LU FACTORIZATION METHOD

by
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$A = \begin{bmatrix} \quad \end{bmatrix}$; $b = \begin{bmatrix} \quad \end{bmatrix}$; $X = \begin{bmatrix} \vdots \end{bmatrix}$

$AX = b$

STEPS

- To solve $AX=b$ (to find X)
- Write $A= LU$
- Then $(LU)X=b$
- $L(UX)=b$
- $LY= b$ (put $UX=Y$)
- Solve $LY = b$ first. (you have Y now)
- Solve $UX=Y$ now

$\downarrow$

x

SOLVE

- $2x-6y+8z=24$
- $5x+4y-3z=2$
- $3x+y+2z=16$

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$Ax = b$

$b = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$A = LU$

$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$

$l_{11} = 2$

$l_{21} = 5$

$l_{11} u_{12} = -6$

$u_{12} = -3$

$l_{21} u_{12} + l_{22} = 4$

$l_{22} = 19$

$l_{11} u_{13} = 8$

$u_{13} = 4$

$l_{21} u_{13} + l_{22} u_{23} = -3$

$u_{23} = -\frac{23}{19}$

$$\begin{bmatrix} 2 & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & -3 & 4 \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\boxed{l_{21} = 5}$$

$$-3 l_{21} + l_{22} = 4$$

$$\boxed{l_{22} = 19}$$

$$4 l_{21} + l_{22} u_{23} = -3$$

$$\boxed{u_{23} = -\frac{23}{19}}$$

$$l_{31} = 3$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 5 & 19 & 0 \\ 3 & 10 & \frac{40}{19} \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -3 & 4 \\ 0 & 1 & -\frac{23}{19} \\ 0 & 0 & 1 \end{bmatrix}$$

$$4 \times 3 + -\frac{23}{19} 10 + l_{33} = 2$$

$$4 l_{31} + u_{23} l_{32} + l_{33} = 2$$

$$\overset{L}{\begin{bmatrix} 2 & 0 & 0 \\ 5 & 19 & 0 \\ 3 & 10 & \frac{40}{19} \end{bmatrix}} \overset{Y}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$$U_{3\times 3} X_{3\times 1} = Y_{3\times 1} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$2y_1 = 24$$

$$y_1 = 12$$

$$5y_1 + 19y_2 = 2$$

$$60 + 19y_2 = 2$$

$$y_2 = -\frac{58}{19}$$

$$3y_1 + 10y_2 + \frac{40}{19}y_3 = 16$$

$$36 + -\frac{580}{19} + \frac{40}{19}y_3 = 16$$

$$y_3 = \left[16 - 36 + \frac{580}{19}\right]\frac{19}{40}$$

$$= 5$$

$$UX = Y$$

$$\begin{bmatrix} 1 & -3 & 4 \\ 0 & 1 & -\frac{23}{19} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -\frac{58}{19} \\ 5 \end{bmatrix} \qquad \begin{cases} x = 1 \\ y = 3 \\ z = 5 \end{cases}$$

$$z = 5$$

$$y - \frac{23}{19} \times z = -\frac{58}{19}$$

$$y = \frac{115}{19} - \frac{58}{19} = \frac{57}{19} = 3$$

$$x - 3y + 4z = 12$$

$$x - 9 + 20 = 12$$

$$x = 1$$

$$(1, 3, 5)$$

$$2x - 3y + 10z = 3$$

$$-x + 4y + 2z = 20$$

$$5x + 2y + z = -12$$

LU

1.8239

25

1.824 | 4

25

1.824

1.8244 | 5

=5

even

odd

1.823

1.824

1.824