

**Question 1** Let  $\{\vec{x}_1, \vec{x}_2\}$  be a basis for a subspace  $W$ . Use the Gram-Schmidt process to find an orthogonal basis  $\{\vec{v}_1, \vec{v}_2\}$  for  $W$ .

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\vec{v}_2 = \frac{1}{7} \begin{bmatrix} -9 \\ 34 \\ -4 \end{bmatrix}$$

**Question 2** Suppose using the Gram-Schmidt process, you found that the set  $\{\vec{v}_1, \vec{v}_2\}$  is an orthogonal basis for some subspace  $W$ . Find an orthonormal basis  $\{\vec{w}_1, \vec{w}_2\}$  for  $W$ ,

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 10 \\ 0 \\ -5 \end{bmatrix}$$

$$\vec{w}_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{w}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

**Question 3** The matrix  $A$  below has three pivot columns.

$$A = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$$

First, find a basis for  $\text{Col } A$ .

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ -1 \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for Col  $A$ .

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{10} \begin{bmatrix} 2 \\ -15 \\ 15 \\ 4 \end{bmatrix} \right\}$$

**Question 4** Let  $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 4 & 10 \\ 0 & 1 & 1 \end{bmatrix}$ . First, find a basis for Col  $A$ .

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for Col  $A$ .

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix} \right\}$$

**Hint:** The column space of  $A$  is only two dimensional in this problem. How do you find a basis for Col  $A$ ?