Question 1 *True/False: If $\mathcal{B}$ and $\mathcal{C}$ are both bases of the vector space V, then the $n \times n$ matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ sends vectors in $\mathcal{B}$-coordinates to the corresponding vectors in $\mathcal{C}$-coordinates.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 2 *Consider two bases $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ for a vector space V. Suppose that $\vec{b}_1 = 3\vec{c}_1 + 2\vec{c}_2$ and $\vec{b}_2 = -\vec{c}_2$.*

(a) *What is the 2×2 change-of-coordinates matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$?*

$$\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} \boxed{3} & \boxed{0} \\ \boxed{2} & \boxed{-1} \end{bmatrix}$$

(b) *If $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ is a vector in $\mathcal{B}$-coordinates, what is the corresponding vector in $\mathcal{C}$-coordinates?*

$$[\vec{x}]_{\mathcal{C}} = \begin{bmatrix} \boxed{9} \\ \boxed{10} \end{bmatrix}$$

(c) *What is the 2×2 change-of-coordinates matrix $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$?*

Hint: $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = \left(\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}\right)^{-1}$

$$\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} = \frac{1}{\boxed{-3}} \begin{bmatrix} \boxed{-1} & \boxed{0} \\ \boxed{-2} & \boxed{3} \end{bmatrix}$$

Question 3 *Consider two bases $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ for a vector space V. Suppose that $\vec{c}_1 = 2\vec{b}_1 - 2\vec{b}_2$ and $\vec{c}_2 = \vec{b}_1 + 3\vec{b}_2$. Which change-of-coordinates matrix is $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$?*

Multiple Choice:

(a) $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$

(b) $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$ ✓

Question 4 *Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ be bases for $\mathbf{R^2}$. Find both change-in-coordinates matrices.*

$$\vec{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, \qquad \vec{c}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \vec{c}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} -\frac{\boxed{1}}{5} & \frac{\boxed{12}}{\boxed{5}} \\ \frac{\boxed{3}}{\boxed{5}} & \frac{\boxed{4}}{\boxed{5}} \end{bmatrix}$$

$$\underset{\mathcal{B}\leftarrow\mathcal{C}}{P} \begin{bmatrix} -\frac{\boxed{1}}{2} & \frac{\boxed{3}}{2} \\ \frac{\boxed{3}}{\boxed{8}} & \frac{\boxed{1}}{8} \end{bmatrix}$$

Hint: $\begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \vec{b}_1 & \vec{b}_2 \end{bmatrix} \sim \begin{bmatrix} I & \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} \end{bmatrix}$
