

Question 1 How do you verify that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is an orthogonal set?

Multiple Choice:

- (a) Show that $\vec{u}_1 \cdot \vec{u}_2 \cdot \vec{u}_3 = \vec{0}$
- (b) Show that $\vec{u}_1 \cdot \vec{u}_2 \cdot \vec{u}_3 = 0$
- (c) Show that $\vec{u}_1 \cdot \vec{u}_2 = 0, \vec{u}_1 \cdot \vec{u}_3 = 0$ and $\vec{u}_2 \cdot \vec{u}_3 = 0$ ✓
- (d) Show that $\vec{u}_1 \cdot \vec{u}_2 = \vec{0}, \vec{u}_1 \cdot \vec{u}_3 = \vec{0}$ and $\vec{u}_2 \cdot \vec{u}_3 = \vec{0}$

Question 2 Use the vectors below to compute the following.

$$\vec{y} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = \boxed{-1} \text{ Simplify.}$$

$$\frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = \boxed{1} \text{ Simplify.}$$

Compute the orthogonal projection of \vec{y} onto $\text{Span}\{\vec{u}_1, \vec{u}_2\}$.

$$\hat{y} = \begin{bmatrix} \boxed{1} \\ \boxed{-1} \\ \boxed{3} \end{bmatrix}$$

Compute \vec{z} such that $\vec{y} = \hat{y} + \vec{z}$ and $\vec{z} \in (\text{Span}\{\vec{u}_1, \vec{u}_2\})^\perp$.

$$\vec{z} = \begin{bmatrix} \boxed{0} \\ \boxed{0} \\ \boxed{0} \end{bmatrix}$$

Question 3 In the previous question, we find that $\vec{y} = \hat{y}$. That is that the vector is its own orthogonal projection onto $\text{Span}\{\vec{u}_1, \vec{u}_2\}$. Which of the following statements must be true?

Multiple Choice:

- (a) \vec{y} is in $\text{Span}\{\vec{u}_1, \vec{u}_2\}$. ✓

(b) \vec{y} is **not** in $\text{Span}\{\vec{u}_1, \vec{u}_2\}$.

Question 4 Use the vectors below to compute the following.

$$\vec{y} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}$$

$$\frac{\vec{y} \cdot \vec{u}_1}{\vec{u}_1 \cdot \vec{u}_1} = \frac{\boxed{2}}{\boxed{13}} \text{ Simplify.}$$

$$\frac{\vec{y} \cdot \vec{u}_2}{\vec{u}_2 \cdot \vec{u}_2} = \frac{\boxed{-3}}{\boxed{5}} \text{ Simplify.}$$

Compute the orthogonal projection of \vec{y} onto $\text{Span}\{\vec{u}_1, \vec{u}_2\}$.

$$\hat{\mathbf{y}} = \frac{1}{65} \begin{bmatrix} \boxed{-69} \\ \boxed{-38} \\ \boxed{205} \end{bmatrix}$$

Compute \vec{z} such that $\vec{y} = \hat{\mathbf{y}} + \vec{z}$ and $\vec{z} \in (\text{Span}\{\vec{u}_1, \vec{u}_2\})^\perp$.

$$\vec{z} = \frac{1}{65} \begin{bmatrix} \boxed{264} \\ \boxed{168} \\ \boxed{120} \end{bmatrix}$$

What vector represents the closest point to \vec{y} in $\text{Span}\{\vec{u}_1, \vec{u}_2\}$?

Multiple Choice:

(a) $\hat{\mathbf{y}}$ ✓

(b) \vec{z}

(c) \vec{y}