Lay 6.2

Math~2210Q

Question 1 True/False: The set of vectors, $\left\{ \begin{bmatrix} 2\\1\\3 \end{bmatrix}, \begin{bmatrix} -1\\1\\-5 \end{bmatrix}, \begin{bmatrix} 0\\-4\\3 \end{bmatrix} \right\}$ is orthogonal.

Multiple Choice:

- (a) True
- (b) False ✓

Hint: Two vectors are orthogonal iff their dot product is zero.

Question 2 True/False: The set of vectors, $\left\{\begin{bmatrix}1\\1\\-2\end{bmatrix},\begin{bmatrix}2\\0\\1\end{bmatrix},\begin{bmatrix}1\\-5\\-2\end{bmatrix}\right\}$ is orthogonal.

Multiple Choice:

- (a) True ✓
- (b) False

Hint: Two vectors are orthogonal if and only if their dot product is zero.

Question 3 True/False: An orthogonal basis for a subspace W of $\mathbf{R^n}$ is a basis for W that is also an orthogonal set.

Multiple Choice:

- (a) True ✓
- (b) False

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Question 4 If $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is a basis for \mathbf{R}^3 , then for any \vec{w} in \mathbf{R}^3 , \vec{w} can be written as a linear combination of $\vec{u}_1, \vec{u}_2, \vec{u}_3$. Given the vectors $\vec{w}, \vec{u}_1, \vec{u}_2$ and \vec{u}_3 below, compute the coefficients in the linear combination.

$$\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$\vec{w} = -\frac{\boxed{1}}{\boxed{6}}\vec{u}_1 + \frac{\boxed{1}}{\boxed{3}}\vec{u}_2 + \frac{\boxed{5}}{\boxed{6}}\vec{u}_3.$$
 (Simplify fractions as much as possible.)

Hint: $c_j = \frac{\vec{w} \cdot \vec{u}_j}{\vec{u}_j \cdot \vec{u}_j}$

Question 5 Let $\vec{y} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$. Find the orthogonal projection of \vec{y} onto \vec{u} .

First, find $\vec{y} \cdot \vec{u} = \boxed{5}$, then find $\vec{u} \cdot \vec{u} = \boxed{37}$

Then the orthogonal projection of \vec{y} onto \vec{u} is $\operatorname{proj}_L \vec{y} = \begin{bmatrix} -5/37\\30/37 \end{bmatrix}$.

Hint: Let L be the line through \vec{u} and $\vec{0}$. The orthogonal projection of \vec{y} onto \vec{u} (or the orthogonal projection of \vec{y} onto L) is $\operatorname{proj}_L \vec{y} = \left(\frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}\right) \vec{u}$

Question 6 True/False: Not every orthogonal set in $\mathbf{R}^{\mathbf{n}}$ is linearly independent.

Multiple Choice:

- (a) True ✓
- (b) False

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Question 7 True/False: Every orthogonal set of non-zero vectors in ${\bf R^n}$ is linearly independent.

Multiple Choice:

- (a) True ✓
- (b) False

Question 8 What is the definition of an orthonormal set?

Multiple Choice:

- (a) An orthogonal set that is linearly independent
- (b) An orthogonal basis
- (c) A basis consisting only of unit vectors
- (d) An orthogonal basis consisting only of unit vectors
- (e) An orthogonal set consisting only of unit vectors \checkmark