CSE 2500 - Homework 3

Mike Medved

September 25th, 2021

1. Exercise Set 3.1

   (a) (5 points) Question 32 part (d)
   Let **R** be the domain of the predicate variable x. Which of the following are true and which are false? Give counter examples for the statements that are false (d) $x^2 > 4 \iff |x| > 2$

      i. The statement is true because for any real number r, since r^2 is greater than four if and only if $|r| > 2$.

   (b) (5 points) Question 33 part (d)
   Let **R** be the domain of the predicate variables a, b, c, and d. Which of the following are true and which are false? Give counterexamples for the statements that are false. (d) $a < b$ and $c < d \Rightarrow ac < bd$

      i. The above statement is can be rewritten as "real numbers a, b, c, and d are such that if $a < b$ and $c < d$, then $ac < bd$." When rewritten, the resultant statement would not need to be true for all values of a, b, c and d. Therefore, it is false.

      A counterexample to prove this finding can be demonstrated if we let $a = 2, b = 5, c = -2$, and $d = -1$.
      In this case, $ac = -4$ and $bd = -5$, which disproves the statement since $-4 \not< -5$.

2. (10 points) Exercise Set 3.2, Question 19
   Write a negation for the following statement.

   $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

   (a) Firstly, negating $\forall$ produces $\exists$, then from analyzing the if-statement, it can be assumed that $\mathbf{p = n}$ **is prime**, $\mathbf{q = n}$ **is odd**, and $\mathbf{r = n = 2}$. Now, the statement can be rewritten in terms of **p, q,** and **r**: $p \to (q \vee r)$. From this, $p \to q$ can be rewritten as $\sim p \vee q$.

   With all of this in mind, the statement can be formally written as $p \to (q \vee r) \equiv \sim p \vee (q \vee r)$. The negation for $\sim p \vee (q \vee r)$ is $\sim (\sim p \vee (q \vee r)) = p \wedge (\sim q \wedge \sim r)$

Therefore, the final negated statement is: $\exists$ an integer n such that n is prime, not odd, and $n \neq 2$.

3. (10 points) Exercise Set 3.2, Question 29.
Write a negation for the following statement.

   $\forall$ integer d, if $6/d$ is an integer then $d = 3$.

   (a) The contrapositive for this statement is: $\forall$ integer d, if $d \neq 3$, then $\frac{6}{d}$ is not an integer.
   (b) The converse for this statement is: $\forall$ integer d, if $d = 3$, then $\frac{6}{d}$ is an integer.
   (c) The inverse of this statement is: $\forall$ integer d, if $\frac{6}{d}$ is not an integer, then $d \neq 3$.

4. (10 points) Exercise Set 3.2, Question 47.
The computer scientists Richard Conway and David Gries once wrote:

   The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for a reasonable [program] correctness.

   Rewrite this statement without using the words necessary or sufficient.

   (a) The statement can be summed up using the following variable substitutions:
   $a =$ a computer program
   $f(x) =$ has no error messages during translation
   $g(x) =$ x is correct

   In this way, $g(x)$ is contingent upon $f(x)$, but $f(x)$ is insufficient to satisfy $g(x)$. Converting this to formal language, we get:

   $\forall x$, if $\sim f(x)$, then $\sim g(x) \wedge \sim (\forall x, \text{ if } f(x), \text{then } g(x))$
   $\forall x$, if $\sim f(x)$, then $\sim g(x) \wedge \exists x$, such that $f(x) \wedge \sim g(x)$

   Finally, with this in mind, the sentence can be rewritten as such: if a computer program has error messages during translation, then the computer program is not correct and there exists a computer program that has no error messages during translation and also not correct.

5. (10 points) Exercise Set 3.3, Question 12.
Let $D = E = -2, -1, 0, 1, 2$. Write negations for each of the following statements are determine which are true, the given statement or its negation.

$\forall x \in D$, $\exists y \in E$ such that $xy \geq y$

(a) The negation of $\forall$ is $\exists$, and the negation of $\geq$ is $<$ by De Morgan's Laws. With this in mind, the full negation of the given statement is: $\exists x \in D$, $\forall y \in E$, such that $xy < y$.

The statement is true, while the negation is not, since by plugging in the provided values from the D and E sets, the negation can quickly be disproved; for example, here are a few instances in which the negation fails to hold true.

i. For $x = -2$ and $y = -1$, $-2(-1) \not< -1$
ii. For $x = -1$ and $y = -2$, $-1(-2) \not< -2$
iii. For $x = 0$ and $y = -2$, $0(-2) \not< -2$
iv. For $x = 1$ and $y = 1$, $1(1) \not< 1$
v. For $x = 2$ and $y = 2$, $2(2) \not< 2$

6. (10 points) Exercise Set 3.3, Question 43.
The following i the definition for $\lim_{x \to a} f(x) = L$
For all real numbers $\epsilon > 0$, there exists a real number $\delta > 0$ such that for all real numbers x, if $a - \delta < x < a + \delta$ and $x \neq a$ then $L - \epsilon < f(x) < L + \epsilon$.
Write what it means for $\lim_{x \to a} f(x) \neq L$. In other words, write the negation of the definition.

(a) The negation of $\forall$ is $\exists$, therefore the two given statements can be rewritten as (1) $\exists x \in D$ such that $\forall y \in E$, $\sim P(x, y)$, and (2) $\forall x \in D$, $\exists y \in E$ such that $\sim P(x, y)$, respectively.

In this way, the negation of the limit is $\exists$ a real number $\epsilon > 0$ such that $\forall$ real numbers $\delta > 0$, $\exists$ a real number x such that $a - \delta < x < a + \delta, x \neq a$ and either $L - \epsilon \geq f(x)$ or $f(x) \geq L + \epsilon$.

7. (10 points) Exercise Set 3.3, Question 56.
Let $P(x)$ and $Q(x)$ be predicates and suppose D is the domain of x. For the statement forms in each air, determine whether (a) they have the same truth value for every choice of $P(x), Q(x)$, and D, or (b) there is a choice of $P(x), Q(x)$, and D for which they have opposite truth values.

$\exists x \in D, (P(x) \wedge Q(x))$, and $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$

(a) If $\exists x \in D$ and $p(x) \wedge q(x)$ is true, then $p(x) \wedge q(x)$ must be true for some $x \in D$. The implication of this statement is that either $\exists x$ such that $p(x)$ is true for $x \in D$, or that $\exists x$ such that $q(x)$ is true for $x \in D$.

Since either $p(x)$ or $q(x)$ is true for some value of $x \in D$, the statement $(\exists x \in D, p(x)) \wedge (\exists x \in D, q(x))$ is thus true.

(b) Conversely, if $(\exists x \in D, p(x)) \wedge (\exists x \in D, q(x))$ is true, either $p(x)$ or $q(x)$ must be true for some $x \in D$.

The implication of this statement is that since either $p(x)$ or $q(x)$ is true for some $x \in D$, then $p(x) \wedge q(x)$ is also true for some $x \in D$. Therefore, the statement $\exists x \in D, (p(x) \wedge q(x))$ must be true.

8. (10 points) Exercise Set 3.4, Question 18
Some of the arguments in 7-18 are valid by universal modus ponens or universal modus tollens; others are invalid and exhibit the converse or the inverse error. State which are valid and which are invalid. Justify your answers.

(a) If an infinite series converges, then its terms go to 0.

(b) The terms of the infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ do not go to 0.

(c) $\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ does not converge.

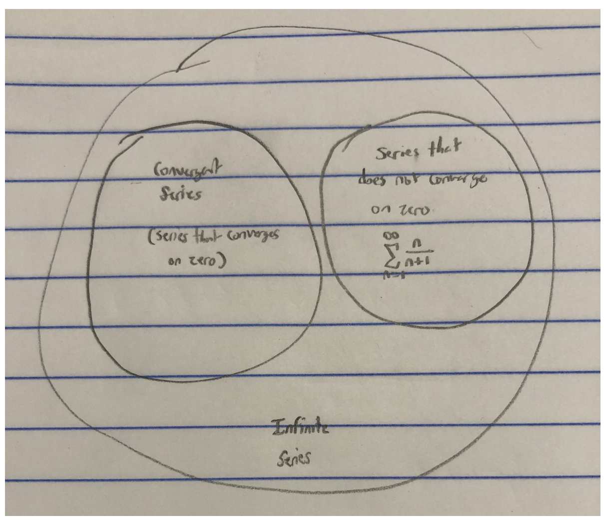

Figure 1: Series Category Diagram

Through the above diagram, it can be seen that the infinite series, $(\sum_{n=0}^{\infty} \frac{n}{n+1}$ is not in the same category as the convergent series, since it is not implied to converge on zero. Similarly, it is completely separate of the convergent category, and as such, it is not convergent, and the argument holds.

The logic supporting this claim is that if a series converges on zero, it is convergent. Conversely, if a series does not converge on zero it is not convergent. Since $(\sum_{n=0}^{\infty} \frac{n}{n+1}$ does not approach zero, it is not convergent.

9. (10 points) Exercise Set 3.4, Question 22.
Indicate whether the argument below is valid or invalid. Support your answers by drawing Venn diagram(s).

   (a) All discrete mathematics students can tell a valid argument from an invalid one.
   (b) All thoughtful people can tell a valid argument from an invalid one.
   (c) $\therefore$ All discrete mathematics students are thoughtful.

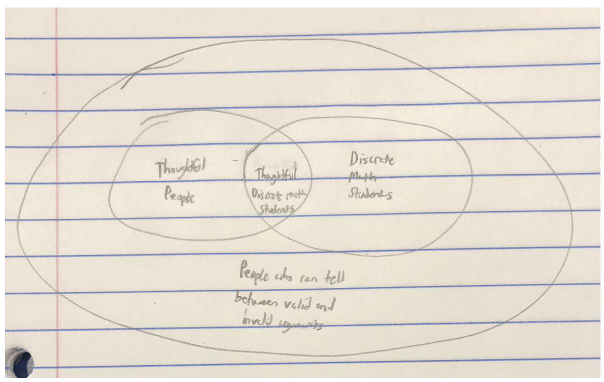

Figure 2: Students Venn Diagram

   Based on the Venn diagram, it can be determined that not all Discrete Math students are thoughtful people, since there is a portion of such students who are not thoughtful people. Therefore, the statement that **all** Discrete Math students are thoughtful is simply invalid.

10. (10 points) Exercise Set 3.4, Question 32.
In exercises 28-32, reorder the premises in each of the arguments to show that the conclusion follows as a valid consequence from the premises. It may be helpful to rewrite the statements in if-then form and replace some statements by their contrapositives.
Exercises 31 and 32 are adapted from the Symbolic Logic by Lewis Carroll.

   (a) When I work a logic example without grumbling, you may be sure it is one I understand.
   (b) The arguments in these examples are not arranged in regular order like the ones I am used to.
   (c) No easy examples make my head ache.

(d) I can't understand examples i the arguments are not arranged in regular order like the ones I am used to.

(e) I never grumble at an example unless it gives me a headache.

$\therefore$ These examples are not easy.

(a) The arguments in these examples are not arranged in regular order like the ones I am used to.

(b) If the arguments are not arranged in regular order like the ones I am used to, then I can't understand the examples.

(c) If I can't understand any logic example, I grumble.

(d) If I grumble at an example, then it gives me a headache.

(e) If I get a headache, then the examples are not easy.

In this case, it can be concluded that the examples are not easy, therefore the argument is valid.