Question 1 *Let $\{\vec{x}_1, \vec{x}_2\}$ be a basis for a subspace W. Use the Gram-Schmidt process to find and orthogonal basis $\{\vec{v}_1, \vec{v}_2\}$ for W.*

$$\vec{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} \boxed{2} \\ \boxed{1} \\ \boxed{4} \end{bmatrix}$$

$$\vec{v}_2 = \frac{1}{7} \begin{bmatrix} \boxed{-9} \\ \boxed{34} \\ \boxed{-4} \end{bmatrix}$$

Question 2 *Let $\{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ be a basis for a subspace W. Use the Gram-Schmidt process to find and orthogonal basis $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ for W.*

$$\vec{x}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{x}_2 = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{v}_1 = \begin{bmatrix} \boxed{3} \\ \boxed{1} \\ \boxed{2} \\ \boxed{1} \end{bmatrix}$$

$$\vec{v}_2 = \frac{1}{15} \begin{bmatrix} \boxed{-6} \\ \boxed{52} \\ \boxed{-16} \\ \boxed{7} \end{bmatrix}$$

$$\vec{v}_3 = \frac{1}{45} \begin{bmatrix} \boxed{4} \\ \boxed{-32} \\ \boxed{-24} \\ \boxed{28} \end{bmatrix}$$

Is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ an orthonormal basis for W?

Multiple Choice:

(a) *Yes*

(b) *No* ✓

Question 3 *Suppose using the Gram-Schmidt process, you found that the set $\{\vec{v}_1, \vec{v}_2\}$ is an orthogonal basis for some subspace W. Find an orthonormal basis $\{\vec{w}_1, \vec{w}_2\}$ i for W,*

$$\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 5 \\ 0 \\ -5 \end{bmatrix}$$

$$\vec{w}_1 = \frac{1}{\sqrt{\boxed{10}}} \begin{bmatrix} \boxed{3} \\ \boxed{1} \\ \boxed{0} \end{bmatrix}$$

$$\vec{w}_2 = \frac{1}{\sqrt{\boxed{2}}} \begin{bmatrix} \boxed{1} \\ \boxed{0} \\ \boxed{-1} \end{bmatrix}$$

Question 4 *Suppose the matrix A below has three pivot columns.*

$$A = \begin{bmatrix} 0 & -2 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \\ 0 & 1 & -1 \end{bmatrix}$$

First, find a basis for $\operatorname{Col} A$.

$$\left\{ \begin{bmatrix} \boxed{0} \\ \boxed{1} \\ \boxed{1} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{-2} \\ \boxed{2} \\ \boxed{2} \\ \boxed{1} \end{bmatrix}, \begin{bmatrix} \boxed{3} \\ \boxed{1} \\ \boxed{4} \\ \boxed{-1} \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for $\operatorname{Col} A$.

$$\left\{ \begin{bmatrix} \boxed{0} \\ \boxed{1} \\ \boxed{1} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{-2} \\ \boxed{0} \\ \boxed{0} \\ \boxed{1} \end{bmatrix}, \begin{bmatrix} \frac{\boxed{45}}{13} \\ \frac{\boxed{-51}}{26} \\ \frac{\boxed{27}}{26} \\ \frac{\boxed{-16}}{13} \end{bmatrix}, \right\}$$

Question 5 *Let* $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 4 & 10 \\ 0 & 1 & 1 \end{bmatrix}$. *First, find a basis for* $\operatorname{Col} A$.

$$\left\{ \begin{bmatrix} \boxed{1} \\ \boxed{3} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{2} \\ \boxed{4} \\ \boxed{1} \end{bmatrix} \right\}$$

Now, use the Gram-Schmidt process to find an orthogonal basis for $\operatorname{Col} A$.

$$\left\{ \begin{bmatrix} \boxed{1} \\ \boxed{3} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{1}/2 \\ \boxed{-1}/2 \\ \boxed{1} \end{bmatrix} \right\}$$

Hint: *The column space of* A *is only two dimensional in this problem. How do you find a basis for* $\operatorname{Col} A$*?*