CSE 2500 - Homework 2

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1. Exercise Set 2.1

   (a) (5 points) Question 28
   Use De Morgan's laws to write negations for the statement "The train is late or my watch is fast."

      i. De Morgan's laws state that given p and q, $(p \vee q) \equiv \sim p \wedge \sim q$, and that $(p \wedge q) \equiv \sim p \vee \sim q$. With this in mind, the statement can be represented as $p \vee q$, and thus can be written as **The train is not late and my watch is not fast.**

   (b) (5 points) Question 37
   Assume x is a particular real number and use De Morgan's laws to write the negation of $0 > x \geq -7$.

      i. The above statement is equivalent to $0 > x$, $x \geq -7$, therefore, by using De Morgan's laws to negate said statement, the result would be $0 \leq x$ or $x < -7$.

2. (10 points) Exercise Set 2.1, Question 46(c).
In Example 2.1.4, the symbol $\oplus$ was introduced to denote exclusive or, so $p \oplus q \equiv (p \vee q) \wedge \sim (p \wedge q)$. Is $(p \oplus q) \wedge r \sim (p \wedge r) \oplus (q \wedge r)$? Justify your answer.

   (a) By constructing a truth table, and observing the columns of both $(p \oplus q) \wedge r$, and $(p \wedge r) \oplus (q \wedge r)$, it is evident that the truth values are the same, thus the two statements are equivalent.

p	q	r	p $\oplus q$	p $\wedge q$	q $\wedge r$	(p $\oplus q) \wedge r$	(p $\wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

3. (10 points) Exercise Set 2.1, Question 52.
   Use Theorem 2.1.1 to verify the logical equivalence:

   $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

   (a) Apply De Morgan's Law: $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv (\sim p \wedge \sim (\sim q)) \vee (\sim p \wedge \sim q)$
   (b) Apply Double Negative Law: $(\sim p \wedge q) \vee (\sim p \wedge \sim q)$
   (c) Apply Distributive Law: $\sim p \wedge (q \vee \sim q)$
   (d) Apply Negation Law for OR: $\sim p \wedge (t)$
   (e) Apply Identity Law for AND: $\sim p$

4. (10 points) Exercise Set 2.2, Question 21.
   Suppose that p and q are statements so that $p \to q$ is false. Find the truth values of each of the following:

   (a) Since $p \to q$ is false, this means p is true, while q is false. As a result, $\sim p$ is false, and q is false. Therefore, $\sim p \to q$ is true.
   (b) Since $p \to q$ is false, this means p is true, while q is false. Therefore $p \vee q$ is true, as p is true.
   (c) Since $p \to q$ is false, this means p is true, while q is false. Additionally, since it is given that $p \to q$ is false, $q \to p$ must be true as a result.

5. (10 points) Exercise Set 2.2, Question 27.
   Use truth tables to establish the truth of the statement "The converse and inverse of a conditional statement are logically equivalent to each other."

p	q	$\sim p$	$\sim q$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

   (a) From the above truth table, it can be determined that $q \to p$ and $\sim p \to \sim q$ have the same truth values, therefore, they are logically equivalent. In this way, the converse and inverse of a conditional statement are logically equivalent to each other.

6. (10 points) Exercise Set 2.2, Question 43.
   Use the contrapositive to rewrite the statements in if-then form in two ways. Doing homework regularly is a necessary condition for Jim to pass the course.

   (a) If Jim passes the course, then it means he did his homework regularly.

(b) If Jim passes the course, then Jim does homework regularly.

7. (10 points) Exercise Set 2.2, Question 50.
Use the logical equivalences $p \to q \equiv \sim p \vee q$ and $p \iff \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol $\to$ or $\iff$, and (b) use the logical equivalence $p \vee q \equiv \sim (\sim p \wedge \sim q)$ to rewrite each statement form using only $\wedge$ and $\sim$.

$$(p \to (q \to r)) \iff ((p \wedge q) \to r)$$

(a) $(\sim p \vee (\sim q \vee r)) \iff (\sim p \vee \sim q) \vee r$
$\Rightarrow \frac{\sim(\sim p \vee (\sim q \vee r))}{\sim p} \vee \frac{((\sim p \vee \sim q) \vee r)}{q}$
$\Rightarrow \wedge(\frac{\sim((\sim p \vee \sim q)}{\sim q}) \vee (\frac{\sim p \vee (\sim q \vee r)}{p})$
$\Rightarrow \sim (\sim p) \wedge \sim (\sim q \vee r) \vee (\sim p \vee \sim q \vee r)$
$\Rightarrow (p \wedge q \wedge \sim r) \vee (\sim p \vee \sim q \vee r)$

(b) $(p \wedge q \wedge \sim r) \vee (\sim p \vee \sim q \vee r) \equiv (p \wedge q \wedge \sim r) \vee (\sim (p \wedge q) \vee r)$
$\Rightarrow (p \wedge q \wedge \sim r) \vee \sim (p \wedge q \wedge \sim r)$
$\Rightarrow \sim (\sim (p \wedge q \wedge \sim r)) \vee \sim (p \wedge q \wedge \sim r)$
$\Rightarrow \sim (\sim (p \wedge q \wedge \sim r) \wedge (p \wedge q \wedge \sim r))$

8. (10 points) Exercise Set 2.3, Question 16
Use truth tables to show that the argument form below is valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid.

$$p \wedge q$$
$$\therefore p$$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(a) In the above truth table, it can be concluded that the p column represents conclusion and the $p \wedge q$ column represents premises. A form of argument is valid if the associated truth table has no conditions under which the conclusion is false for a true premises. Through this definition, and by analyzing the truth table, there is no case in which a conclusion is false, which a premise is true. As a result, this form of argument is valid.

9. (10 points) Exercise Set 2.3, Question 28.
Use symbol to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise,

state whether the converse or the inverse error is made.

If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.

Neither of these two numbers is divisible by 6.

$\therefore$ The product of these two numbers is not divisible by 6.

(a) With the two provided statements, it can be deduced that at least one of the numbers given is divisible by six (p), and that the product of the two numbers is divisible by six (q). Therefore, a relationship can be established such that $p \to q$. Given the other two arguments, it can be claimed that:

$$\begin{gathered} p \to q \\ \sim p \\ \therefore \sim q \end{gathered}$$

This matches the inverse error, which invalidates the given argument.

10. (10 points) Exercise Set 2.3, Question 40.
Sharky, the leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

(a) Socko: Lefty killed Sharky.

(b) Fats: Muscles didn't kill Sharky

(c) Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.

(d) Muscles: Lefty didn't kill Sharky.

Who did kill Sharky?

(a) Given that only one of the statements by Sharky's henchmen is true, either Socko or Muscles have to be telling the truth. This can be deduced since their statements are inverses of one another. In this way, we can assume that both Fats and Lefty are lying, since Fats claimed that "Muscles didn't kill Sharky," we can conclude that Muscles did in fact kill Sharky.