

**Question 1** How do you verify that  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$  is an orthogonal set?

**Multiple Choice:**

- (a) Show that  $\vec{u}_1 \cdot \vec{u}_2 \cdot \vec{u}_3 = \vec{0}$
- (b) Show that  $\vec{u}_1 \cdot \vec{u}_2 \cdot \vec{u}_3 = 0$
- (c) Show that  $\vec{u}_1 \cdot \vec{u}_2 = 0, \vec{u}_1 \cdot \vec{u}_3 = 0$  and  $\vec{u}_2 \cdot \vec{u}_3 = 0$  ✓
- (d) Show that  $\vec{u}_1 \cdot \vec{u}_2 = \vec{0}, \vec{u}_1 \cdot \vec{u}_3 = \vec{0}$  and  $\vec{u}_2 \cdot \vec{u}_3 = \vec{0}$

**Question 2** Use the vectors below to compute the following.

$$\vec{y} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = \boxed{-1} \text{ Simplify.}$$

$$\frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = \boxed{1} \text{ Simplify.}$$

Compute the orthogonal projection of  $\vec{y}$  onto  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ .

$$\hat{y} = \begin{bmatrix} \boxed{1} \\ \boxed{-1} \\ \boxed{3} \end{bmatrix}$$

Compute  $\vec{z}$  such that  $\vec{y} = \hat{y} + \vec{z}$  and  $\vec{z} \in (\text{Span}\{\vec{u}_1, \vec{u}_2\})^\perp$ .

$$\vec{z} = \begin{bmatrix} \boxed{0} \\ \boxed{0} \\ \boxed{0} \end{bmatrix}$$

**Question 3** In the previous question, we find that  $\vec{y} = \hat{y}$ . That is that the vector is its own orthogonal projection onto  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ . Which of the following statements must be true?

**Multiple Choice:**

- (a)  $\vec{y}$  is in  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ . ✓

(b)  $\vec{y}$  is **not** in  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ .

**Question 4** Use the vectors below to compute the following.

$$\vec{y} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$$

$$\frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = \frac{\boxed{2}}{\boxed{13}} \text{ Simplify.}$$

$$\frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = \frac{\boxed{-3}}{\boxed{5}} \text{ Simplify.}$$

Compute the orthogonal projection of  $\vec{y}$  onto  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ .

$$\hat{\mathbf{y}} = \frac{1}{65} \begin{bmatrix} \boxed{-69} \\ \boxed{-38} \\ \boxed{205} \end{bmatrix}$$

Compute  $\vec{z}$  such that  $\vec{y} = \hat{\mathbf{y}} + \vec{z}$  and  $\vec{z} \in (\text{Span}\{\vec{u}_1, \vec{u}_2\})^\perp$ .

$$\vec{z} = \frac{1}{65} \begin{bmatrix} \boxed{264} \\ \boxed{168} \\ \boxed{120} \end{bmatrix}$$

What vector represents the closest point to  $\vec{y}$  in  $\text{Span}\{\vec{u}_1, \vec{u}_2\}$ ?

**Multiple Choice:**

- (a)  $\hat{\mathbf{y}}$  ✓
- (b)  $\vec{z}$
- (c)  $\vec{y}$