MATH3160 — Portfolio 1.1-2.1

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1 Deliverables

1.1 Sample Space

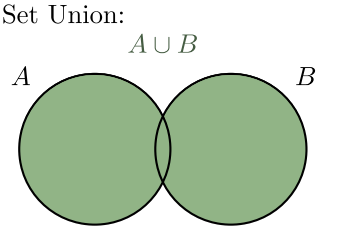
Definition: The Sample Space S with respect to an Experiment E is the set of all possible outcomes for the given experiment.

1.1.1 Examples

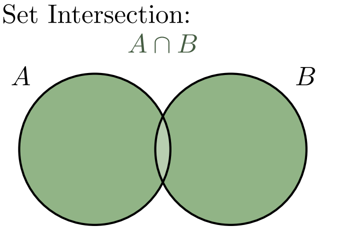
- A coin flip has a sample space of $\{H, T\}$.
- A die roll has a sample space of $\{1, 2, 3, 4, 5, 6\}$.

1.2 Set Operations

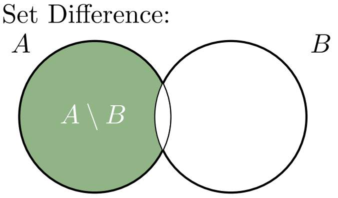
- Set Union:



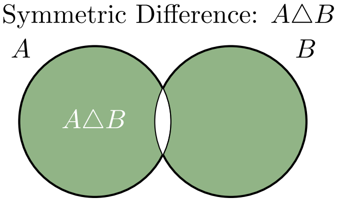
- Set Intersection:



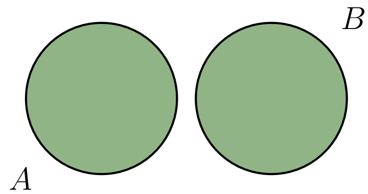
- Set Difference:



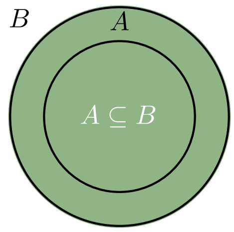
- Symmetric Difference: $A \triangle B$



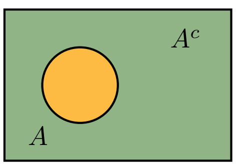
- Set Disjoint: $A \perp B$



- Set Inclusion:



- Set Compliment:



1.3 Events as Sets

Event	**Set**
Sure Event	**S** (Entire Sample Space)
Impossible Event	$\emptyset$ (Empty Set)
Event A	The subset all of favorable outcomes to A
Event A or B	A $\cup$ B
Event A and B	A $\cap$ B
Event A implies B	Set B is included in Set A
Event A, B incompatible	Set A and Set B are disjoint
Contrary of Event A	A^c with respect to the sample space S

1.4 Combinatorics: n-choose-k

Definition: The number of ways of choosing k objects from a set of n objects is denoted by $\binom{n}{k}$.

This can be accomplished by utilizing the following formula, where n represents the number of elements in the set, and k represents the amount being chosen.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

1.4.1 Examples

- Given the set $N = \{1, 2, 3, 4, 5, 6, 7\}$, how many ways are there to choose 3 *ordered* objects from N?
  - For the first entry, there are seven possibilities, for the second entry, there are six possibilities, and so on. Therefore, according to the definition of **n-choose-k**, there are $7 * 6 * 5$ different choices for choosing three ordered elements from the seven element set.

- Given the same set N, how many ways are there to choose 3 *different* objects from N?
  - There are $\frac{7!}{4!*3!}$ ways to choose three different objects from the set.

1.5 Binomial Theorem

Definition: The binomial theorem is a formula that allows us to expand $(a+b)^n$ into a polynomial.

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

1.5.1 Example

- $(x+y)^4$
  - $\sum_{k=0}^{4} \binom{4}{k} x^{4-k} y^k$
  - $\binom{4}{0} x^4 y^0 + \binom{4}{1} x^3 y^1 + \binom{4}{2} x^2 y^2 + \binom{4}{3} x^1 y^3 + \binom{4}{4} x^0 y^4$
  - $x^4 + 4x^3 y + 6x^2 y^2 + 4x^1 y^3 + y^4$

1.6 Classical Definition of Probability

Let S be a *finite* sample space in which all outcomes are equally likely to occur, and let A be an event in S.

Definition: The classical definition of probability only applies to *finite* sample spaces where all events are equally likely to occur. Under these conditions, the probability of an event A is defined as the number of favorable outcomes divided by the total number of outcomes in the sample space.

$$\frac{\text{\# of favorable outcomes}}{\text{total number of outcomes}}$$

1.6.1 Examples

1. An example of these circumstances is the probability of flipping heads on a coin. There is one favorable outcome, and there are two total outcomes, so the probability of landing heads is $\frac{1}{2}(50\%)$.

2. Another example is the probability of rolling a 6 on a die. There is one favorable outcome, and there are six total outcomes, so the probability of rolling a 6 is $\frac{1}{6}(16.67\%)$.

1.7 Axiomatic Definition of Probability

Let S be a sample space, and let A be an event in S. Let $P(A)$ be the probability of A.

Definition: The axiomatic definition of probability always applies, as opposed to the classical definition. The axiomatic definition of probability is based on the following axioms:

1. The probability of the sample space is 1.

2. The union of n disjoint sets can be represented as the sum of the probabilities of all the sets:

$$P(A_1 \cup A_2 \cup ... A_n) = \sum_{i=1}^{n} P(A_i) = 1$$

1.7.1 Examples

Since the axiomatic definition of probability covers all other definitions of probability, namely the classical and frequentist definitions, we can use the axiomatic definition to find the probability of any event. As such, an example of the classical definition can be used as an example of the axiomatic definition.

1. For example, a classical problem is finding the probability of flipping heads on a coin, which ends up being $\frac{1}{2}$(50%) since there are two outcomes, and only one is favorable.

   - This satisfies both the axioms of probability, since the probability of the sample space is 1, and there are two distinctly disjoint outcomes (heads or tails) that are each half of the total probability.

2. Another example is the probability of rolling a 6 on a die. This ends up being $\frac{1}{6}$(16.67%) since there are six outcomes, and only one is favorable.

   - This satisfies both the axioms of probability, since the probability of the sample space is 1, and there are six distinctly disjoint outcomes (1, 2, 3, 4, 5, and 6) that are each $\frac{1}{6}$ of the total probability.

The intuition behind the axiomatic definition is that the probability P acts like a measure of total mass one.