Question 1 *For any $n \times n$ matrix A and any vector $\vec{b} \in \mathbf{R^n}$, $A_i(\vec{b})$ is the matrix obtained from A be replacing column i by $\vec{b}$.*

Given that $A = \begin{bmatrix} 1 & 7 \\ -2 & -3 \end{bmatrix}$ *and* $\vec{b} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ *compute the following.*

$$A_1(\vec{b}) = \begin{bmatrix} \boxed{2} & \boxed{7} \\ \boxed{9} & \boxed{-3} \end{bmatrix}$$

$$A_2(\vec{b}) = \begin{bmatrix} \boxed{1} & \boxed{2} \\ \boxed{-2} & \boxed{9} \end{bmatrix}$$

Question 2 *Use Cramer's rule to determine the unique solution $\vec{x}$ to the equation:*

$$\begin{bmatrix} -4 & 1 \\ 2 & 12 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

$$x_1 = -\frac{\boxed{37}}{\boxed{50}}$$

$$x_2 = \frac{\boxed{1}}{\boxed{25}}$$ *(reduce the fraction)*

Question 3 *Use Cramer's rule to determine the unique solution $\vec{x}$ to the equation:*

$$\begin{bmatrix} 1 & -5 \\ 7 & -8 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$x_1 = \frac{\boxed{13}}{\boxed{27}}$$

$$x_2 = -\frac{\boxed{2}}{\boxed{9}}$$ *(reduce the fraction)*

Question 4 *For which of the following matrices can you use cramer's rule to find the solution to $A\vec{x} = \vec{b}$ for some vector $\vec{b}$ of the appropriate dimension?*

Multiple Choice:

(a) $A = \begin{bmatrix} 1 & 7 \\ 3 & 4 \\ 0 & 7 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -9 \\ 0 & 1 \end{bmatrix}$ ✓

(c) $A = \begin{bmatrix} 1 & 7 & 2 & 5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ }*vspace5pt*

Question 5 *For which of the following matrices can you use cramer's rule to find the solution to $A\vec{x} = \vec{b}$ for some vector $\vec{b}$ of the appropriate dimension?*

Multiple Choice:

(a) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 7 & 1 \\ 0 & 3 & 1 \end{bmatrix}$ ✓

(b) $A = \begin{bmatrix} 2 & -6 \\ -3 & 9 \end{bmatrix}$

Question 6 *Theorem: Let A be an invertible $n \times n$ matrix. Then*

$$A^{-1} = \frac{1}{\det A} \text{ adj } A.$$

Let $A = \begin{bmatrix} 2 & 4 & 7 \\ 1 & 3 & 1 \\ -1 & -1 & 2 \end{bmatrix}$. *Compute the following.*

$\det A = \boxed{6}$

$$adj\ A = \begin{bmatrix} \boxed{7} & \boxed{-15} & \boxed{-6} \\ \boxed{-3} & \boxed{11} & \boxed{5} \\ \boxed{2} & \boxed{-2} & \boxed{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \boxed{7}/\boxed{6} & -\boxed{5}/\boxed{2} & -\boxed{1} \\ -\boxed{1}/\boxed{2} & \boxed{11}/\boxed{6} & \boxed{5}/\boxed{6} \\ \boxed{1}/\boxed{3} & -\boxed{1}/\boxed{3} & \boxed{1}/\boxed{3} \end{bmatrix}$$