CSE 2500 - Introduction to Discrete Systems Fall 2021

Midterm Exam 2

For full credit, please adhere to the following:

- There are 3 problems with total 100 points.
- Unsupported answers receive no credit.
- All answers can be typed or handwritten, and should be readable.
- Submit the assignment in one file (.pdf, .doc, etc.) via HuskyCT.
- You have 15 minutes to upload your solutions after the exam.

1. (a) (15 points) Assume that m and n are particular integers.
   - Is $(6m + 8n)$ even? Why?
   - Is $10mn + 7$ odd? Why?
   - If $m > n > 0$, is $m^2 - n^2$ composite? Why?

   (b) (5 points) Is 605.83 a rational number? Justify your answer

   (c) (10 points) Prove the following statement directly from the definitions of the terms. Do not use any other facts previously proved in class or in the text or in the exercises.

   For all integers a, b, and c, if $a|b$ and $a|c$, then $a|(5b + 3c)$.

2. (a) (20 points) Prove directly from the definitions that for every integer n, $n^2 - n + 3$ is odd. Use division into two cases: n is even and n is odd.

   (b) (10 points) Consider the following statement: For all integers n, if n^3 is even then n is even. Prove the statement by **contradiction**.

   (c) (10 points) Consider the following statement: For all integers n, if n^3 is even then n is even. Prove the statement by **contraposition**.

3. (30 points) (a) 10 points; (b) 10 points; (c) 10 points

For each integer $n \geq 3$, let $P(n)$ be the equation

$$\boxed{3 + 4 + 5 + \cdots + n = \frac{(n-2)(n+3)}{2}.} \leftarrow P(n)$$

(Recall that by definition $3 + 4 + 5 + \cdots + n = \sum_{i=3}^{n} i$.)

(a) Is $P(3)$ true? Justify your answer.

(b) In the inductive step of a proof that $P(n)$ is true for all integers $n \geq 3$, we suppose $P(k)$ is true (this is the inductive hypothesis), and then we show that $P(k+1)$ is true. Fill in the blanks below to write what we suppose and what we must show for this particular equation.

Proof that for all integers $k \geq 3$, if $P(k)$ is true then $P(k+1)$ is true:

Let k be any integer that is greater than or equal to 3, and suppose that ______.
We must show that ______.

(c) Finish the proof started in (b) above.