MATH3160 — Portfolio 5.5, 5.6

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November 9th, 2022

1 Deliverables

1.1 Geometric Random Variable

A geometric random variable represents the number of Bernoulli trials, $Bern(p)$, until the first successful trial occurs.

Definition: Let X be a geometric random variable with parameter $p \in (0, 1)$ written as $X \sim Geom(p)$.

$Im\ X$	pmf	$E[X]$	$Var(X)$	mgf
$\mathbb{N}$	$p * q^{k-1}$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1 - qe^t}$

Table 1: Properties of $X \sim Geom(p)$

The mgf of a geometric random variable takes values t such that $t \in \left(-\infty, ln\left(\frac{1}{q}\right)\right)$.

1.1.1 Geometric Random Variable Mass Plot

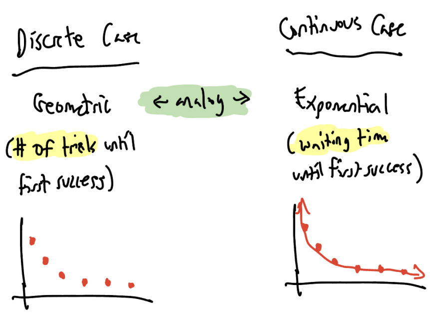

Figure 1: Discrete vs Continuous Plot for $X \sim Geom(p)$, $X \sim Exp(p)$

1.2 Memoryless Property

The memoryless property states that the probability of a geometric random variable is independent of the number of trials that have already occurred.

$$P(X > n + m | x > m) = P(X > n)$$

$$\text{for all } m, n \in \mathbb{N}; m, n > 1.$$

The geometric random variable is the only discrete random variable for which the memoryless property holds true. Additionally, we have found that $P(X > n) = q^n$

1.3 Poisson Random Variable

A poisson random variable represents the number of events that occur in a fixed interval of time.

Definition: Let X be a poisson random variable with parameter $\lambda \in \mathbb{R}^+$ written as $X \sim \rho(\lambda)$.

Im X	Taylor(e^x)	*pmf*	*E[X]*	*Var(X)*	*mgf*
$\mathbb{N}$	$\sum_{n=0}^{\mathbb{R}} \frac{x^n}{n!}$	$\frac{\lambda^k}{k!} \cdot e^{-\lambda}$	λ	λ	$e^{\lambda(e^t-1)}$

Table 2: Properties of $X \sim \rho(\lambda)$

1.3.1 Poisson Random Variable Mass Plot

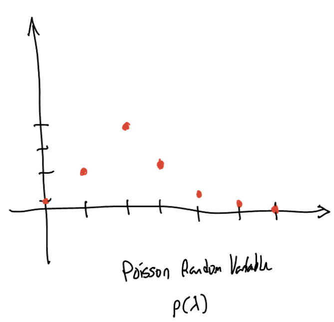

Figure 2: pmf plot for $X \sim \rho(\lambda)$

1.3.2 Special Properties

There are several special properties associated with the Poisson random variable:

1. Given two independent Poisson random variables X, Y, the sum of $M_{X+Y} = M_{\alpha+\beta}$, where $\alpha, \beta \sim \rho(\lambda)$.
2. Poisson random variables have time homogeneity, meaning the interval on which I resides can be scaled by a constant c to create another Poisson random variable Y. For example, given a Poisson random variable X, we can double the time interval to create Y such that $Y \sim \rho(2\lambda)$.

1.3.3 Typical Examples

1. The number of cars that pass a certain point in a given hour.
2. The number of customers that arrive at a bank in a given hour.
3. The number of defective parts that are produced in a given hour.
4. The number of radioactive particles that decay in a given hour.