

linear

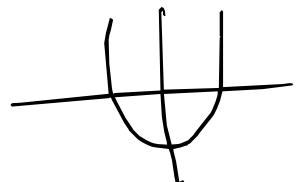
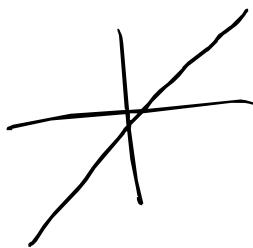
if:

additive:

$$F(x+y) = F(x) + F(y)$$

$$F(ax) = a F(x)$$

if $a \in \mathbb{R}$



higher order polynomials

$$x^2, x^3,$$

w/ degree ≥ 1

lin eq

$$b = a_1 x_1 + a_2 x_2 + \dots + a_n x_n$$

a_1, \dots, a_n parameters

x_1, \dots, x_n variables

$$\begin{matrix} b_1 \\ b_2 \end{matrix} = \left\{ \begin{array}{l} \text{linear system} \end{array} \right.$$

tanks purchased
allies by Ukraine

from NATO

$\downarrow i \dots n$

y_i

$$y_i = 20$$

$x_i = \# \text{ of visits by country } i \text{ to UK}$

$$y_i = \boxed{20} + \boxed{2} x_i$$

$$\hookrightarrow y = \underline{m} x + \underline{b} \rightarrow y = \beta_0 + \beta_1 x$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

SPS: france has 1 visit, 3

germany has 2, 4

$$\left\{ \begin{array}{l} 3 = \beta_0 + \beta_1 \cdot \boxed{1} \\ 4 = \beta_0 + \beta_1 \cdot 2 \end{array} \right.$$

$$4 - 3 = \cancel{\beta_0} + \beta_1 + 2\beta_1 - \cancel{\beta_1}$$

$$1 = \beta_1$$

$$3 = \beta_0 + 1 \cdot 1$$

$$\beta_0 = 2$$

Stored as:

final grades	HW 1	HW 2	HW 3	Final
95	5	10	100	45
30	0	10	13	40
100	100	0	100	100

Diagram illustrating the storage of the matrix:

- The matrix is shown as a 3x5 grid of values.
- On the left, vertical brackets group the rows: [95], [30], and [100].
- On the right, horizontal brackets group the columns: [HW 1], [HW 2], [HW 3], and [Final].
- A large oval encloses the entire 3x5 grid of values.
- Arrows point from the row labels [95], [30], and [100] to the first three rows of the grid.
- Arrows point from the column labels [HW 1], [HW 2], [HW 3], and [Final] to the first four columns of the grid.

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

$$4 = \beta_0 + 2\beta_1$$

$$3 = \beta_0 + \beta_1$$

matrices

$m \times n$ matrices have m rows
 $\downarrow \quad \downarrow$
 $\mathbb{R}^m \times \mathbb{C}^n$ n columns

Roman Catholic
Rum & coke

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{bmatrix} & 1 & 2 & 3 & \dots & & \\ & a_{11} & & & - & - & a_{1n} \\ & \vdots & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & a_{m1} & & & - & - & a_{mn} \end{bmatrix}$$

$M \leftarrow c(1, 2, 3), c(1, 4, 5)$, $\text{row_wise} = T$

$$\begin{array}{l} \text{1) } \left[\begin{array}{ccccc} 1 & -1 & 0 & \dots \\ 2 & -1 & 0 & \dots \\ 3 & -1 & 0 & \dots \\ 4 & 5 & 0 & \dots \\ 5 & 0 & 0 & \dots \end{array} \right] \\ \text{2) } \left[\begin{array}{ccccc} 1 & -1 & 0 & \dots \\ 2 & -1 & 0 & \dots \\ 3 & -1 & 0 & \dots \\ R_1 \rightarrow R_1 + R_2 \\ 4 & 0 & 0 & \dots \\ 5 & 0 & 0 & \dots \end{array} \right] \end{array}$$

$$R^T = R' = [1 \dots -n]$$

$$R = \begin{bmatrix} 1 \\ \vdots \\ n \end{bmatrix}$$

$$\begin{bmatrix} R \\ \vdots \\ R_n \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \end{bmatrix}$$
$$\sum \not\in [x] = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\beta_0 + 2\beta_1 \\ 1\beta_0 + 1\beta_1 \end{bmatrix}$$

$A \cdot \beta$

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$