

**Question 1** Determine the characteristic polynomial of  $A = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$ .

$$\boxed{1}\lambda^2 + \boxed{1}\lambda + \boxed{-5}$$

**Hint:** The characteristic polynomial of a square matrix  $A$  is  $\det(A - \lambda I)$ .

**Question 2** Let  $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}$ .

(a) Determine the characteristic polynomial of  $A$ .

$$\boxed{1}\lambda^2 + \boxed{-4}\lambda + \boxed{-21}$$

(b) What are the real eigenvalues of  $A$ ?

$$\text{List from smallest to largest: } \lambda_1 = \boxed{-3}, \lambda_2 = \boxed{7}$$

**Question 3** Let  $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & -4 \\ 0 & 0 & -2 \end{bmatrix}$ .

(a) Determine the characteristic polynomial of  $A$ .

**Multiple Choice:**

- (i) -6
- (ii)  $(\lambda - 1)(\lambda - 3)(\lambda + 2)$
- (iii)  $(1 - \lambda)(3 - \lambda)(-2 - \lambda)$  ✓
- (iv)  $(1 - \lambda)(3 - \lambda)(2 - \lambda)$

(b) What are the real eigenvalues of  $A$ ?

$$\text{List from smallest to largest: } \lambda_1 = \boxed{-2}, \lambda_2 = \boxed{1}, \lambda_3 = \boxed{3}$$

**Hint:** If  $A$  is triangular, then so is  $A - \lambda I$  and there is an easy way to determine the determinant of triangular matrices.

**Question 4** Let  $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -3 & 0 \\ 0 & 1 & -2 \end{bmatrix}$ .

(a) Determine the characteristic polynomial of  $A$ .

$$\boxed{-2}\lambda^2 + \boxed{-6}\lambda + \boxed{18}$$

(b) What are the real eigenvalues of  $A$ ?

$$\lambda = \frac{\boxed{3} \pm \boxed{3}\sqrt{5}}{-2}$$

**Hint:** Try cofactor expansion. Also, quadratic formula.

**Question 5** In the characteristic polynomial,  $(3 - \lambda)(4 - \lambda)^3(-3 - \lambda)(3 - \lambda)$  what is the multiplicity of...

(a)  $\lambda = 3$ ?  $\boxed{2}$

(b)  $\lambda = 4$ ?  $\boxed{3}$

(c)  $\lambda = -3$ ?  $\boxed{1}$

**Question 6** True/False: Let  $A$  be an  $n \times n$  matrix.  $A$  is invertible if and only if zero is an eigenvalue of  $A$ .

**Multiple Choice:**

(a) True

(b) False ✓