Question 1 *Compute the maximum and minimum values attained by the quadratic form* $Q(\vec{x}) = \vec{x}^T A\vec{x}$*, where* $A = \begin{bmatrix} -2 & 4 \\ 4 & 4 \end{bmatrix}$ *subject to the constraint* $\vec{x}\vec{x}^T = 1$*, and determine a unit vector where each extremum is attained.*

Maximum $= \boxed{6}$ *is attained at* $\vec{u} = \dfrac{1}{\sqrt{\boxed{5}}} \begin{bmatrix} \boxed{1} \\ \boxed{2} \end{bmatrix}$

Minimum $= \boxed{-4}$ *is attained at* $\vec{v} = \dfrac{1}{\sqrt{\boxed{5}}} \begin{bmatrix} \boxed{2} \\ \boxed{-1} \end{bmatrix}$

Question 2 *Find the maximum value for the quadratic form corresponding to the matrix* A *and the vector* $\vec{u}$ *you computed above, subject to* $\vec{x}\vec{x}^T = 1$ *and* $\vec{x}^T \vec{u} = 0$.

Maximum $= \boxed{-4}$ *is attained at* $\vec{v} = \dfrac{1}{\sqrt{\boxed{5}}} \begin{bmatrix} \boxed{2} \\ \boxed{-1} \end{bmatrix}$

Question 3 *Find the maximum value for the quadratic form:* $Q(\vec{x}) = 7x_1^2 + 3x_2^2 + 8x_1x_2$*, subject to* $\|\vec{x}\| = 1$.

Maximum $= \boxed{5} + \boxed{2}\sqrt{\boxed{5}}$.

Question 4 *Given the orthogonal diagonalization of* A *below:*

$$A = PDP^{-1} \text{ where } P = \begin{bmatrix} -2/3 & -1/3 & 2/3 \\ -2/3 & 2/3 & -1/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix} \text{ and } D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

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Find the unit vector $\vec{u}$ *at which the quadratic form* $Q(\vec{x}) = \vec{x}^T A\vec{x}$ *attains its maximum subject to* $\|x\| = 1$*. Then find the unit vector* $\vec{v}$ *at which* Q *attains its maximum subject to both* $\|x\| = 1$ *and* $\vec{x} \cdot \vec{u} = 0$.

$\vec{u} = \dfrac{1}{\boxed{3}} \begin{bmatrix} \boxed{2} \\ \boxed{-1} \\ \boxed{2} \end{bmatrix} \qquad \vec{v} = \dfrac{1}{\boxed{3}} \begin{bmatrix} \boxed{-1} \\ \boxed{2} \\ \boxed{2} \end{bmatrix}$.
