MATH3160 — Portfolio 4.2b

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1 Deliverables

1.1 Moment Generating Function (mgf)

The moment generating function of a discrete random variable can be represented as $M_X(t) \to [0, \infty]$ where

$$M_X(t) = E\left[e^{tX}\right]$$

1.1.1 Example of computing mgf, E[X], and *Var(X)*

X = The number of days/week student comes to UCom

The pmf(x):

$x \in \text{Im } X$	0	1	2
$P(X=x)$	0.04	0.32	0.64

a) $M_X(t) = ?$ b) $E[X] = ?$ c) $Var(X) = ?$

a) $M_X(t) = \sum_{x \in \text{Im } X} e^{t \cdot x} \cdot \text{pmf}(x) \Rightarrow e^{t \cdot 0} \cdot 0.04 + e^{t \cdot 1} \cdot 0.32 + e^{t \cdot 2} \cdot 0.64$

$1 \cdot 0.04 + e^{t} \cdot 0.32 + e^{2t} \cdot 0.64$

$\Rightarrow 0.04 + (0.32)e^t + (0.64)e^{2t}$

Domain $M_X \to \mathbb{R}$

b) $M'_X(t) = 0.32e^t + 1.28e^{2t}$

$\hookrightarrow$ for $t = 0 \to E[X] = M'_X(0)$

$\Rightarrow 0.32e^0 + 1.28e^0 = 1.6$

c) $E[X^2] = M''_X(t) = 0.32e^t + \underbrace{(1.28) \cdot 2}_{2.56}e^{2t}$

$\Rightarrow 0.32e^t + 2.56e^{2t}$

$\hookrightarrow$ for $t = 0$, $0.32 + 2.56 = 2.88$

$Var(X) = E[X^2] - (E[X])^2 = 2.88 - 1.6^2 = 0.32$

1.2 The sum of mgf

There is a property that permits the sum of two independent random variables to be taken with respect to their mgf. Generally speaking, $M_X + M_Y \neq M_{X+Y}$, however, for independent X and Y, it holds true. The proof of this property is shown below:

Theorem. *Let X and Y be independent random variables with mgf $M_X(t)$ and $M_Y(t)$, respectively. Then, $M_{X+Y}(t) = M_X(t)M_Y(t)$.*

Proof. We will show that given two independent random variables, X and Y, the mgf of $X+Y$ is equal to the product of the mgf of X and Y. We will do this by showing that the mgf of $X+Y$ is equal to the mgf of X times the mgf of Y.

$$\begin{aligned} M_{X+Y}(t) &= E\left[e^{t(X+Y)}\right] \\ &= E\left[e^{tX} \cdot e^{tY}\right] \\ &= E\left[e^{tX} \cdot E\left[e^{tY}\right]\right] \\ &= E\left[e^{tX} \cdot M_Y(t)\right] \\ &= M_X(t) \cdot M_Y(t) \end{aligned}$$

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