

Question 1 True/False: $n \times n$ matrices A and B are said to be similar if $A = PBP^{-1}$ for some invertible matrix P .

Multiple Choice:

- (a) True ✓
- (b) False

Question 2 Let $D = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ and let $k \geq 1$. What is D^k ?

Multiple Choice:

- (a) $D^k = \begin{bmatrix} 2^k & 0 \\ 0 & 5^k \end{bmatrix}$ ✓
- (b) $D^k = \begin{bmatrix} 2k & 0 \\ 0 & 5k \end{bmatrix}$
- (c) $D^k = \begin{bmatrix} 2 & k \\ k & 5 \end{bmatrix}$
- (d) $D^k = \begin{bmatrix} 2^k & k \\ k & 5^k \end{bmatrix}$

Question 3 Let $D = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ and let $k \geq 1$. What is D^k ?

Multiple Choice:

- (a) $D^k = \begin{bmatrix} 2^k & 0 \\ 0 & 5^k \end{bmatrix}$ ✓
- (b) $D^k = \begin{bmatrix} 2k & 0 \\ 0 & 5k \end{bmatrix}$
- (c) $D^k = \begin{bmatrix} 2 & k \\ k & 5 \end{bmatrix}$
- (d) $D^k = \begin{bmatrix} 2^k & k \\ k & 5^k \end{bmatrix}$

Question 4 Let $A = PDP^{-1}$ such that $P = \begin{bmatrix} 3 & -1 \\ -8 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$. Compute the following:

$$A^4 = \begin{bmatrix} \boxed{16} & \boxed{0} \\ \boxed{0} & \boxed{16} \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \boxed{136} & \boxed{48} \\ \boxed{-384} & \boxed{-136} \end{bmatrix}$$

Question 5 True/False: An $n \times n$ matrix A is diagonalizable if and only if A has exactly n eigenvectors.

Multiple Choice:

- (a) True
- (b) False ✓

Hint: Read the Diagonalization Theorem.

Question 6 True/False: If a 4×4 matrix A has a linearly independent set of four eigenvectors, then A is diagonalizable.

Multiple Choice:

- (a) True ✓
- (b) False

Hint: Read the Diagonalization Theorem.

Question 7 True/False: It is possible for an $n \times n$ matrix A to have a linearly independent set of more than n eigenvectors.

Multiple Choice:

- (a) *True*
- (b) *False* ✓

Hint: A has n eigenvalues (counting multiplicities). For each eigenvalue, the eigenspace has dimension less than or equal to the multiplicity of the eigenvalue.

