Question 1 *Compute the quadratic form* $Q(\vec{x}) = \vec{x}^T A\vec{x}$ *given* $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ *and* $A = \begin{bmatrix} 2 & 6 \\ 6 & 1 \end{bmatrix}$.

$Q(\vec{x}) = \boxed{2}x_1^2 + \boxed{1}x_2^2 + \boxed{12}x_1x_2$

Question 2 *Compute the matrix of the quadratic form:*
$Q(\vec{x}) = 7x_1^2 + 3x_2^2 + 8x_1x_2$. *Assume* $\vec{x} \in \mathbf{R^2}$.

$$A = \begin{bmatrix} \boxed{7} & \boxed{4} \\ \boxed{4} & \boxed{3} \end{bmatrix}$$

Question 3 *Compute the matrix of the quadratic form:*
$Q(\vec{x}) = x_1^2 - x_2^2 + 6x_3^2 + 4x_1x_2 - 10x_1x_3$. *Assume* $\vec{x} \in \mathbf{R^3}$.

$$A = \begin{bmatrix} \boxed{1} & \boxed{2} & \boxed{-5} \\ \boxed{2} & \boxed{-1} & \boxed{0} \\ \boxed{-5} & \boxed{0} & \boxed{6} \end{bmatrix}$$

Question 4 *Suppose* P *orthogonally diagonalizes* A. *Then* $A = PDP^{-1}$ *for some diagonal matrix* D. *Which of the following is equivalent to* P^TAP?

Multiple Choice:

(a) D ✓

(b) PDP^{-1}

(c) P^TDP

(d) *cannot be simplified*

Hint: *If* P *orthogonally diagonalizes* A, *then* P *is an orthogonal matrix which means* $P^T = P^{-1}$.

Question 5 *Suppose P orthogonally diagonalizes A. Substitute $\vec{x} = P\vec{y}$ into the Qudratic form $Q(\vec{x}) = \vec{x}^T A\vec{x}$ and simplify as much as possible. This substitution is called a change of variable.*

Multiple Choice:

(a) $P\vec{y}^T AP\vec{y}$

(b) $\vec{y}^T D\vec{y}$ ✓

(c) $P^T\vec{y}^T AP\vec{y}$

(d) $A\vec{y}$

(e) $D\vec{y}$

Hint: *The previous question might be helpful.*

Question 6 *The orthogonal diagonalization of A is given below. Substitute the change of variable $\vec{x} = P\vec{y}$ into the Quadratic form $Q(\vec{x}) = \vec{x}^T A\vec{x}$ and give the new quadratic form.*

$$A = PDP^{-1} \text{ where } P = \begin{bmatrix} -2/3 & -1/3 & 2/3 \\ -2/3 & 2/3 & -1/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix} \text{ and } D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

Hint: *The previous question might be helpful.*

The new quadratic form is $\boxed{3}y_1^2 + \boxed{9}y_2^2 + \boxed{15}y_3^2$. *Notice that it has no cross product terms. i.e. no $y_i y_j$ where $i \neq j$.*
