MATH3160 — Discrete Case Portfolio

Mike Medved

December 7th, 2022

1 Deliverables

1.1 Joint Distribution Function

Let X, Y be two discrete random variables for the sake of the below definitions and exercises.

1.1.1 Joint Distribution Function

The joint distribution function of X and Y is defined on $F\colon \mathbb{R}^2 \to [0,1]$, with the following formula:

$$F(x,y) = P(X \leq x, Y \leq y)$$

1.1.2 Joint Probability Mass Function

The joint pmf of X and Y is defined on $f\colon Im\ X \times Im\ Y \to [0,1]$, and takes the following formula:

$$f(x,y) = P(X = x, Y = y) \quad \forall (x,y) \in Im\ X \times Im\ Y$$

1.1.3 Independence of X and Y

X and Y are said to be independent if their pmf functions hold the following: $f(x,y) = f_X(x) \cdot f_Y(y)$. However, in order to be independent, this property must hold $\forall (x,y) \in Im\ X \times Im\ Y$.

Equivalently, X and Y are also said to be independent if their distribution functions hold the following property, $F(x,y) = F_X(x) \cdot F_Y(y)$.

1.1.4 Examples

Let us throw two fair dice, the random variable X will represent the number of sixes rolled between the two dice, and Y will represent the number that appeared on die 1.

The sample space S for an experiment containing two dice rolls is the following set S, with the pmf table:

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),...,(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

	1	**2**	**3**	**4**	**5**	**6**	**pmf X**
0	$\frac{5}{32}$	$\frac{5}{32}$	$\frac{5}{32}$	$\frac{5}{32}$	$\frac{5}{32}$	0	$\frac{25}{32}$
1	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$
2	0	0	0	0	0	$\frac{1}{32}$	$\frac{1}{32}$
pmf Y	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	

According to the definition of joint independence, we can see that X and Y are independent, since the following property holds: $f(0,1) = \frac{25}{32} \cdot \frac{1}{6} = \frac{5}{32}$, and $f(1,1) = \frac{10}{32} \cdot \frac{1}{6} = \frac{1}{32}$. Therefore X and Y must be independent.