$$(a, \beta) = \arg\min E[(Y - a + bX)^2]$$

step 1
take partials
w/r/t α and
β

$$E\left[\frac{\partial}{\partial a}(Y - a + bX)^2\right]$$
$$= E[2(Y - a + bX)\cdot(-1)]$$
$$= E[-2(Y - a + bX)]$$

for a

$$E\left[\frac{\partial}{\partial b}(Y - a + bX)^2\right]$$
$$= E[2(Y - a + bX)\cdot(-X)]$$
$$= E[-2X(Y - a + bX)]$$

for b

---

step 2
set partials
to $\emptyset$ and
solve

$$E[-2(Y - a + bX)] = 0$$
$$\Rightarrow E[Y - a + bX] = 0$$
$$\Rightarrow a = E[Y] - bE[X]$$

for a

$$E[-2X(Y-a+bX)] = 0$$

$$\Rightarrow E[X(Y-(a+bX))] = 0$$

group + expand
linearity of $E[\cdot]$

$$\Leftrightarrow E[XY] - E[X(a+bX)] = 0$$

linearity of $E[\cdot]$
note sign

$$E[XY] - aE[X] + bE[X^2] = 0$$

substitute a

$$E[XY] - (E[Y] - bE[X])E[X] + bE[X^2] = 0$$

rearrange

all algebra

$$E[XY] = (E[Y] - bE[X])E[X] + bE[X^2]$$

$$E[XY] = E[Y]E[X] - bE[X]^2 + bE[X^2]$$

rearrange

$$E[XY] - E[Y]E[X] = b\left(E[X^2] - E[X]^2\right)$$

for B

$$\frac{E[XY] - E[Y]E[X]}{E[X^2] - E[X]^2} = b$$

defs. of cov and var

$$\frac{Cov(X,Y)}{Var(X)} = b$$