# MATH3160 — Portfolio 2.2-3.3

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## 1 Deliverables

### 1.1 Properties of Probability

- The probability of an Impossible Event must be zero.
  - Let $A \subseteq S, P(A) = P(A \cup \emptyset) \Rightarrow P(A) = P(A) + P(\emptyset) \Rightarrow P(\emptyset) = 0$
- The probability of a Contrary Event, $P(A^c)$ is one minus the probability of $P(A)$.
  - Let $A$ be an event, $P(A^c) = 1 - P(A)$
- If $A$ is contained within $B$, then $P(A) \leq P(B)$.
  - Let $A \subseteq B$, then $P(B) = P(A) + P(B - A) \Rightarrow P(B) - P(A) = P(B - A) \geq 0$

### 1.2 Famous Problems

1. Strange Dice
   (a) The interesting idea behind the strange dice problem is that given a set of three dice, where each die is a three-faced die with three distinct values, we can show that the probability of Die A beating Die B is 55%, and the probability of Die B beating Die C is also 55%. You would think that the probability of Die A beating Die C would be higher than 55%, but it is actually 44.4%.

   This is significant because Player 2 has a substantial advantage over Player 1, since Player 1 merely chooses a die from the three, but Player 2 can mathematically determine which die to choose in order to beat Player 1.
2. Chevalier de Mere
   (a) The interesting idea behind the Chevalier de Mere problem is that given a fair die, the probability of rolling a single six in six rolls is approximately 0.5177, however rolling a double six on two dies in 24-rolls is always slightly lower, approximately 0.4914. This is significant because it shows that the probability of rolling a double six is lower than the probability of rolling a single six, which is a bit counter-intuitive.
3. Birthday Paradox
   (a) The interesting idea behind the Birthday Paradox problem is that given a group of $n$ people, the probability that two people share the same birthday is approximately $1 - \frac{365!}{(365-n)!365^n}$. This is significant because it shows that the probability of two people sharing the same birthday is not zero, and is actually quite high.

   The reason why the probability is actually quite high is because when we compute the contrary event $A^c$, we must take $A = 1 - A^c$, and since the denominator of $A^c$ grows exponentially with respect to $n$, when we subtract this value from 1, we get a high percentage.

## 1.3 Conditional Probability

The Conditional Probability of an event refers to the probability of an event occurring given that another event has already occurred, this may or may not influence the final probability, as some events are independent of one-another.

**Definition:** Let $S$ be a sample space, $P$ be a probability resting within the sample space, and events $A, B \in S$ such that $P(B) > 0$. The conditional probability $P(A|B)$ can be represented by the formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now that we know what the definition of conditional probability is, we can take a closer look at some of it's properties.

**Properties:**

1. Product Rule (2 Sets)
   (a) The product rule for two sets states that given $P(A \cap B)$, you can rewrite it as $P(A) * P(B|A)$.
2. Product Rule (3 sets)
   (a) The product rule for three sets is similar, but instead of $P(A \cap B)$, we have $P(A \cap B \cap C)$, and as such, we can rewrite it as $P(A) * P(B|A) * P(C|A \cap B)$.
3. Generalized Product Rule
   (a) The generalization of the two above rules goes as follows: Given $n$ sets, $P(A_1 \cap A_2 ... A_{n-1} \cap A_n)$ can be rewritten as $P(A_1) * P(A_2|A_1) * P(A_3|A_1 \cap A_2) * ... * P(A_{n-1}|A_1 \cap A_2 ... A_{n-2}) * P(A_n|A_1 \cap A_2 ... A_{n-1})$.
4. Law of Total Probability
   (a) The Law of Total Probability states that given a sample space $S$, $A_1, A_2, ...A_n$ are partitions of $S$ that are mutually disjoint, and the sum of the partitions' probabilities must add up to the probability of the entire sample space, $S = \sum_{i=1}^{n} P(A_i)$.
5. Bayes' Rule
   (a) Bayes' Rule states that given $P(A|B)$, it can be rewritten in the form of $\frac{P(A)*P(B|A)}{P(B)}$.

## 1.4 Trees

Trees can be used to solve a complex probability problem by breaking it down into smaller, more manageable problems. Below is an example of what a tree looks like for a generic two-step experiment.

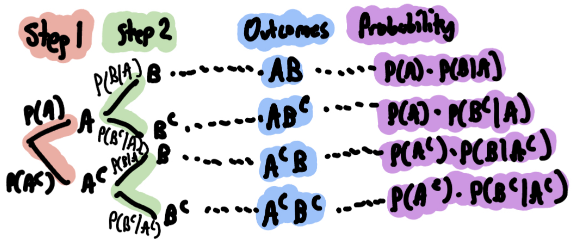


Figure 1: Tree visualization for a generic two-step probability experiment

In this experiment, there are two starting outcomes, with two outcomes for each initial starting outcome. The above diagram explores visually how each outcome is related, and how the probability of each outcome can be calculated in the end. We can use a tree to visualize these outcomes, and further, visually see how we trace the probabilities through the tree, in order to help us compute the final probability for a given path.