MATH3160 — Portfolio 4.2b

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1 Deliverables

1.1 Moment Generating Function (mgf)

The moment generating function of a discrete random variable can be represented as $M_X(t) \to [0, \infty]$ where

$$M_X(t) = E\left[e^{tX}\right]$$

1.1.1 Example of 1.1

X = The number of days/week student comes to UConn

The pmf(x):

$x \in \text{Im } X$	0	1	2
$P(X=x)$	0.04	0.32	0.64

a) $M_X(t) = ?$ b) $E[X] = ?$ c) $Var(X) = ?$

a) $M_X(t) = \sum_{x \in \text{Im } X} e^{t \cdot X} \cdot \text{pmf}(x) \Rightarrow e^{t \cdot 0} \cdot 0.04 + e^{t \cdot 1} \cdot 0.32 + e^{t \cdot 2} \cdot 0.64$

$1 \cdot 0.04 + e^t \cdot 0.32 + e^{2t} \cdot 0.64$

$\Rightarrow 0.04 + (0.32)e^t + (0.64)e^{2t}$

Domain $M_X \to \mathbb{R}$

b) $M_X'(t) = 0.32e^t + 1.28e^{2t}$

↳ for $t=0 \to E[X] = M_X'(0)$

$\Rightarrow 0.32e^0 + 1.28e^0 = 1.6$

c) $E[X^2] = M_X''(t) = 0.32e^t + (1.28) \cdot 2e^{2t}$

2.56

$\Rightarrow 0.32e^t + 2.56e^{2t}$

↳ for $t=0$, $0.32 + 2.56 = 2.88$

$Var(X) = E[X^2] - (E[X])^2 = 2.88 - 1.6^2 = 0.32$