Question 1 *True/False: If $\hat{x}$ is a least-squares solution of $A\vec{x} = \vec{b}$, then $A\hat{x}$ is a point in the column space of A closest to $\vec{b}$.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 2 *When does $A\vec{x} = \vec{b}$ have exactly one least-squares solution? Select all that apply.*

Select All Correct Answers:

(a) *When A is $n \times n$.*

(b) *When $A\vec{x} = \vec{b}$ is consistent.*

(c) *When the columns of A are linearly independent.* ✓

(d) *When $A\vec{x} = \vec{b}$ is has exactly one solution.* ✓

Question 3 *Let* $A = \begin{bmatrix} 4 & 1 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$ *and* $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

Compute A^TA. $A^TA = \begin{bmatrix} \boxed{16} & \boxed{4} \\ \boxed{4} & \boxed{6} \end{bmatrix}$

Compute $A^T\vec{b}$. $A^T\vec{b} = \begin{bmatrix} \boxed{4} \\ \boxed{1} \end{bmatrix}$

Solve the normal equations $A^TA\vec{x} = A^T\vec{b}$ to determine the set of least-squares solutions.

$$\vec{x} = \begin{bmatrix} 1/4 \\ \boxed{0} \end{bmatrix}$$

Why is there only one least-squares solution?

Multiple Choice:

(a) *Because the columns of A are linearly independent.* ✓

(b) *Because $A\vec{x} = \vec{b}$ is has exactly one solution.*

Question 4 *Let* $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 0 \\ 0 & 5 & 0 \end{bmatrix}$ *and* $\vec{b} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$.

Compute A^TA. $A^TA = \begin{bmatrix} \boxed{1} & \boxed{-1} & \boxed{2} \\ \boxed{-1} & \boxed{30} & \boxed{-2} \\ \boxed{2} & \boxed{-2} & \boxed{4} \end{bmatrix}$

Compute $A^T\vec{b}$. $A^T\vec{b} = \begin{bmatrix} \boxed{3} \\ \boxed{10} \\ \boxed{6} \end{bmatrix}$

Solve the normal equations $A^TA\vec{x} = A^T\vec{b}$ to determine the set of least-squares solutions.

$$\vec{x} = x_3 \begin{bmatrix} \boxed{-2} \\ \boxed{0} \\ \boxed{1} \end{bmatrix} + \begin{bmatrix} \boxed{100}/29 \\ \boxed{13}/29 \\ \boxed{0} \end{bmatrix}$$

Question 5 *When will the method of solving the normal equations, $A^TA\vec{x} = A^T\vec{b}$, work for finding the least-squares solutions?*

Multiple Choice:

(a) *Always* ✓

(b) *Only when the columns of A are linearly independent.*

(c) *Only when the columns of A are orthogonal.*

Question 6 *When will the method of finding the orthogonal projection of $\vec{b}$ work for finding the least-squares solutions?*

Multiple Choice:

(a) *Always*

(b) *Only when the columns of A are linearly independent.*

(c) *Only when the columns of A are orthogonal.* ✓

Question 7 *When will the method of usng the QR factorization work for finding the least-squares solutions?*

Multiple Choice:

(a) *Always*

(b) *Only when the columns of A are linearly independent.* ✓

(c) *Only when the columns of A are orthogonal.*
