

Problem 1 (10)

Prove or disprove that the following inference rule for functional dependencies.

If $\{A \rightarrow B, C \rightarrow D\}$, then $\{AC \rightarrow BD\}$

Proof can be made by using inference rules IR1 through IR3. Disproof should be done by showing a relational instance that refutes the rule.

Proof. Given $\{A \rightarrow B, C \rightarrow D\}$, we can use

IRE (Augmentation) to add C to both sides of $A \rightarrow B$ giving $AC \rightarrow BC$ (1)

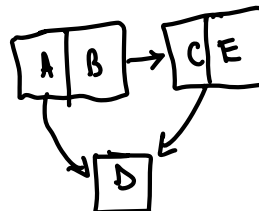
Similarly, add A to both sides of $C \rightarrow D$ to get $AC \rightarrow AD$ (2).

Now, we apply IR1 (Reflexivity) + know $AC \rightarrow AC$ holds. By IR3 (Transitivity), we know (1) and (2) are $AC \rightarrow ABC$, and that $AC \rightarrow ABCD$, which includes $AC \rightarrow BD$.

Thus, $AC \rightarrow BD$ can be inferred from $A \rightarrow B$ and $C \rightarrow D$ proving the inference rule holds. \square

Problem 2 (10)

Show that $AB \rightarrow D$ is in the closure of $\{AB \rightarrow C, CE \rightarrow D, A \rightarrow E\}$



$$(AB)^+ = \{A, B, C, D, E\}$$

Since $D \in (AB)^+$, $AB \rightarrow D$ is in the closure of $\{AB \rightarrow C, CE \rightarrow D, A \rightarrow E\}$

Problem 3 (60)

Consider the relation schema $R = \{A, B, C, D, E, F, G, H, I, J\}$ and the set of functional dependencies $F = \{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, B \rightarrow G, F \rightarrow GH, D \rightarrow IJ\}$ that hold true for R .

- Find a minimal cover for F .
- Suppose R is decomposed into three tables $R_1 = \{A, B, C, D, E\}$, $R_2 = \{B, F, G, H\}$, and $R_3 = \{D, I, J\}$. Show if this decomposition satisfies the dependency preservation property or not.
- Suppose R is decomposed into three tables $R_1 = \{A, B, C, D, E\}$, $R_2 = \{B, F, G, H\}$, and $R_3 = \{D, I, J\}$. Show if this decomposition satisfies the lossless join property or not.
- Suppose R is decomposed into three tables $R_1 = \{A, B, C, D, E\}$, $R_2 = \{B, F, G, H\}$, and $R_3 = \{D, I, J\}$. What is the key of R_1 and what best normal form is R_1 ?
- Suppose R is decomposed into three tables $R_1 = \{A, B, C, D, E\}$, $R_2 = \{B, F, G, H\}$, and $R_3 = \{D, I, J\}$. What is the key of R_2 and what best normal form is R_2 ?
- Suppose R is decomposed into three tables $R_1 = \{A, B, C, D, E\}$, $R_2 = \{B, F, G, H\}$, and $R_3 = \{D, I, J\}$. What is the key of R_3 and what best normal form is R_3 ?

a) Minimal Cover of F :

$$\{AB \rightarrow C, CE \rightarrow D, A \rightarrow E\}$$

d) Key/Best NF for R_1 :

Prime elements: $\{A, B\}$

Key: $\{A, B\}^+$

Best NF: 1NF (1st Normal form)

e) Key/Best NF for R_2 :

Prime Elements: $\{B\}$

Key: B^+

Best NF: 2NF (2nd Normal form)

f) Key/Best NF for R_3 :

Prime Element: $\{D\}$

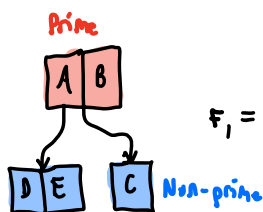
Key: $\{D\}^+$

Best NF: BCNF

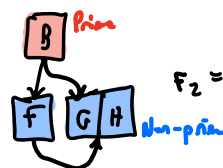
$$R_1 = \{A, B, C, D, E\}$$

$$R_2 = \{B, F, G, H\}$$

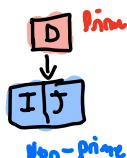
$$R_3 = \{D, I, J\}$$



$$F_1 = \{AB \rightarrow C, A \rightarrow DE\}$$



$$F_2 = \{B \rightarrow F, B \rightarrow G, F \rightarrow GH\}$$



$$F_3 = \{D \rightarrow IJ\}$$

b) Dependency Preservation Check

$$F_1 \cup F_2 \cup F_3 = F$$

All dependencies are preserved after decomposition.

c) Lossless Decomposition Check

$$R_1 \cup R_2 \cup R_3 = \{A \dots J\}$$

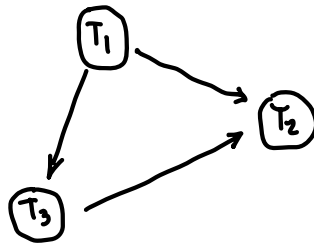
$$R_1 \cap R_2 \cap R_3 = \emptyset$$

The decomposition is lossy as there is no intersection between R_1, R_2, R_3 . There is an intersection between R_1, R_2 but none for R_3 .

Problem 4 (20)

Consider the schedule given next that involves three transactions T1, T2, and T3. Draw the serializability (precedence) graph for the schedule and state whether this schedule is serializable or not. If the schedule is serializable, write down the equivalent serial schedule(s).

Transaction T ₁	Transaction T ₂	Transaction T ₃
		read_item(Y); read_item(Z);
read_item(X); write_item(X);	read_item(Z);	
read_item(Y); write_item(Y);	read_item(Y); write_item(Y); read_item(X); write_item(X);	
		write_item(Y); write_item(Z);



The schedule is serializable since there are no cycles. Thus, the equivalent schedule is T₁, then T₃, then T₂.

Problem 5 (20)

The figure given below shows the log corresponding to a particular schedule at the point of a system crash for the four transactions T1, T2, T3, and T4.

- Suppose that we use the immediate update protocol with checkpointing. Describe the recovery process from the system crash. Specify which transactions are rolled back, which operations in the log are redone and which (if any) are undone, and whether any cascading rollback takes place.
- What should be the values of A, B, C, D and E once the recovery process is completed.

```

1 [start_transaction, T1]
2 [read_item, T1, A]
3 [read_item, T1, B]
4 [write_item, T1, B, 15, 50]
5 [commit, T1]
6 [start_transaction, T2]
7 [read_item, T2, C]
8 [write_item, T2, C, 20, 40]
9 [start_transaction, T3]
10 [write_item, T3, E, 70, 35]
11 [checkpoint]
12 [write_item, T2, D, 15, 40]
13 [commit, T2]
14 [start_transaction, T4]
15 [read_item, T4, D]
16 [write_item, T4, D, 40, 10]
17 [read_item, T3, B]
18 [write_item, T3, B, 50, 25]
19 [read_item, T2, A]
20 [write_item, T2, A, 80, 20]
21 [commit, T4]
  
```

System crash

a) The following operations will need to be redone:

3 - Redone for durability reason even though committed before checkpoint
 5 } Committed after checkpoint
 7 }
 9 }
 13 - Committed just before crash

The following operations will need to be undone:

6 } Uncommitted at time of the crash
 11 }

No cascading rollback is required since none of the transactions that were committed depended on uncommitted changes of another transaction.

b)

Object	Value
A	80
B	15
C	20
D	10
E	N/A

E is unknown because the T₃ transaction never committed, thus the write operation performed on E in T₃ is undone. We do not know the initial value of E so N/A is put.