MATH3160 — Portfolio 6.4

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1 Deliverables

1.1 Independent Identically Distributed Random Variables

It is said that a set of random variables are IID if the following conditions are met if both the random variables are independent of each other, and are distributed identically.

1.1.1 Example

An example of ten IID random variables is rolling 10 independent fair dice. The probability of rolling a given number is $\frac{1}{6}$ regardless of which die you are tracking, and the probability of rolling a given number on any die is independent of the other dice. Thus, the ten dice are IID with respect to one another.

We can compute the probability of the total sum of the ten dice being between 30 and 40 (inclusive) as follows:

As each roll is IID, let X_i be the number rolled on the ith die. We can see that $X = \{X_1 \cdots X_{10}\} \sim \mathit{Unif}\{1...6\}$ with parameters $\mu = 3.5, \sigma^2 = 2.91$. We can now compute the final probability:

$$
\begin{aligned}
P(30 \leq \sum_{i=1}^{10} X_i \leq 40) &\xRightarrow{\text{Cntrl Lim Thrm}} \frac{X_1 + \cdots + X_10}{10} \approx \mathcal{N}\left(3.5, \frac{2.91}{10}\right) \\
&\xRightarrow{\text{Expansion}} P(30 \leq X_1 + X_2 + \cdots + X_{10} \leq 40) \\
&\xRightarrow{\text{Simplify}} P(\frac{30}{10} \leq \frac{X_1 + X_2 + \cdots + X_{10}}{10} \leq \frac{40}{10}) \\
&\xRightarrow{\text{Simplify}} P(3 \leq \frac{X_1 + X_2 + \cdots + X_{10}}{10} \leq 4) \\
&\xRightarrow{\text{Std Norm}} P\left(\frac{3 - 3.5}{\sqrt{0.291}} \leq \frac{X_1 + \cdots + X_10}{10} \leq \frac{4 - 3.5}{\sqrt{0.291}}\right) \\
&\Rightarrow P(-0.92 \leq Z \leq 0.92) \\
&\Rightarrow \phi(0.92) - \phi(-0.92) \\
&\Rightarrow 2\phi(0.92) = 2(0.82) - 1 \\
&= 0.64
\end{aligned}
$$

1.2 Central Limit Theorem

The Central Limit Theorem is used to approximate the distribution of a sum of IID random variables.

Definition: Let $X_1, ..., X_n$ be IID random variables of expectation μ and variance σ^2, then:

$$\frac{X_1 + ... + X_n}{n} \approx \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right).$$