<u>The Invertible Matrix Theorem:</u> Let A be a square $n \times n$ matrix. Then the following statements are equivalent.

(a) A is an invertible matrix.

(b) A is row equivalent to the $n \times n$ identity matrix.

(c) A has n pivot positions.

(d) The equation $A\vec{x} = \vec{0}$ has only the trivial solution.

(e) The columns of A form a linearly independent set.

(f) The linear transformation $\vec{x} \mapsto A\vec{x}$ is one-to-one.

(g) The equation $A\vec{x} = \vec{b}$ has at least one solution for each $\vec{b}$ in $\mathbf{R^n}$.

(h) The columns of A span $\mathbf{R^n}$.

(i) The linear transformation $\vec{x} \mapsto A\vec{x}$maps $\mathbf{R^n}$ onto $\mathbf{R^n}$.

(j) There is an $n \times n$ matrix C such that $CA = I$.

(k) There is an $n \times n$ matrix D such that $AD = I$.

(l) A^T is an invertible matrix.

Question 1 *Determine if the matrices below are invertible.*

A square matrix that is row equivalent to I_8.

Multiple Choice:

(a) *Invertible* ✓

(b) *Not Invertible*

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix}.$$

Multiple Choice:

(a) *Invertible* ✓

(b) *Not Invertible*

$$A = \begin{bmatrix} 1 & 8 & 0 & 5 \\ 0 & 6 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Multiple Choice:

(a) *Invertible*

(b) *Not Invertible* ✓

$$A = \begin{bmatrix} 1 & 8 & 0 & 5 \\ 0 & 6 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Multiple Choice:

(a) *Invertible*

(b) *Not Invertible* ✓

$$A = \begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix}.$$ *(Hint: A is not in echelon form)*

Multiple Choice:

(a) *Invertible* ✓

(b) *Not Invertible*