CS-E4740 - Federated Learning

FL Algorithms

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Spring 2025

Playlist

Glossary

Course Site

Outline

Recap and Learning Goals

Applying Gradient Methods to GTVMin

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Asynchronous FL Algorithms

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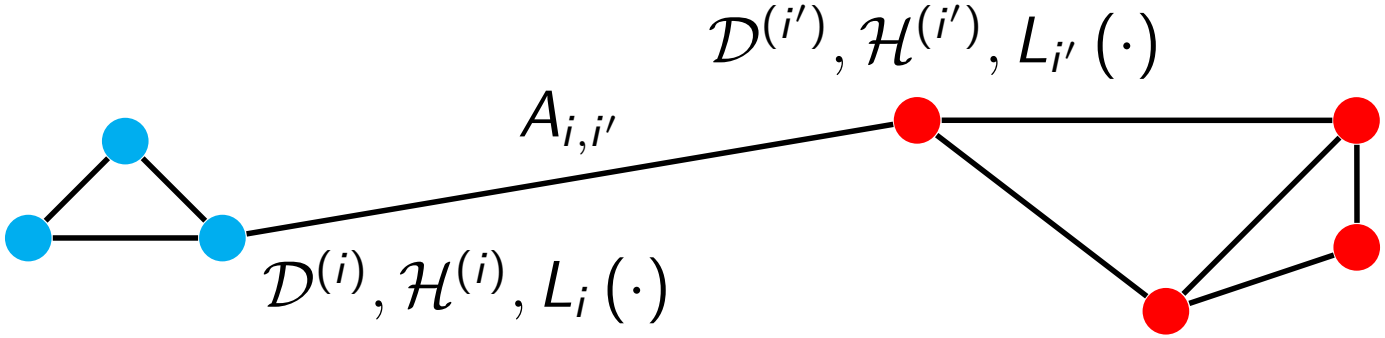
FL Network as Mathematical Model for FL



- An FL network consists of devices $i = 1, \ldots, n$.
- Some i, i' connected by edge with the weight $A_{i,i'} > 0$.
- Device i **generates data** $\mathcal{D}^{(i)}$ and **trains model** $\mathcal{H}^{(i)}$.
- Data $\mathcal{D}^{(i)}$ used to construct loss func. $L_i(\cdot)$.

GTV Minimization (for Parametric Models)

We train local models in a collaborative fashion by solving

$$\min_{\mathbf{w}^{(1)},\ldots,\mathbf{w}^{(n)}} \sum_{i=1}^{n} L_i\left(\mathbf{w}^{(i)}\right) + \alpha \sum_{\{i,i'\}\in\mathcal{E}} A_{i,i'} \left\|\mathbf{w}^{(i)} - \mathbf{w}^{(i')}\right\|_2^2 \quad \text{(GTVMin)}.$$

- Solution consists of learnt modelparams. $\widehat{\mathbf{w}}^{(i)}$.
- Tuning parameter $\alpha \geq 0$ controls clustering of $\widehat{\mathbf{w}}^{(i)}$.
- For $\alpha = 0$, GTVMin reduces to separate ERM for each i.
- Increasing α makes $\widehat{\mathbf{w}}^{(i)}$ more similar across nodes i.

Learning Goals

After completing this module, you know how

- FL alg. can be obtained from **gradient descent**,
- to implement GD as **message passing**,
- to generalize GD to handle **non-parametric models**,
- to implement **asynchronous FL algos.**

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Gradient Step for GTVMin

Starting from initial local params. $\mathbf{w}^{(i,0)}$, repeat grad. steps

$$\mathbf{w}^{(i,k+1)} = \mathbf{w}^{(i,k)} - \eta_{k,i} \left[\nabla L_i \left(\mathbf{w}^{(i,k)} \right) + 2\alpha \sum_{i' \in \mathcal{N}^{(i)}} A_{i,i'} \left(\mathbf{w}^{(i,k)} - \mathbf{w}^{(i',k)} \right) \right]$$

- The learn. rate $\eta_{k,i}$ determines extent of update.
- $\nabla L_i \left(\mathbf{w}^{(i,k)} \right)$ **steers** update towards **min. local loss**.
- $\left(\mathbf{w}^{(i,k)} - \mathbf{w}^{(i',k)} \right)$ **steers** update to **agree with neigh.**
- $\alpha A_{i,i''}$ balances those two steering effects.

Synchronous Operation

The gradient step

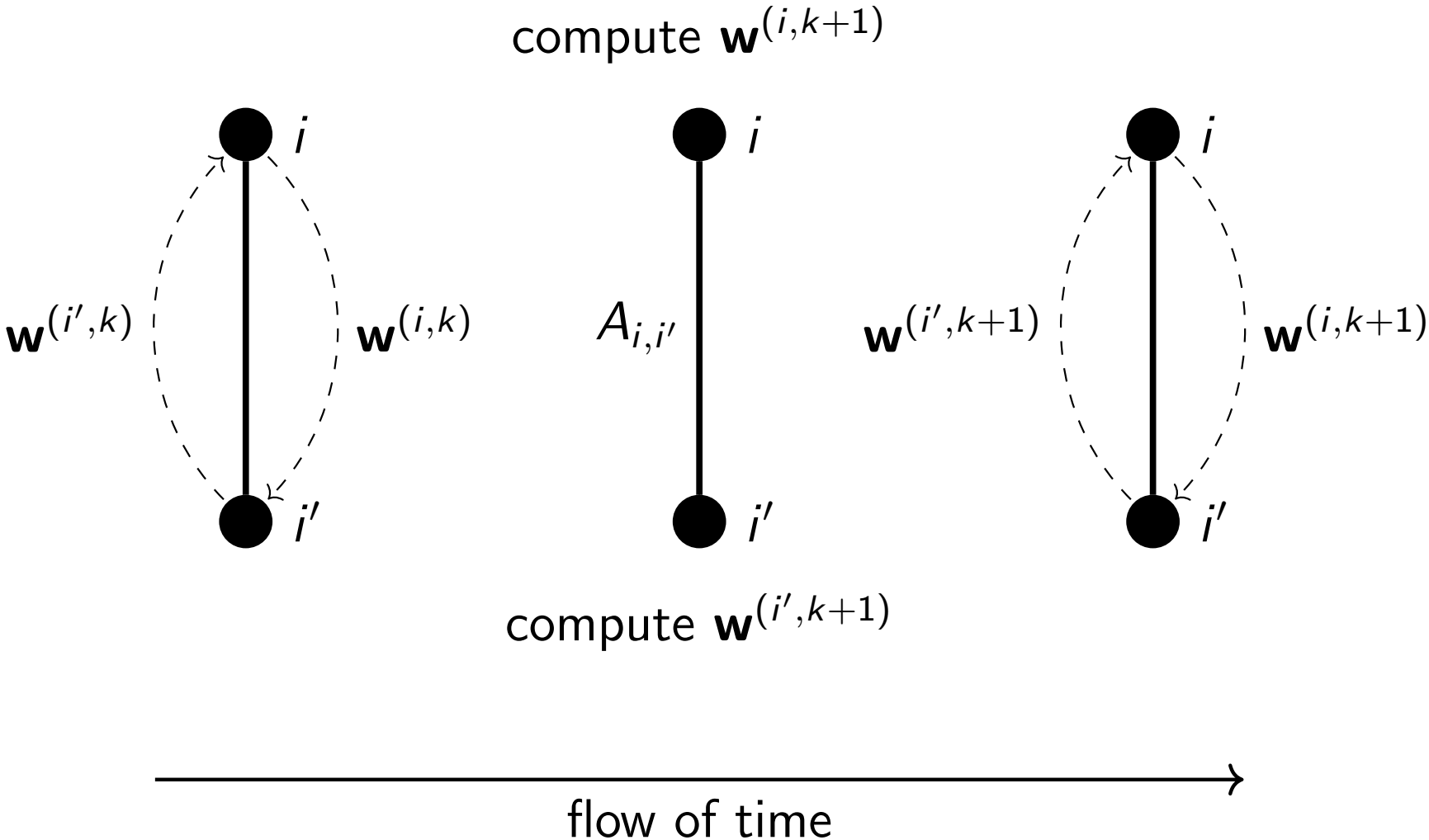
$$\mathbf{w}^{(i,k+1)} = \mathbf{w}^{(i,k)} - \eta_{k,i} \left[\nabla L_i \left(\mathbf{w}^{(i,k)} \right) + 2\alpha \sum_{i' \in \mathcal{N}^{(i)}} A_{i,i'} \left(\mathbf{w}^{(i,k)} - \mathbf{w}^{(i',k)} \right) \right]$$

has to be carried out by all nodes $i = 1, \ldots, n$.

When these (local) gradient steps are completed, each node shares its new model params. with its neighbours.

After sharing the model params., start new iteration $k := k+1$.

Message Passing Implementation



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Federated Gradient Descent (FedGD)

Each node $i = 1, \ldots, n$ initializes

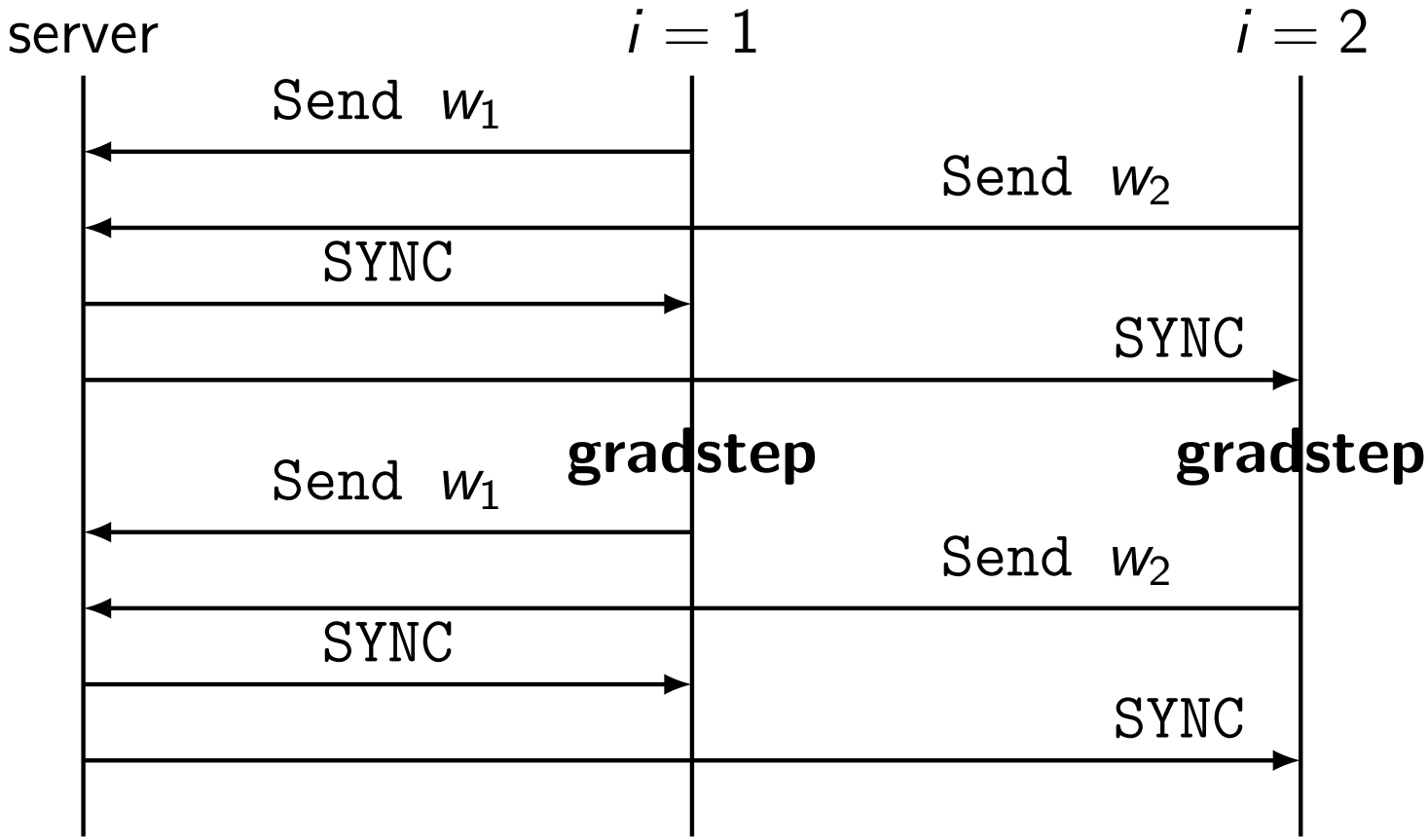
- local model params. $\widehat{\mathbf{w}}_0^{(i)} := \mathbf{0}$, and
- iteration counter $k := 0$.

Repeat the following steps at each node i:

- Send $\widehat{\mathbf{w}}_k^{(i)}$ to all neighbours $\mathcal{N}^{(i)}$.
- Do a gradient step.
- Increment iteration counter $k := k + 1$.

CAUTION: Nodes must execute steps synchronously!

Implementing FedGD with a Sync-Server



Python demo

Federated Stochastic Gradient Descent (FedSGD)

- Consider FL network with node i carrying local dataset

$$\mathcal{D}^{(i)} = \left\{ \left(\mathbf{x}^{(i,1)}, y^{(i,1)}\right), \ldots, \left(\mathbf{x}^{(i,m_i)}, y^{(i,m_i)}\right)\right\}.$$

- Node i uses local loss function

$$L_i\left(\mathbf{w}^{(i)}\right) := (1/m_i) \sum_{r=1}^{m_i} \left(y^{(i,r)} - \left(\mathbf{w}^{(i)}\right)^T \mathbf{x}^{(i,r)} \right)^2.$$

- FedGD requires to compute gradient,

$$\nabla L_i\left(\mathbf{w}^{(i)}\right) = (-2/m_i) \sum_{r=1}^{m_i} \mathbf{x}^{(i,r)} \left(y^{(i,r)} - \left(\mathbf{w}^{(i)}\right)^T \mathbf{x}^{(i,r)} \right).$$

Stochastic Gradient Approximation

For some applications, the computation of

$$\sum_{r=1}^{m_i} \mathbf{x}^{(i,r)} \left(y^{(i,r)} - \left(\mathbf{w}^{(i)}\right)^T \mathbf{x}^{(i,r)} \right)$$

is intractable, e.g., too many data points or too slow access.

$\Rightarrow$ Use instead a sum over random subset $\mathcal{B} \subseteq \{1, \ldots, m_i\}$,

$$\sum_{r \in \mathcal{B}} \mathbf{x}^{(i,r)} \left(y^{(i,r)} - \left(\mathbf{w}^{(i)}\right)^T \mathbf{x}^{(i,r)} \right).$$

We refer to $\mathcal{B}$ as batch with batch size $|\mathcal{B}|$.

Federated Averaging (FedAvg)

- Some FL applications use common model at all nodes,

$$\mathbf{w}^{(i)} = \mathbf{w}^{(i')} \quad \forall\, i, i' \in \mathcal{V}.$$

- GTVMin becomes constrained optimization problem:

$$\min_{\mathbf{w}^{(1)},\ldots,\mathbf{w}^{(n)}} \sum_{i \in \mathcal{V}} L_i\left(\mathbf{w}^{(i)}\right) \text{ s.t. } \quad \mathbf{w}^{(i)} = \mathbf{w}^{(i')} \quad \forall\, i, i' \in \mathcal{V}.$$

- For diff. $L_i\left(\mathbf{w}^{(i)}\right)$ we can apply projected GD.

- Projection step amounts to averaging $(1/n)\sum_{i=1}^{n} \mathbf{w}^{(i)}$.

(Almost) FedAvg

Init. counter (clock) $k := 0$ and model params $\widehat{\mathbf{w}} := \mathbf{0}$.

1. **Broadcast.** Server sends $\widehat{\mathbf{w}}$ to all nodes $i \in \mathcal{V}$.
2. **Local Gradient Step.** Each node computes

$$\mathbf{w}^{(i,k)} = \widehat{\mathbf{w}} - \eta_{k,i} \nabla L_i \left(\widehat{\mathbf{w}} \right).$$

3. **Collect.** Nodes send $\mathbf{w}^{(i,k)}$ back to server.
4. **Aggregate.** Server computes $\widehat{\mathbf{w}} := (1/n) \sum_{i=1}^{n} \mathbf{w}^{(i,k)}$.
5. **Clock Tick.** Server increments $k := k + 1$. Go to step 1.

FedAvg

We obtain FedAvg via the following modifications:[1]

- Use (stochastic) approximations of gradients.
- Instead of single GD step, compute several GD steps.
- Only a subset of nodes compute local updates during each iteration.

[1] B. McMahan et.al., Communication-Efficient Learning of Deep Networks from Decentralized Data, PMLR, 2017

FedProx

FedProx replaces GD steps in FedAvg with[2]

$$\mathbf{w}^{(i)} := \underset{\mathbf{v}\in\mathbb{R}^d}{\operatorname{argmin}} \left[L_i(\mathbf{v}) + (1/\eta) \left\| \mathbf{v} - \widehat{\mathbf{w}}^{(\text{global})} \right\|_2^2 \right].$$

Empirical studies found FedProx to result in more robust FL systems compared to FedAvg.

FedProx seems to handle well varying computational power of devices.

[2] T. Li, et.al, Federated Optimization in Heterogeneous Networks, Proc. of Machine Learning and Systems 2, 2020.

Federated Relaxation (FedRelax)

- Consider GTVMin objective function

$$f\left(\mathbf{w}^{(1)},\ldots,\mathbf{w}^{(n)}\right)=\sum_{i=1}^{n} L_i\left(\mathbf{w}^{(i)}\right)+\alpha\sum_{\{i,i'\}\in\mathcal{E}} A_{i,i'}\left\|\mathbf{w}^{(i)}-\mathbf{w}^{(i')}\right\|_2^2.$$

- Complicated due to coupling terms $A_{i,i'}\left\|\mathbf{w}^{(i)}-\mathbf{w}^{(i')}\right\|_2^2$.

- Without coupling, GTVMin would be much easier.

- Optimize $f(\cdot)$ w.r.t. $\mathbf{w}^{(i)}$, holding $\left\{\mathbf{w}^{(i')}\right\}_{i'\in\mathcal{V}\setminus\{i\}}$ fixed![3]

[3] Similar idea is used in the Jacobi method for solving linear equations.

FedRelax for Parametric Models

- **Init.** Set counter $k := 0$, local model params. $\widehat{\mathbf{w}}_0^{(i)} := \mathbf{0}$.
- **Repeat until stopping criterion:**
  - Each node i shares $\widehat{\mathbf{w}}_k^{(i)}$ with neighbours $\mathcal{N}^{(i)}$.
  - **Local Update.** Each node i computes

$$\widehat{\mathbf{w}}_{k+1}^{(i)} := \underset{\mathbf{w}^{(i)} \in \mathbb{R}^d}{\operatorname{argmin}} \, L_i\left(\mathbf{w}^{(i)}\right) + \alpha \sum_{i' \in \mathcal{N}^{(i)}} A_{i,i'} \left\| \mathbf{w}^{(i)} - \widehat{\mathbf{w}}_k^{(i')} \right\|_2^2 .$$

  - **Clock Tick.** $k := k + 1$.

Note the similarity of local update with ridge regression!

FedRelax for Non-Parametric Models

- **Init.** $k := 0$, construct test-set $\mathcal{D}^{\{i,i'\}}$ for each $\{i, i'\} \in \mathcal{E}$
- **Repeat until stopping criterion:**
  - Each i shares $\widehat{h}_k^{(i)}(\mathbf{x})$ for each $\mathbf{x} \in \mathcal{D}^{\{i,i'\}}$ and $i' \in \mathcal{N}^{(i)}$.
  - **Local Update.** Each node i computes

$$\widehat{h}_{k+1}^{(i)} \in \underset{h^{(i)} \in \mathcal{H}^{(i)}}{\operatorname{argmin}} L_i\left(h^{(i)}\right) + \alpha \sum_{i' \in \mathcal{N}^{(i)}} A_{i,i'} D\left(h^{(i)}, \widehat{h}_k^{(i')}\right).$$

  - **Clock Tick.** $k := k + 1$.

Here, we use the discrepancy measure

$$D\left(h^{(i)}, h^{(i')}\right) := (1/m') \sum_{\mathbf{x} \in \mathcal{D}^{\{i,i'\}}} \left[h^{(i)}(\mathbf{x}) - h^{(i')}(\mathbf{x})\right]^2.$$

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FL Algorithms are Fixed-Point Iterations

- Consider FL net. with parametric local models.
- FL algorithms presented so far can be written as

$$\mathbf{w}^{(i,k+1)} = \mathcal{F}^{(i)}\left(\mathbf{w}^{(1,k)}, \ldots, \mathbf{w}^{(n,k)}\right).$$

- This is a synchronous fixed-point iteration with operators

$$\mathcal{F}^{(i)} : \mathbb{R}^{nd} \to \mathbb{R}^{d} \text{ , for each node } i = 1, \ldots, n.$$

Challenges of Synchronous FL

Implementing synchronous fixed-point iteration is challenging.

- Devices might have limited computational resources.
- The evaluation of local loss might require data collection.
- Message passing over unreliable wireless links..
- Devices might spontaneously join or drop out.

Asynchronous FL Algorithms

- Consider a FL algorithms with fixed point operators $\mathcal{F}^{(i)}$.
- We obtain an asynchronous variant by the update

$$\mathbf{w}^{(i,k+1)} = \mathcal{F}^{(i)}\left(\mathbf{w}^{(1,k_{i,1})}, \ldots, \mathbf{w}^{(n,k_{i,n})}\right).$$

- k is not a clock tick but only some event index.
- $k - k_{i,i'}$ represents communication delay from i' to i.