MATH3160 — Continuous Case Portfolio

Mike Medved

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1 Deliverables

1.1 Joint Continuous Random Variables

Let X, Y be two discrete random variables for the sake of the below definitions and exercises.

1.1.1 Joint Distribution Function

The joint distribution function of X and Y is defined on $F\colon \mathbb{R}^2 \to [0,1]$, with the following formula:

$$F(x,y) = P(X \leq x, Y \leq y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,v)\,\mathrm{d}v\,\mathrm{d}u$$

1.1.2 Joint Density Function

The joint density of X and Y is defined on $f\colon \mathbb{R}^2 \to [0,\infty]$, and takes the following formula:

$$f(x,y) = P((x,y) \in \mathbb{R}^2) = \iint_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y\ \forall D \in \mathbb{R}^2$$

1.1.3 Independence of X and Y

X and Y are said to be independent if their marginal densities hold equal their joint density, as such: $f(x,y) = f_X(x) \cdot f_Y(y)$. However, in order to be independent, this property must hold $\forall (x,y) \in \mathbb{R}^2$.

Equivalently, X and Y are also said to be independent if their marginal distributions hold the following property, $F(x,y) = F_X(x) \cdot F_Y(y)\ \forall (x,y) \in \mathbb{R}^2$.

1.1.4 Importance of the Density Function

The density function for a pair of continuous random variables presents an remarkable property, that it's integral over the entire space is equal to 1.

This property holds because the integral $\iint_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y$ over the domain D represents the volume of the sample space, and due to the law of total probability, the volume of the sample space must be equal to 1.

1.1.5 Marginal Densities

The marginal density of X is defined as $f_X(x) = \int_x^1 f(x,y)\,\mathrm{d}y$, and the marginal density of Y is defined as $f_Y(y) = \int_y^1 f(x,y)\,\mathrm{d}x$.