Question 1 *True/False: The set of vectors,* $\left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix} \right\}$ *is orthogonal.*

Multiple Choice:

(a) *True*

(b) *False* ✓

Hint: *Two vectors are orthogonal iff their dot product is zero.*

Question 2 *True/False: The set of vectors,* $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix} \right\}$ *is orthogonal.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Hint: *Two vectors are orthogonal if and only if their dot product is zero.*

Question 3 *True/False: An orthogonal basis for a subspace W of $\mathbf{R^n}$ is a basis for W that is also an orthogonal set.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 4 *If* $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ *is a basis for* $\mathbf{R^3}$*, then for any* $\vec{w}$ *in* $\mathbf{R^3}$*,* $\vec{w}$ *can be written as a linear combination of* $\vec{u}_1, \vec{u}_2, \vec{u}_3$*. Given the vectors* $\vec{w}, \vec{u}_1, \vec{u}_2$ *and* $\vec{u}_3$ *below, compute the coefficients in the linear combination.*

$$\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \quad \vec{u_2} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$\vec{w} = -\frac{\boxed{1}}{\boxed{6}}\vec{u}_1 + \frac{\boxed{1}}{\boxed{3}}\vec{u}_2 + \frac{\boxed{5}}{\boxed{6}}\vec{u}_3.$$ *(Simplify fractions as much as possible.)*

Hint: $c_j = \dfrac{\vec{w} \cdot \vec{u}_j}{\vec{u}_j \cdot \vec{u}_j}$

Question 5 *Let* $\vec{y} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$*. Find the orthogonal projection of* $\vec{y}$ *onto* $\vec{u}$*.*

First, find $\vec{y} \cdot \vec{u} = \boxed{5}$*, then find* $\vec{u} \cdot \vec{u} = \boxed{37}$*.*

Then the orthogonal projection of $\vec{y}$ *onto* $\vec{u}$ *is* $\mathrm{proj}_L \vec{y} = \begin{bmatrix} -\boxed{5}/\boxed{37} \\ \boxed{30}/\boxed{37} \end{bmatrix}$.

Hint: *Let* L *be the line through* $\vec{u}$ and $\vec{0}$*. The orthogonal projection of* $\vec{y}$ *onto* $\vec{u}$ *(or the orthogonal projection of* $\vec{y}$ *onto* L *) is* $\mathrm{proj}_L \vec{y} = \left(\dfrac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \right) \vec{u}$

Question 6 *True/False: Not every orthogonal set in* $\mathbf{R^n}$ *is linearly independent.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 7 *True/False: Every orthogonal set of non-zero vectors in* $\mathbf{R^n}$ *is linearly independent.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 8 *What is the definition of an orthonormal set?*

Multiple Choice:

(a) *An orthogonal set that is linearly independent*

(b) *An orthogonal basis*

(c) *A basis consisting only of unit vectors*

(d) *An orthogonal basis consisting only of unit vectors*

(e) *An orthogonal set consisting only of unit vectors* ✓