Combined Noise in CMOS Image Sensors

Josh Johnson

April 2025

Abstract

In this short paper, we summarize the results of the in-depth noise analysis of CMOS image sensing given by **CMOS**. This is the type of image sensor in the tracking camera that we are planning to use, so it is useful to have a model of the noise we expect to see. For thermal and shot noise, we will model the probability distribution of noise voltage as Gaussian curves. However we are unable to produce a closed form distribution of $1/f$ noise without testing the specific hardware we are going to use.

Recall that a Gaussian curve with mean μ and variance σ^2 has the form

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

1 Thermal and Shot Noise

At low illumination, the dominant sources of noise in reset and readout transistors are thermal and shot noise. The mean square noise voltage at the end of reset is

$$\sigma_{th}^2 := \overline{V_{n,th}^2(t_r)} \approx \frac{kT}{2C_{pd}},$$

where

- t_r is the reset noise
- k is the Boltzmann constant
- T is temperature
- C_{pd} is the photodiode capacitance.

The thermal noise voltage then has probability distribution

$$f_{th}(v_{th}) = \frac{1}{\sqrt{2\pi\sigma_{th}^2}} \exp\left(-\frac{v_{th}^2}{2\sigma_{th}^2}\right)$$

The mean squared of the shot noise at the end of integration is

$$\sigma_{sh}^2 := \overline{V_{n,sh}^2(t_{int})} = \frac{q(i_{ph}+i_{dc})}{C_{pd}^2(v_{pd}(0))} t_{int} \left(1 - \frac{1}{2(v_{pd}(0)+\phi)} \frac{i_{ph}+i_{dc}}{C_{pd}(v_{pd}(0))} t_{int}\right)^2,$$

where

- t_{int} is the integration time
- q is the charge of an electron (C)
- i_{ph} is the photocurrent
- i_{dc} is the dark current
- C_{pd} is the photodiode capacitance
- v_{pd} is the photodiode voltage
- ϕ is the built-in junction potential.

The shot noise voltage then has probability distribution

$$f_{sh}(v_{sh}) = \frac{1}{\sqrt{2\pi\sigma_{sh}^2}} \exp\left(-\frac{v_{sh}^2}{2\sigma_{sh}^2}\right).$$

During readout, there is a component of thermal noise from transistors M2, M3, M4. The mean square noise voltages are given by

$$\begin{aligned}
\sigma_{M2}^2 &:= \overline{V_{n,M2}^2} = \frac{2}{3}\frac{kT}{C_0}\frac{1}{1+g_{m2}/g_{d3}} \\
\sigma_{M3}^2 &:= \overline{V_{n,M3}^2} = \frac{kT}{C_0}\frac{1}{g_{d3}(1/g_{d3}+1/g_{m2})} \\
\sigma_{M4}^2 &:= \overline{V_{n,M4}^2} = \frac{2}{3}\frac{kT}{C_0} g_{m4}\left(\frac{1}{g_{d3}}+\frac{1}{g_{m2}}\right),
\end{aligned}$$

where

- k is the Boltzmann constant
- T is temperature
- C_0 is the column storage capacitace
- g_{m2} is the transconductance of M2
- g_{d3} is the channel conductance of M3
- g_{m4} is the transcondcutance of M4.

From this, we obtain the probability distributions of the variables v_{M2}, v_{M3}, v_{M4},

$$f_{M2}(v_{M2}) = \frac{1}{\sqrt{2\pi\sigma_{M2}^2}} \exp\left(-\frac{v_{M2}^2}{2\sigma_{M2}^2}\right)$$
$$f_{M3}(v_{M3}) = \frac{1}{\sqrt{2\pi\sigma_{M3}^2}} \exp\left(-\frac{v_{M3}^2}{2\sigma_{M3}^2}\right)$$
$$f_{M4}(v_{M4}) = \frac{1}{\sqrt{2\pi\sigma_{M4}^2}} \exp\left(-\frac{v_{M4}^2}{2\sigma_{M4}^2}\right)$$

We want the distribution which tells us the probability of the sum of these random variables exceeding a certain amount, so we convolve the distributions. Fortunately, combining Gaussians is easy, because the means and variances simply add. Hence, the combined distribution would just be the convolution of the 5 listed above:

$$F(x) = \frac{\exp\left(-\frac{x^2}{2(\sigma_{th}^2+\sigma_{sh}^2+\sigma_{M2}^2+\sigma_{M3}^2+\sigma_{M4}^2)}\right)}{\sqrt{2\pi(\sigma_{th}^2+\sigma_{sh}^2+\sigma_{M2}^2+\sigma_{M3}^2+\sigma_{M4}^2)}}.$$

2 1/f Noise

There is a contribution to $1/f$ noise during readout from the follower transistor M2 and the access transistor M3. If we let

$$C_{M2} = \left(1+\frac{g_{m2}}{g_{d3}}\right)C_0, \quad C_{M3} = \left(1+\frac{g_{d3}}{g_{m2}}\right)C_0,$$

then the mean square $1/f$ noise voltage due to each source is

$$\overline{V_{M2}^2(t)} = \left(\frac{qg_{m2}}{AC_{0x}}\right)^2 \frac{e^{-\frac{2g_{m2}}{C_{M2}}t}}{C_{M2}^2} \int_0^t \int_0^t \int_{\lambda_L}^{\lambda_H} g(\lambda)\mathcal{C}_\lambda(s_1, |s_2-s_1|) e^{\frac{g_{m2}}{C_{M2}}(s_1+s_2)} d\lambda ds_1 ds_2$$

and

$$\overline{V_{M3}^2(t)} = \left(\frac{qg_{m3}}{AC_{0x}}\right)^2 \frac{e^{-\frac{2g_{d3}}{C_{M3}}t}}{C_{M3}^2} \int_0^t \int_0^t \int_{\lambda_L}^{\lambda_H} g(\lambda)\mathcal{C}_\lambda(s_1, |s_2-s_1|) e^{\frac{g_{d3}}{C_{M3}}(s_1+s_2)} d\lambda ds_1 ds_2,$$

where

$$\mathcal{C}_\lambda(t,\tau) = \frac{1}{4}e^{-2\lambda\tau}\left(1-e^{-4\lambda t}\right), \quad g(\lambda) = \frac{4kTAt_{ox}N_t}{\lambda \log(\lambda_H/\lambda_L)},$$

and

- q is the charge of an electron (C)
- A is the channel area

- C_{ox} is the gate capacitance density
- λ is the rate of trapped electrons
- λ_H is the fastest transistion rate or high corner frequency
- λ_L is the slowest transistion rate or low corner frequency.

Moreover, there is $1/f$ noise due to reset with mean square voltage

$$\overline{V_{out}^2(t_r)} = \left(\frac{q}{AC_{ox}}\right)^2 \frac{1}{(t_r+\delta)^2} \int_0^{t_r} \int_0^{t_r} \int_{\lambda_L}^{\lambda_H} \mathcal{C}_\lambda(s_1, |s_2 - s_1|) g(\lambda) d\lambda ds_1 ds_2,$$

where

- t_r is reset time
- δ is thermal time.

Unfortunately, it is difficult to produce a closed form probability distribution of $1/f$ noise, and the most accurate model will require physical testing and numerical methods. As such, the most we say at this time is the variance of the distributions, but we are unable to give the full distribution a neat expression.

References

[1] Hui Tian *Noise Analysis in CMOS Image Sensors*, Stanford University, 2000.