# Social Science Inquiry II Week 9: Beyond linear regression

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# Loading packages for this class

- > set.seed(60637)
- > library(ggplot2)

► Housekeeping

What is it?

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- ► Somehow part of *artificial intelligence*...(basically, how computers perform tasks)
- ► In general, a flexible, *data-driven* approach to make predictions, classify data, or take decisions.

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- ► Is there overlap between the two?

▶ In linear regression, propose a model:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \dots + \hat{\beta}_K X_{Ki}$$

► Select  $\hat{\beta}_0 \dots \hat{\beta}_K$  to minimize

$$\sum_{i=1}^{N} \hat{\varepsilon_i}^2 = \sum_{i=1}^{N} \left( \hat{Y}_i - Y_i \right)^2$$

For prediction tasks, we could use the same model,

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▶ But select  $\hat{\beta}_0 \dots \hat{\beta}_K$  to minimize squared prediction error for the next observation:

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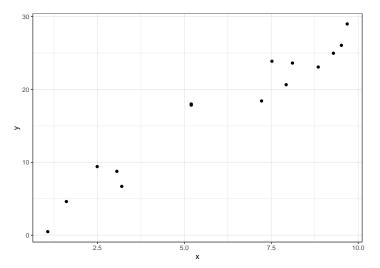
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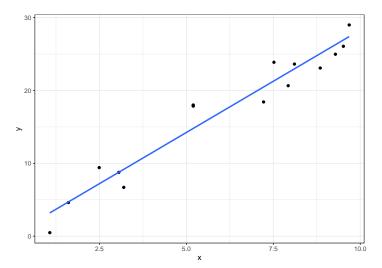
#### Some ML tools

- ► A major concern of ML: *overfit* 
  - ▶ If your model fits the data *too* perfectly, it's not useful for prediction

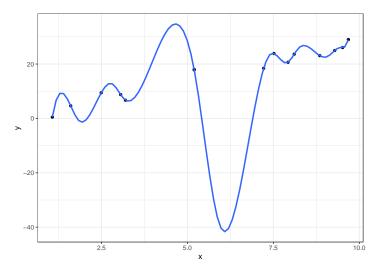
## Suppose we would like to fit a model to the following data:



## We could use a single line:



## Or we could fit a curve that goes between every point:



▶ If we were to draw another observation from the joint distribution of (Y, X), which one do you think would do a better job of prediction?

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- ➤ You can fit different models on the training set, and then see which one does the best job of predicting response in the test set. (This is not a new idea.)
- ► There are some different ways to do this:
  - ► Leave-k-out
  - ► Leave-one-out
  - ▶ k-fold cross validation

▶ Given some data  $(Y_1, X_1), ..., (Y_N, X_N)$ , we fit a model,  $\hat{f}(X)$ .

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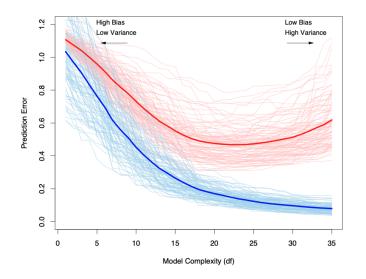
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▶ We may be interested in  $Err_{\mathcal{T}}$ , in practice most estimating methods will give us estimates of Err.



Hastie et al. (2009)

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- And, once we have selected a version of a model, we may want to assess how a selected model performs.

▶ We can't measure expected test error directly.

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Training	Validation	Test
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- ► Select model with minimum error in validation set.
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▶ We can potentially get more out of our data by cross-validating.

Version 1	Training	Validation
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$$\widehat{\mathrm{Err}}_{CV} = \sum_{i=1}^{N} L\left(y_i, \hat{f}^{-k(i)}(x_i)\right)$$

 $\hat{f}^{-k(i)}$  are the fits from the folds k that do not contain i.

### K-fold cross validation.

Version 1	Training	Training	Training	Training	Validation
Version 2	Training	Training	Training	Validation	Training
Version 3	Training	Training	Validation	Training	Training
Version 4	Training	Validation	Training	Training	Training
Version 5	Validation	Training	Training	Training	Training

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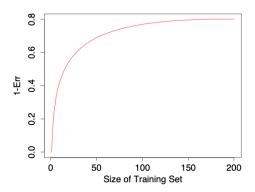
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▶ Rule of thumb is often 5 or 10.

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- ▶ This is a common problem when we think about an industry setting, where for every customer a business might have a large number of measurements. Which ones should they use to predict an outcome?

► Consider a model:

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- ▶ With regularization, we shrink some of the  $\hat{\beta}_k$  nearly all the way or all the way to zero.
- ▶ For ridge regression or lasso, we select the  $\hat{\beta}_k$  using:

$$\underset{\beta}{\operatorname{argmin}} \left\{ \sum_{i=1}^{N} \left( Y_i - \beta_0 + \sum_{k=1}^{K} X_{ki} \beta_k \right)^2 + \lambda \sum_{k=1}^{K} |\beta_k|^q \right\}$$

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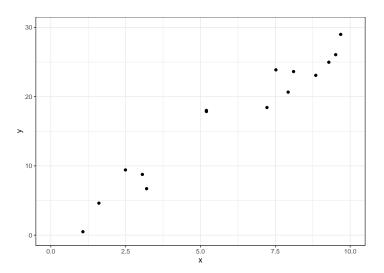
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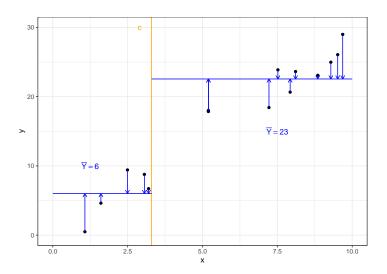
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- ightharpoonup . . . and for each sub-sample, predict  $\hat{Y}$  as the mean of the  $Y_i$  within each sample.
- ► We want to pick *c* to minimize:

$$Q = \sum_{i: X_i \leq c} (Y_i - \bar{Y}_{\mathsf{lower}})^2 + \sum_{i: X_i > c} (Y_i - \bar{Y}_{\mathsf{upper}})^2$$





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- We will do the same approach to finding thresholds to minimize prediction error, but we'll want to pick which  $X_k$  we use for thresholding, as well.
- ▶ Generally, we'll define the depth of the tree as 2 or three variables; first we'll split on  $X_k$ , then we'll split on  $X_i$ ...

## Honesty

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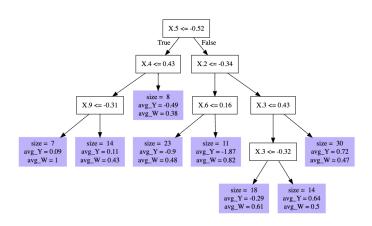
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- ▶ This can result in some leaves being empty. Prune them?
- ► This procedure reduces bias relative to those proposed by Breiman (2001).

```
> library(grf)
> set.seed(60637)
> n <- 500
> p <- 10
> X <- matrix(rnorm(n * p), n, p)
> W <- rbinom(n, 1, 0.5)
> Y <- pmax(X[, 1], 0) * W + X[, 2] +
+ pmin(X[, 3], 0) + rnorm(n)
> c.forest <- causal_forest(X, Y, W)</pre>
```

#### An honest tree



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- ► The question: how to best recommend to them movies that they have not yet rated?
- ► The challenge: come up with the best recommendation algorithm, winner gets \$1 million.
- ► This can be framed as a matrix completion problem: put users on rows, movies on columns, predict all of the missing rankings.

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  - ► Fit prediction models separately to treatment and control, so we can do a better job of estimating treatment effects at different covariate values.
  - Learn which covariates to include in a (causal) regression model.
  - ► For observational data, predict propensity to be in treatment vs. control group, based on covariates.

### Causal inference: no free lunch

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- ► Machine learning does not solve the fundamental problem of causal inference.
- ► Causal interpretations are based on assumptions about the data generating process, or knowledge of assignment procedures. These are outside the realm of machine learning methods.

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- ► ML methods may do a good job of producing estimates, but how do we account for inference?
- Cross-validation
- ► Bootstrapping
- ► Applying these solutions to prediction under multiple linear regression

### References I

Athey, S. and Imbens, G. W. (2019). Machine learning methods that economists should know about. <u>Annual Review of Economics</u>, 11:685–725.

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