

4.1 - 7.1 Portfolio

(Exam 2 Preparation Portfolio)

jeana.vatamanelu

March 2023

1. Question 1

- (a) Write the definitions of lemma, theorem, corollary.
- (b) Write an example, from whatever book, of lemma followed by theorem followed by corollary.

2. Question 2

- (a) Write the outline of a direct proof.
- (b) Let a, b, c be three integers. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
- (c) Prove that the union of two bounded sets is bounded.

3. Question 3

- (a) Write the outline of a proof by contrapositive.
- (b) Let n be a natural number. Prove that if n is a $M_4 + 2$ or n is a $M_4 + 3$, then n is not a perfect square.
- (c) Let x and y be two integers. Prove that if xy is not divisible by 11, then x is not divisible by 11 and y is not divisible by 11.

4. Question 4

- (a) Check for clarity the proofs you have written above. Which one seems the most clear?
- (b) Give seven examples in which you use the words as, since, therefore, consequently, hence, so, thus.

5. Question 5

- (a) Write the outline of a proof by contradiction of a statement P .
- (b) Prove by contradiction that $\sqrt{5}$ is an irrational number.
- (c) Let n be an even integer that is not a M_4 . Prove by contradiction that that we cannot find two integers x and y such that $x^2 + 3y^2 = n$.

6. Question 6

- (a) Write the outline of a proof by contradiction of a statement $P \Rightarrow Q$.
- (b) Specify one feature that the proof by contradiction of $P \Rightarrow Q$ has in common with the proof by contrapositive of $P \Rightarrow Q$.
- (c) Specify one feature that is different in the proof by contradiction of $P \Rightarrow Q$ with respect to the proof by contrapositive of $P \Rightarrow Q$.

7. Question 7

- (a) Write four ways in which you can read the statement $P \Leftrightarrow Q$.
- (b) Write the two possible outlines for the proof of $P \Leftrightarrow Q$.

8. Question 8

- (a) Let x and y be two integers. Prove that $4 \mid x^2 - y^2$ iff x and y have the same parity.
- (b) What kind of proof did you use to prove $P \Rightarrow Q$ in the above proof?
- (c) What kind of proof did you use to prove $Q \Rightarrow P$ in the above proof?

9. Question 9

- (a) Let x and y be two real numbers. Prove that $x^2 = y^2$ iff $x = y$ or $x = -y$.
- (b) What kind of outline of an "iff proof" have you used to prove the above statement?