

Question 1 For any $n \times n$ matrix A and any vector $\vec{b} \in \mathbf{R}^n$, $A_i(\vec{b})$ is the matrix obtained from A by replacing column i by \vec{b} .

Given that $A = \begin{bmatrix} 1 & 7 \\ -2 & -3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ compute the following.

$$A_1(\vec{b}) = \begin{bmatrix} \boxed{2} & \boxed{7} \\ \boxed{9} & \boxed{-3} \end{bmatrix}$$

$$A_2(\vec{b}) = \begin{bmatrix} \boxed{1} & \boxed{2} \\ \boxed{-2} & \boxed{9} \end{bmatrix}$$

Question 2 Use Cramer's rule to determine the unique solution \vec{x} to the equation:

$$\begin{bmatrix} -4 & 1 \\ 2 & 12 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

$$x_1 = -\frac{\boxed{37}}{\boxed{50}}$$

$$x_2 = \frac{\boxed{1}}{\boxed{25}} \text{ (reduce the fraction)}$$

Question 3 Use Cramer's rule to determine the unique solution \vec{x} to the equation:

$$\begin{bmatrix} 1 & -5 \\ 7 & -8 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$x_1 = -\frac{\boxed{1}}{\boxed{9}} \text{ (reduce the fraction)}$$

$$x_2 = -\frac{\boxed{2}}{\boxed{9}} \text{ (reduce the fraction)}$$

Question 4 For which of the following matrices can you use cramer's rule to find the solution to $A\vec{x} = \vec{b}$ for some vector \vec{b} of the appropriate dimension?

Multiple Choice:

(a) $A = \begin{bmatrix} 1 & 7 \\ 3 & 4 \\ 0 & 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix} \checkmark$

(c) $A = \begin{bmatrix} 1 & 7 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 5 \end{bmatrix} \}$ vspace5pt

Question 5 For which of the following matrices can you use cramer's rule to find the solution to $A\vec{x} = \vec{b}$ for some vector \vec{b} of the appropriate dimension?

Multiple Choice:

(a) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 7 & 1 \\ 0 & 3 & 1 \end{bmatrix} \checkmark$

(b) $A = \begin{bmatrix} 2 & -6 \\ -3 & 9 \end{bmatrix}$

Question 6 Theorem: Let A be an invertible $n \times n$ matrix. Then

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A.$$

Let $A = \begin{bmatrix} 2 & 4 & 7 \\ 1 & 3 & 1 \\ -1 & -1 & 2 \end{bmatrix}$. Compute the following.

$$\det A = \boxed{16}$$

$$\operatorname{adj} A = \begin{bmatrix} \boxed{7} & \boxed{15} & \boxed{-17} \\ \boxed{3} & \boxed{11} & \boxed{-5} \\ \boxed{2} & \boxed{2} & \boxed{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \boxed{7} & \boxed{15} & \boxed{17} \\ \boxed{16} & \boxed{16} & \boxed{16} \\ \boxed{3} & \boxed{11} & \boxed{5} \\ \boxed{16} & \boxed{16} & \boxed{16} \\ \boxed{1} & \boxed{1} & \boxed{1} \\ \boxed{8} & \boxed{8} & \boxed{8} \end{bmatrix}$$