CSE 2500 - Fall 2021 Exam I Solutions

1. (a) [20 points] Write the converse, inverse and contrapositive of the following statement.
$$(p \wedge r \rightarrow z) \rightarrow (q \leftarrow (\neg z \vee r))$$

> **Solution:**
> Converse: (7 points) $(q \leftarrow (\neg z \vee r)) \rightarrow (p \wedge r \rightarrow z)$
> Inverse: (7 points) $\neg(p \wedge r \rightarrow z) \rightarrow \neg(q \leftarrow (\neg z \vee r))$
> Contrapositive: (6 points) $\neg(q \leftarrow (\neg z \vee r)) \rightarrow \neg(p \wedge r \rightarrow z)$

(b) [20 points] Use a truth table to determine whether the following argument form is valid or invalid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. (Hint: an argument form is valid if and only if the conclusion is true for every critical line.)

$$p \rightarrow (q \rightarrow r)$$
$$q \leftarrow p \vee r$$
$$\therefore p \rightarrow r$$

> **Solution:**
>
> | points: | | | 3 | 3 | 3 | 3 | 5 |
> |---|---|---|---|---|---|---|---|
> | p | q | r | $q \rightarrow r$ | $p \rightarrow (q \rightarrow r)$ | $p \vee r$ | $q \leftarrow (p \vee r)$ | $p \rightarrow r$ |
> | T | T | T | T | T | T | T | T |
> | T | T | F | F | F | T | T | |
> | T | F | T | T | T | T | F | |
> | T | F | F | T | T | T | F | |
> | F | T | T | T | T | T | T | T |
> | F | T | F | F | T | F | T | T |
> | F | F | T | T | T | T | F | |
> | F | F | F | T | T | F | T | T |
>
> 3 points: Therefore, the argument is valid.

2. (a) [15 points] Express the statement "No white elephants are carnivores" as a *formal* logical expression involving predicates, quantifiers, and logical connectives with a domain consisting of all *animals* (not all elephants). Also, your answer should properly take into consideration that not all elphants are white.

> **Solution:**
> Let A be the set of all animals, the predicate, Elephant(a) be "a is an elephant", the

predicate, White(a) be "a is white", and the predicate, Carnivore(a) be "a is a carnivore". Therefore,
(15 points):

$$\forall a \in A, \text{Elephant}(a) \land \text{White}(a) \rightarrow \neg\text{Carnivore}(a) \tag{1}$$

or (12 points):

$$\forall a \in A, \text{WhiteElephant}(a) \rightarrow \neg\text{Carnivore}(a) \tag{2}$$

(b) [15 points] Write the negation for the statement in part a, above. Please give both the informal statement as well as the formal statement with predicates and quantifiers.

Solution: 10 points:

$$\exists a \in A, \neg(\text{Elephant}(a) \land \text{White}(a) \rightarrow \neg\text{Carnivore}(a))$$
$$\equiv \exists a \in A, (\text{Elephant}(a) \land \text{White}(a)) \land \text{Carnivore}(a) \tag{3}$$

5 points: "There exists a white elephant that is a carnivore." or "There exists a carnivorous white elephant."

3. (a) [15 points] Is the following argument valid or invalid? Justify your answer.
Every nonzero real number has a reciprocal.
Zero does not have a reciprocal.
Therefore, zero is not a nonzero real number.

Solution: Valid (10 points). (Modus Tollens) (5 points).

(b) [15 points] Consider the statement, "Somebody is older than everybody." Rewrite this statement in the form "$\exists$ a person x such that $\forall$ _____.

Solution: 15 points: $\exists$ a person x such that $\forall$ persons y, older(x, y), or $\exists$ a person x such that $\forall$ persons y, age(x) > age(y)