CSE 2500 - Homework 1

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1. (10 points) Exercise Set 1.1, Question 4.
   Fill in the blanks using a variable or variables to rewrite the given statement.
   Given any real number, there is a real number that is greater.

   (a) Given any real number r, there is a real number s such that $s > r$.

   (b) For any real number r, there exists a real number s such that $s > r$.

2. (10 points) Exercise Set 1.1, Question 11.
   Fill in the blanks to rewrite the given statement.
   Every positive number has a positive square root.

   (a) All positive numbers are greater than zero.

   (b) For every positive number e, there exists a $\sqrt{e} > 0$ for e.

   (c) For every positive number e, there exists a positive number r such that $r = \sqrt{e} > 0$.

3. (10 points) Exercise Set 1.2, Question 4.

   (a) $2 \in \{2\}$

   (b) $\{2,2,2,2\}$ has 1 element.

   (c) $\{2,\{2\}\}$ has 2 elements.

   (d) $\{0\} \in \{\{0\},\{1\}\}$

   (e) $0 \notin \{\{0\},\{1\}\}$

4. (10 points) Exercise Set 1.2, Question 7.
   Use the set-roster notation to indicate the elements in each of the following sets.

   (a) $S = \{1, -1\}$

   (b) $T = \{0, 2\}$

   (c) $U = \emptyset$

   (d) $V = Z$

   (e) $W = \emptyset$

(f) $X = Z$

5. (10 points) Exercise Set 1.2, Question 8.
Let $A = \{c, d, f, g\}$, $B = \{f, j\}$, $C = \{d, g\}$. Answer each of the following questions. Give reasons for your answers.

   (a) $B \not\subseteq A$ - since $j \notin A$, not all elements of B are elements of A.

   (b) $C \subseteq A$ - since all elements of C are elements of A.

   (c) $C \subseteq C$ - since comparing C's elements to itself will always yield C being a subset of itself.

   (d) $C \subset A$ - since, by definition, every element of C is also an element of A, however there are at least one element of A that is not an element of C.

6. (10 points) Exercise Set 1.2, Question 12.
Let $S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set.

   (a) $S \times T = \{(2,1), (2,3), (2,5), (4,1), (4,3), (4,5), (6,1), (6,3), (6,5)\}$
   $S \times T$ has 9 elements.

   (b) $T \times S = \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$
   $T \times S$ has 9 elements.

   (c) $S \times S = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$
   $S \times S$ has 9 elements.

   (d) $T \times T = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)\}$
   $T \times T$ has 9 elements.

7. (10 points) Exercise Set 1.2, Question 14.
Let $R = \{a\}$, $S = \{x, y\}$, and $T = \{p, q, r\}$.
Find each of the following sets.

   (a) $R \times (S \times T) = \{(a, (x, p)), (a, (x, q)), (a, (x, r)), (a, (y, p)), (a, (y, q)), (a, (y, r))\}$

   (b) $(R \times S) \times T = \{((a, x), p), ((a, x), q), ((a, x), r), ((a, y), p), ((a, y), q), ((a, y), r)\}$

   (c) $R \times S \times T = \{(a, x, p), (a, x, q), (a, x, r), (a, y, p), (a, y, q), (a, y, r)\}$

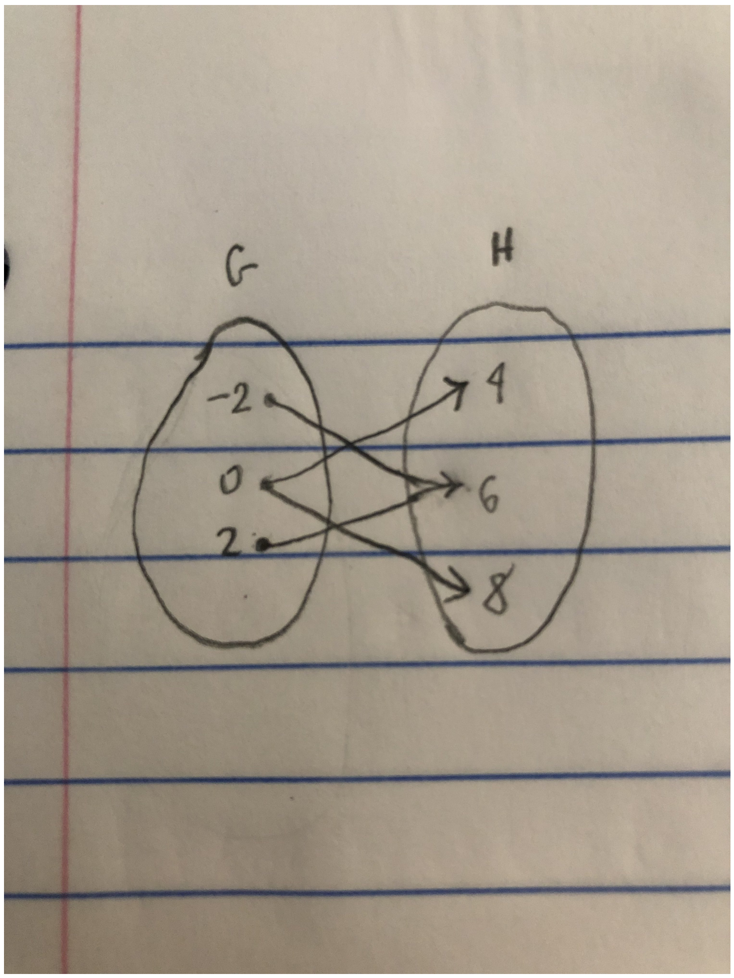
8. (10 points) Exercise Set 1.3, Question 4.
Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows: For all $(x, y) \in G \times H, (x, y) \in V$, means that $\frac{x-y}{4}$ is an integer.

   (a) $2V6$ since $\frac{2-6}{4} = -1$, which is an integer and $(2, 6) \in V$.
   $(-2)V(-6)$ is not true: $-6 \notin H$, therefore $(-2, -6) \notin V$.
   $(0, 6) \notin V$ because $\frac{0-6}{4} = \frac{-2}{3} \notin Z$, therefore $(0,\ 6) \notin V$.
   $(2, 4) \notin V$ because $\frac{2-4}{2} = \frac{-1}{2} \notin Z$, therefore $(2,\ 4) \notin V$.

(b) $V = \{(0,4), (2,6), (-2,6), (0,8)\}$, since elements of V must be in the set $G \times H$ and $\frac{x-y}{4} \in Z$.

(c) Since $G \times H = \{(-2,4), (-2,6), (-2,8), (0,4), (0,6), (0,8), (2,4), (2,6), (2,8)\}$, the domain of V $= \{-2, 0, 2\}$, and the co-domain of V $= \{4, 6, 8\}$.

(d) Arrow Diagram of V



9. (10 points) Exercise Set 1.3, Question 8.
Let $A = \{2, 4\}$ and $B = \{1, 3, 5\}$ and define relations U, V, and W from A to B as follows:
For every $(x, y) \in A \times B$:

- $(x, y) \in U \Rightarrow y - x > 2$
- $(x, y) \in V \Rightarrow y - 1 = \frac{x}{2}$
- $W = \{(2,5), (4,1), (2,3)\}$

(a) Arrow diagrams for U, V , and W.

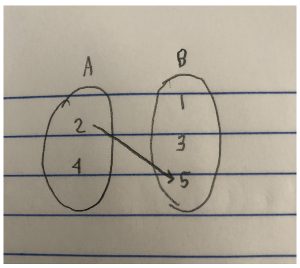


Figure 1: Diagram of U

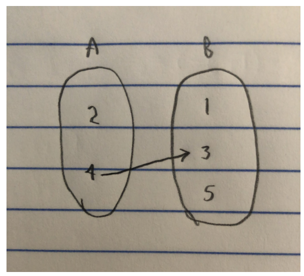


Figure 2: Diagram of V

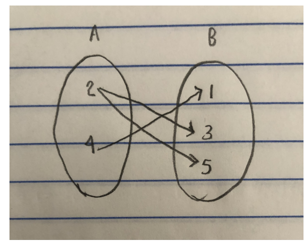


Figure 3: Diagram of W

(b) Neither U, V, or W are functions. This is because U and V only contain a single point which satisfies the predicate, therefore not having a y-value that maps to each x-value.

By definition, a function cannot have two distinct ordered pairs that have the same first element - this is why W is not a function - it has two y-values that satisfy $x = 2$.

10. (10 points) Exercise Set 1.3, Question 20.
Define functions H and K from R to R by the following formulas: For every x $\in R$, $H(x) = (x - 2)$ and $K(x) = (x - 1)(x - 3) + 1$. Does $H = K$? Explain.

(a) $H = K$ because $H(x) = (x - 2)^2$ and $K(x) = (x - 1)(x - 3) + 1$
$\Rightarrow x^2 - 4x + 3 + 1$
$\Rightarrow x^2 - 4x + 4$
$\Rightarrow (x - 2)^2$
$\Rightarrow H(x)$.

Therefore, $H(x) = K(x) \Rightarrow H = K$.