Question 1 *Determine the characteristic polynomial of $A = \begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}$.*

$\boxed{1}\lambda^2 + \boxed{1}\lambda + \boxed{-5}$

Hint: *The characteristic polynomial of a square matrix A is $\det(A - \lambda I)$.*

Question 2 *Let $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}$.*

(a) *Determine the characteristic polynomial of A.*

$\boxed{1}\lambda^2 + \boxed{-4}\lambda + \boxed{-21}$

(b) *What are the real eigenvalues of A?*

List from smallest to largest: $\lambda_1 = \boxed{-3}$, $\lambda_2 = \boxed{7}$

Question 3 *Let $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & -4 \\ 0 & 0 & -2 \end{bmatrix}$.*

(a) *Determine the characteristic polynomial of A.*

Multiple Choice:

(i) *-6*

(ii) $(\lambda - 1)(\lambda - 3)(\lambda + 2)$

(iii) $(1 - \lambda)(3 - \lambda)(-2 - \lambda)$ ✓

(iv) $(1 - \lambda)(3 - \lambda)(2 - \lambda)$

(b) *What are the real eigenvalues of A?*

List from smallest to largest: $\lambda_1 = \boxed{-2}$, $\lambda_2 = \boxed{1}$, $\lambda_3 = \boxed{3}$

Hint: *If A is triangular, then so is $A - \lambda I$ and there is an easy way to determine the determinant of triangular matrices.*

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Question 4 *Let* $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -3 & 0 \\ 0 & 1 & -2 \end{bmatrix}$.

(a) *Determine the characteristic polynomial of A.*

$$\boxed{-2}\lambda^2 + \boxed{-6}\lambda + \boxed{18}$$

(b) *What are the real eigenvalues of A?*

$$\lambda = \frac{\boxed{3} \pm \boxed{3}\sqrt{5}}{-2}$$

Hint: *Try cofactor expansion. Also, quadratic formula.*

Question 5 *In the characteristic polynomial,* $(3 - \lambda)(4 - \lambda)^3(-3 - \lambda)(3 - \lambda)$ *what is the multiplicity of...*

(a) $\lambda = 3?$ $\boxed{2}$

(b) $\lambda = 4?$ $\boxed{3}$

(c) $\lambda = -3?$ $\boxed{1}$

Question 6 *True/False: Let A be an $n \times n$ matrix. A is invertible if and only if zero is an eigenvalue of A.*

Multiple Choice:

(a) *True*

(b) *False* ✓