

Question 1 Let $\{\vec{x}_1, \vec{x}_2\}$ be a basis for a subspace W . Use the Gram-Schmidt process to find an orthogonal basis $\{\vec{v}_1, \vec{v}_2\}$ for W .

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\vec{v}_2 = \frac{1}{7} \begin{bmatrix} -9 \\ 34 \\ -4 \end{bmatrix}$$

Question 2 Suppose using the Gram-Schmidt process, you found that the set $\{\vec{v}_1, \vec{v}_2\}$ is an orthogonal basis for some subspace W . Find an orthonormal basis $\{\vec{w}_1, \vec{w}_2\}$ for W ,

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 10 \\ 0 \\ -5 \end{bmatrix}$$

$$\vec{w}_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{w}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

Question 3 The matrix A below has three pivot columns.

$$A = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$$

First, find a basis for $\text{Col } A$.

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \\ -1 \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for Col A .

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{10} \begin{bmatrix} 45 \\ -51 \\ 27 \\ -16 \end{bmatrix} \right\}$$

Question 4 Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 4 & 10 \\ 0 & 1 & 1 \end{bmatrix}$. First, find a basis for Col A .

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for Col A .

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix} \right\}$$

Hint: The column space of A is only two dimensional in this problem. How do you find a basis for Col A ?