MATH3160 — Discrete Case Portfolio

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1 Deliverables

1.1 Joint Distribution Function

Let X, Y be two discrete random variables for the sake of the below definitions and exercises.

1.1.1 Joint Distribution Function

The joint distribution function of X and Y is defined on $F\colon \mathbb{R}^2 \to [0,1]$, with the following formula:

$$F(x,y) = P(X \leq x, Y \leq y)$$

1.1.2 Joint Probability Mass Function

The joint pmf of X and Y is defined on $f\colon \mathit{Im}\ X \times \mathit{Im}\ Y \to [0,1]$, and takes the following formula:

$$f(x,y) = P(X = x, Y = y) \quad \forall (x,y) \in \mathit{Im}\ X \times \mathit{Im}\ Y$$

1.1.3 Independence of X and Y

X and Y are said to be independent if their pmf functions hold the following: $f(x,y) = f_X(x) \cdot f_Y(y)$. However, in order to be independent, this property must hold $\forall (x,y) \in \mathit{Im}\ X \times \mathit{Im}\ Y$.

Equivalently, X and Y are also said to be independent if their distribution functions hold the following property, $F(x,y) = F_X(x) \cdot F_Y(y)$.

1.1.4 Examples

Let us throw two fair dice, the random variable X will represent the number of sixes rolled between the two dice, and Y will represent the number that appeared on die 1.

In this way, the sample space S for this experiment is:

$$\begin{aligned} S &= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ &= \dots \\ &= (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{aligned}$$

In this way, the joint pmf table for X, Y is: