

# Data Generating Process

## Objective for DGPs:

- Create multiple clusters, where distributions of covariates are different across clusters
- Each cluster has a different arm that produces highest reward
- Generate "lumpy" reward functions that cannot be straightforwardly recovered by a linear model
- Allow levers to move:
  - ▶ number of covariates used to define clusters
  - ▶ relative size of clusters
  - ▶ *Heterogeneity ratio* (value of best contextual/best fixed policy)

# Data Generating Process

## Requirements for DGPs:

- The difference between the best contextual policy & the control is fixed across DGPs
  - Differences in power curves between DGPs are based on ability of agent to learn the DGP, not differences in effect sizes
- The best fixed arm is always the same arm across DGPs

# Data Generating Process

## Generate Baseline Dataset ( $N = 10000, p = 15$ )

- Parameter
  - ▶ Number of useful covariates  $p' \in \{3, 5, 10\}$
  - ▶ Largest cluster size ratio  $c \in \{0.4, 0.6, 0.8\}$
  - ▶ Heterogeneity ratio  $h \in \{1.05, 1.5, 1.95\}$
- Generate large covariate matrix  $X$  using correlated multivariate normal distribution, covariance matrix generated from  $\frac{\text{Beta}(2,2)-0.5}{2}$
- Use iterative KNN to group covariates observed into  $k = 3$  clusters with cluster size  $[N * c, \frac{N*(1-c)}{2}, \frac{N*(1-c)}{2}]$ , where  $c =$  largest cluster size ratio

# Data Generating Process

## Reward Generation

- Generate reward for best arm for each cluster  $c_i, i = 1, \dots, 3$ ,

$$R_{w_{best,i},c_i} = 0.6 - X_{1,c_i}$$

- Reward for 2nd best arm for each cluster  $c_i, i = 1, \dots, 3$ ,

$$R_{w_{best_2,i},c_i} = R_{w_{best,i},c_i} + \epsilon$$

where  $\epsilon \sim \mathcal{N}(\mu, 0.01), \mu \leq 0$ .

- Reward for the rest of the arm

$$R_{w,c_i} = -X_{1,c_i} \text{ for } w \notin \{w_{best,i}, w_{best_2,i}\}, i = 1, \dots, 3,$$

- We vary  $\mu$  to search for the level of desired heterogeneity ratio  $h$ .

## Note

- 0.6 was chosen to simulate an average treatment effect between best-fixed policy and control policy; assume control policy is not the best arm for any cluster. (See PAP for justification of magnitude.)
- In the simulations, arm 0 is chosen as the best arm for the largest cluster. It is also chosen as the 2nd best arm for the other two clusters to ensure it is the best-fixed policy.