Question 1 *True/False: Let $T : V \to W$ be a linear transformation. The matrix for T relative to the bases $\mathcal{B}$ and $\mathcal{C}$ for V and W respectively is given by:*

$$M = \begin{bmatrix} [T(\vec{b}_1)]_{\mathcal{C}} & [T(\vec{b}_2)]_{\mathcal{C}} & \cdots & [T(\vec{b}_n)]_{\mathcal{C}} \end{bmatrix}$$

where $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n\}$.

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 2 *True/False: Let $T : V \to W$ be a linear transformation. Let $\mathcal{B}$ and $\mathcal{C}$ be bases for V and W respectively. Let M be the matrix for T relative to $\mathcal{B}$ and $\mathcal{C}$. Then which of the following equations is true?*

Multiple Choice:

(a) $[T(\vec{x})]_{\mathcal{C}} = M[\vec{x}]_{\mathcal{B}}$ ✓

(b) $[T(\vec{x})]_{\mathcal{B}} = M[\vec{x}]_{\mathcal{C}}$

Question 3 *Let$\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ be bases for the vector spaces V and W respectively. Let $T : V \to W$ be a linear transformation. Given the equations below, find the matrix for T relative to $\mathcal{B}$ and $\mathcal{C}$.*

$$T(\vec{b}_1) = 4\vec{c}_1 + 2\vec{c}_2 \quad T(\vec{b}_2) = -3\vec{c}_2$$

$$M = \begin{bmatrix} \boxed{4} & \boxed{0} \\ \boxed{2} & \boxed{-3} \end{bmatrix}$$

Question 4 *Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ be bases for the vector spaces V and W respectively. Let $T : V \to W$ be a linear transformation. Given the equations below, find the matrix for T relative to $\mathcal{B}$ and $\mathcal{C}$.*

$$T(\vec{b}_1) = -\vec{c}_1 + 2\vec{c}_2 \quad T(\vec{b}_2) = \vec{c}_1 + \vec{c}_2 \quad T(\vec{b}_3) = 5\vec{c}_1$$

$$M = \begin{bmatrix} \boxed{-1} & \boxed{1} & \boxed{5} \\ \boxed{2} & \boxed{1} & \boxed{0} \end{bmatrix}$$

Question 5 *Let $\mathcal{B}$ be a basis for some vector space V. If the linear transformation $T : V \to V$ sends vectors written with respect to the basis $\mathcal{B}$ to vectors written with respect to the basis $\mathcal{B}$, then the matrix for T relative to $\mathcal{B}$ (or the $\mathcal{B}$-matrix for T) satisfies:*

$$[T(\vec{x})]_{\mathcal{B}} = [T]_{\mathcal{B}}[\vec{x}]_{\mathcal{B}}$$

Multiple Choice:

(a) *True* ✓

(b) *False*

Question 6 *True/False. Suppose $A = PDP^{-1}$ where D is a diagonal $n \times n$ matrix. If $\mathcal{B}$ is the basis for $\mathbf{R^n}$ formed fromt he columns of P, then D is the $\mathcal{B}$-matrix for the transformation $\vec{x} \mapsto A\vec{x}$.*

Multiple Choice:

(a) *True* ✓

(b) *False*

Hint: *See the diagonal matrix representation theorem on page 291 of Lay.*

Question 7 *Suppose $A = PDP^{-1}$ where P, D, P^{-1} are given below. Let the linear transformation $T : \mathbf{R^3} \to \mathbf{R^3}$ be defined by $T(\vec{x}) = A\vec{x}$. Which of the following gives a basis $\mathcal{B}$ for $\mathbf{R^3}$ with the property that $[T]_{\mathcal{B}}$ is diagonal.*

$$P = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & -3 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -3 & 1 & 9 \\ -1 & 0 & 3 \end{bmatrix}$$

Multiple Choice:

(a) $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix}$ ✓

Hint: *See the diagonal matrix representation theorem on page 291 of Lay.*