Question 1 *True/False: The standard basis for* $\mathbf{R}^2$ *is* $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$.

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 2 *True/False: Let* $\vec{x}$ *be in some vector space* V *and let* $\mathcal{B} = \{\vec{b}_1, \dots, \vec{b}_n\}$ *be a basis for* V. *Then* $\vec{x}$ *can be written in two different ways:*

$$\vec{x} = c_1\vec{b}_1 + \cdots + c_n\vec{b}_n \quad \text{and} \quad \vec{x} = d_1\vec{b}_1 + \cdots + d_n\vec{b}_n$$

where not all of the c_i*'s are equal to the corresponding* d_i*'s.*

Multiple Choice:

(a) *True*

(b) *False* ✓

Hint: *Check out the unique representation theorem.*

Question 3 *Suppose* $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ *is a basis for some vector space* V *and* $\vec{x} = 3\vec{b}_1 - 2\vec{b}_2 + 8\vec{b}_3$ *is a vector in* V. *What is* $[\vec{x}]_{\mathcal{B}}$*?*

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} \boxed{3} \\ \boxed{-2} \\ \boxed{8} \end{bmatrix}$$

Question 4 $\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$ *is a basis for* $\mathbf{R}^2$ *and* $\vec{x} = \begin{bmatrix} 8 \\ 10 \end{bmatrix}$ *is a vector in* $\mathbf{R}^2$. *Find the coordinate vector of* $\vec{x}$ *relative to* $\mathcal{B}$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} \boxed{3} \\ \boxed{2} \end{bmatrix}$$

Question 5 $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \end{bmatrix} \right\}$ *is a basis for* $\mathbf{R^2}$ *and* $\vec{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ *is a vector in* $\mathbf{R^2}$. *Find the coordinate vector of* $\vec{x}$ *relative to* $\mathcal{B}$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} \dfrac{\boxed{31}}{\boxed{15}} \\ -\dfrac{\boxed{13}}{\boxed{15}} \end{bmatrix}$$

Question 6 *Let* $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\}$ *be a basis for* $\mathbf{R^3}$. *If* $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, *what is* $\vec{x}$, *that is the same vector in* $\mathbf{R^3}$, *but written in terms of the standard basis of* $\mathbf{R^3}$?

$$\vec{x} = \begin{bmatrix} \boxed{12} \\ \boxed{2} \\ \boxed{-7} \end{bmatrix}$$

Hint: $\vec{x} = P_{\mathcal{B}}[\vec{x}]_{\mathcal{B}}$

Question 7 *Let* $\mathcal{B}$ *be a basis for* $\mathbf{R^2}$. *If* $\vec{x} = \begin{bmatrix} -17 \\ 5 \end{bmatrix}$ *and* $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$, *then which of the following is the basis* $\mathcal{B}$?

Multiple Choice:

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} -17 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\}$ ✓

Hint: $\vec{x} = P_{\mathcal{B}}[\vec{x}]_{\mathcal{B}}$