Question 1 *Determine the characteristic polynomial of* $A = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$.

$\boxed{1}\lambda^2 + \boxed{1}\lambda + \boxed{-5}$

Hint: *The characteristic polynomial of a square matrix* A *is* $\det(A - \lambda I)$.

Question 2 *Let* $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}$.

(a) *Determine the characteristic polynomial of* A.

$\boxed{1}\lambda^2 + \boxed{-4}\lambda + \boxed{-21}$

(b) *What are the real eigenvalues of* A*?*

List from smallest to largest: $\lambda_1 = \boxed{-3}$, $\lambda_2 = \boxed{7}$

Question 3 *Let* $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & -4 \\ 0 & 0 & -2 \end{bmatrix}$.

(a) *Determine the characteristic polynomial of* A.

Multiple Choice:

(i) -6
(ii) $(\lambda - 1)(\lambda - 3)(\lambda + 2)$
(iii) $(1 - \lambda)(3 - \lambda)(-2 - \lambda)$ ✓
(iv) $(1 - \lambda)(3 - \lambda)(2 - \lambda)$

(b) *What are the real eigenvalues of* A*?*

List from smallest to largest: $\lambda_1 = \boxed{-2}$, $\lambda_2 = \boxed{1}$, $\lambda_3 = \boxed{3}$

Hint: *If* A *is triangular, then so is* $A - \lambda I$ *and there is an easy way to determine the determinant of triangular matrices.*

Question 4 *Let* $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -3 & 0 \\ 0 & 1 & -2 \end{bmatrix}$.

(a) *Determine the characteristic polynomial of A.*

$(-2-\lambda)(\boxed{1}\lambda^2 + \boxed{2}\lambda + \boxed{-9})$

(b) *What are the real eigenvalues of A?*

$\lambda = \boxed{-2}, \lambda = \boxed{-1} \pm \sqrt{\boxed{10}}$

Hint: *Try cofactor expansion. Also, quadratic formula.*

Question 5 *In the characteristic polynomial,* $(3-\lambda)(4-\lambda)^3(-3-\lambda)(3-\lambda)$ *what is the multiplicity of...*

(a) $\lambda = 3$? $\boxed{2}$

(b) $\lambda = 4$? $\boxed{3}$

(c) $\lambda = -3$? $\boxed{1}$

Question 6 *True/False: Let A be an* $n \times n$ *matrix. A is invertible if and only if zero is an eigenvalue of A.*

Multiple Choice:

(a) *True*

(b) *False* ✓