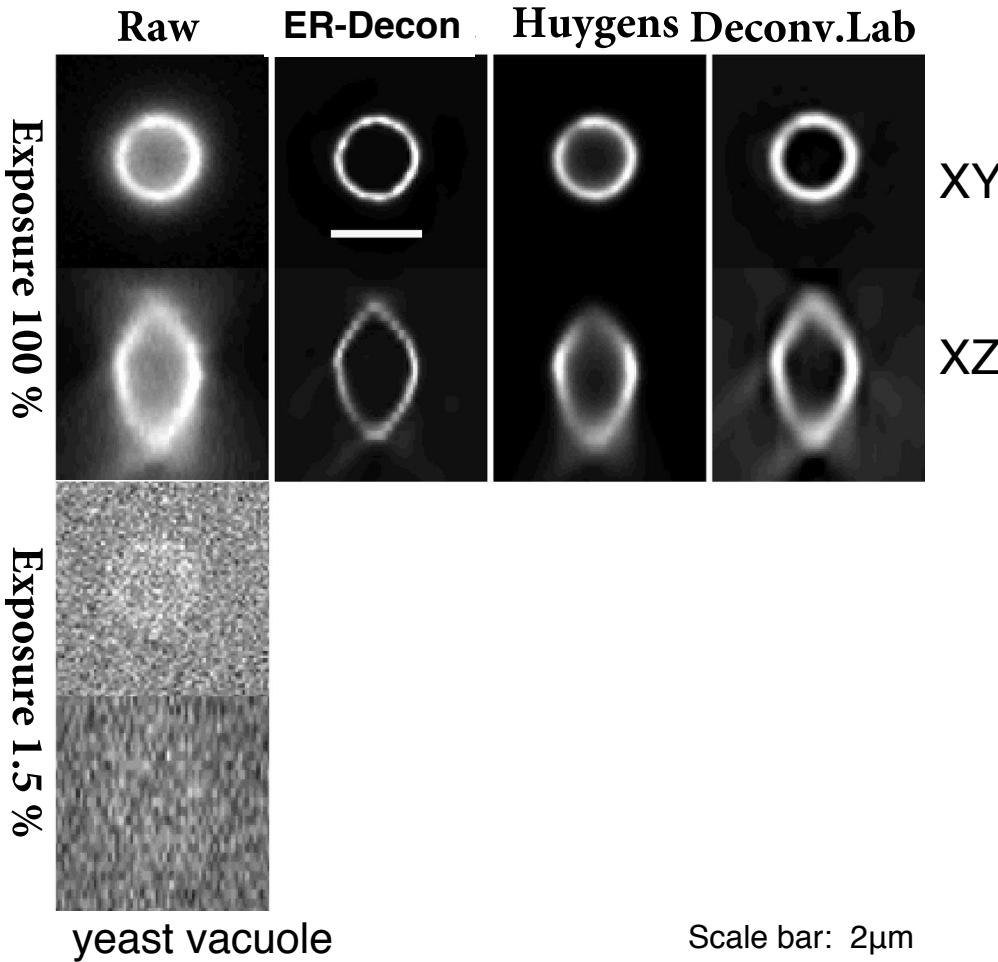
# Validate by quantitative comparison to high dose image



Compare our method (ER-Decon), with two existing methods

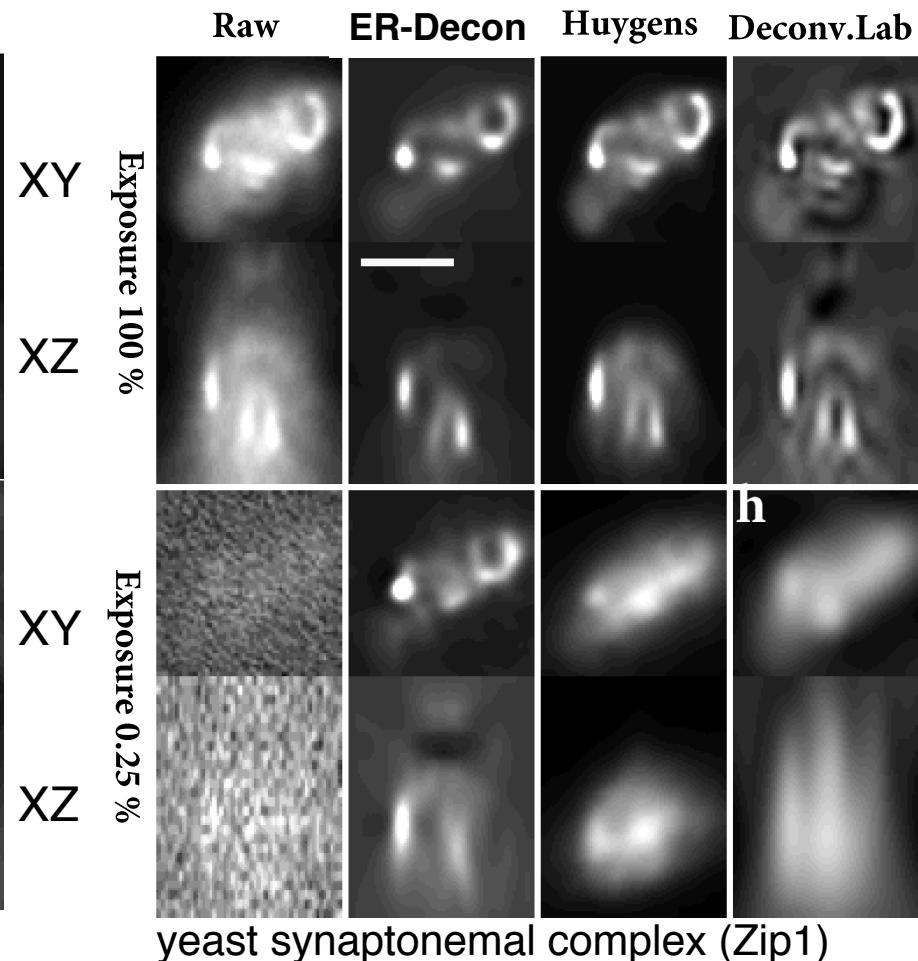
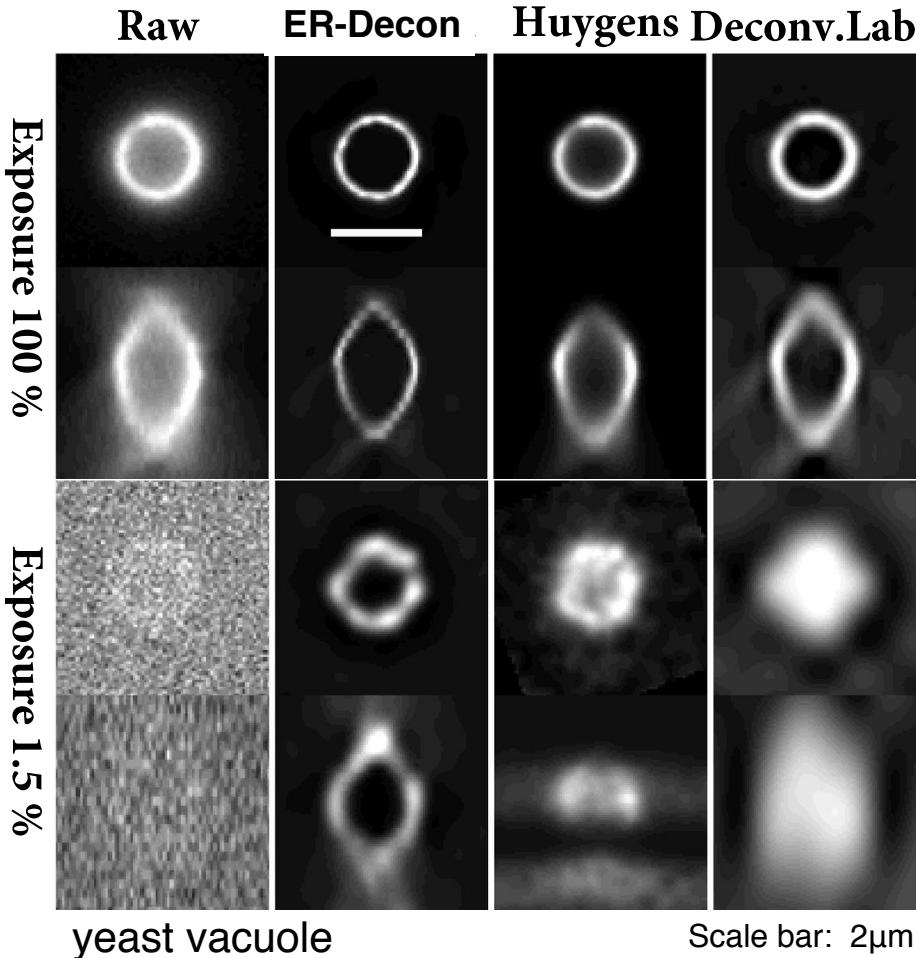
- (1) **Huygens** – commercially available method  
(uses MLE approach)
- (2) **DeconvolutionLab (DeconLAB)** from Unser's group at EPFL -  
best among free packages (uses wavelet approach)

# *Remarkable signal recovery simultaneous with deconvolution*



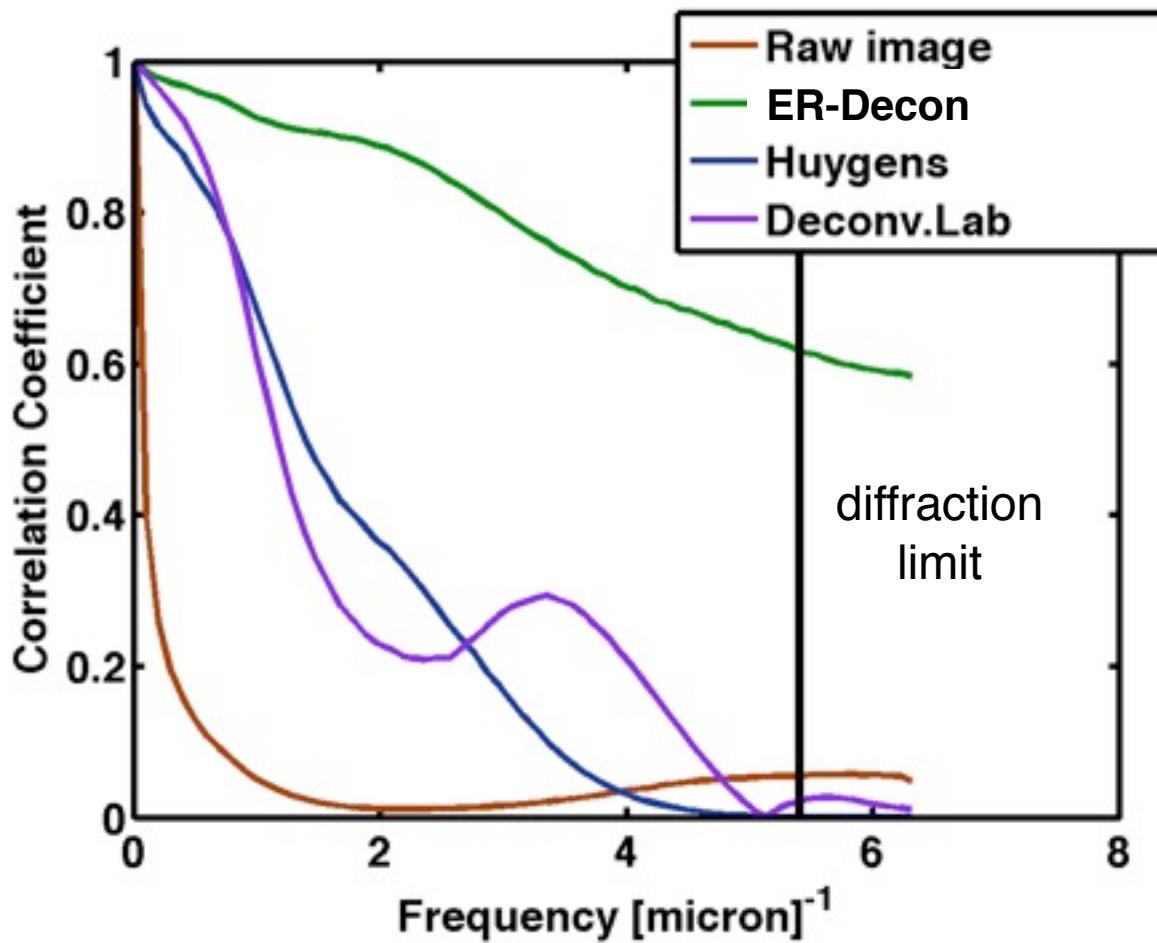
M. Arigovindan

# Remarkable signal recovery simultaneous with deconvolution



M. Arigovindan

# *Fourier shell analysis confirms signal recovery (400-fold reduced intensity)*



*enabling technology for *in vivo* imaging*

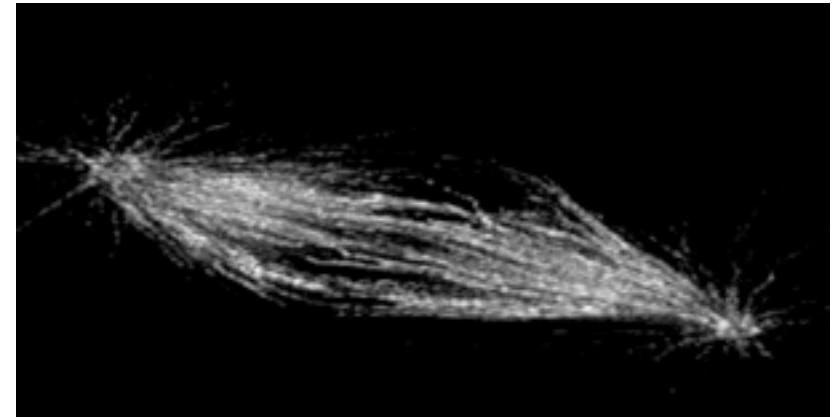
M. Arigovindan

# *ER-Decon deconvolution of SIM data*

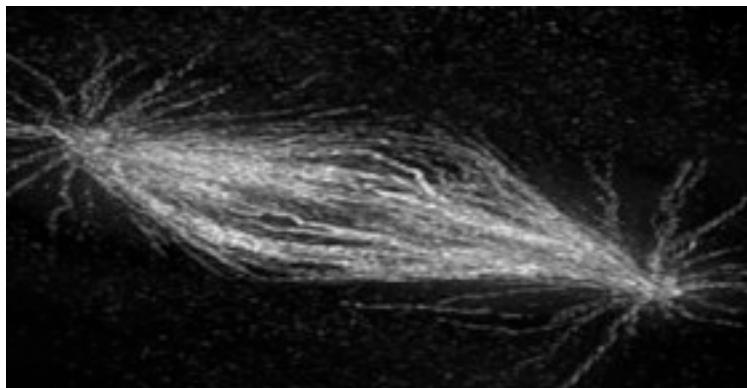
Widefield



SIM-Wiener filtering

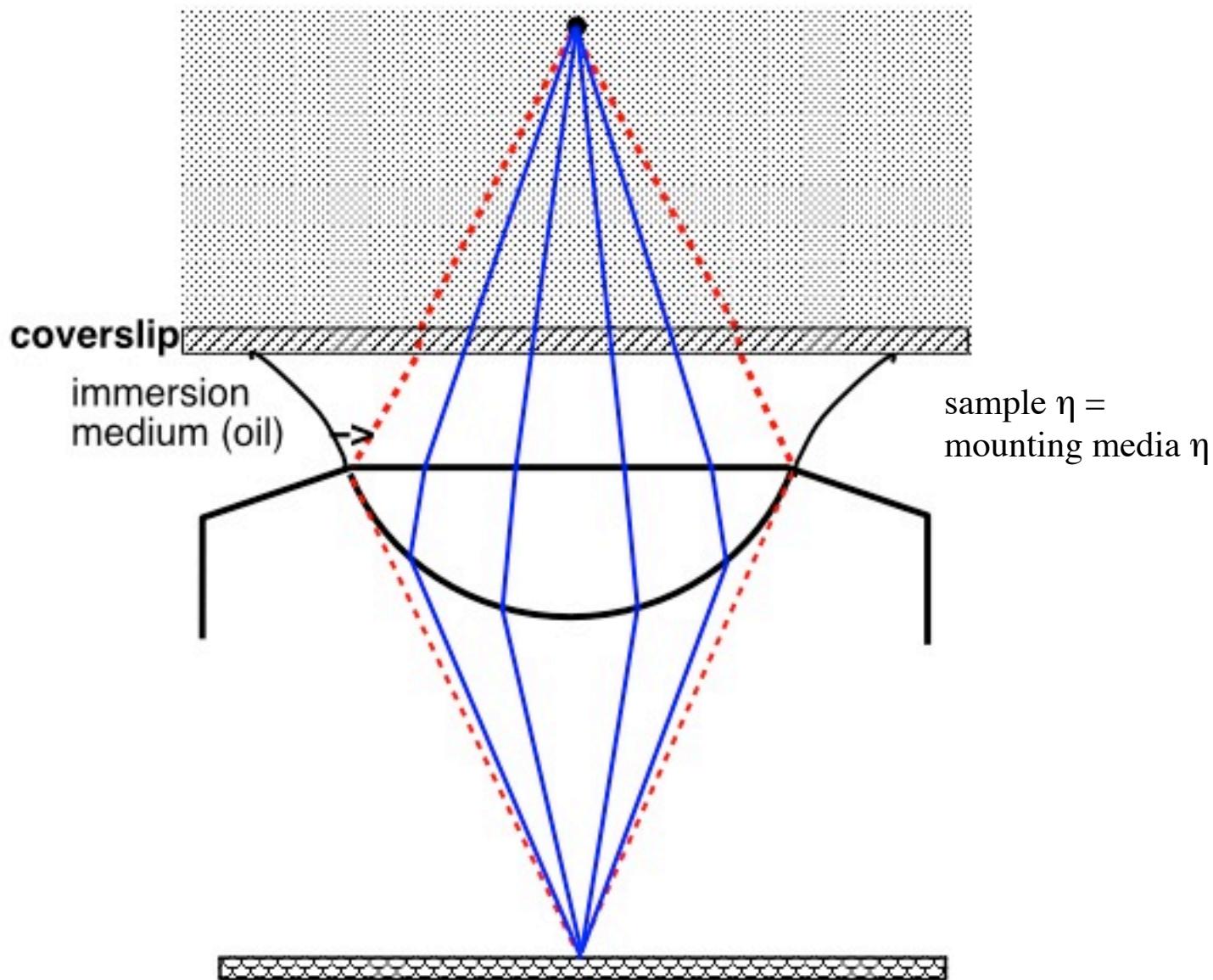


SIM + ER-Decon

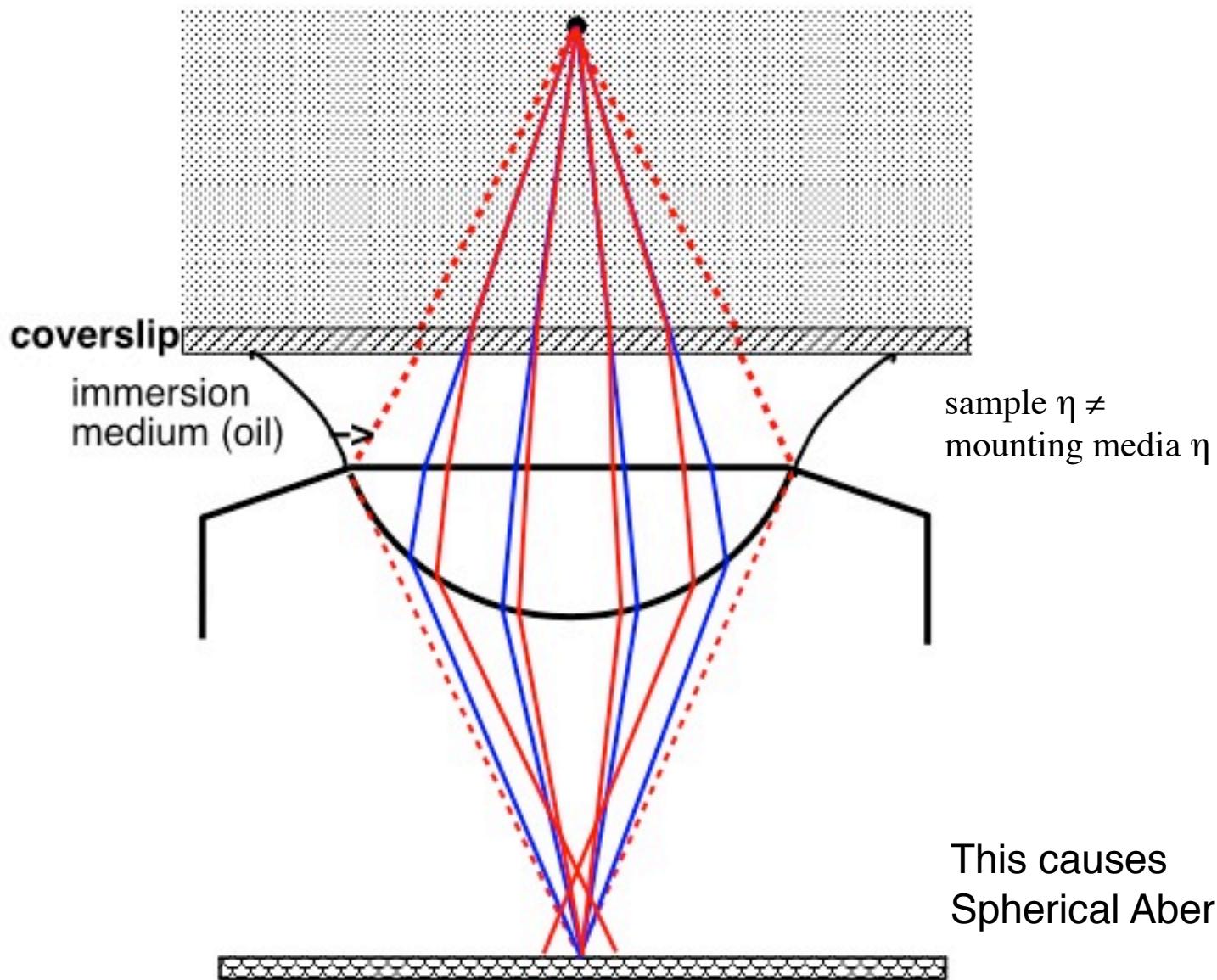


M. Arigovindan

# Properties of the Sample Affect Image Formation in Microscopy

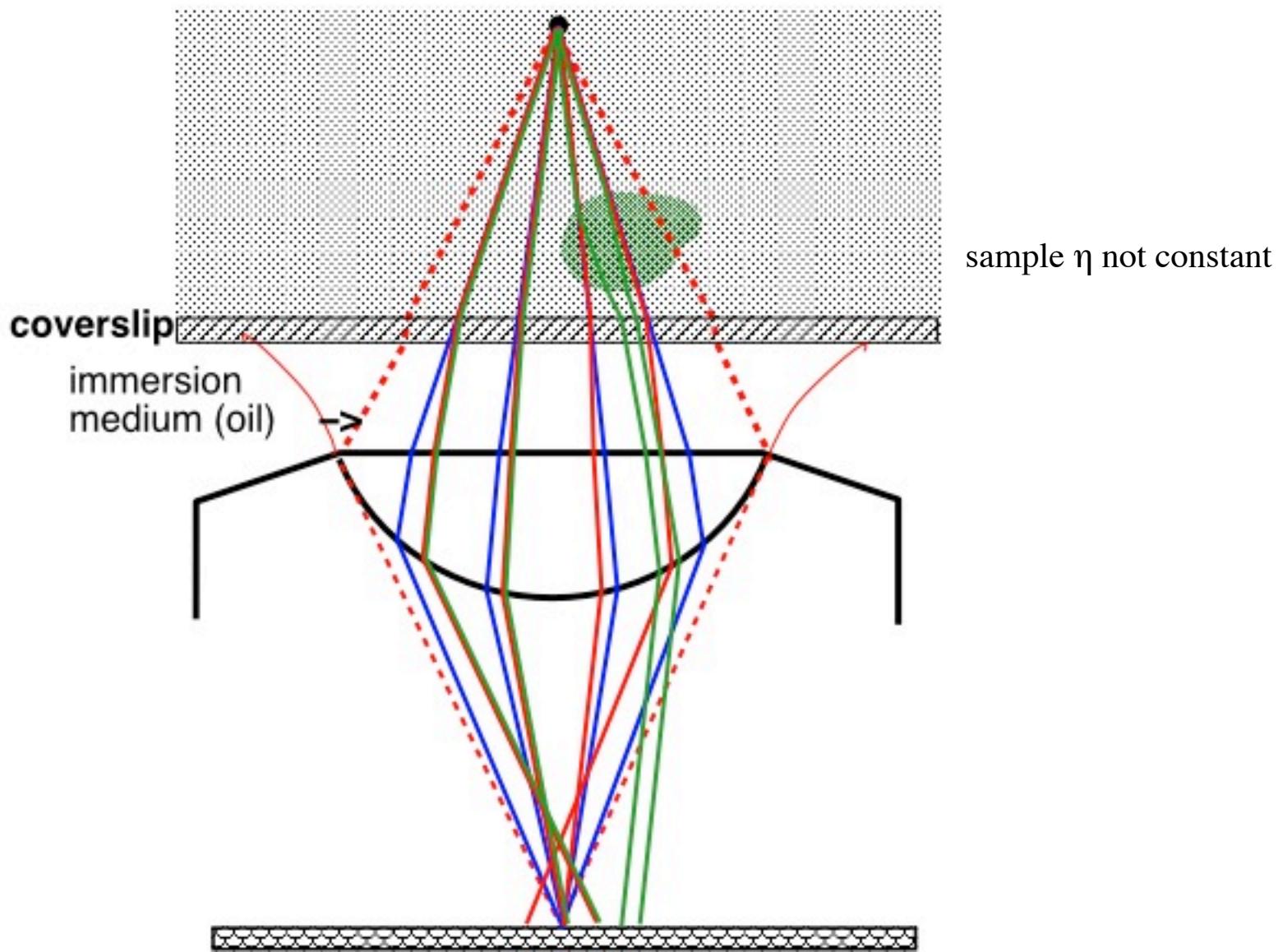


# Properties of the Sample Affect Image Formation in Microscopy

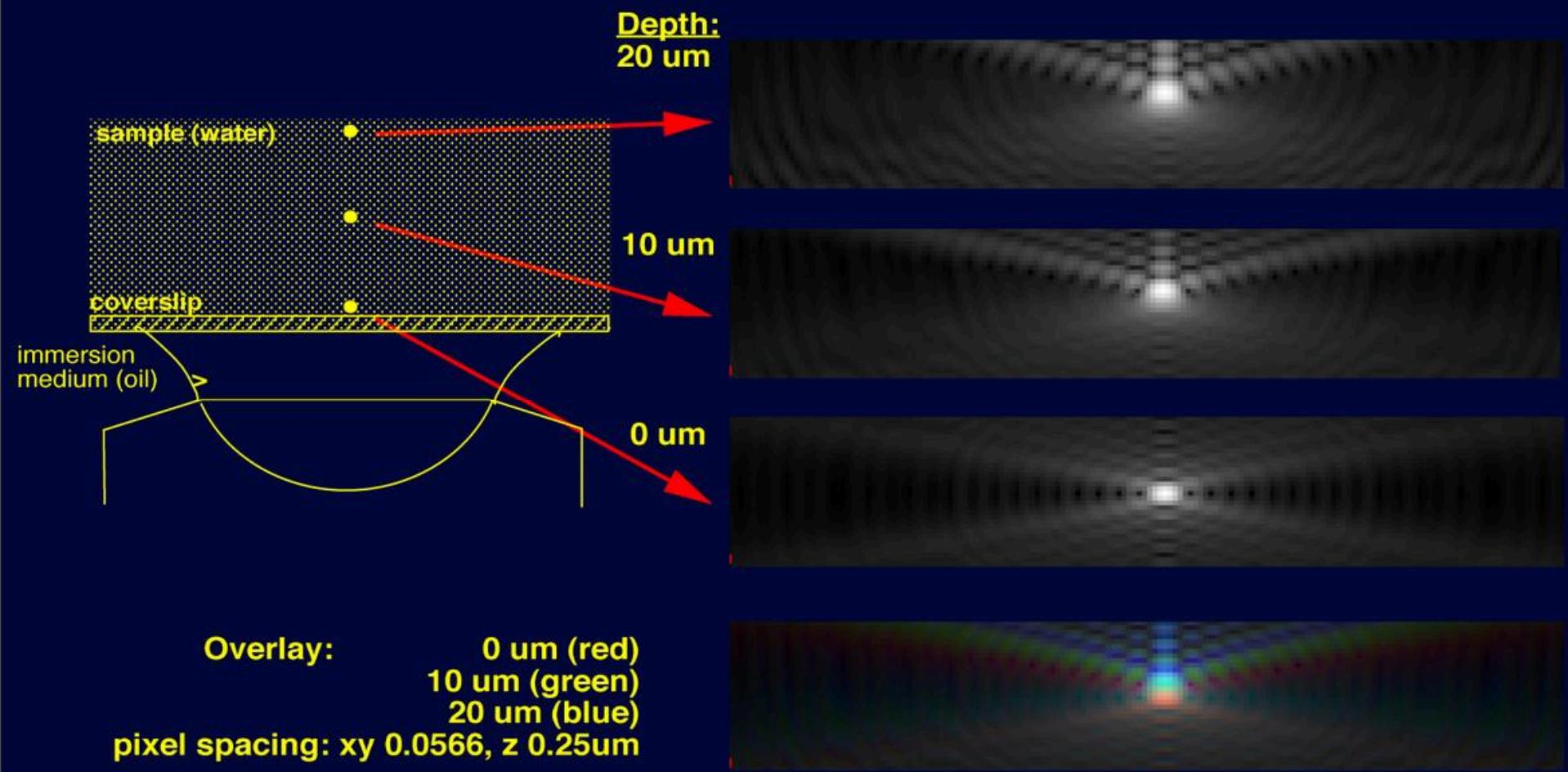


This causes  
Spherical Aberration

# Properties of the Sample Affect Image Formation in Microscopy



# Depth-Dependent Effects in Water: Simulations of PSFs



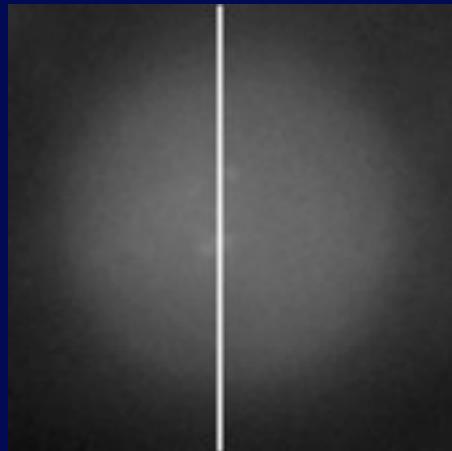
Hansel

# Depth Dependent Deconvolution: Biological Data

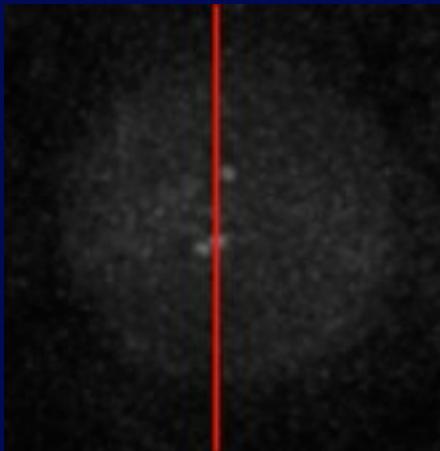
Line Profile Through 1 Spot in Nucleus

Section ~7 .6  $\mu\text{m}$  into sample

**Measured Data**

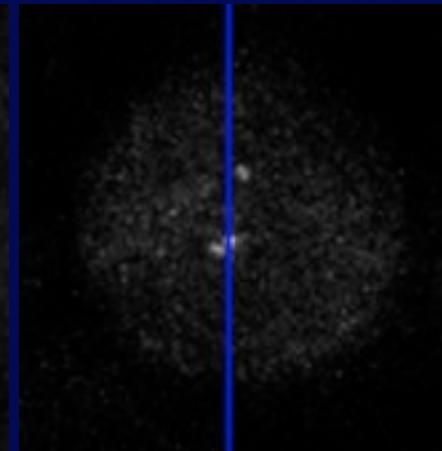


Spatially-Invariant



**Post-Deconvolution**

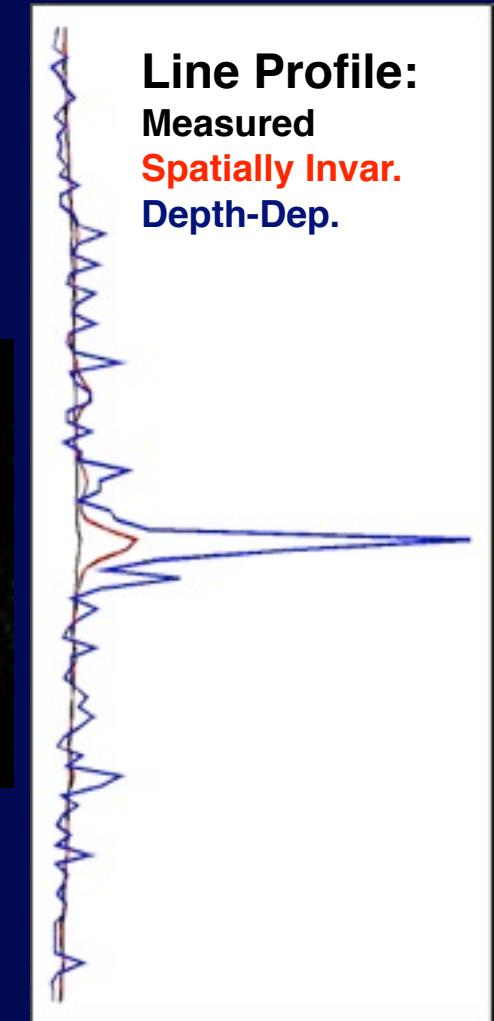
Depth-Dependent



Spot FWHM:

Measured = 0.28  $\mu\text{m}$   
Spatially-Invariant = 0.20  $\mu\text{m}$   
Depth-Dependent = 0.11  $\mu\text{m}$

**Line Profile:**  
Measured  
**Spatially Invar.**  
Depth-Dep.



Hanser

# Adaptive optics can correct for depth dependent effects (reshape optical wavefront)

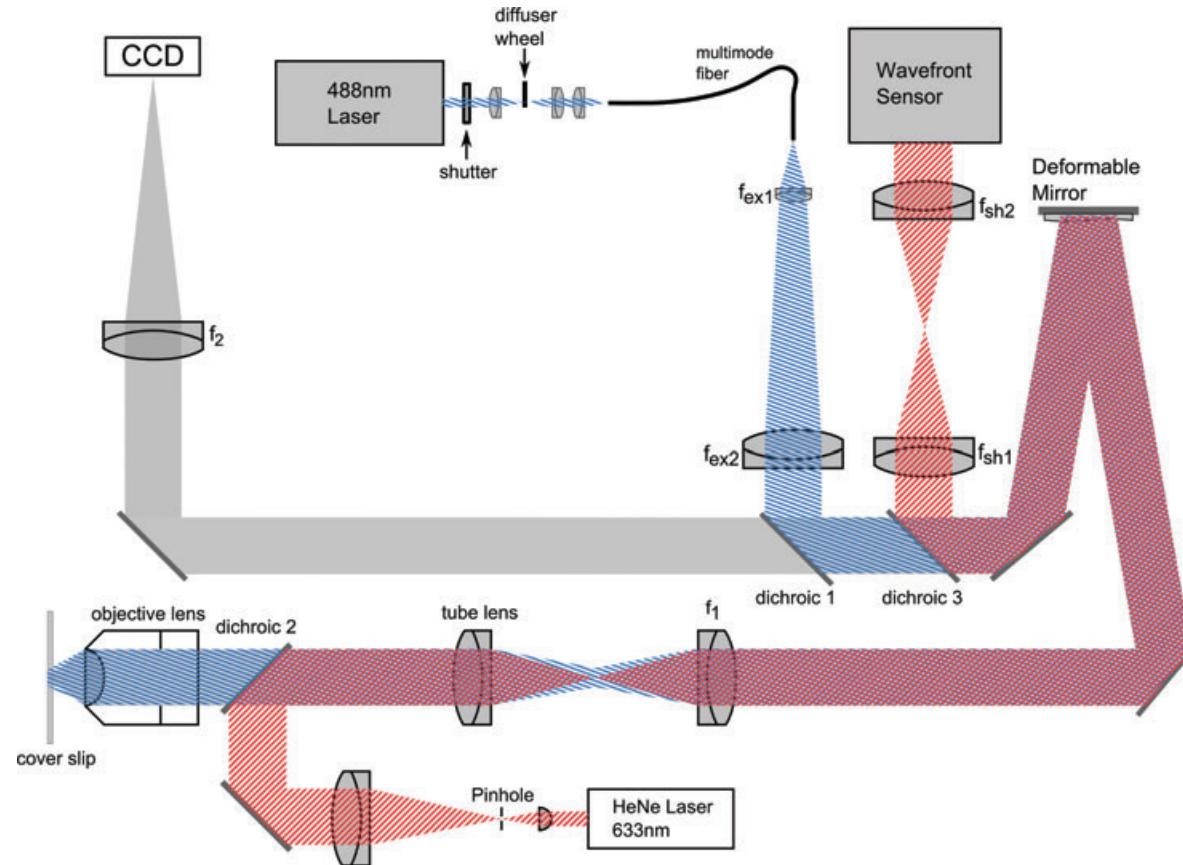
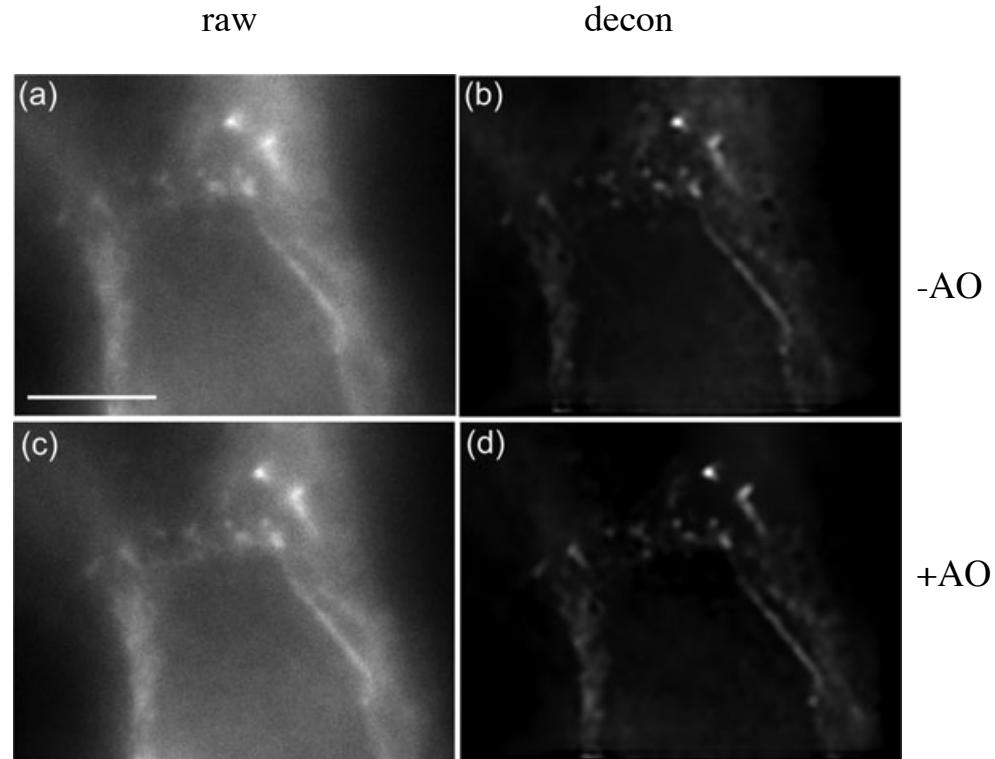
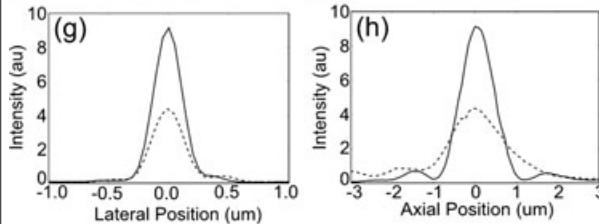
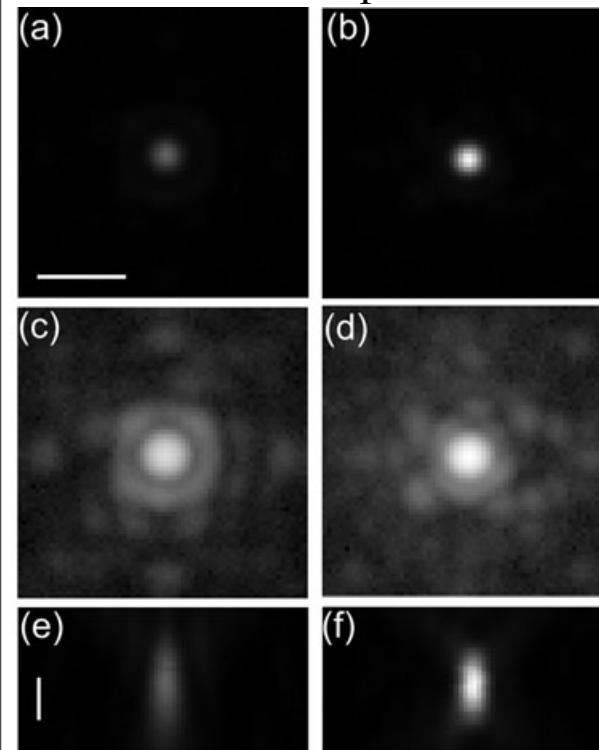


Fig. 2. Microscope layout. Grey represents the emission path. The blue striped beam is the excitation light and the red striped beam is the reference beam. See text for details.

# Adaptive optics can correct for depth dependent effects

flat mirror      mirror shaped to correct  
                  spherical aberration



Alexa 488-phalloidin stained mouse cells:  
images at  $4.4\mu\text{m}$  below cover slip

# Structured illumination microscopy: the idea

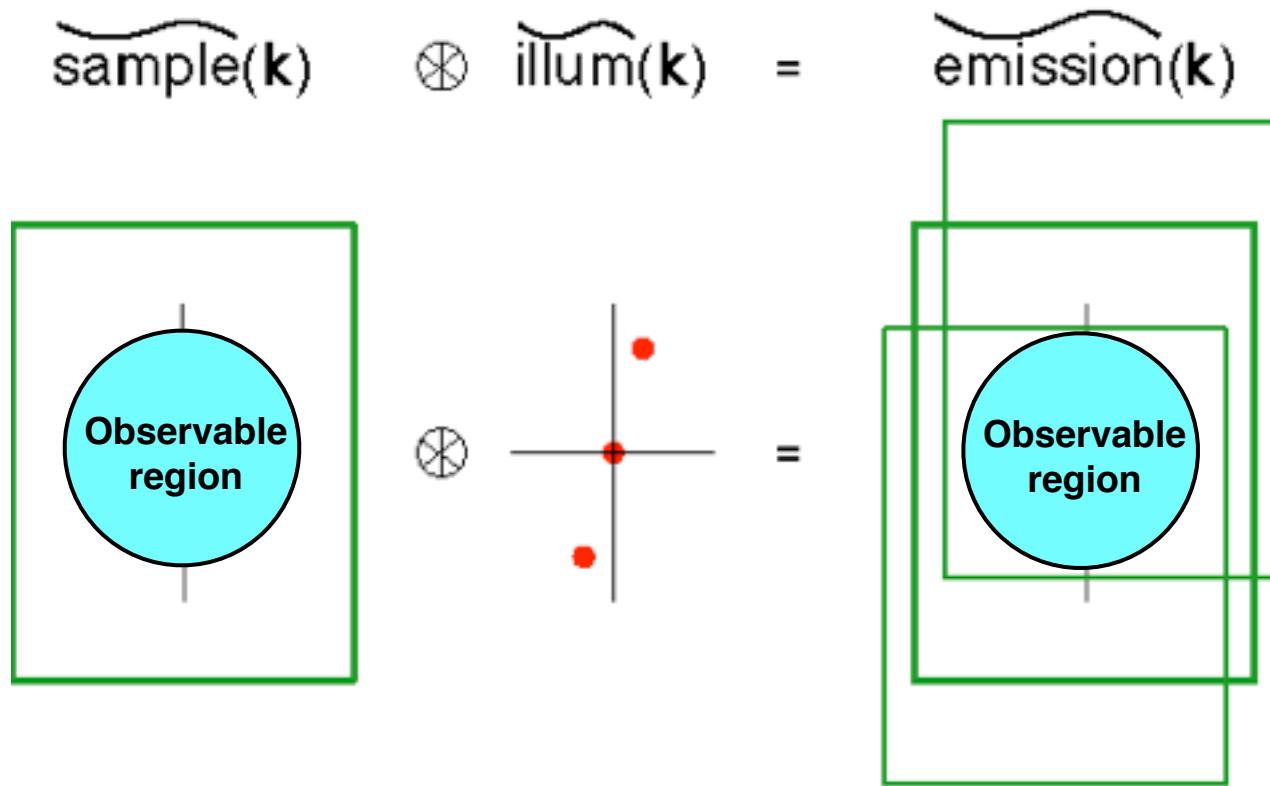
Two patterns  
superposed  
multiplicatively  
give rise to  
**moiré fringes**



The moiré fringes  
may be coarse enough  
to resolve  
even if neither  
original pattern is

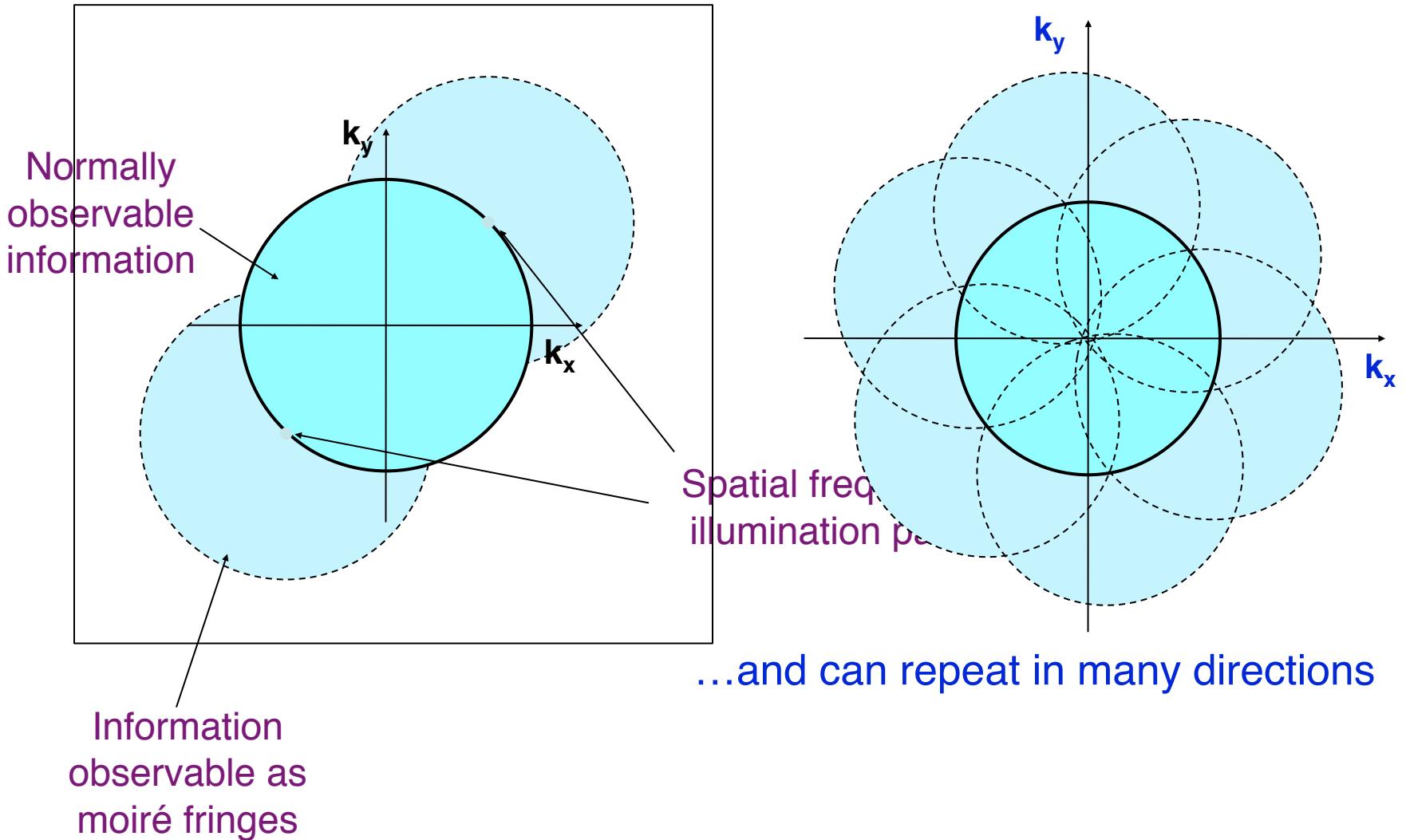
- *Illuminate the sample with a light pattern*
- *Observe moiré fringes between the pattern and the sample structure*
- *Deduce otherwise unresolvable information about the sample*

# *Illuminate sample with parallel stripes*



*The emitted light contains 3 superimposed information components shifted by  $0, \pm$  the inverse stripe spacing  
Record 3 images ( $0^\circ, 120^\circ, 240^\circ$  shifts) to sort out 3 components*

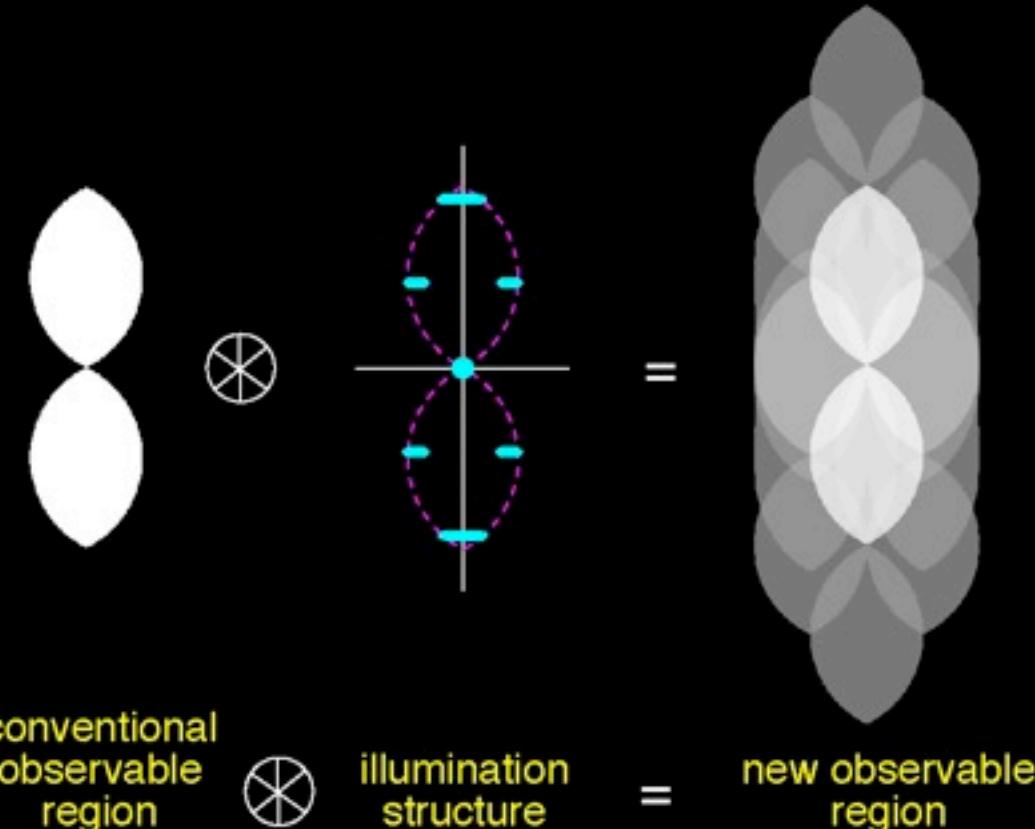
# *Resolution extension by Structured Illumination*



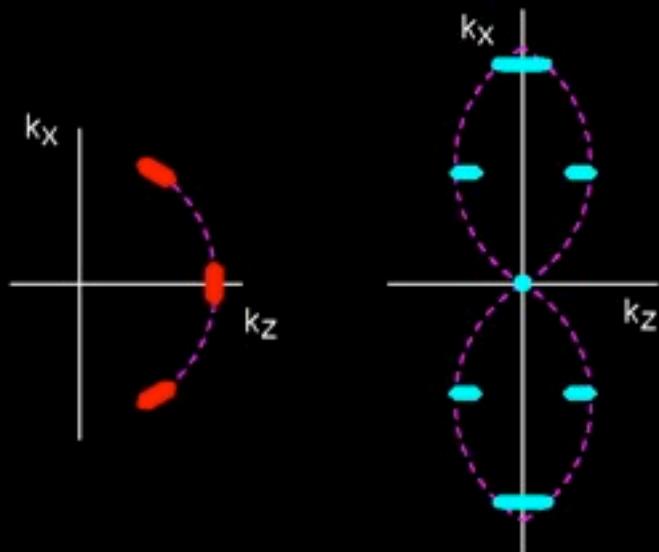
Q: What about 3D?

A: Use 3 beams

(admit diffraction orders 1, 0 and +1 )



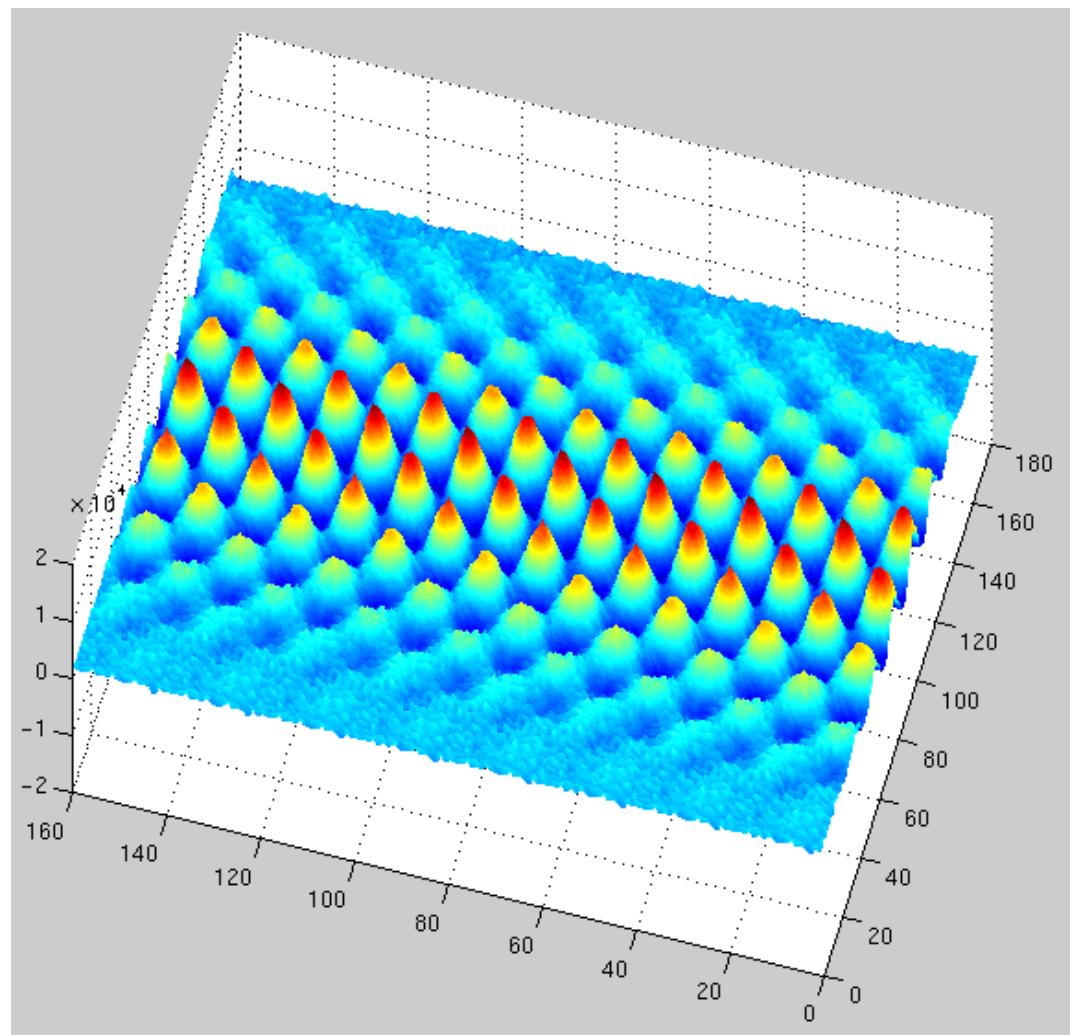
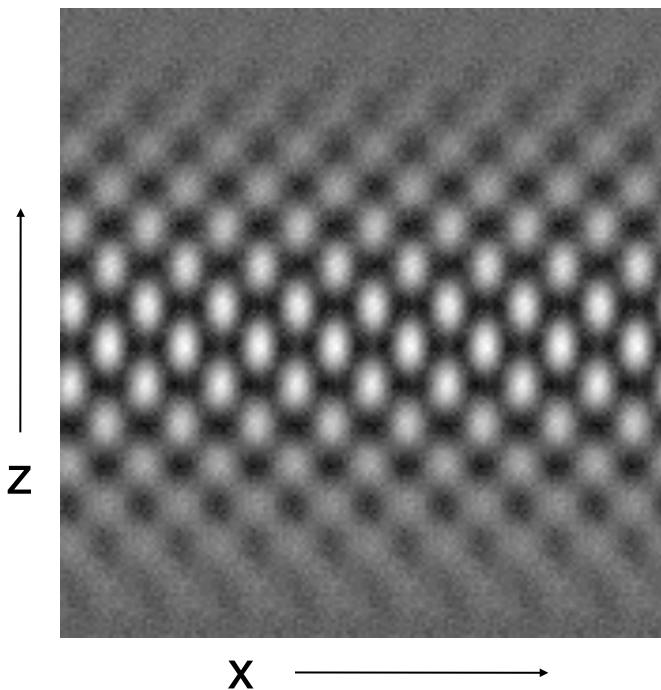
3 coherent beams → 7 intensity components



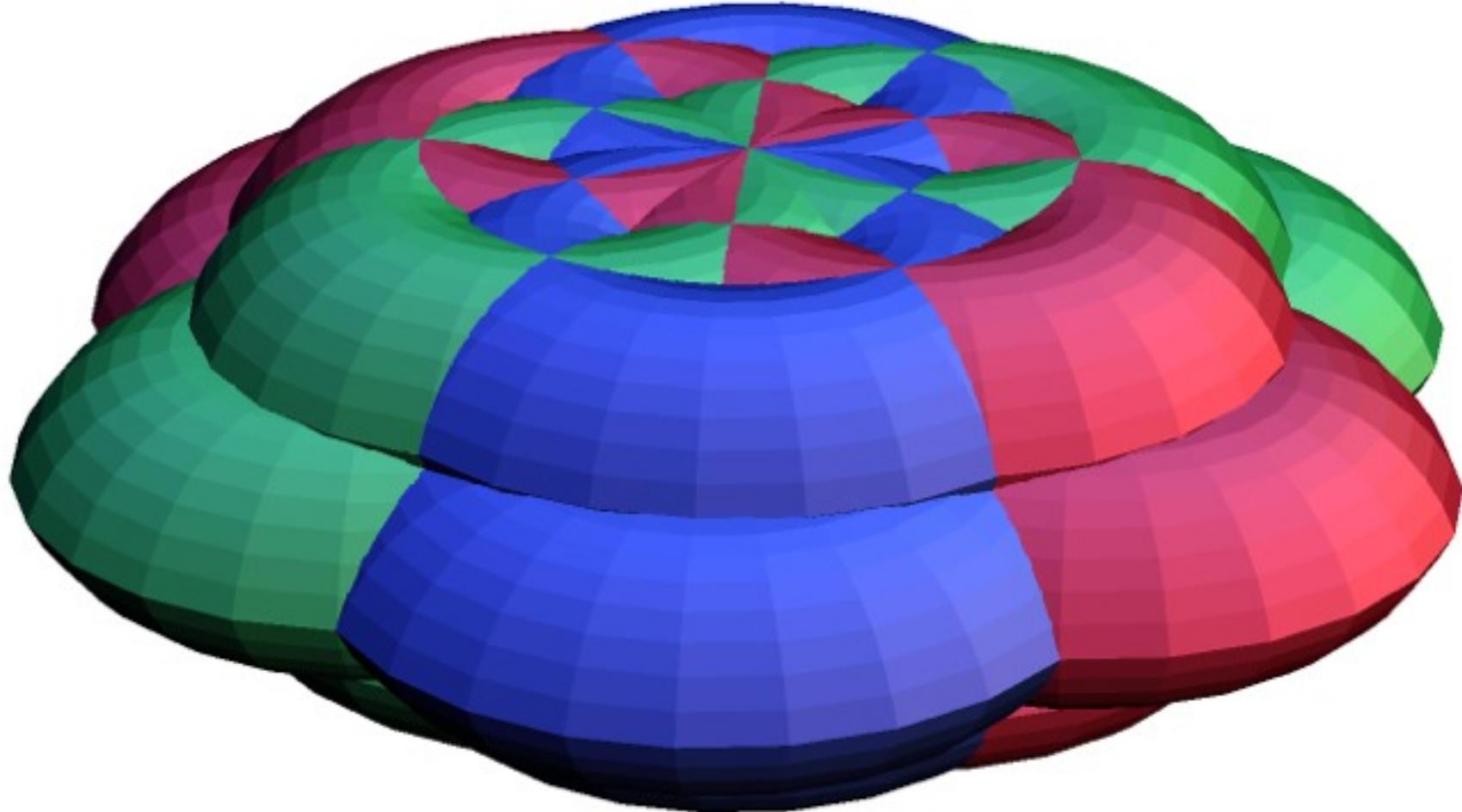
Covers full "confocal" OTF support without discarding light

Same procedure as for 2D, except need 5 phases

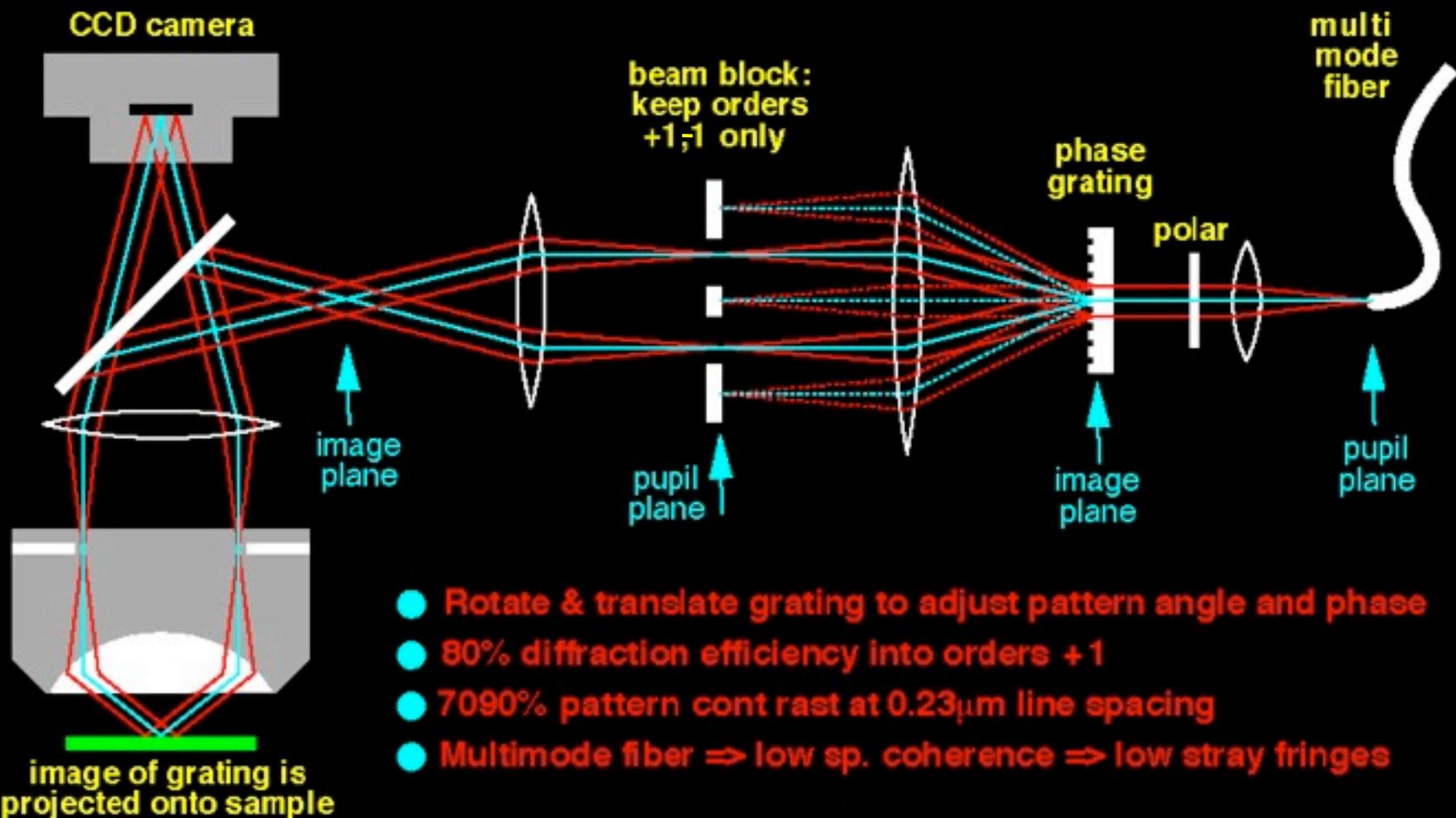
## illumination intensity in real space



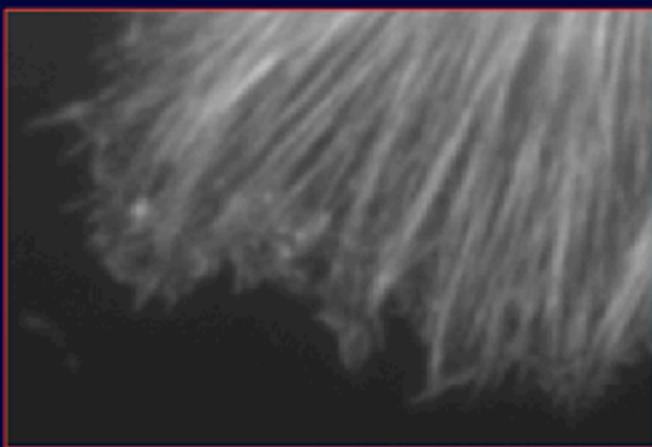
# Observable Region Structured Illumination (3 orientations)



# Structured illumination apparatus

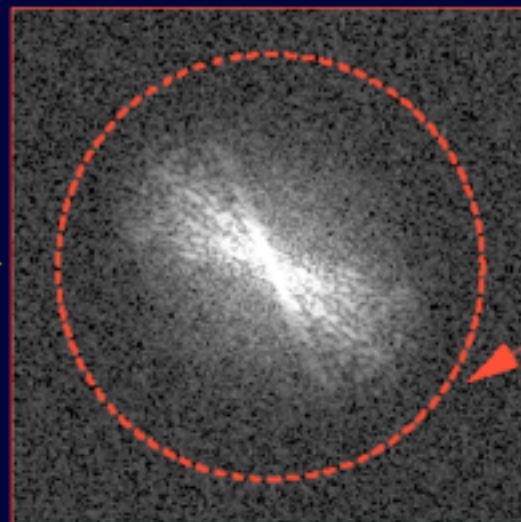


Real space images  
(raw data)



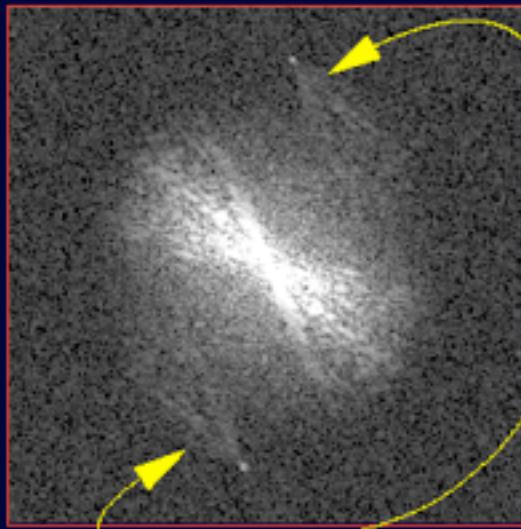
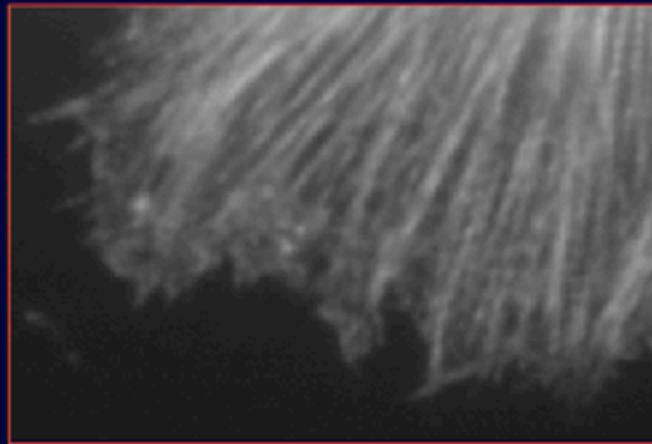
under  
normal  
illumination

Fourier transforms



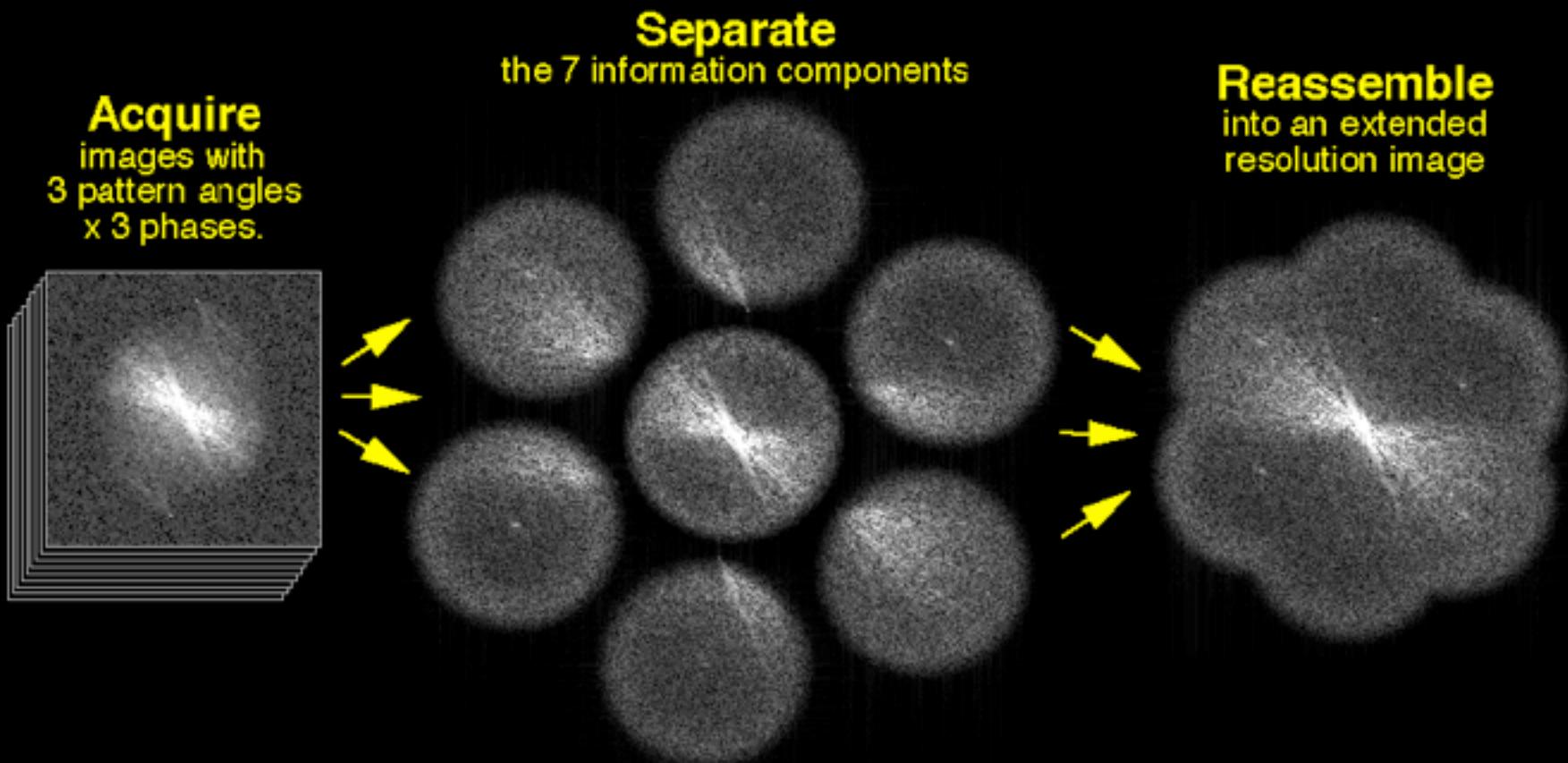
(theoretical  
resolution  
limit)

under  
structured  
illumination



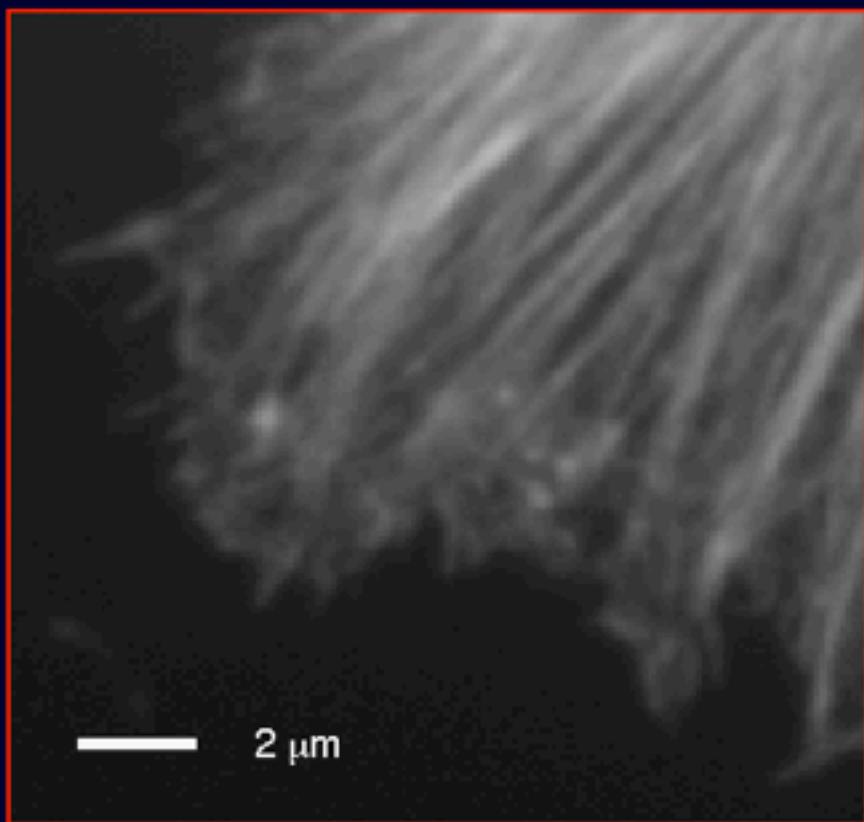
displaced  
information  
(i.e. moire  
fringes)

## Reconstruction in reciprocal space

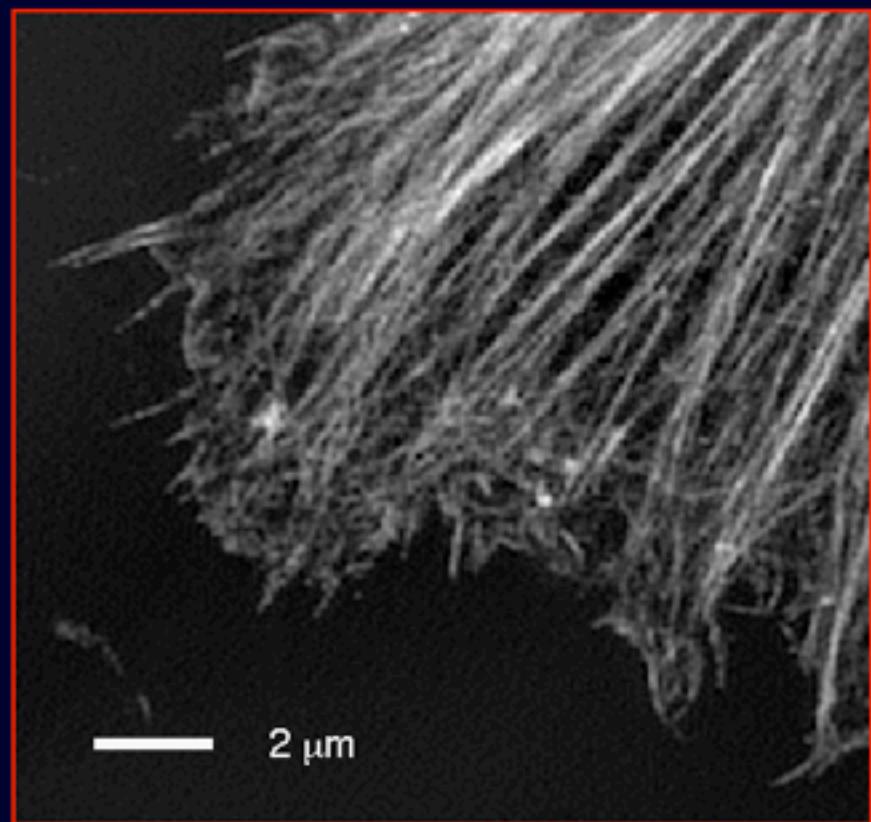


## Resolution comparison Actin in a HeLa cell

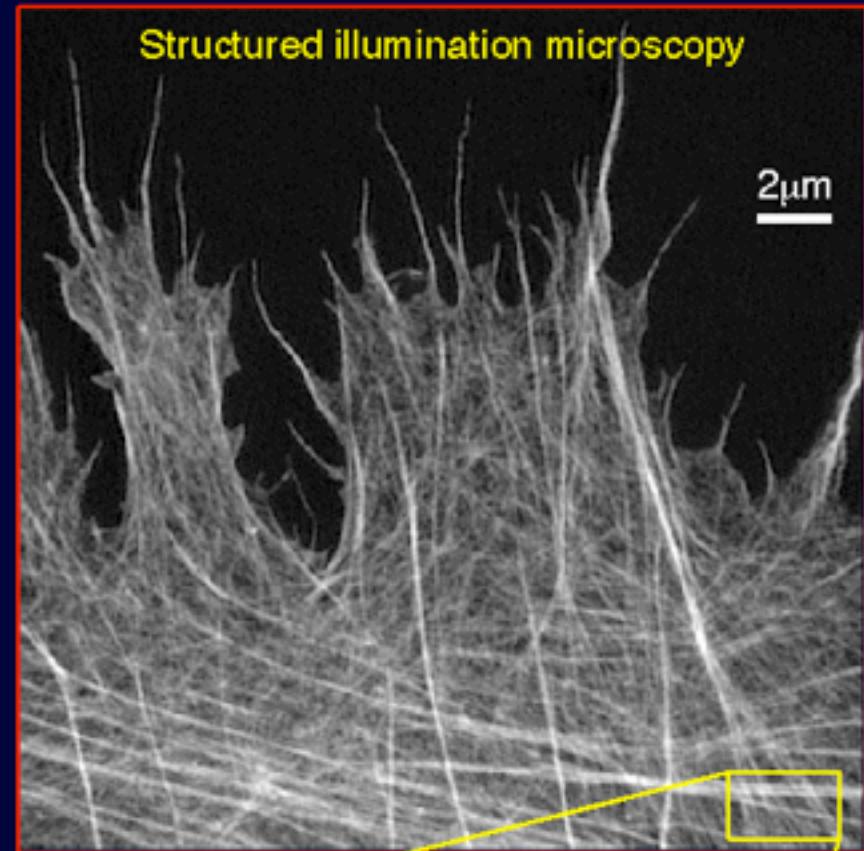
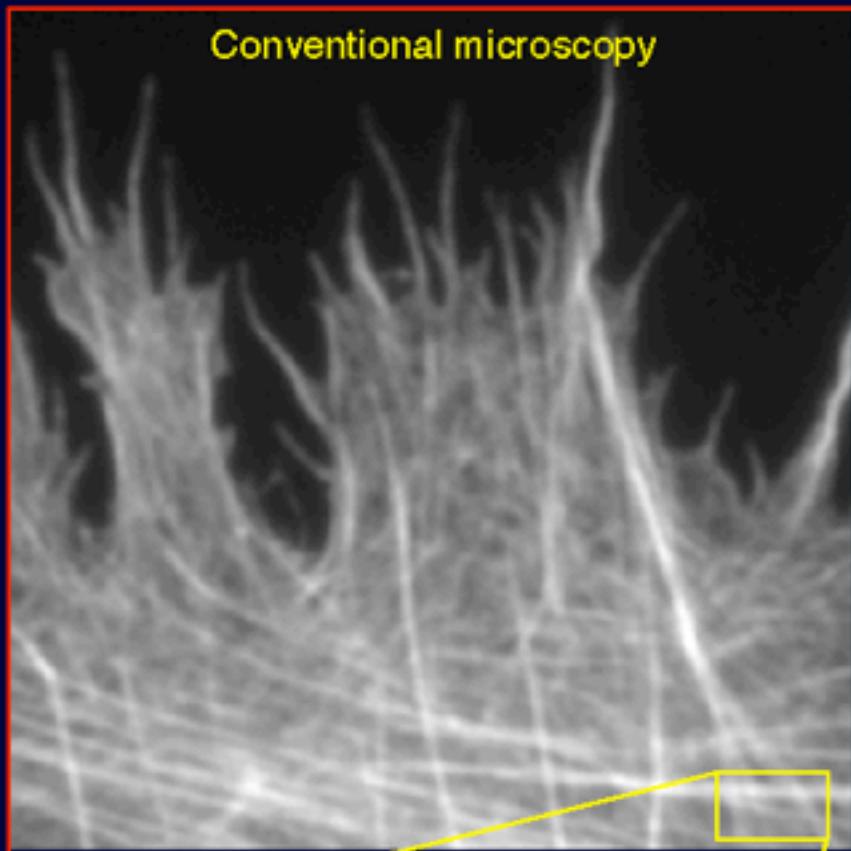
Conventional microscopy



Structured illumination  
final reconstruction

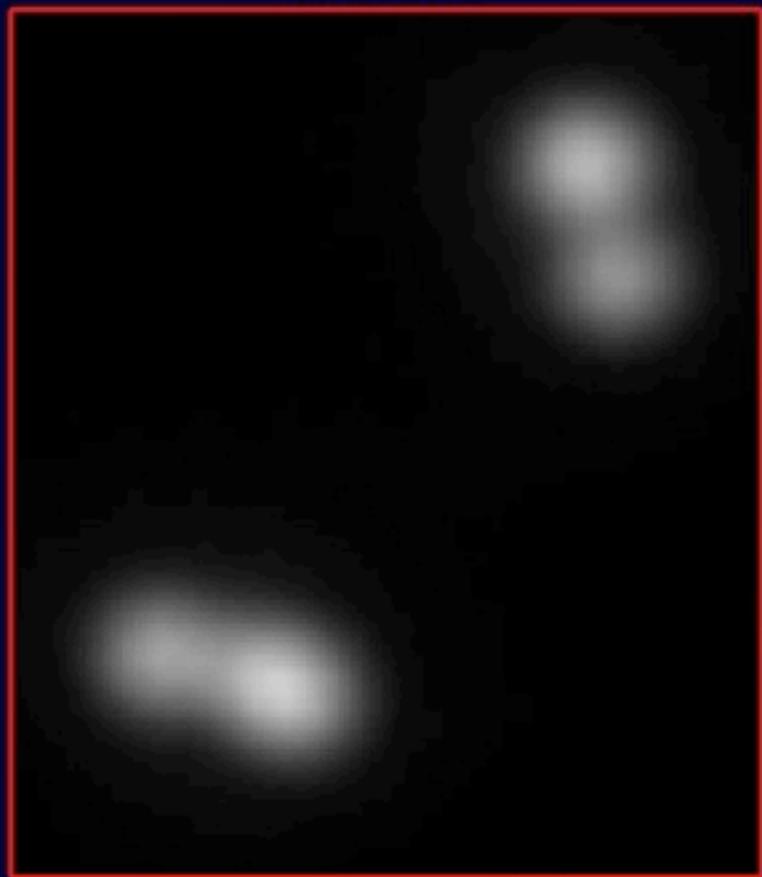


# Actin in a HeLa cell

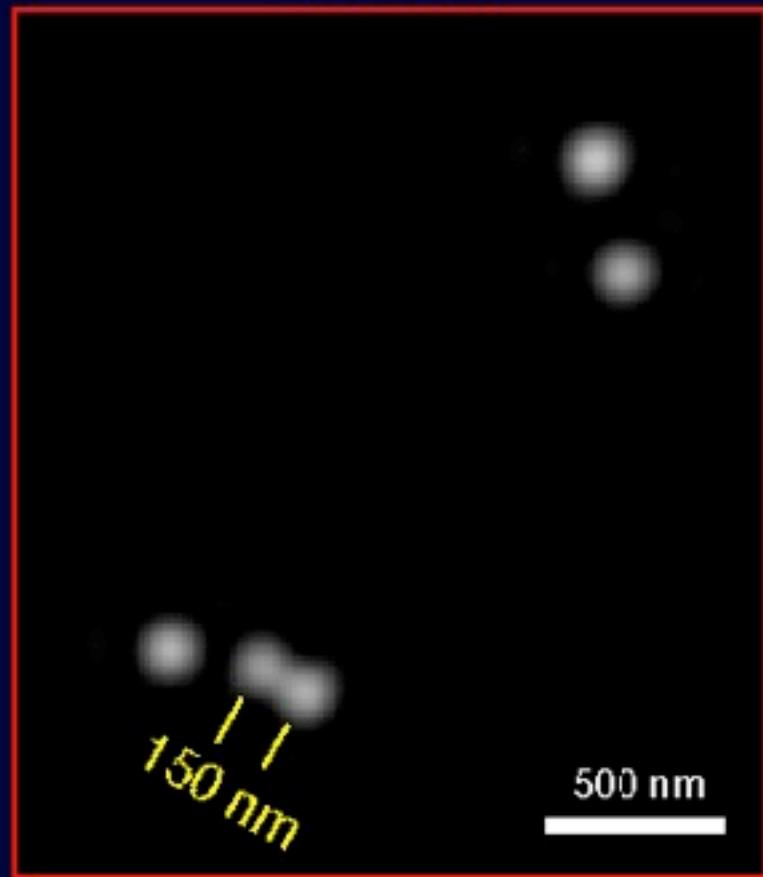


# 120 nm microspheres

Conventional  
microscopy

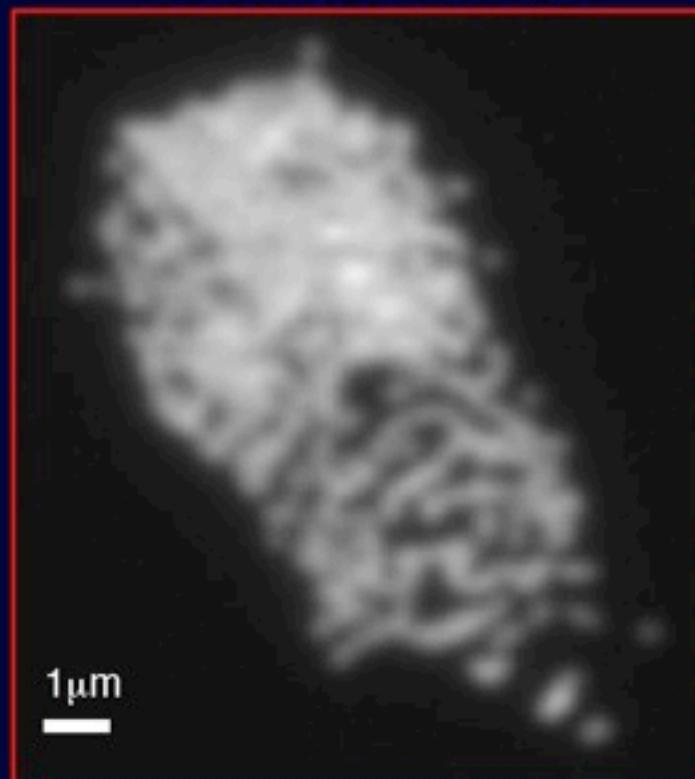


Structured illumination  
microscopy

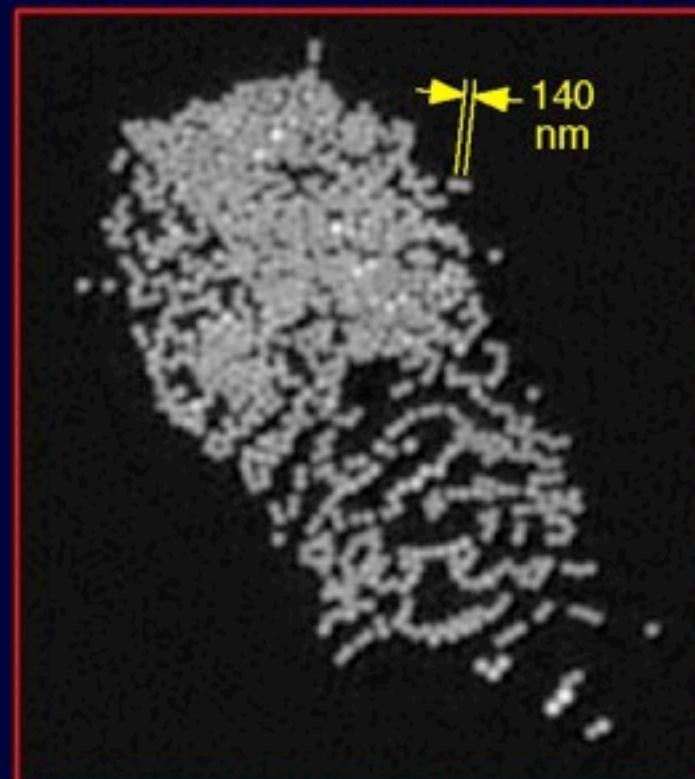


## 120 nm microspheres

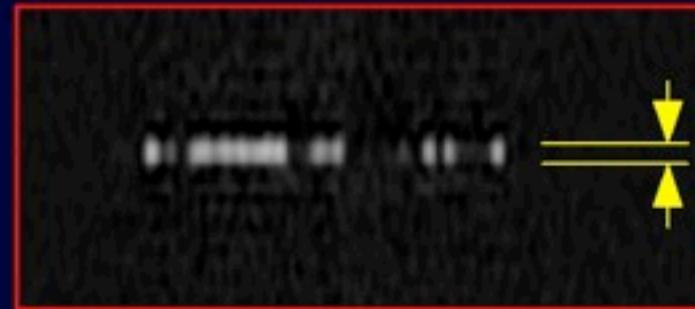
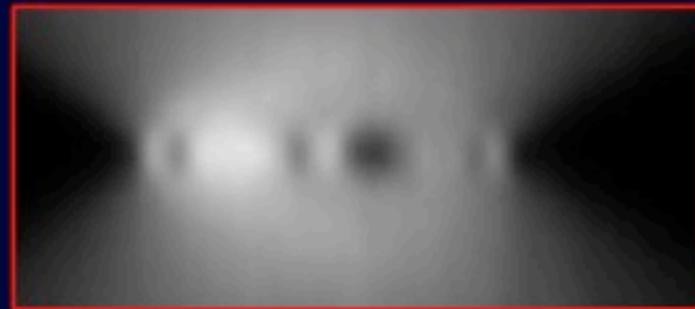
Conventional microscopy

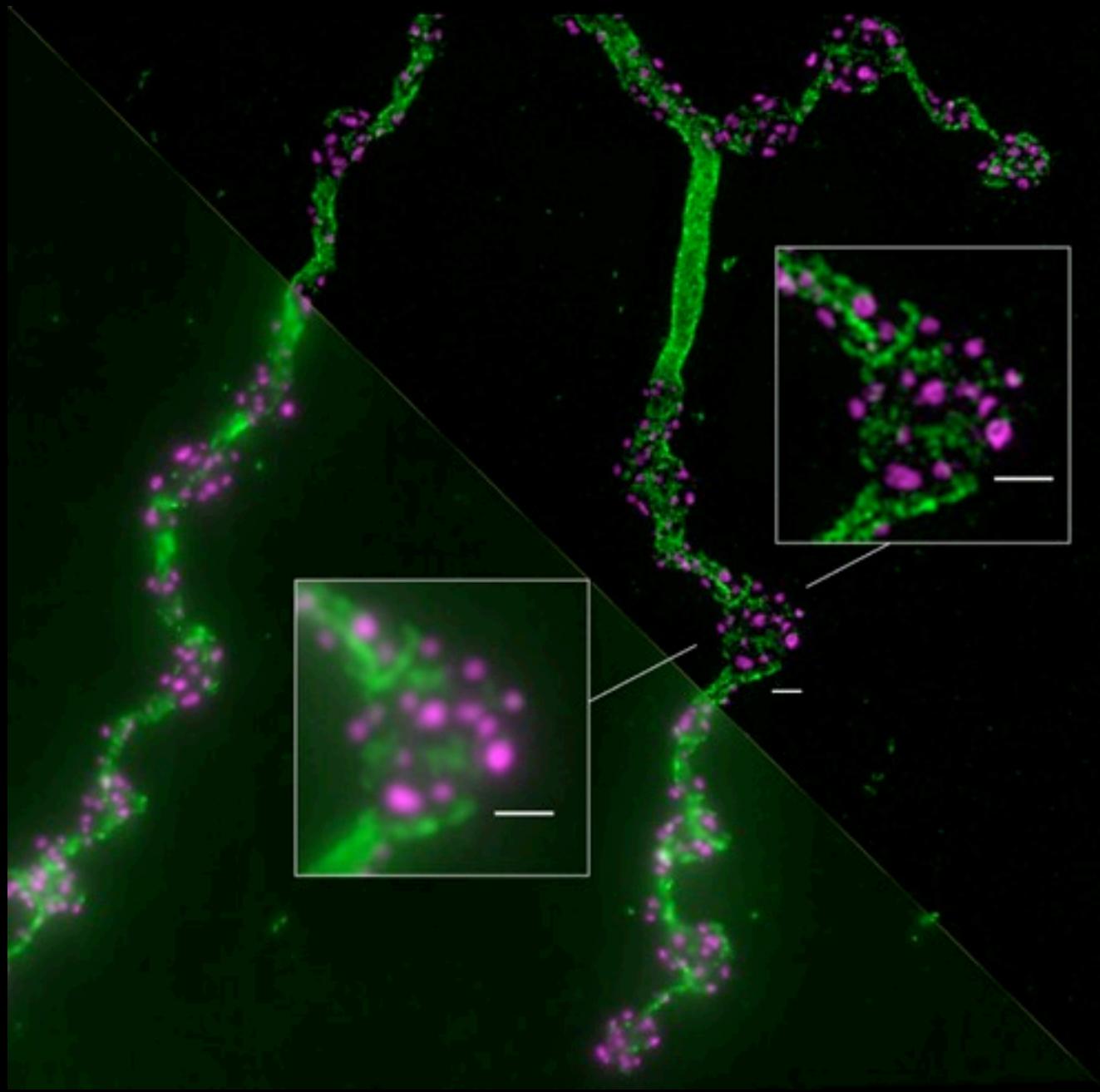


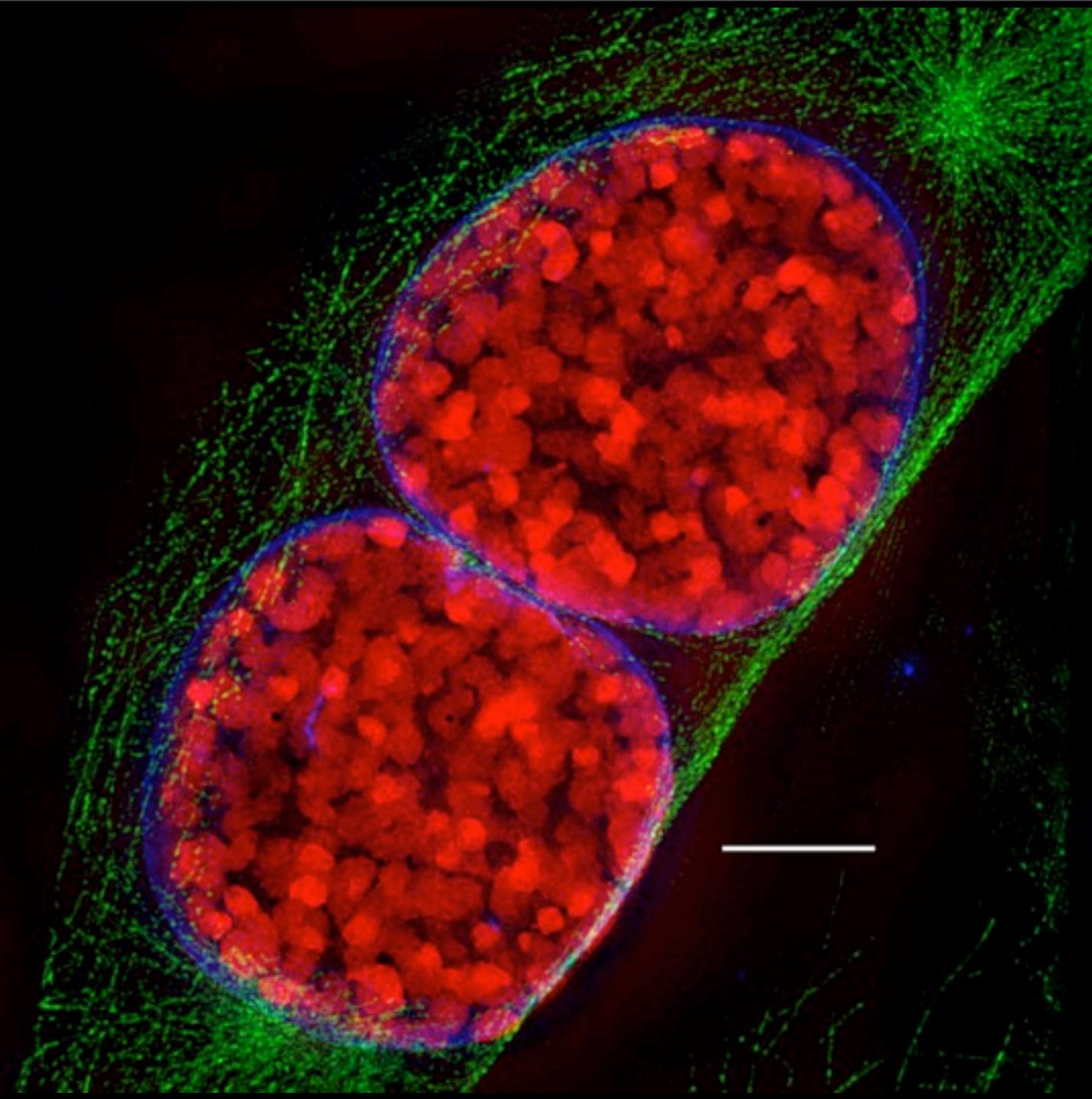
Structured illumination microscopy



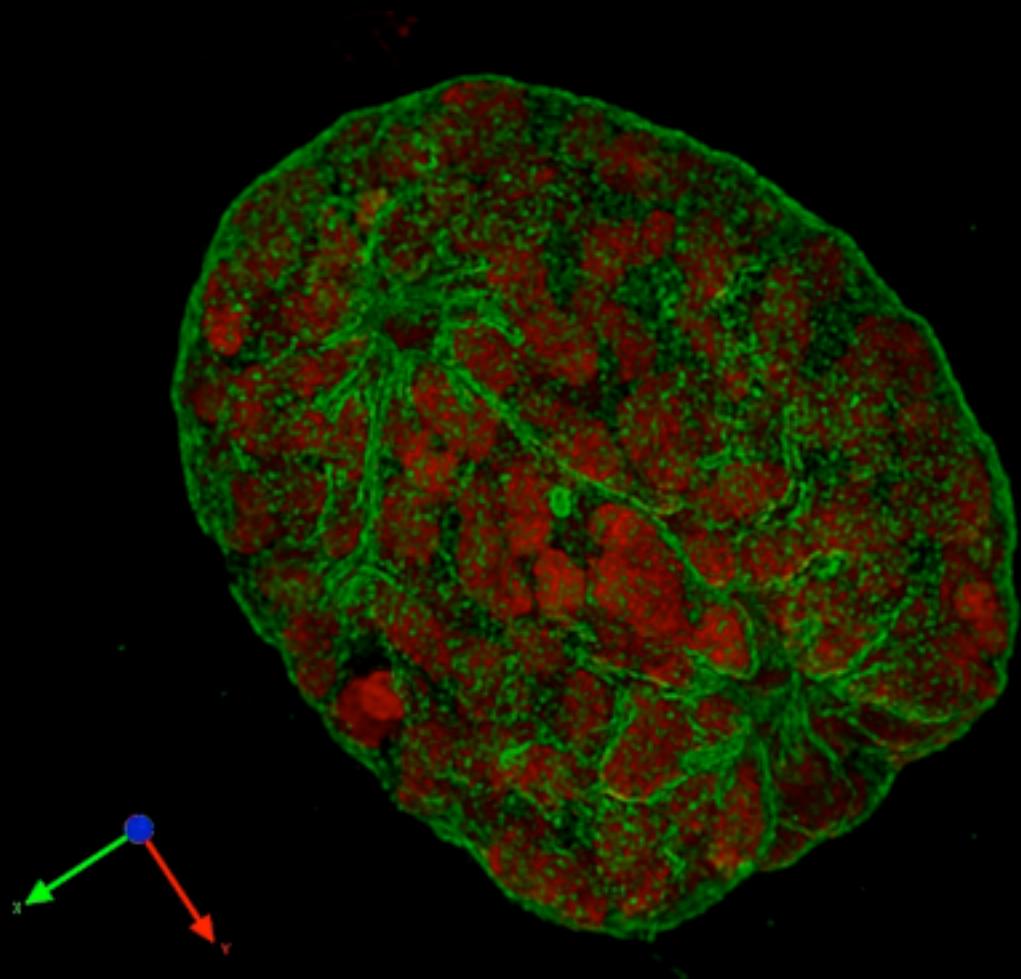
XZ



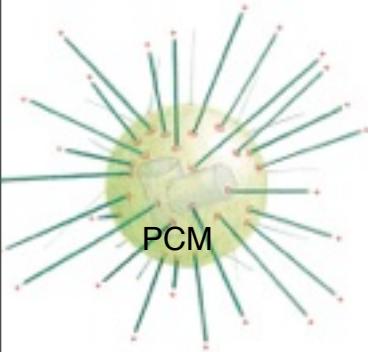




Thursday, May 9, 13

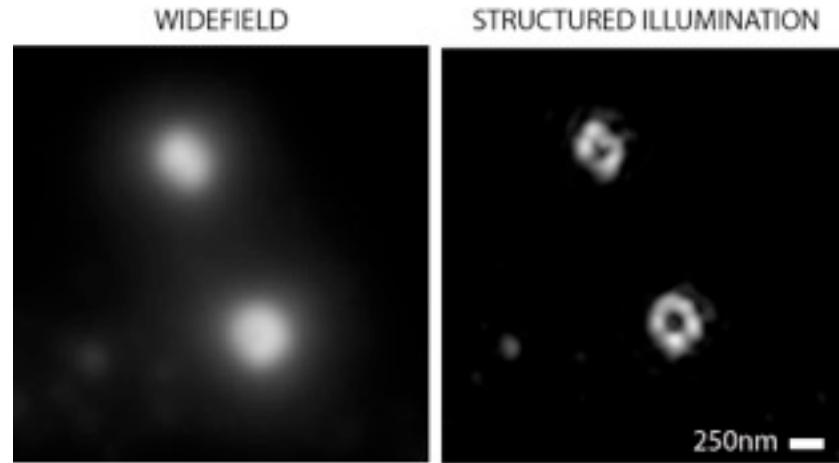


## *PCM long considered amorphous (decades of EM)*



- systematically map key components in PCM
- light microscopy provides required selectivity
- challenge: small size (200 - 500nm)

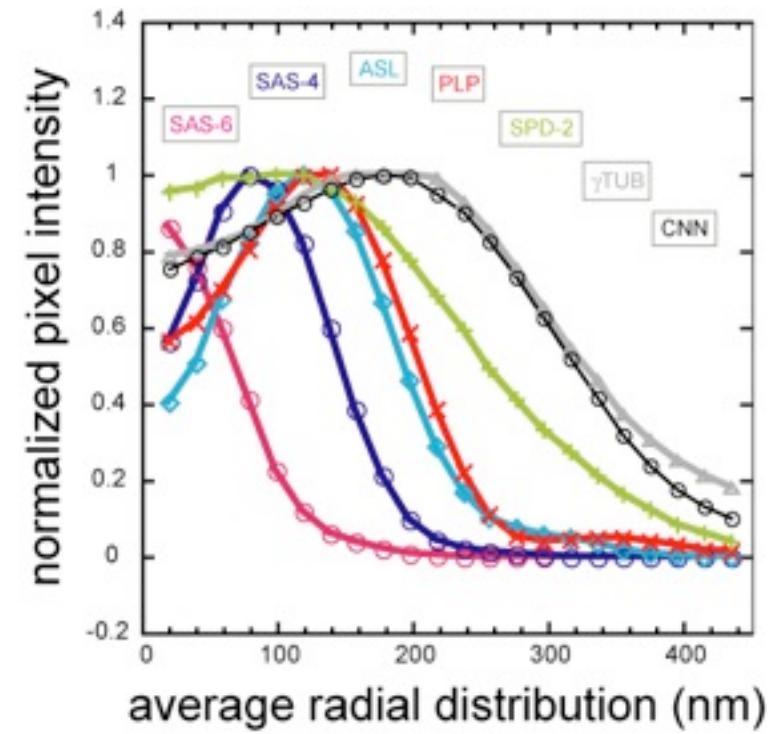
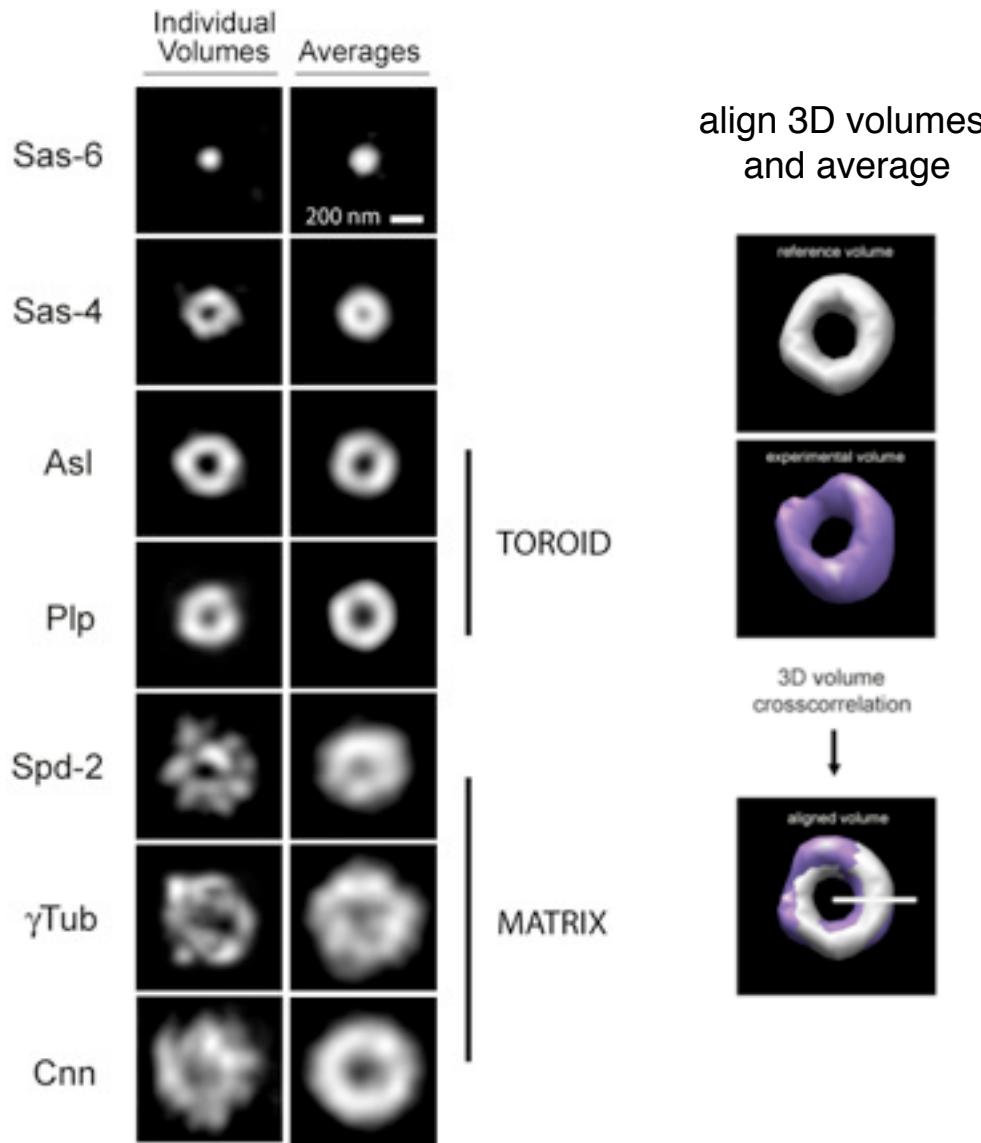
*=> only tractable with sub-diffraction microscopy*



*combination of SIM/STORM provides maximal confidence/resolution*

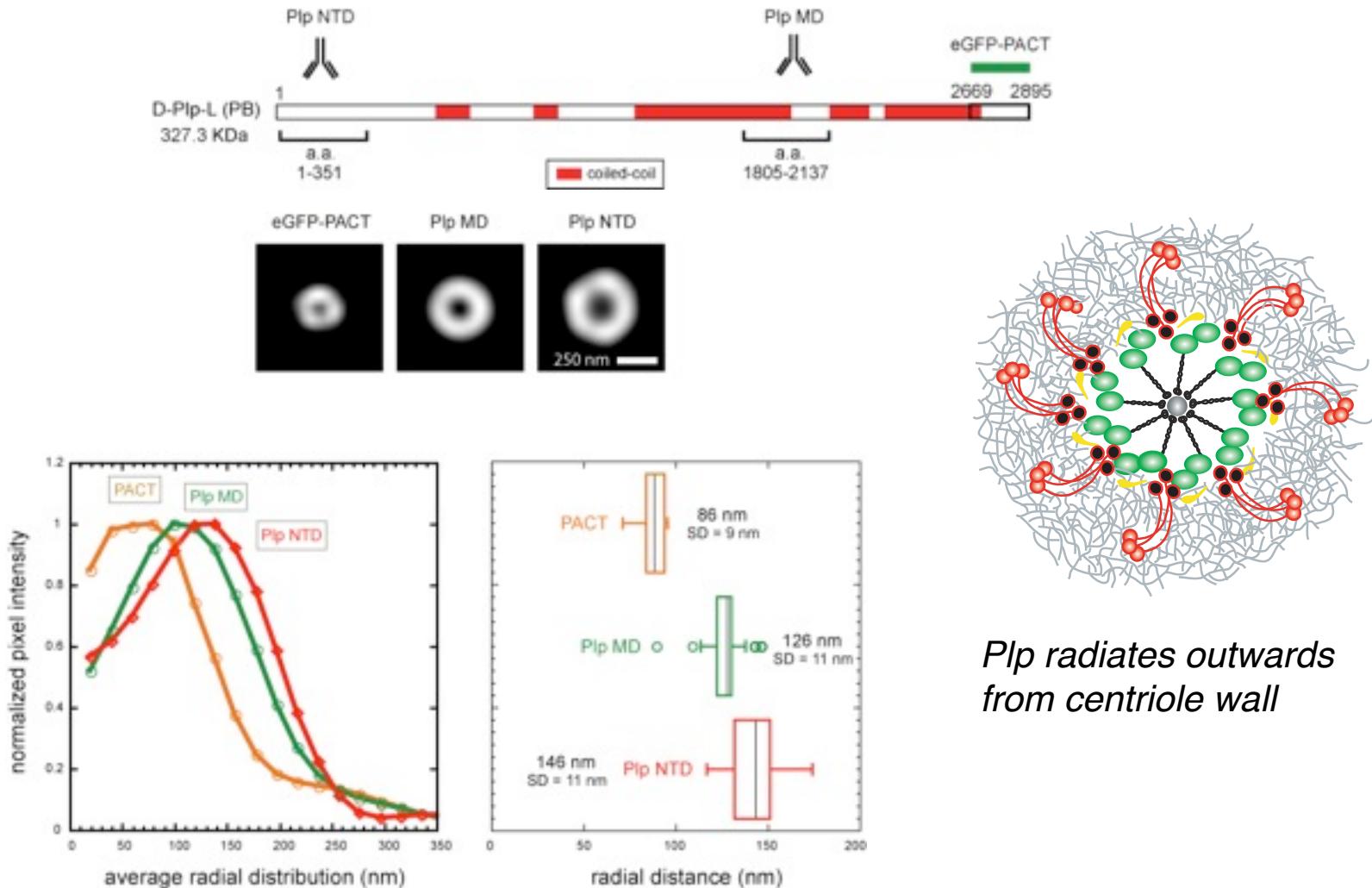
V. Mennella

# 3DSIM + 3D Averaging + Quantitative Analysis: PCM is organized in well-defined architectural subdomains



V. Mennella, B. Keszthelyi

# What is the molecular architecture of Plp?

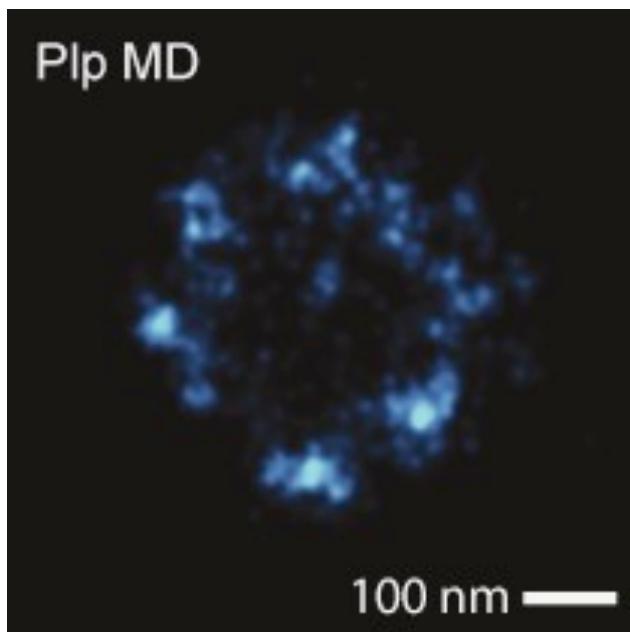


*Plp radiates outwards from centriole wall*

Molecular Structural information in cells!

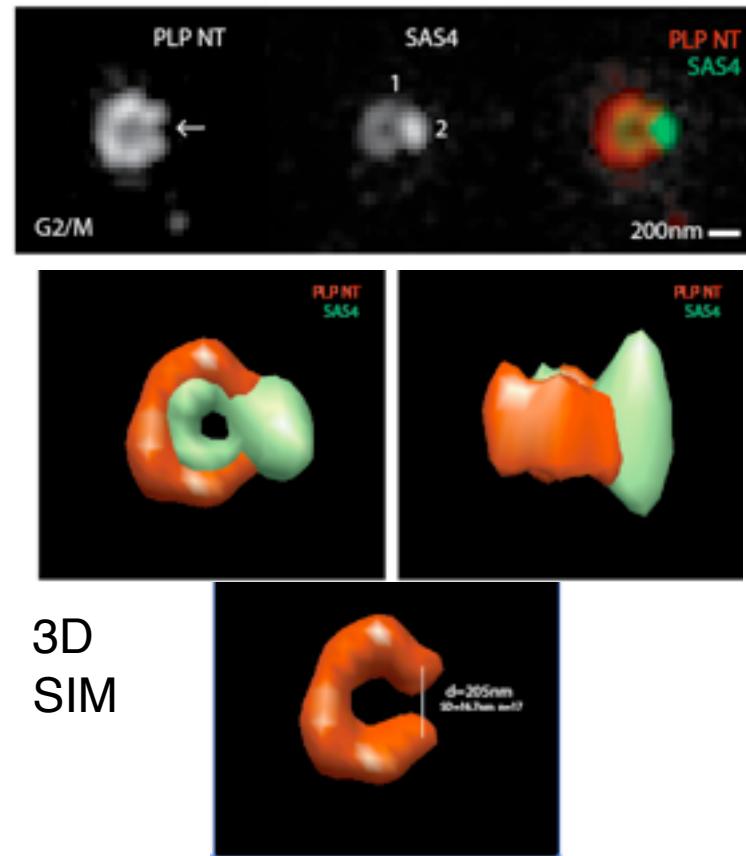
V. Mennella

# *PLP organized in symmetrical clusters that open for new procentriole*



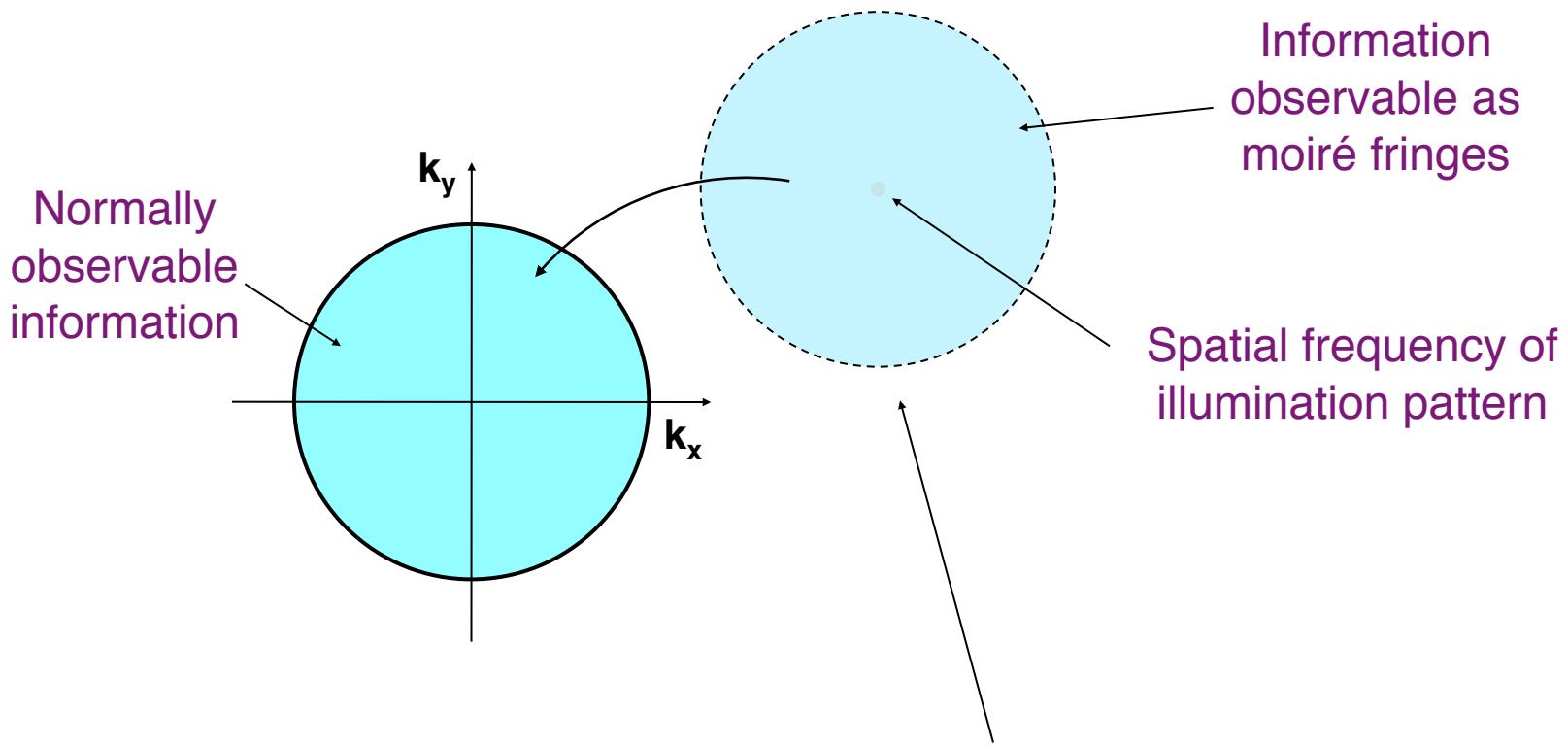
STORM ~ 9 fold symmetry

XYZ resolution:  
32nm,37nm,59nm



V. Mennella, B. Huang

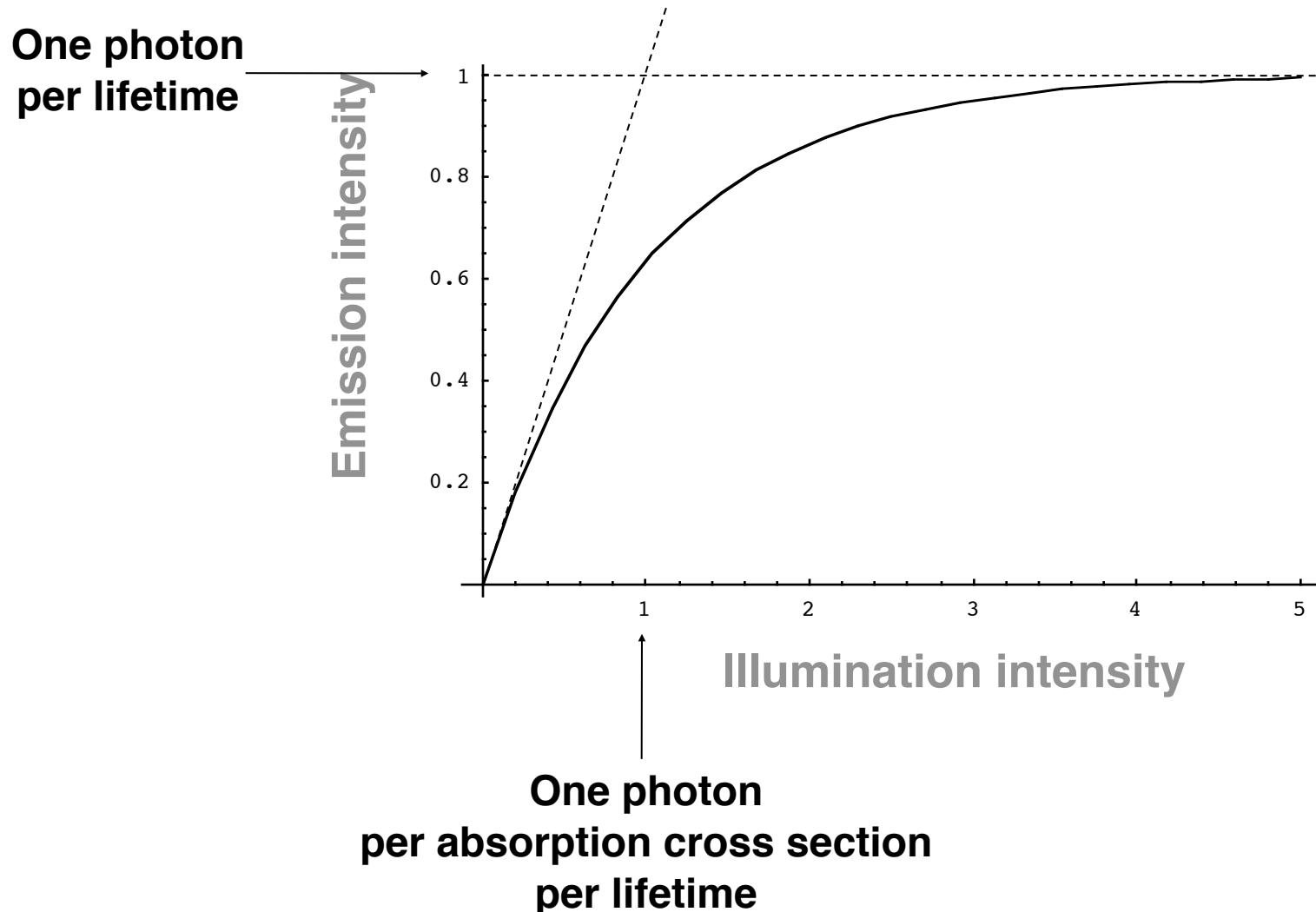
# *What are the limits of Structured Illumination?*



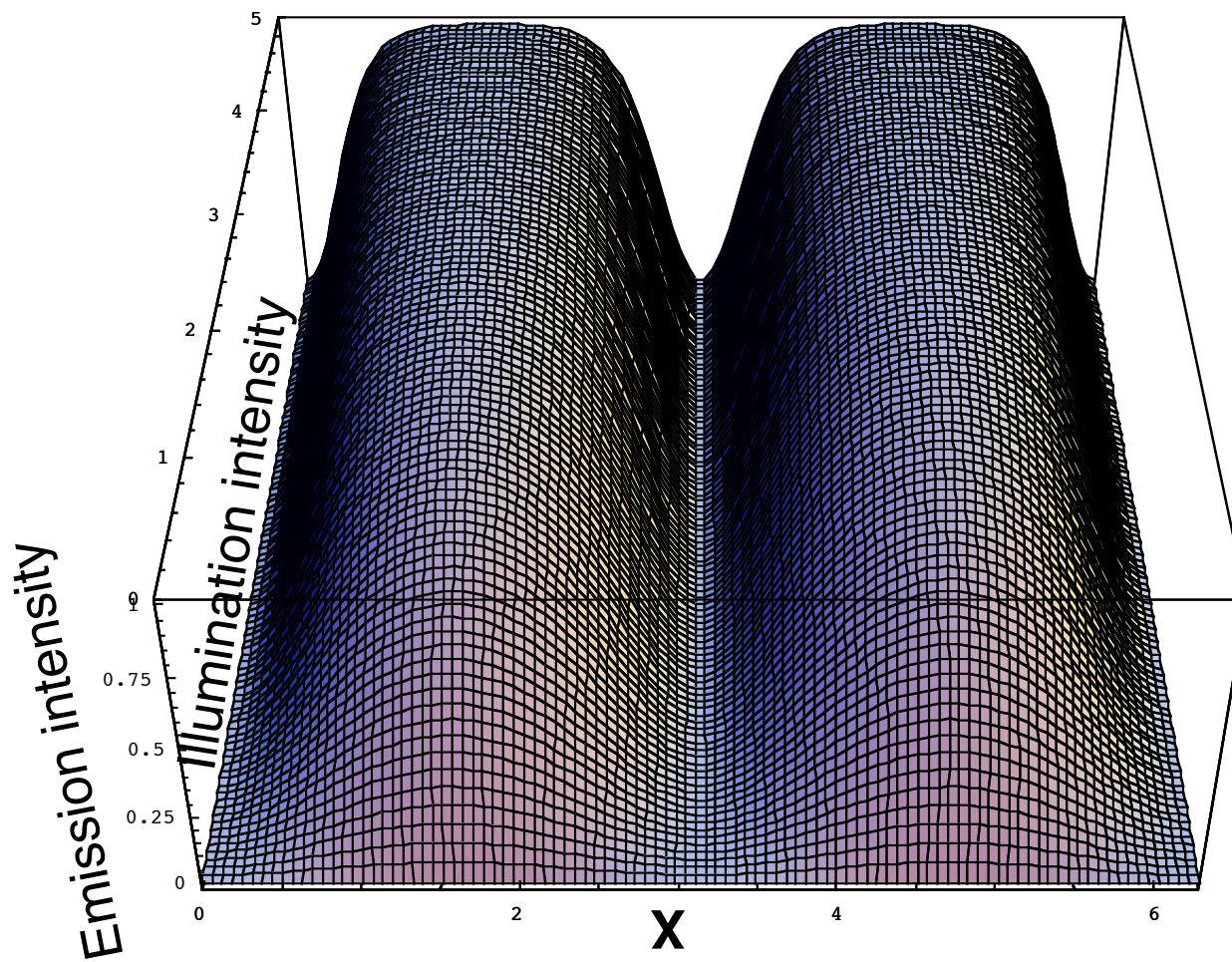
- Linear theory says this is impossible
- How about exploiting non-linear processes?

# **A simple source of nonlinearity: saturation**

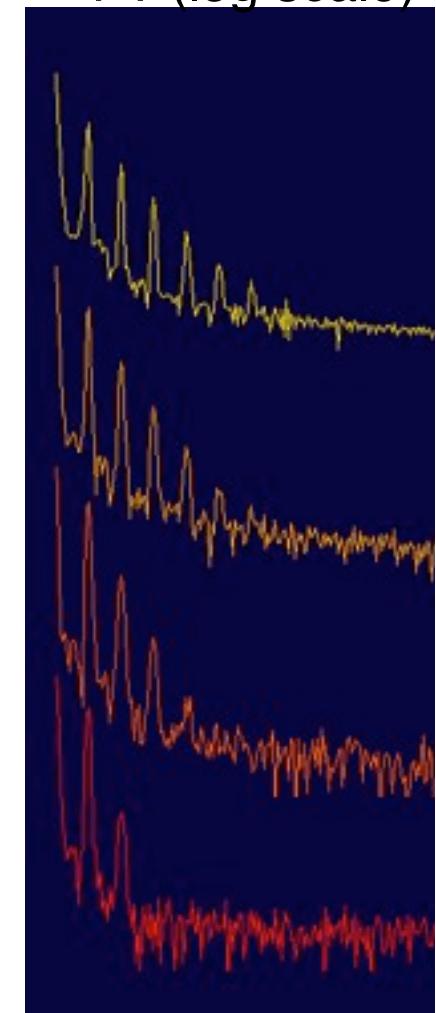
(R. Heintzmann Max-Planck Gottingen)



# *Saturated Structured Illumination*



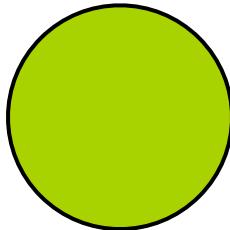
FT (log scale)



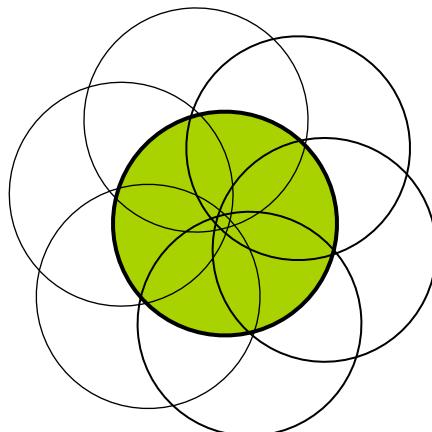
# *Resolution extension by nonlinear structured illumination*

Effective observable regions

Conventional  
microscopy



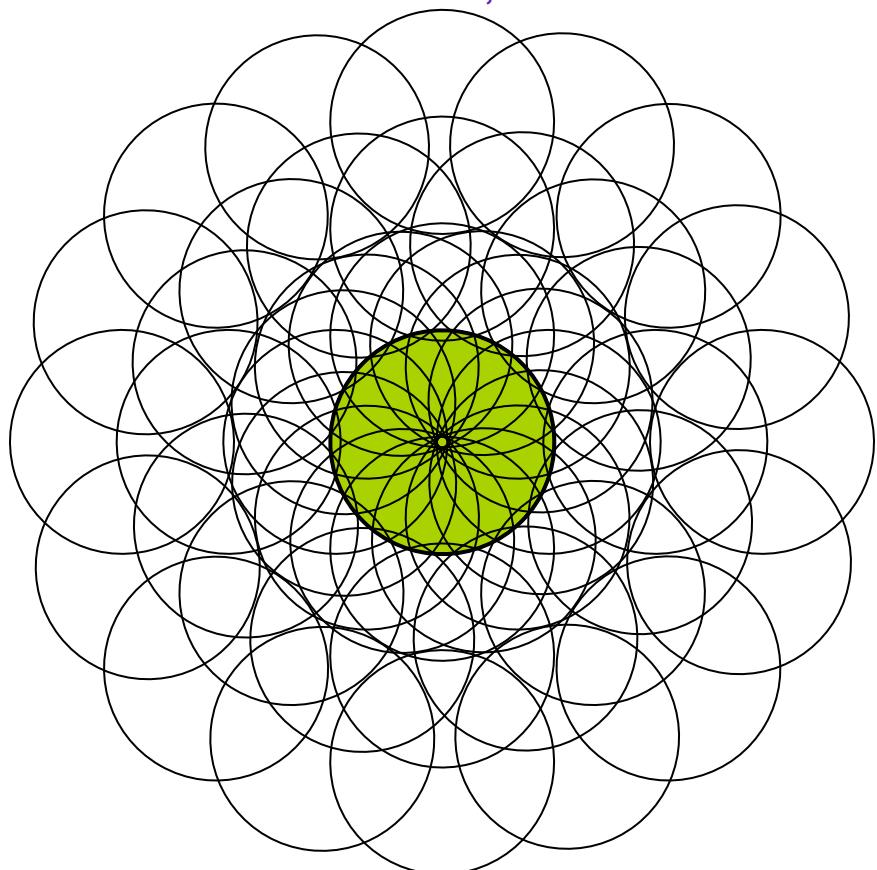
200 nm res.



100 nm res.

Linear  
Structured  
illumination  
3 directions

Nonlinear structured illumination  
2 new harmonics, 8 directions



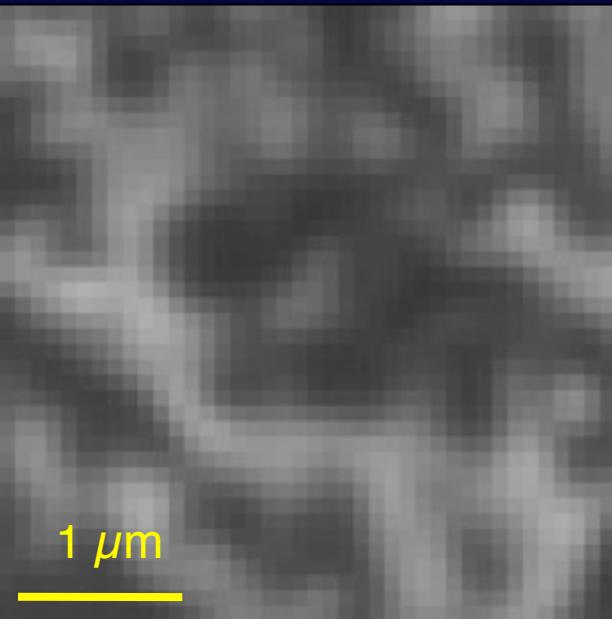
50 nm res.

# *Non-linear structured illumination*

Conventional  
microscopy

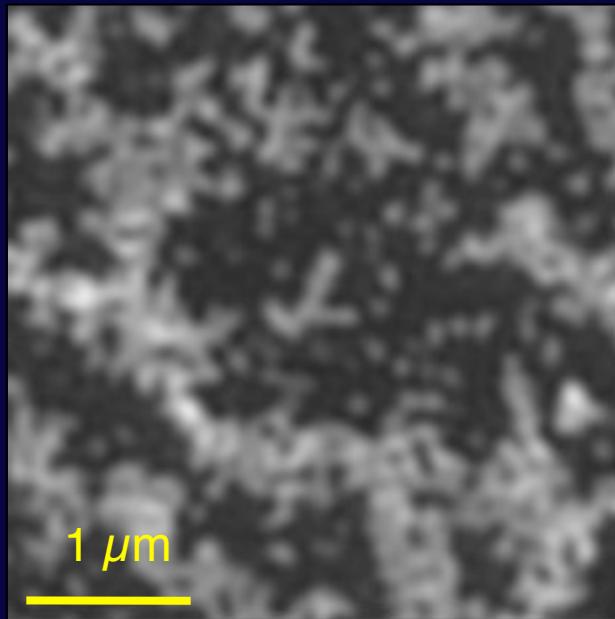
Linear  
structured  
illumination

Saturated  
structured  
illumination



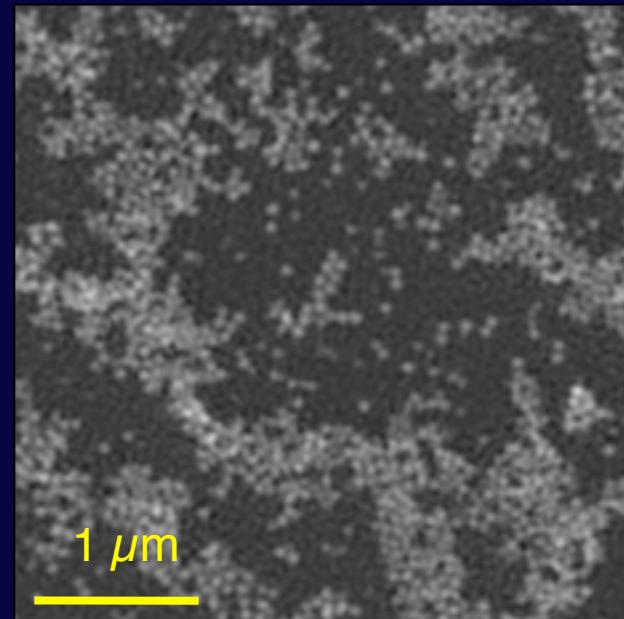
1  $\mu\text{m}$

**~250 nm resolution  
(diffraction limit)**



1  $\mu\text{m}$

**~120 nm resolution**



1  $\mu\text{m}$

**46 nm resolution!**

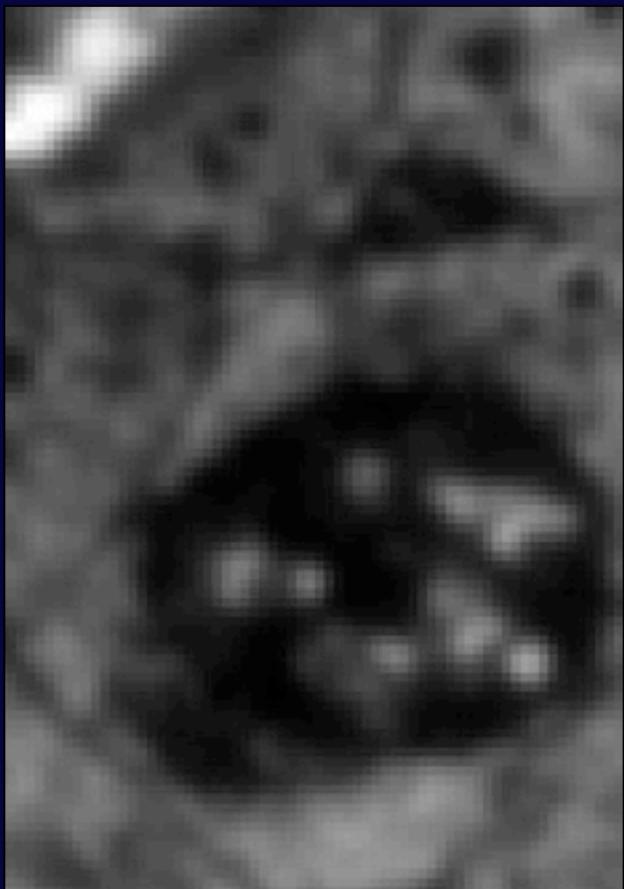
**50 nm microspheres**

# *Drosophila* embryo section DNA/RNA stain

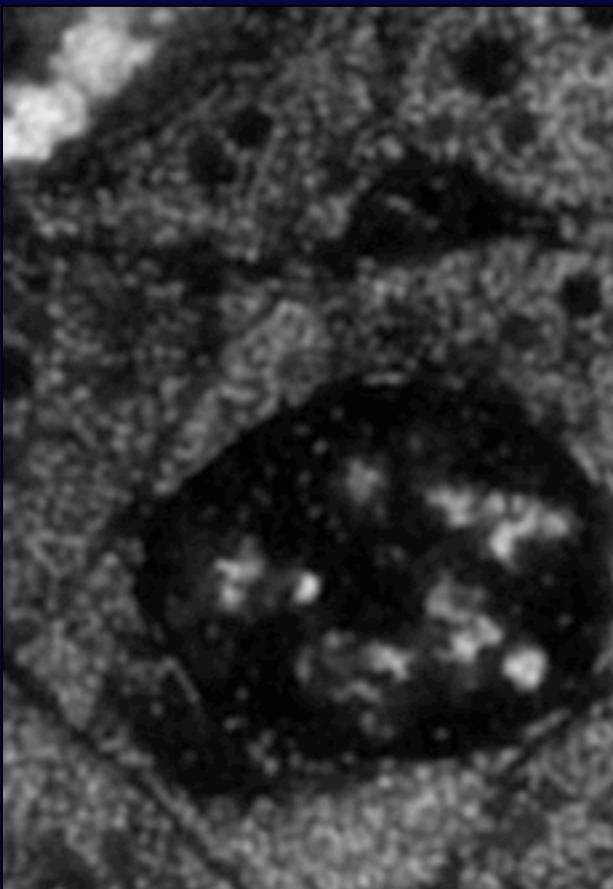
Conventional  
microscopy

Linear  
structured  
illumination

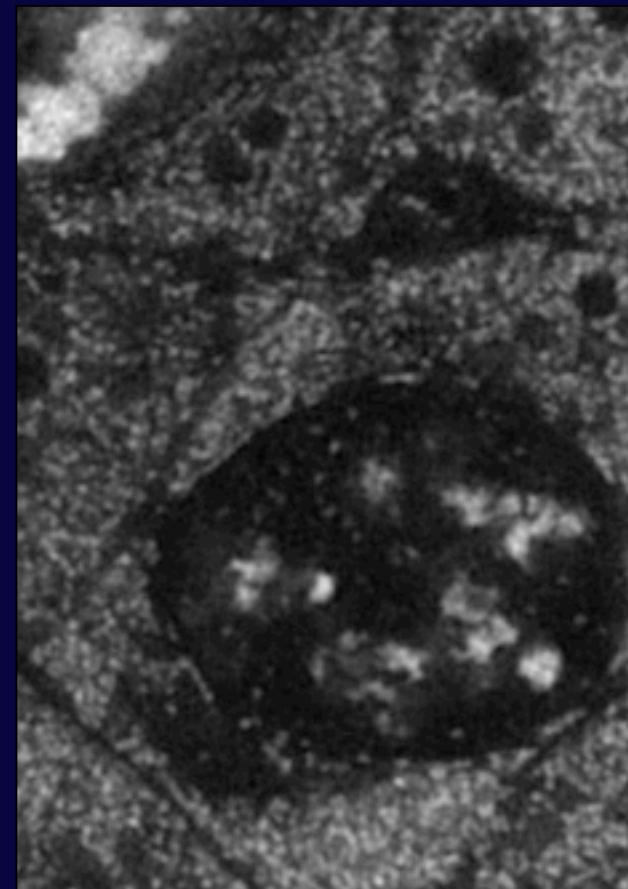
Saturated  
structured  
illumination  
(1 new harmonic)



Min. FWHM  $\approx$  280 nm



Min. FWHM  $\approx$  110 nm



Min. FWHM  $\approx$  80 nm

# Going beyond the diffraction limit: more light collection angles

We know:

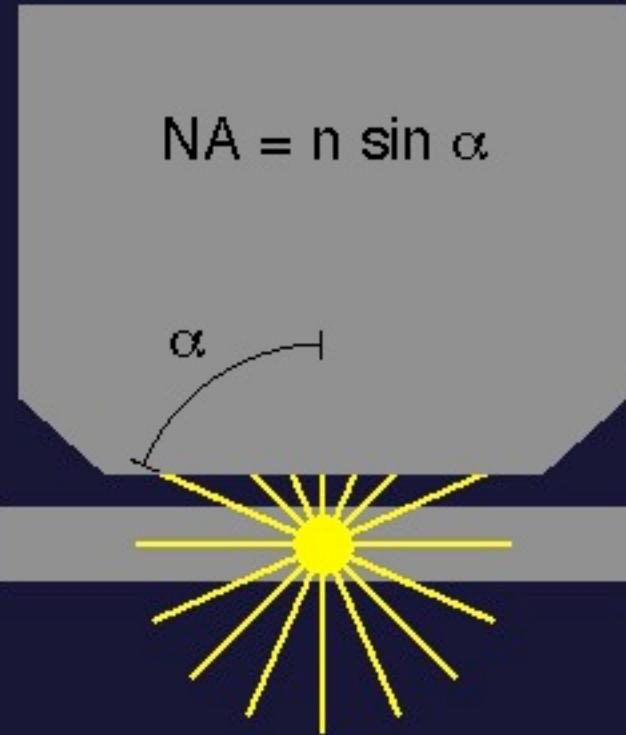
Higher NA



Gathering light over  
larger set of angles

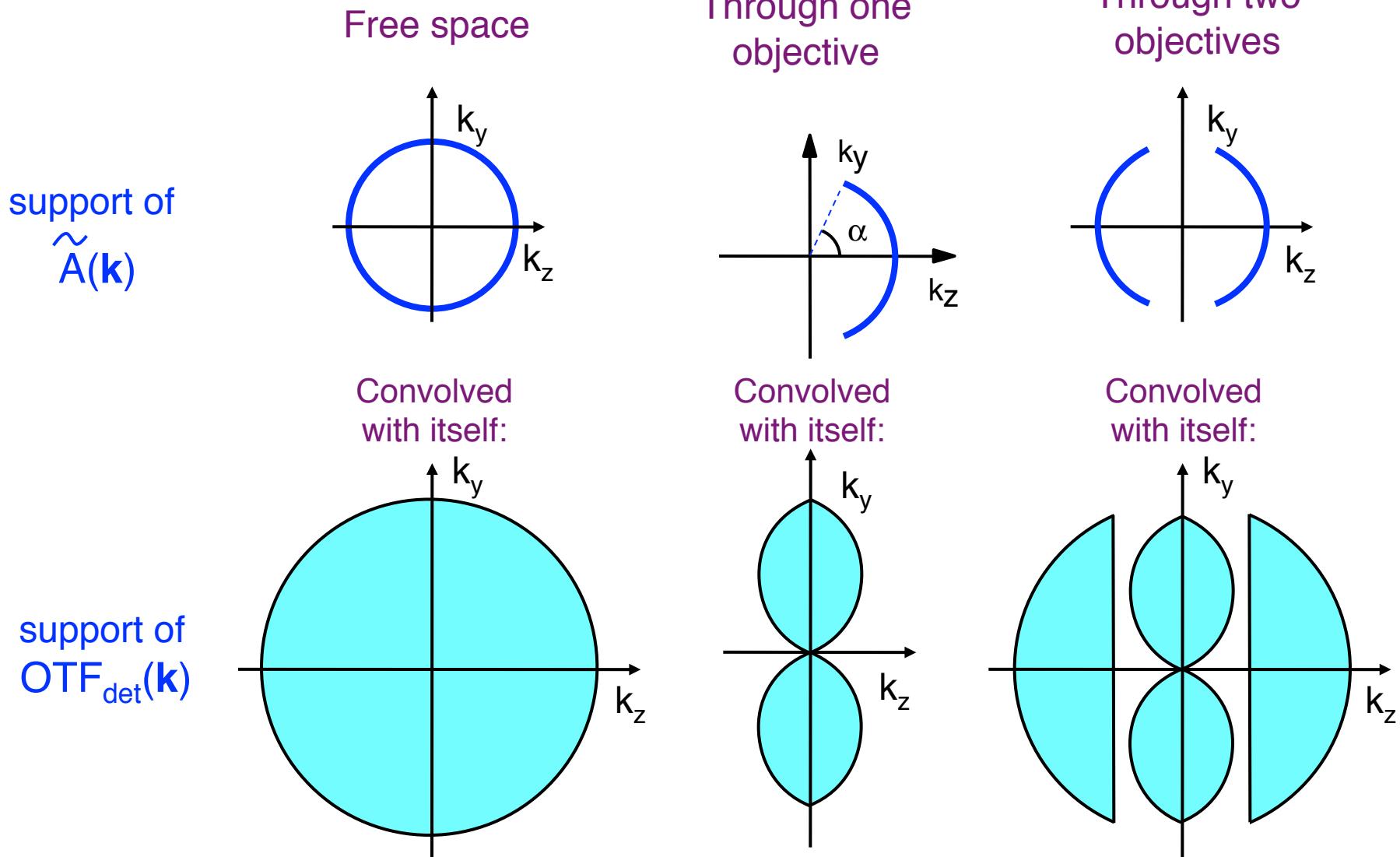
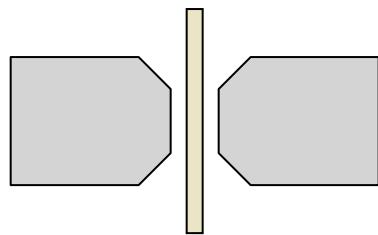


Higher resolution

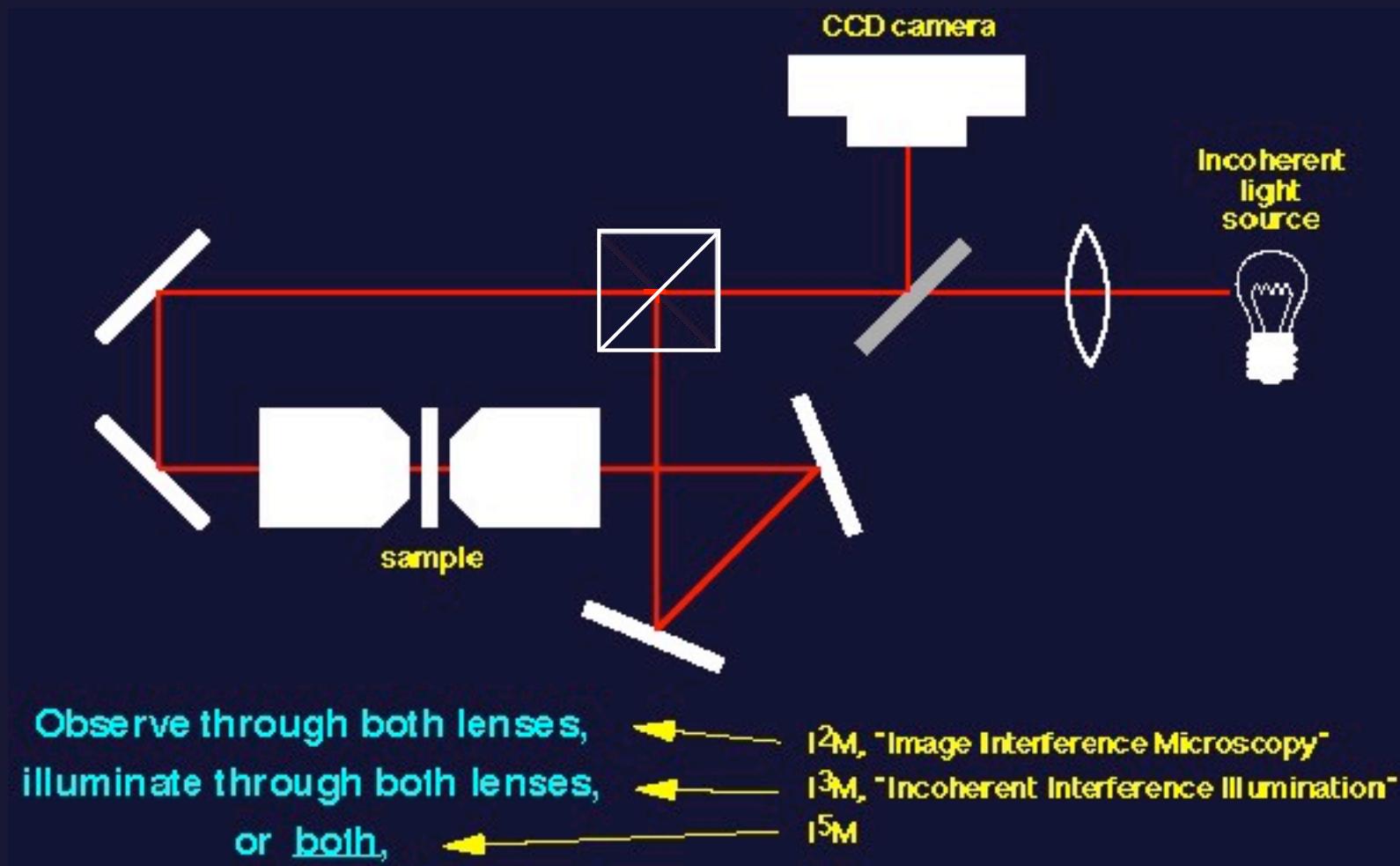


...So what about gathering the light  
emitted toward the back side?

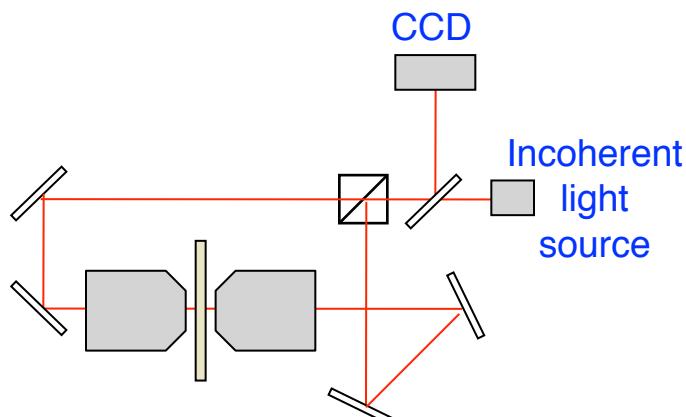
# OTF when detecting through two lenses



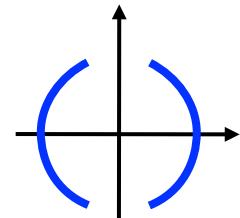
# I5M concept



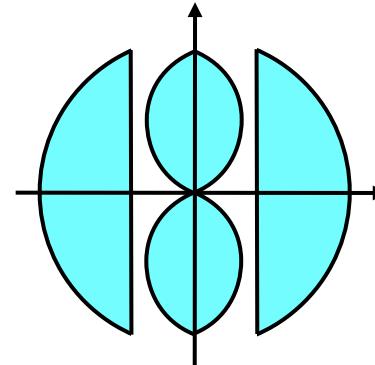
# I<sub>5</sub>M OTF



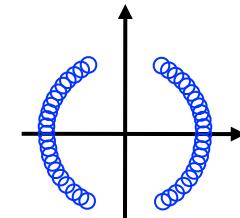
Detection angles



OTF<sub>det</sub>



Illum. angles

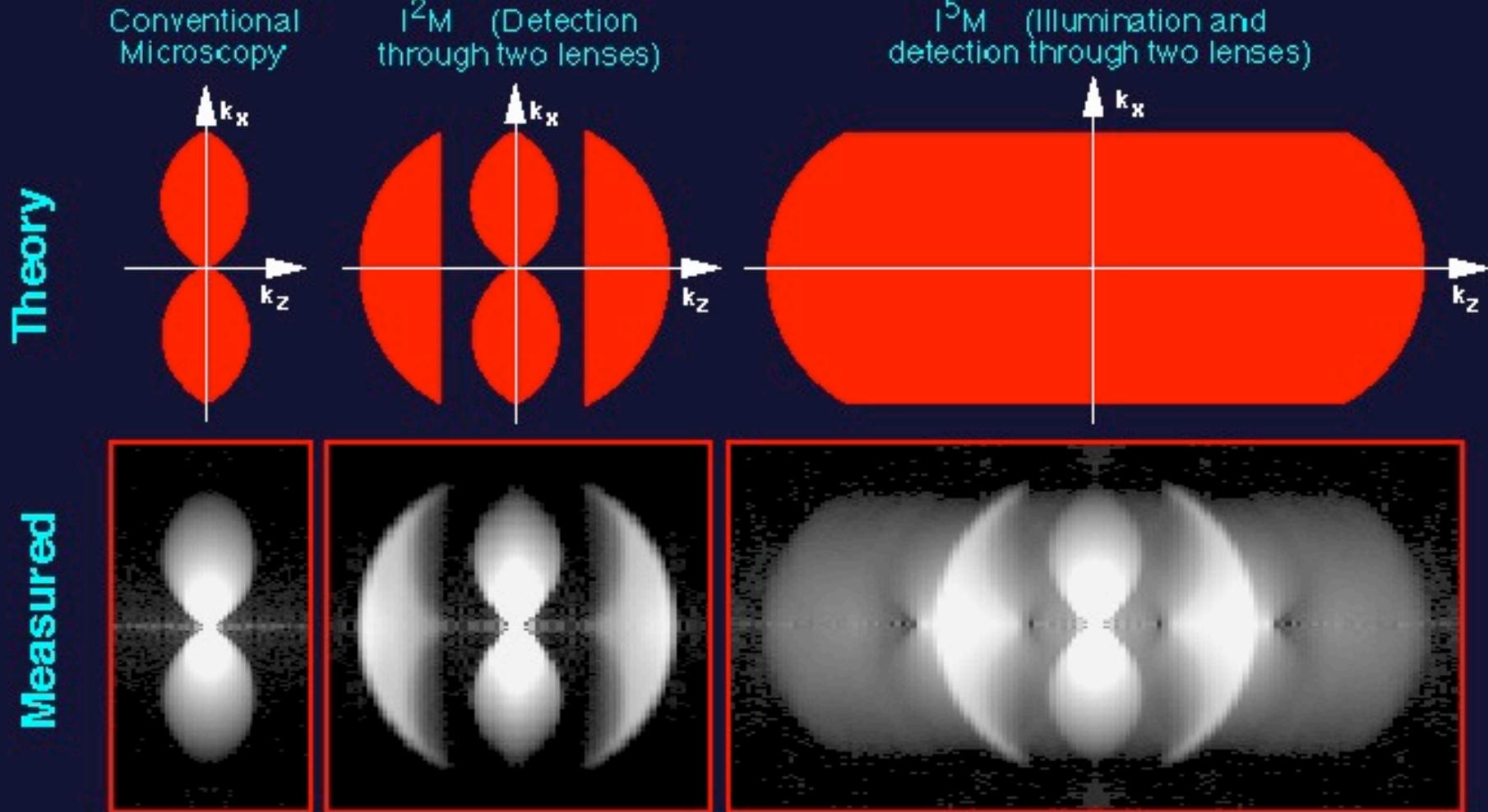


$\tilde{\text{illum}}$

$$\text{OTF}_{\text{eff}} = \text{OTF}_{\text{det}} \otimes \tilde{\text{illum}} = \text{OTF}_{\text{eff}}$$

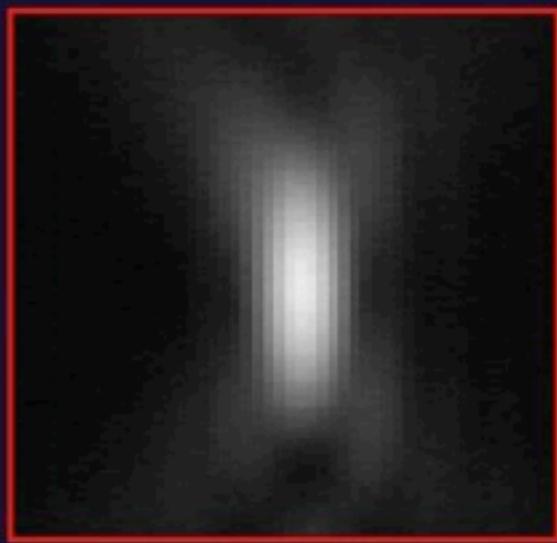
Diagram illustrating the calculation of the effective OTF ( $\text{OTF}_{\text{eff}}$ ) as the product of the detected OTF ( $\text{OTF}_{\text{det}}$ ) and the illumination function ( $\tilde{\text{illum}}$ ). The detected OTF is shown as four overlapping cyan circles, and the illumination function is shown as two overlapping cyan arcs. The result is a uniform cyan oval representing the effective OTF.

## Observable Information [ Optical Transfer Functions ]

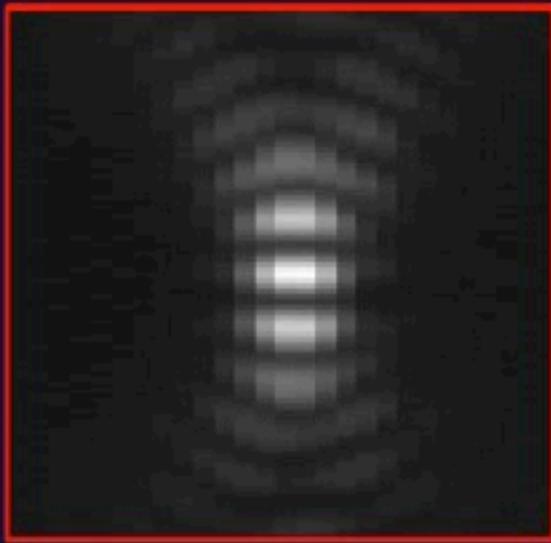


## Deconvolution of bead data

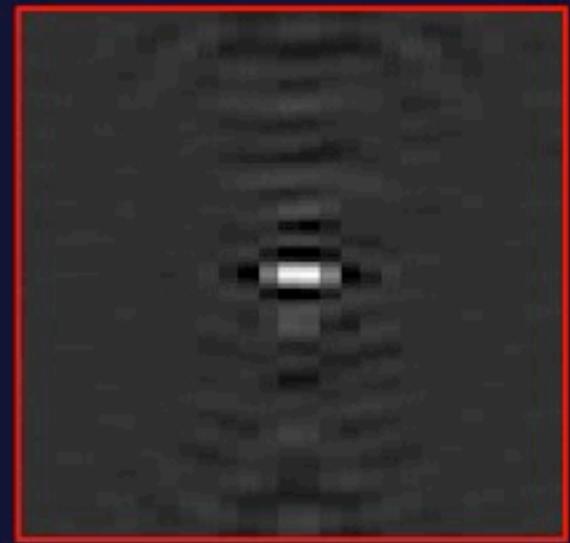
One lens



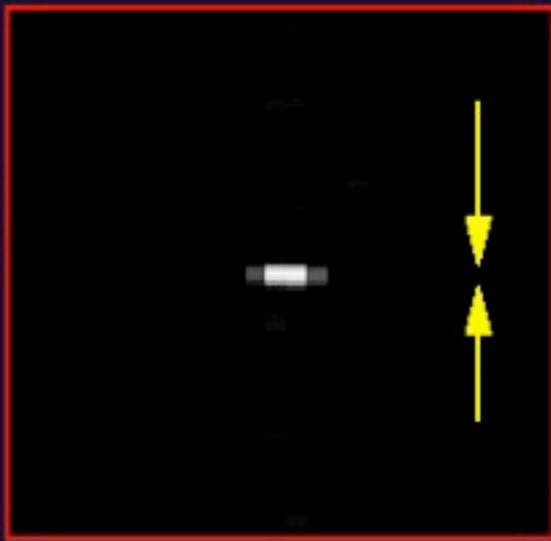
Two lens det. & illum.



After Wiener filtering



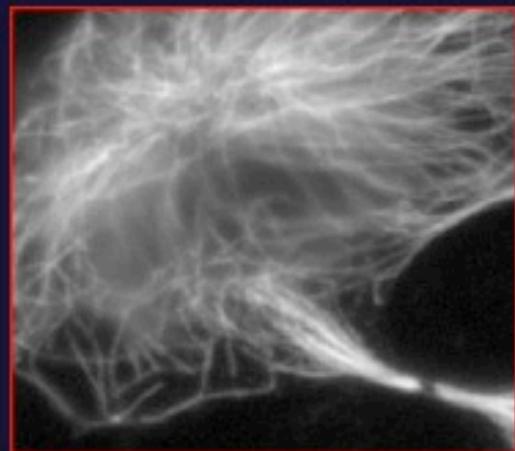
After iterative  
deconvolution  
using  
positivity  
constraint



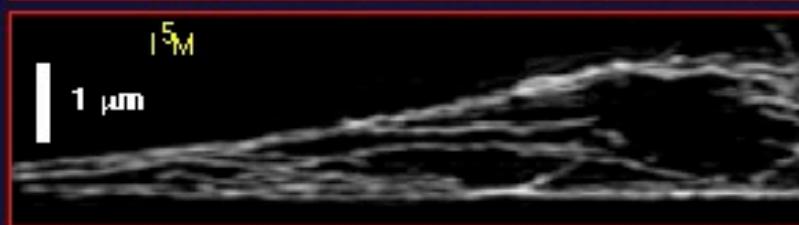
FWHM  
~70nm

# Microtubules in a recently divided PtK2 cell

XY  
view  
of area.

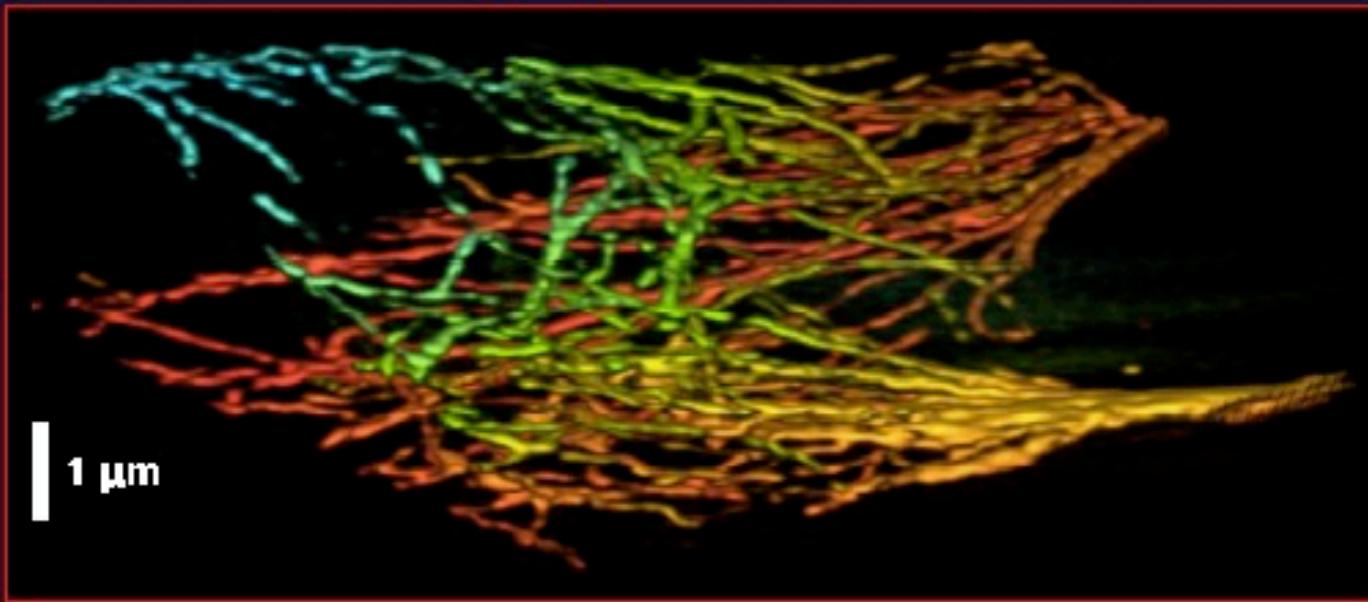


conventional microscopy  
+ deconvolution

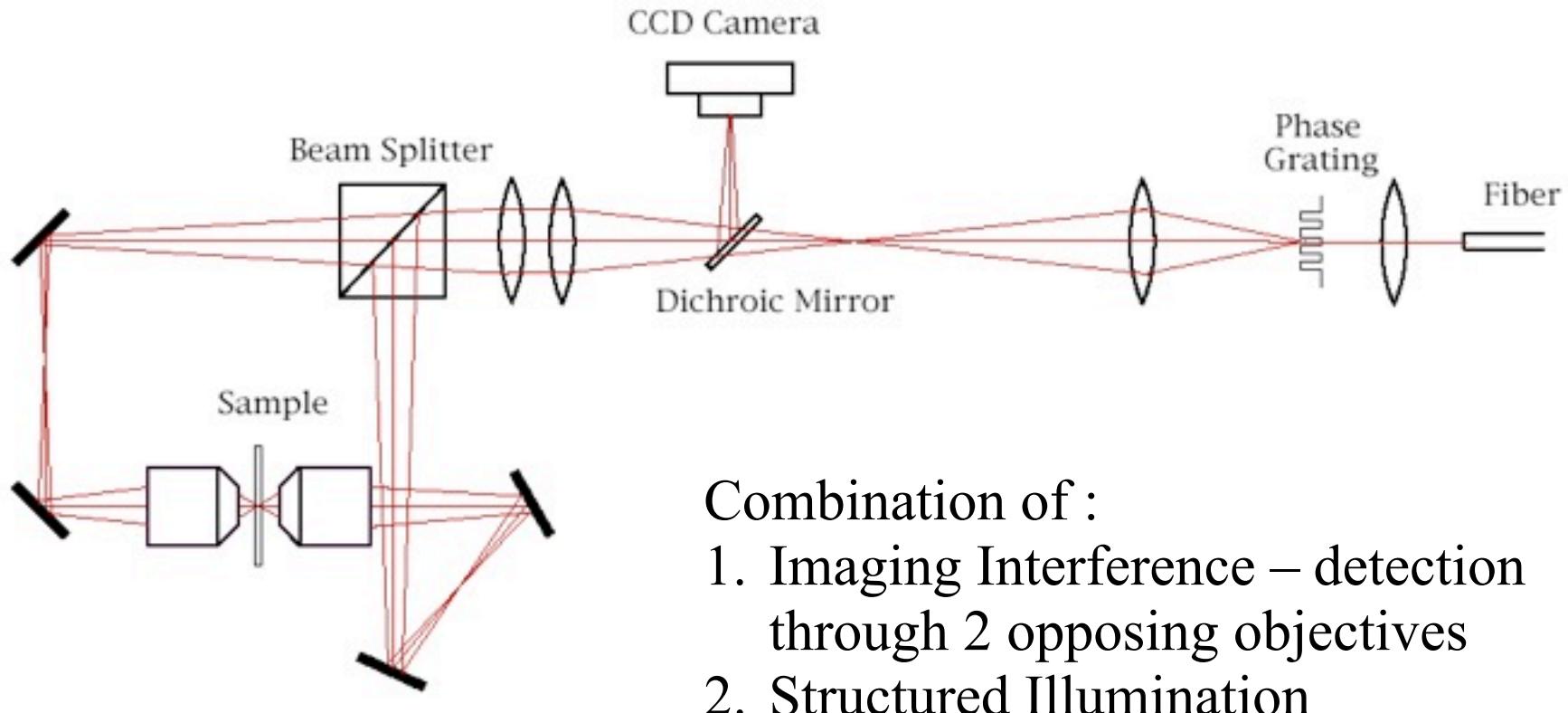


XZ  
projections  
1 μm slices

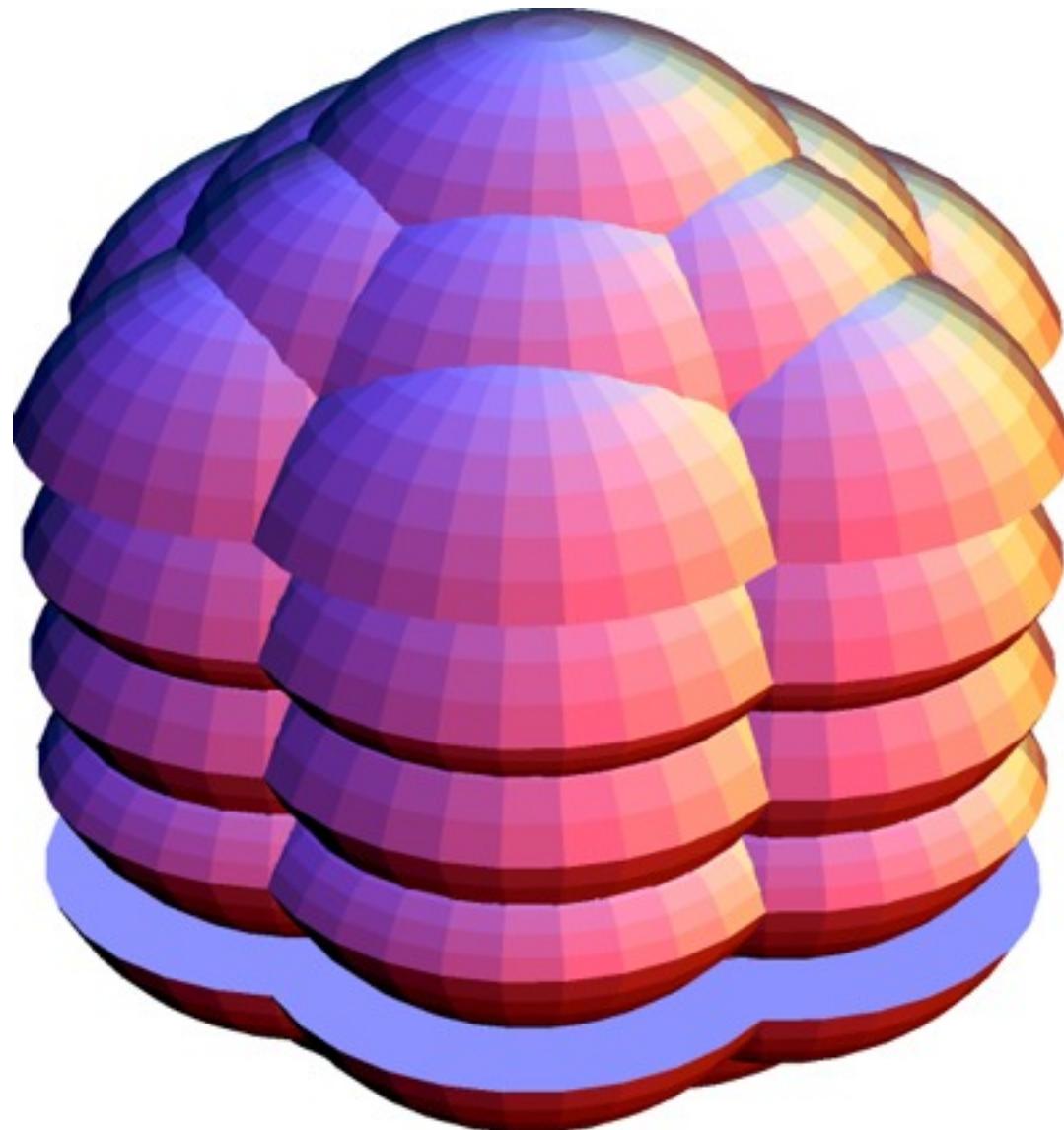
15  
3D view



# *Combine methods: I<sup>5</sup>S use two lenses to collect more angles*

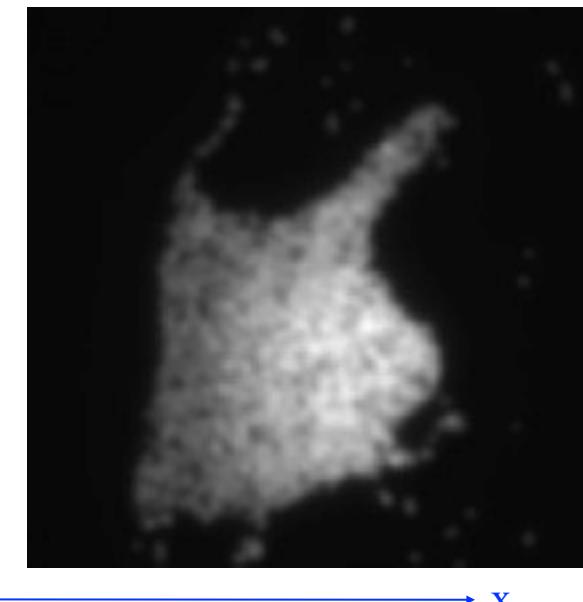


Observable Region through 2 opposing objectives  
Structured Illumination (3 orientations) – I<sup>5</sup>S

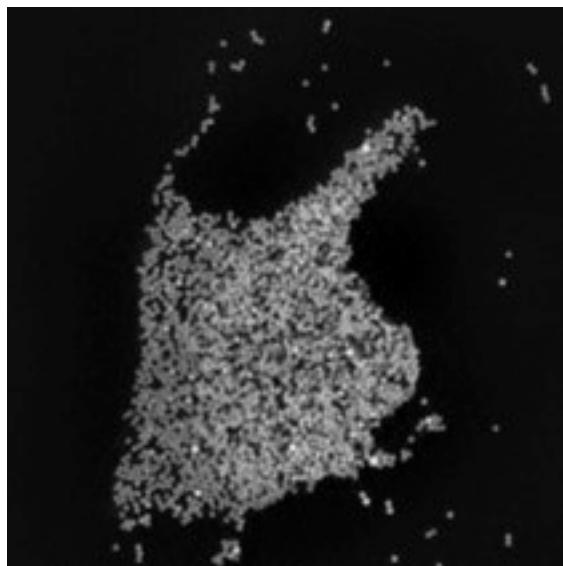


# *Comparison of Resolving Power*

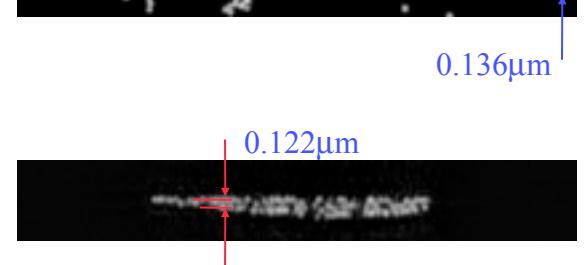
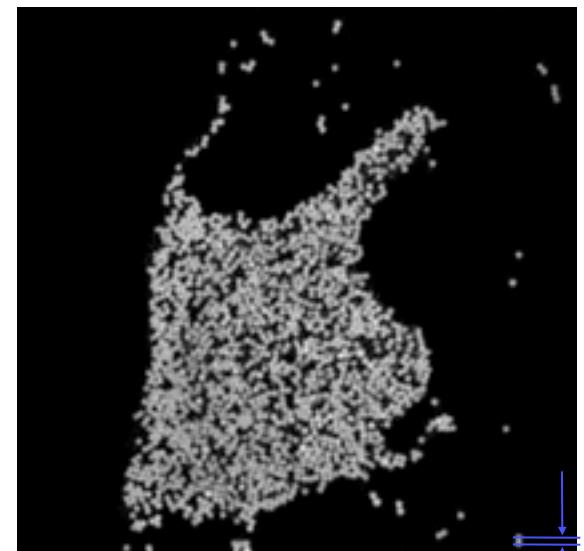
Conventional



Structured Illum.



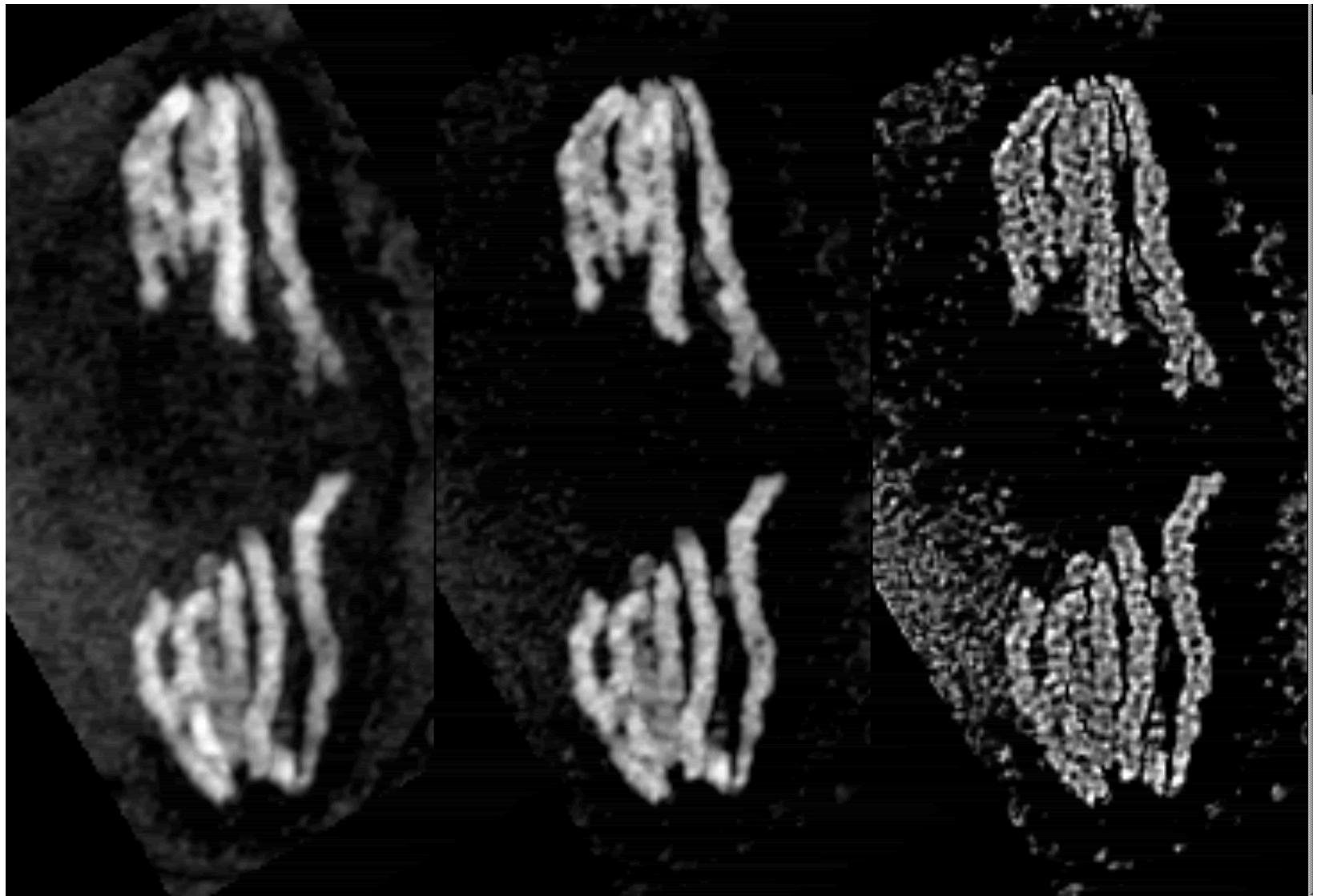
I<sup>5</sup>S



Sample: 0.12μm red-fluorescence microspheres

**Lin Shao, Mats Gustafsson**

## *Drosophila Anaphase Chromosomes (.2μm wide)*



**Lin Proc.**

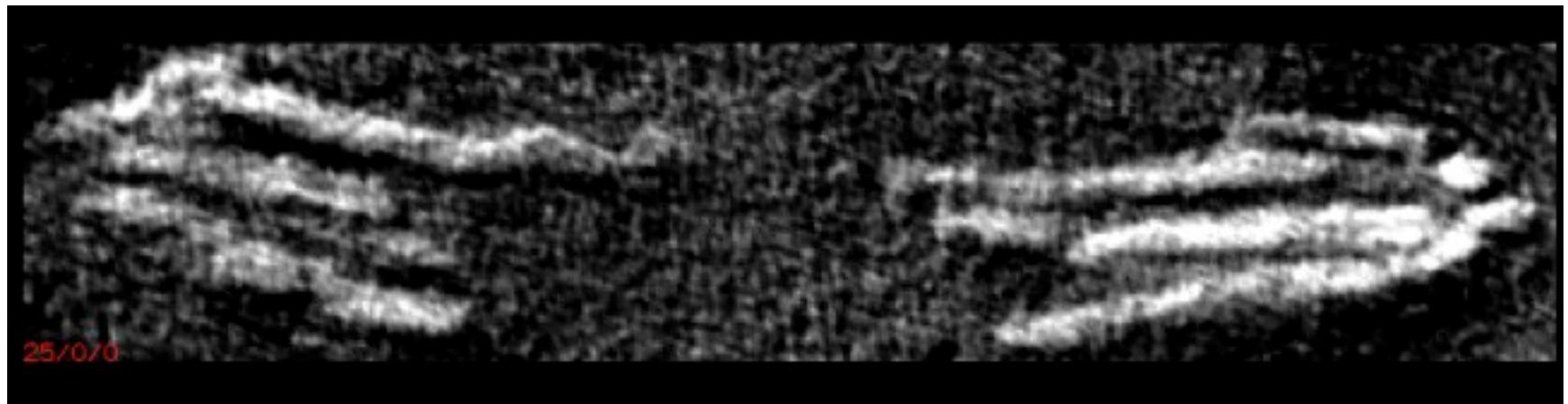
**Decon**

**Decon + LCE**

**Lin Shao**

# *Comparison between OM and EM-Tomo*

EM



OM

