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REGULAR ARTICLE

Application of a real-world university-course timetabling model solved by integer programming

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Abstract In this case study, we describe an integer programming (IP) approach, which has been implemented at the School of Economics and Management at Hannover University, Germany, to create the complete timetable of all courses for a term. Approximately 150 different weekly lectures, tutorials and seminars ranging from 5 to 650 students are taught by about 100 teachers. The decision problem is to assign these teaching groups to time slots and rooms so that several soft and hard constraints are met. It is modeled as an assignment problem with numerous types of constraints and about 100,000 binary or integer variables. An open source mixed-integer solver can be used to solve the problem to optimality within minutes whereas the commercial CPLEX solver takes only seconds. We also describe the implementation process and report results from an anonymous satisfaction survey among the faculty with respect to the new planning approach.

Keywords University timetabling · Academic timetabling · Integer programming

1 Introduction

University course timetabling is the problem of assigning periodically repeated teaching groups to rooms and time slots of a week. The resulting weekly

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schedule is used to organize the teaching process at a university in time and space during the course of a term (also called a semester). A teaching group is defined by a subject or topic, the teacher(s) and in some cases a specific set of students. The number of teaching groups and the assignment of teachers to the different groups is often given to the course timetabling problem. Both the objectives and the constraints of this problem are usually highly institution-specific. It is therefore not surprising, that there is not a single commonly used tool to solve this planning problem, even though a rich body of literature on academic, school and university timetabling exists.

The solution of the timetabling problem is important to very different stakeholders of the teaching and study process. It determines the schedules of both students and faculty who have very different criteria to evaluate the quality of a schedule. It also determines the required room capacity that can constitute a major bottleneck in the teaching process. The relative power of faculty, students, school and university administration therefore determines how the planning process is organized and to which schedule it eventually leads.

This paper reports our experience from the implementation of a formalized planning process, based on a complex integer programming model at the School of Economics and Management at Hannover University in Germany. The second author of this paper at that time was “Associate Dean for Study Programs” and therefore responsible for the development of degree programs, as well as the organization and quality assurance of the school-wide teaching and examination process. This also includes the responsibility for the course timetables which became more important due to the recent introduction of tightly structured BSc/MSc-programs and tuition at our institution.

Introductions to the basic elements of (automated) timetabling, approaches to solve the problem and surveys of course and examination timetabling can be found, e.g. in Carter and Laporte (1998), De Werra (1985) and Schaerf (1999). For a review of recent developments in timetabling we refer to the surveys in Asratian and De Werra (2002), Baker et al. (2002), Burke and Petrovic (2002), Lange (2005) and Petrovic and Burke (2004). Methods to solve the problem are primarily based on three different approaches: local search (see e.g. Di Gaspero and Schaerf 2003), constraint logic programming (see e.g. Rudová and Murray 2003) and integer or mixed-integer programming, as presented in this paper.

While there are numerous publications on university course timetabling models and methods, the number of papers describing actual implementations of automated timetabling systems appears to be rather limited. In many cases, numerical results are reported for data sets that are “inspired” by a real-world setting, but it remains unclear to which extent an automated timetabling system has actually been implemented. The performance of an automated timetabling system, however, can only be assessed when it is used. This includes teacher, student and/or administration satisfaction which is rarely studied and reported in detail. In Martin (2004), an assignment-type integer programming model is presented that is used at Ohio University’s College of Business to schedule classes. At the Athens University of Economics and Business an integer programming model based on the aggregation of both teaching subjects and time is used, see Dimopoulou and Miliotis (2001). A different integer programming model for a Greek engineering department is tested on real-world data in Daskalaki et al. (2004). While that paper presents a rather sophisticated model, it does not become clear whether the approach has actually been implemented and what the results were. The application of constraint logic programming to schedule courses at the Computer Science Department of Munich University is described in Abdennadher and Marte (2000). Another approach based on constraint logic programming and tested on real-world data is presented in Deris et al. (1997).

The model presented in this paper was inspired by the general assignment-type integer programming approach in Haase et al. (2004). Their paper focusses on algorithmic aspects of a column-generation based heuristic tailored to a relatively lean special case of our more general and application-oriented model.

Since the focus of our work was on the actual implementation of a formalized decision support system, it reflects the situation at our university. Our model (in contrast to many others) contains many constraints and objectives that are relevant for large universities like those in Germany where the powerful position of the professors determines the schedule to a large extent. For this reason, it is neither possible nor necessary to model individual students or classes of students and their particular preferences.

The remainder of this case study is organized as follows: In Sect. 2, we present the School of Economics and Management at Hannover University, its degree programs and the previously used timetabling approach in more detail. Section 3 is devoted to the optimization model, that formally states our timetabling problem. Section 4 describes the problem instance solved in Hannover and a scenario analysis with respect to the room requirements. In Sect. 5, we report our experience with the implementation of this approach and present results from a satisfaction survey among the teachers at the School of Economics and Management.

2 Timetabling at the School of Economics and Management in Hannover

We now briefly describe the School of Economics and Management at Hannover University which is in many ways typical for public universities in Germany. Hannover University has today approximately 24,000 students in nine schools, ranging from architecture to law. With a body of about 2,800 students, the 30 -year-old School of Economics and Management builds one of the largest schools of the university with 22 institutes, each responsible for a field such as Marketing, Banking and Finance, Business Taxation, Microeconomics or Public Finance.

The established approach to timetabling at our school was as follows: One term in advance, the various institutes used an intra-net application to report to the school administration, which courses in single or multiple groups, at which time and in which room they planned to teach. Very often the plan of the previous year served as a starting point, i.e. the teachers tended to replicate this plan. The institutes would also report the expected number of students, based on the experience from previous years. In this process, a single room might have been requested for several different teaching groups at the same time by teachers from different institutes. A junior member of the administrative staff then identified conflicts between core courses and with respect to room availability. In a tedious and time-consuming bargaining process, he contacted those professors or assistants who had been cooperative in earlier years and asked them to accept a different room and/or time slot in order to eliminate the most important conflicts with respect to the most important courses. This approach led to a timetable which reflected teachers' preferences much more than students' needs or wishes.

Our school schedules teaching groups in 2-h time grids throughout the day. Teaching groups are assigned to time slots between 08:15–09:45, 10:00–11:30, 12:30–14:00, 14:15–15:45, 16:15–17:45 and 18:15–19:45. Since the standard duration of a teaching group is only 90 minutes, there is always enough time for transitions.

3 Modelling the timetabling problem as an assignment problem

3.1 Assumptions

The assumptions of our model can be separated into four classes: assumptions concerning time, rooms, courses and teaching groups and teachers' preferences. The assumptions are labelled to facilitate the explanation of the model. The symbols and parameters are first introduced as they are related to these assumptions and then presented jointly in a separate subsection on the notation of the model.

Time

- A1 Time is divided into discrete slots consisting of days t (Monday to Friday) and intra-day periods h (e.g. 90-min intervals).
- A2 Transition times are not considered.

Rooms

- B1 Rooms differ only with respect to their size. Rooms of the same size are combined to a room type r.
- B2 Room types are classified according to decreasing size.
- B3 The number of regularly available rooms CR_{rth} of room type r depends on day t and intra-day time slot h.
- B4 If more than CR_{rth} rooms of type r are required at day t and intra-day time slot h, it is assumed that those additional rooms can be rented. However, this causes extraordinary high costs.

Courses and teaching groups

- C1 Each course v is taught in one or multiple teaching groups $g \in GV_v$. Multiple teaching groups for a course are installed to limit class sizes.

C2 Each group g requires one or more teachers d. If there is a team of teachers required for a given teaching group, all of them have to be assigned to the same time slot and room (type).

C3 A teaching group must be assigned to a single time slot.

C4 Linked teaching groups must be assigned to the same room type t and consecutive time slots of the same day.

C5 For each year of study several core courses may not be scheduled in parallel. This also holds for very attractive combinations of electives. All those are gathered in conflict groups. Courses $v \in VK_k$ in conflict group k may not be scheduled in parallel. This holds for all teaching groups $g \in GV_v$ of course $v \in VK_k$.

C6 Teaching groups $g \in GP_p$ in parallelism group p should be scheduled in parallel.

C7 Teaching groups $g \in GI_i$ in the same-time-of-day-group i should be assigned to the same intra-day time slot h of one or several days of the week.

C8 For each sequence a the single teaching group af_a has to precede the single teaching group ef_a.

C9 Teaching groups in set GTH_{th} are preassigned to day t and intra-day time slot h.

C10 For each teaching group g there is a preferred room type pr_g. Assigning this teaching group to a room type with smaller rooms is heavily penalized, while assigning this teaching group to a room type with larger rooms is penalized only very slightly.

Teacher's preferences

D1 Teachers $d \in DG_g$ teaching group g have preferences pt_{thg} for each time slot h of day t (a lower value indicating a higher preference).

D2 They may request breaks between two of their teaching groups g assigned to the same day d.

D3 They can state a maximum number of teaching groups per day.

D4 They can state a maximum number of teaching days per week.

D5 They may ask for a certain amount of breaks (days without teaching) between teaching days.

D6 They may ask for consecutive days of teaching in order to use the rest of the week for research.

D7 They may wish to have certain teaching groups g distributed to different days d.

D8 They may wish to have certain teaching groups assigned to the same day.

D9 They may state time slots during which they may not be scheduled for any teaching at all.

3.2 Notation

Sets and indices

$a \in A$	sequence groups, used to establish time sequences between teaching groups (e.g. lecture before exercise)
af_a	first teaching group of sequence group a
al_l	first teaching group of link l
$d, \tilde{d} \in D$	teachers
DG_g	set of teachers of teaching group g
ef_a	second teaching group of sequence group a
el_l	second teaching group of link l
$et \in ET$	same-day groups, used to assign teaching groups to the same day of the week
$f \in F$	distribution groups, used to spread teaching groups over different days
$g, \tilde{g} \in G$	teaching groups (defined by content and teacher)
GD_d	set of teaching groups of teacher d
GET_{et}	set of teaching groups of same-day group et
GF_f	set of teaching groups of distribution group f
GI_i	set of teaching groups of same-time-of-day-group i (to be taught during the same time of day of one or several days)
GP_p	set of teaching groups of parallelism group p
GTH_{th}	set of teaching groups exogenously assigned to day t and time slot h
GV_v	set of teaching groups of course v
$h, \tilde{h} \in H$	intra-day periods (time slots) of a day (ordered set)
$i \in I$	same-time-of-day-groups (to be scheduled in identical time slots possibly at different days)
$k \in K$	conflict groups, used to avoid parallel teaching of (e.g. mandatory) courses
$l \in L$	links between teaching groups (to be scheduled in consecutive time slots)
$p \in P$	parallelism groups, used to enforce parallel scheduling of teaching groups
$r \in R$	room types of different sizes (ordered set)
$t \in T$	days of the week (ordered set)
$(t, h) \in T \times H = TH$	combination of day and intra-day time slot
$\overline{THD}_d$	set of time slots of the week a teacher d may not be assigned to

$v \in V$	courses (defined by content), taught in one or more teaching groups
VK_k	set of courses of conflict group k

The "card(.)"-operator gives the cardinality of a set, i.e. the number of elements in the set. The "ord(.)"-operator gives the position of an element of a set that forms a sequence.

Input data

CR_{rth}	number of available rooms of type r during time slot h of day t
$disth_d$	minimum break (in time slots) between two teaching groups of teacher d assigned to the same day
$distt_d$	minimum break between two teaching days of teacher d (in days)
$ltok_d$	parameter which equals 1, if teacher d does not mind days without teaching (gaps) between teaching days, (0 otherwise)
mxg_d	maximum number of groups per day teacher d may be scheduled for
mxt_d	maximum number of days per week teacher d may be scheduled for
pr_g	preferred room type for teaching group g ($1 \leq pr_g \leq \mathrm{card}(R)$)
pt_{thg}	teacher's preference value for teaching group g at day t during time slot h
π_{rthdg}	preferences of teacher d for teaching group g in a room of type r, at day t and time slot h (computed from pr_g and pt_{thg})
sr_r	penalty for the violation of the capacity CR_{rth} of rooms of type r (cost of renting additional rooms)
sf_f	penalty for the violation of distribution group f
si_i	penalty for the violation of same-time-of-day-group i
sk_k	penalty for the violation of conflict group k
sp_p	penalty for the violation of parallelism group p
st_{et}	penalty for the violation of same-day group et

Decision variables

b_{ft}	magnitude of the violation of distribution group f during day t
e_{dt}	binary variable which equals 1, if teacher d teaches during day t, (0 otherwise)
ft_{dt}	auxiliary binary variable to compute a bound on the difference between $e_{d,t-1}$ and e_{dt}

m_{grth} binary variable which equals 1, if teaching group g is scheduled for day t in time slot h in room type r, (0 otherwise)

n_{ih} binary variable which equals 1, if at least one teaching group g of same-time-of-day-group i is scheduled for any day t during time slot h, (0 otherwise)

o_{ih} magnitude of the violation of same-time-of-day-group i during time slot h

q_{pth} magnitude of the violation of parallelism group p during day t in time slot h

$qe_{et,t}$ magnitude of the violation of same-day group et during day t

s_{vth} binary variable which equals 1, if at least one teaching group g of course v is scheduled for day t and time slot h, (0 otherwise)

u_{rth} number of additional rooms of room type r required at day t during time slot h

w_{kth} magnitude of the violation of conflict group k during day t in time slot h, equals 1, if two teaching groups of courses in conflict group k are scheduled for day t during time slot h; equals 2 in case of three groups, etc.

x_{rthdg} binary variable which equals 1, if teacher d teaches teaching group g at day t during time slot h in a room of type r, (0 otherwise)

z_{pth} binary variable which equals 1, if at least one teaching group g of parallelism group p is scheduled for day t during time slot h, (0 otherwise)

$ze_{et,t}$ binary variable which equals 1, if at least one teaching group g of same-day group et is scheduled for day t, (0 otherwise)

3.3 Formulation

The *Model Timetable* consists of the objective function and 20 sets of constraints. The restrictions are classified into four categories: teachers' basic assignment restrictions, school-specific requirements, constraints resulting from the institutes' perspective and restrictions related to the teachers' preferences. The objective function (1) sums

- the preferences of all teachers d referring to the room and time slot combination of a group the teachers teaches ($g \in GD_d$) given by the actual schedule in x_{rthdg},
- the violation of room type capacity CR_{rth} during any time slot counted by the integer variable u_{rth} multiplied with the penalty sr_r;
- the violation w_{kth} of conflict group k during any time slot weighted with the penalty sk_k;
- the violation q_{pth} of parallelism group p in any time slot multiplied with the penalty sp_p;
- the violation o_{ih} of same-time-of-day-group i for all intra-day periods multiplied with the penalty si_i;
- the violation b_{ft} of the distribution groups f at any day t weighted with the penalty sf_f; and
- the violation $qe_{et,t}$ of same-day group et weighted with the penalty st_{et}.

$$\begin{aligned}\text{Min } Z = &\sum_{r\in R}\sum_{t\in T}\sum_{h\in H}\sum_{d\in D}\sum_{g\in GD_d} \pi_{rthdg}\cdot x_{rthdg}\\ &+\sum_{t\in T}\sum_{h\in H}\left(\sum_{r\in R} sr_r\cdot u_{rth}+\sum_{k\in K} sk_k\cdot w_{kth}+\sum_{p\in P} sp_p\cdot q_{pth}\right)\\ &+\sum_{i\in I}\sum_{h\in H} si_i\cdot o_{ih}+\sum_{t\in T}\left(\sum_{f\in F} sf_f\cdot b_{ft}+\sum_{et\in ET} st_{et}\cdot qe_{et,t}\right) \end{aligned} \tag{1}$$

Teachers' basic assignment restrictions. Constraints (2) ensure that every teacher d is assigned to one of his allowed time slots (all possible time slots ($(t,h) \in TH$) without the forbidden ones ($\overline{THD}_d$)) for each of these teaching groups g (assumption D9). Constraints (3) guarantee that a teacher can be scheduled only for one of his teaching groups g at any time. If more than one teacher teaches group g, all card(DG_g) of them have to be assigned simultaneously (4) to the same room of type r, see assumption C2. Constraints (5) measure the extent u_{rth} to which the number CR_{rth} of regularly available rooms of type r is exceeded for all days and time slots. Even if there is more than one teacher assigned to a room for the same teaching group nevertheless only one room is needed. Therefore, the sum of the decision variables has to be divided by the number of teachers card(DG_g) teaching group g.

$$\sum_{r\in R}\sum_{(t,h)\in TH\setminus\overline{THD}_d} x_{rthdg}=1 \qquad d\in D, g\in GD_d \tag{2}$$

$$\sum_{r\in R}\sum_{g\in GD_d} x_{rthdg}\le 1 \qquad d\in D, t\in T, h\in H \tag{3}$$

$$\sum_{d \in DG_g} x_{rthdg} = \text{card}(DG_g) m_{grth} \qquad g \in G, \text{card}(DG_g) > 1, \; r \in R, t \in T, h \in H \tag{4}$$

$$\sum_{d \in D} \sum_{g \in GD_d} \frac{x_{rthdg}}{\text{card}(DG_g)} - u_{rth} \leq CR_{rth} \quad r \in R, t \in T, h \in H \tag{5}$$

School-related constraints. In this class of restrictions we handle constraints and preferences of the school as a whole. Conflict groups are defined for courses v taught in possibly multiple teaching groups g. To get the connection of courses v to teaching groups g we use the auxiliary variables s_{vth}. In constraints (6), the binary variables s_{vth} are forced to 1, if one of the teaching groups of course v is scheduled at day t during time slot h. In constraints (7) this information is used to calculate the violation w_{kth} of conflict group k at any time.

$$\sum_{r \in R} \sum_{d \in DG_g} \frac{x_{rthdg}}{\text{card}(DG_g)} - s_{vth} \leq 0 \qquad v \in V, g \in GV_v, t \in T, h \in H \tag{6}$$

$$\sum_{v \in V_k} s_{vth} - w_{kth} \leq 1 \qquad k \in K, t \in T, h \in H \tag{7}$$

Constraints (8) ensure that the *linked teaching groups* al_l and el_l related to link l are scheduled in consecutive time slots in the same room type r at the same day t, see assumption C4.

$$x_{rthd,al_l} = x_{rt,h+1,\tilde{d},el_l} \qquad l \in L, \; d \in DG_{al_l}, \tilde{d} \in DG_{el_l}, r \in R, \; t \in T, h \in H, \text{ord}(h) < \text{card}(H) \tag{8}$$

The *fixed teaching groups* are scheduled for their exogenously given day t and a time slot h in constraints (9). If there is more than one teacher in such a group, the cardinality of the respective set of teachers has to be taken into account.

$$\sum_{r \in R} \sum_{d \in DG_g} \frac{x_{rthdg}}{\text{card}(DG_g)} = 1 \quad t \in T, h \in H, g \in GTH_{th} \tag{9}$$

Institutes' restrictions. Constraints (10) guarantee that teaching group af_a related to **sequence group** a (e.g. a lecture dealing with operations research), is scheduled before teaching group ef_a (e.g. a tutorial for operations research), see assumption C8. For this purpose, it is necessary to convert the two-dimensional composed time scheme (t, h) into consecutive time slots $1, \ldots, \text{card}(T) \cdot \text{card}(H)$.

$$\sum_{r\in R}\sum_{(t,h)\in TH}\sum_{d\in DG_{ef_a}}(\text{ord}(h)+(\text{ord}(t)-1)\cdot\text{card}(H))\cdot\frac{x_{rthd,ef_a}}{\text{card}(DG_{ef_a})}$$
$$-\sum_{r\in R}\sum_{(t,h)\in TH}\sum_{d\in DG_{af_a}}(\text{ord}(h)+(\text{ord}(t)-1)\cdot\text{card}(H))\cdot\frac{x_{rthd,af_a}}{\text{card}(DG_{af_a})}$$
$$\geq 1 \quad a\in A \tag{10}$$

According to assumption C6 teaching groups of *parallelism group* should be scheduled in parallel, if possible (11). Violations are measured via the integer variable q_{pth} and penalized in the objective function.

$$\sum_{r\in R}\sum_{g\in GP_p}\sum_{d\in DG_g}\frac{x_{rthdg}}{\text{card}(DG_g)}+q_{pth}=\text{card}(GP_p)z_{pth} \quad \begin{array}{l}p\in P,\\ t\in T,\\ h\in H\end{array} \tag{11}$$

Constraints (12) assign teaching groups of a *same-time-of-day-group* i to a single intra-day time slot h of one or several days (assumption C7). Violations are counted via the integer variable o_{ih} and penalized in the objective function. The reasoning is similar to that for the parallelism groups with the exception, that an assignment to different days is allowed.

$$\sum_{r\in R}\sum_{t\in T}\sum_{g\in GI_i}\sum_{d\in DG_g}\frac{x_{rthdg}}{\text{card}(DG_g)}+o_{ih}=\text{card}(GI_i)n_{ih} \quad i\in I,h\in H \tag{12}$$

Teachers' preferences. Besides the temporal preferences for a particular teaching group, teachers have further preferences with respect to teaching hours (e.g. breaks between teaching groups), teaching days (e.g. the number of days dedicated to teaching) and combinations of teaching days and teaching groups. A first set of constraints is related to *teaching hours*. Constraints (13) ensure that the favored temporal distance (or break) $disth_d$ between two time slots of teaching is respected. The sum of the decision variable x_{rthdg} in the time interval $[h, h+disth_d]$ may not exceed 1 unless the latter of two consecutive groups is linked to the first via a link l in equation (8). Constraints (14) limit the maximum number of teaching groups per day for teacher d.

$$\sum_{r\in R}\left(\sum_{g\in GD_d}\sum_{\substack{\tilde{h}\in H\\ ord(h)\leq ord(\tilde{h})\\ ord(\tilde{h})\leq ord(h)+disth_d}}x_{rt\tilde{h}dg}-\sum_{l\in L}x_{rt,h+1,d,el_l}\right)\leq 1 \quad \begin{array}{l}d\in D,\\ disth_d>0,\\ t\in T,h\in H,\\ \text{ord}(h)<\text{card}(H)\end{array} \tag{13}$$

$$\sum_{r\in R}\sum_{h\in H}\sum_{g\in GD_d}x_{rthdg}\leq mxg_d \quad d\in D,t\in T \tag{14}$$

A second set of restrictions is related to *teaching days*. In constraints (15) and (16) the binary variable e_{dt} are forced to a value of 1 if teacher d teaches on day t. While this is not the most compact formulation of these constraints, we found that the given form is computationally more efficient than a more compact formulation. Constraints (17) assure that the number of teaching days does not exceed the maximum number mxt_d, see assumption D4. According to assumption D5 some teachers would like to have breaks (distance in days, $distt_d$ between their teaching days (18)). Others do not want to have days without teaching (gaps) between their days of teaching. Equation (19) calculates the auxiliary binary variables ft_{dt} used in constraints (20) to monitor if the preferences concerning gaps are met: If teacher d wants his teaching days arranged consecutively, the sum of the auxiliary variable plus the value of $e_{d,\text{card}(T)}$ may at most equal 1 because $ltok_d$ is 0. Otherwise there may be breaks between the teaching days.

$$\sum_{g\in GD_d}\sum_{r\in R}\sum_{h\in H} x_{rthdg} \leq \text{card}(H)\cdot e_{dt} \qquad t\in T, d\in D \tag{15}$$

$$\sum_{g\in GD_d}\sum_{r\in R}\sum_{h\in H} x_{rthdg} \geq e_{dt} \qquad t\in T, d\in D \tag{16}$$

$$\sum_{t\in T} e_{dt} \leq mxt_d \qquad d\in D \tag{17}$$

$$\sum_{\substack{\tilde{t}\in T\\ ord(t)\leq ord(\tilde{t})\leq ord(t)+distt_d}} e_{d\tilde{t}} \leq 1 \qquad d\in D, t\in T \tag{18}$$

$$e_{d,t-1} - e_{dt} \leq ft_{dt} \qquad t\in T, \text{ord}(t)>1, d\in D \tag{19}$$

$$\sum_{\substack{t\in T\\ ord(t)>1}} ft_{dt} + e_{d,\text{card}(T)} \leq 1 \qquad d\in D, ltok_d = 0 \tag{20}$$

The combination of teaching days and teaching groups is treated in a third set of restrictions. Constraints (21) monitor whether the teaching groups $g\in GET_{et}$ in the same-day group et are assigned to the same day. The violations are counted in the variable $q_{et,t}$ and penalized in the objective function. As stated in assumption D7, the teaching groups $g\in GF_f$ in distribution group f should be assigned to different days, see constraints (22).

$$\sum_{r\in R}\sum_{h\in H}\sum_{g\in GET_{et}}\sum_{d\in DG_g} \frac{x_{rthdg}}{\text{card}(DG_g)} + qe_{et,t}$$

$$= \text{card}(GET_{et})ze_{et,t} \qquad et\in ET, t\in T \tag{21}$$

$$\sum_{r\in R}\sum_{h\in H}\sum_{g\in GF_f}\sum_{d\in DG_g} \frac{x_{rthdg}}{\text{card}(DG_g)} - b_{ft} \leq 1 \quad f\in F, t\in T \tag{22}$$

Domain of variables

$$x_{rthdg}, s_{vth}, z_{pth}, ze_{et,t}, m_{grth}, n_{ih}, e_{dt}, ft_{dt} \quad \in \{0,1\} \tag{23}$$

$$u_{rth}, w_{kth}, q_{pth}, qe_{et,t}, o_{ih}, b_{ft} \quad \geq 0 \text{ and integer} \tag{24}$$

All decision variables are either integer or binary.

4 Computational results

4.1 Input data

The model was tested by solving instances from the School of Economics and Management at Hannover University. The model was initially implemented in the General Algebraic Modeling System (GAMS IDE 2.0.31.8 Rev. 143) and solved with the CPLEX solver 9.1.2 [alternatively the open source solver Coin-Cbc (Forrest and Lougee-Heimer 2005)] on a 3 GHz Intel Pentium 4 processor with 4 GB RAM.

In our attempt to test the model by creating a schedule for the summer semester 2006, we had to respect the exogenously given timing of the engineering courses that our students in the Management and Engineering program have to take. We included these teaching groups as pre-assigned to their respective time slots and added a room type for these and other teaching groups taking place in "external" rooms, that are not regularly available to our teaching groups. In our case the input data consisted of

- five working days, $T = \{$Monday, Tuesday, …, Friday$\}$;
- six intra-day time slots, $H = \{$08:15–09:45, 10:00–11:30, 12:30–14:00, 14:15–15:45, 16:15–17:45, 18:15–19:45$\}$; and
- 11 room types r ordered by the number of seats, starting with the largest room as reported in Table 1. Only 10 of 11 room types represented rooms belonging to the School of Economics and Management. In two cases, there were two rooms in a room type whereas in the other eight cases a room type represented only a single room. The 10th room class ("PC") represents the school's computer labs and the 11th room type ("Other") represents "external" rooms under the administration of other schools of the university used for exogenously determined teaching groups.

Table 1 Description of room classes

Room type	1	2	3	4	5	6	7	8	9	PC	Oth.
# rooms	1	1	2	1	1	1	1	1	1	2	n/a
Seats/room	450	200	150	99	62	44	36	30	24	24	n/a
Slots (supply)	16	29	58	26	30	30	30	30	30	58	n/a
Slots (demand)	13	23	23	13	20	19	13	11	11	10	24
Utilization (%)	81	79	40	50	67	63	43	37	37	17	n/a

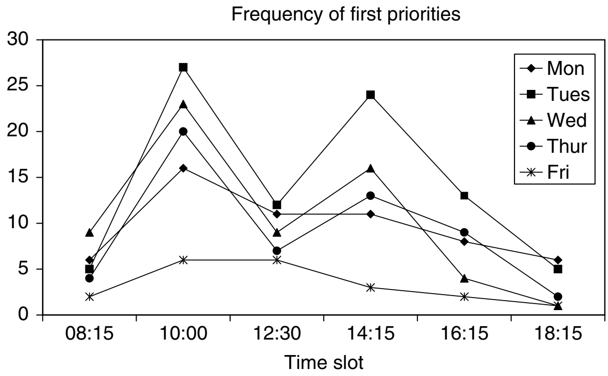

Fig. 1 Frequency of first priorities

We considered

- 99 teachers d;
- 156 courses v in
- 181 teaching groups g (24 of those taught and scheduled by other schools, see assumption C9);
- 29 conflict groups k with 2 to 26 courses (8.6 courses per conflict group on average);
- seven links l between teaching groups;
- seven sequence groups a;
- 31 distribution groups f; and
- six same-day groups et.

The problem instance did not include a parallelism group p or a same-time-of-day group i, as for this particular summer semester 2006 nobody requested us to enforce such a constraint.

The preference parameter π_{rthdg} used in the objective function (1) contains, on the one hand, the teachers' temporal preference pt_{thg} with integer values between 1 ("most preferable") and 5 ("least preferable") concerning each time slot (t,h) for a specific teaching group g. On the other hand, it also reflects the preference pr_g concerning the room type r for teaching group g with $1 \leq pr_r \leq 11$ $(= \text{card}(R))$. Figure 1 displays for each day t and intra-day time slot h, the number of teaching groups with a temporal preference value $pt_{thg} = 1$. Teaching during lunch time, in the evenings and on Friday afternoons is apparently not very attractive from most teachers' point of view. However, several teachers set their first priority for these apparently less attractive time slots.

The combined parameter π_{rthdg} was calculated as follows:

$$\pi_{rthdg} = pt_{thg} + \begin{cases} 0.05(pr_g - \text{ord}(r)), & \text{ord}(r) \leq pr_g \\ 100(\text{ord}(r) - pr_g), & \text{otherwise} \end{cases} \tag{25}$$

Table 2 Penalty costs

Violation of ...	Penalty cost parameter	Value
Room capacity	sr_r	10,000
Conflict groups	sk_k	1,000
Parallelism groups	sp_p	100
Distribution groups	sf_f	10
Same-time-of-day-groups	si_i	1
Same-day groups	st_{et}	100

Remember that room types were ordered by decreasing size (assumption B2). Assigning a teaching group to a room type r of rooms that are larger than required ($\text{ord}(r) < pr_g$) is only mildly punished in the first case of equation (25). However, the opposite assignment to a too small room type is practically unacceptable and therefore heavily punished in the second case of (25), see assumption C10.

The other penalty costs were assigned the values shown in Table 2.

From the modeling perspective, renting additional rooms is formally feasible. But in our practical setting, these rooms were simply not available. Therefore, we set the penalty cost parameter sr_r to a prohibitively high value to enforce a timetable respecting the capacity limits. The other objectives were treated as less important and therefore the respective penalty cost parameters were assigned lower values.

4.2 Results

The input data led to a GAMS model requiring 36 MB of RAM, consisting of 35,611 rows, 91,768 columns (i.e. 91,767 binary or integer variables) and 662,824 non-zeroes in the coefficient matrix. Despite the impressive size of the problem instance, we obtained a proven optimal integer solution after a computation time of 10.0 s. using CPLEX and 248.34 s. using the open-source solver Coin-Cbc. The determined schedule respected all of the "soft constraints" related to conflict groups k, distribution groups f, same-day-groups et and the limit on the "regularly" available room capacity, so that our solution was perfectly feasible with respect to all constraints that had been articulated. In particular, the schedule was conflict-free to an extent which our administrative staff found simply unthinkable. Figure 2 displays the number of scheduled teaching groups per time slot.

Now the question was to which extent this result was bought at the expense of the teachers' temporal preferences. Surprisingly, in only 28 cases teaching groups were not scheduled according to teachers' highest preferences. In 17 cases out of those 28, teachers were scheduled according to their second priority, in eight cases according to their third priority and in only three cases according to their fifth priority. In 14 cases, the teaching groups were shifted to bigger rooms than originally requested. This lead to a minor penalty which is reflected in the computation of the parameter π_{rthdg}, see Eq. (25).

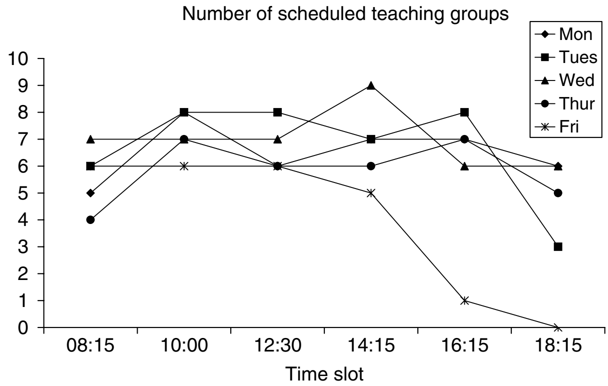

Fig. 2 Number of scheduled teaching groups

Table 3 Scenario analysis 1: successive elimination of large rooms

Scenario	Base case (1121111112X)	Scenario A1 (1111111112X)	Scenario A2 (1110111112X)	Scenario A3 (1110110112X)	Scenario A4 (1110110102X)
Util. 1 (%)	93.75	100.00	100.00	100.00	100.00
Util. 2 (%)	72.41	75.86	96.55	100.00	100.00
Util. 3 (%)	44.83	80.00	100.00	100.00	100.00
Util. 4 (%)	65.38	65.38	n/a	n/a	n/a
Util. 5 (%)	56.67	56.67	60.00	80.00	86.67
Util. 6 (%)	53.33	53.33	66.67	83.33	80.00
Util. 7 (%)	43.33	43.33	43.33	n/a	n/a
Util. 8 (%)	33.33	33.33	33.33	36.67	70.00
Util. 9 (%)	36.67	36.67	36.67	36.67	n/a
Util. PC (%)	17.24	17.24	17.24	17.24	17.24
Util. Oth. (%)	n/a	n/a	n/a	n/a	n/a
CPU (s)	10.00	9.66	31.11	98.34	92.97
Temp. obj.	1.27	1.28	1.42	1.56	1.56

The decision support system presented in this paper was not only useful to create a high-quality timetable for the summer semester 2006. It is also an extremely powerful instrument to analyze the use of resources in the teaching process. Lecture halls and smaller teaching rooms very often constitute a bottleneck in the teaching operation of a university. We asked for the consequences of reducing the number of available rooms on the quality of the schedule. The rooms in room classes 2–9 were at that time entirely under the control of our school.

In Table 3, we first present a base case using all the available room capacity. A series of scenarios A1–A4 reflects a reduced availability of large rooms. For each scenario, a string of the form "1121111112X" is given where the digits indicate the number of available rooms per room type, starting with the first room type.

Table 4 Scenario analysis 2: successive elimination of small rooms

Scenario	Base case (1121111112X)	Scenario B1 (1121111102X)	Scenario B2 (1121111002X)	Scenario B3 (1121110002X)	Scenario B4 (1121100002X)
Util. 1 (%)	93.75	93.75	93.75	93.75	100.00
Util. 2 (%)	72.41	72.41	72.41	72.41	89.66
Util. 3 (%)	44.83	44.83	46.55	65.52	86.21
Util. 4 (%)	65.38	65.38	69.23	84.62	96.15
Util. 5 (%)	56.67	56.67	66.67	80.00	96.67
Util. 6 (%)	53.33	53.33	73.33	86.67	n/a
Util. 7 (%)	43.33	50.00	76.67	n/a	n/a
Util. 8 (%)	33.33	63.33	n/a	n/a	n/a
Util. 9 (%)	36.67	n/a	n/a	n/a	n/a
Util. PC (%)	17.24	17.24	17.24	17.24	17.24
Util. Oth. (%)	n/a	n/a	n/a	n/a	n/a
CPU (s)	10.00	9.27	12.81	13.59	30.25
Temp. obj.	1.27	1.27	1.27	1.28	1.32

If the rooms in room classes 1 and 2, as presented in Table 1, are completely eliminated, it is not possible to create a feasible schedule. However, one can reduce the number of rooms in room type 3 from two to one. This scenario A1 still has a feasible solution that is presented in the third column of Table 3. The last column for scenario A4 shows, that four rooms could be eliminated. The average value of the temporal part of the objective function (teachers satisfaction with the timing of teaching groups) rose from a value of 1.27 in the base case to 1.56 (on a scale from 1 to 5). The computation times also increased.

In Table 4, we studied the opposite approach of successively eliminating small rooms, starting from the same base case. In this second series of scenarios B1–B4, it was also possible to eliminate four (small) rooms and still find a feasible timetable. Since large rooms can substitute small rooms, but not vice versa, the increase of the temporal part of the objective function from 1.27 to 1.32 is significantly smaller than in the first analysis in Table 3.

That is, from the teachers' point of view the school should rather give small rooms away than large rooms. In the actual implementation of the base case at our school, several changes were requested by the teachers. These were possible as in the base case there was ample free room capacity. In the rigid scenarios A4 and B4 such a change would have been impossible. In addition, several non-recurring events during a semester have to be considered. After the timetable has been generated, these events are scheduled manually using free rooms and time slots.

While we used CPLEX for most of our computations in this study as the solver, it should be noted that the open source solver CoinCbc (see Forrest and Lougee-Heimer (2005)) can be used to solve the base case within about four minutes to optimality. In our case it is therefore not necessary to use the much more powerful but also costly CPLEX solver, even though this is much more convenient, in particular when the utilization of rooms becomes tighter and the computation times increase.

5 Implementation and evaluation

It is a major decision to introduce a completely new timetabling approach for a relatively large university school. In the beginning, members of the faculty administration were extremely sceptical whether the planning process could be formalized at all and whether our model-based and centralized planning approach would be accepted by the faculty, because some were perceived as influential and/or stubborn. In this section, we describe the measures we took to avoid these problems and the experience we gained in the process of creating a schedule for the summer semester 2006.

Before the decision to invest a substantial amount of time and money in the development of a web-based decision support system was made, we wanted to ensure that our system yields superior timetables. When we used our system for the first time, we therefore worked with written questionnaires to gather the necessary information from our colleagues.

In this process we had to make sure that the faculty actually cooperated in the planning process. In particular, they had to reveal their preferences and restrictions to a much larger extent than they had previously been used to. In addition, they had to accept that they would no longer essentially decide themselves when to teach, but would be told when to teach. This loss of individual (temporal) freedom was certainly no incentive to cooperate. In a meeting of the professors, the general approach was briefly announced and explained and the professors were asked to cooperate. This did not include the technical details of the model. Afterwards, a letter containing the above-mentioned questionnaires was sent to all the institutes. It explained the reasons for this approach as well as the questionnaires, but not the model itself.

After solving the model, we were surprised by the high extent to which the teachers could be scheduled according to their individual top priorities. In only 28 out of 181 cases, a teaching group was scheduled during a time slot that was not the teachers' first priority. In 25 out of these 28 cases, a time slot with a second or third priority (out of a total of five) was assigned. We conjectured, that teachers' preferences, to some extent, already reflected implicit knowledge of constraints and possible conflicts. It seemed that they tended to repeat a schedule, that had "worked" in previous years.

In order to promote the acceptance of our approach, we sent the computed schedule to all institutes and asked them to point out any major problems. Faced with this tentative schedule, several teachers asked to be assigned to different rooms and/or time slots, even in cases were this led to a deterioration with respect to their originally reported preferences! However, as the schedule also reported the results for all of the conflict groups, it was easily possible to ensure that such a change would not create a conflict. We were in a position to grant almost all of these wishes, which helped a lot to achieve acceptance for the model-based planning approach.

To assess the satisfaction with this formalized approach, we initiated an anonymous satisfaction survey. We sent a questionnaire to all the professors and assistants who had been scheduled and asked them to report their satisfaction

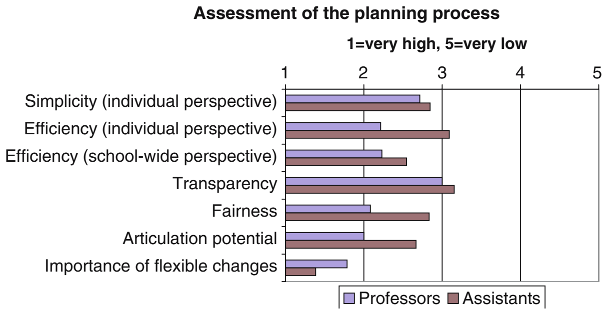

Fig. 3 Faculty's assessment of the planning process (averages)

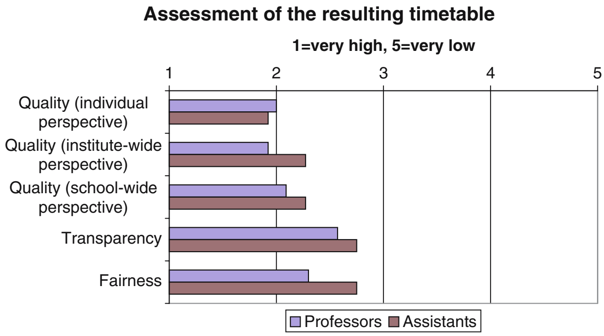

Fig. 4 Faculty's assessment of the resulting timetable (averages)

with the planning process and with the result. Approximately two-thirds of the professors and half of the assistants responded in this survey. (Being clearly biased, we did not answer our own questionnaires.) Figure 3 indicates that on average the professors were happier with the process than the assistants. This was not surprising, as the second group had only been informed about this test of a new timetabling approach in writing whereas professors had been asked in a meeting to cooperate. In general, the response was positive. Both professors and assistants were least satisfied with the transparency of the process. The reason might be that they had not been given any precise technical information as to how their preferences and restrictions would be reflected in the decision model. Both groups strongly agreed, that it should be possible to flexibly change the automatically computed timetable in a subsequent bargaining process.

The satisfaction with the resulting timetable, as reported in Fig. 4, was even higher. We conjectured, that introducing a centralized planning approach had initially created substantial fear of unattractive individual schedules. With the benefit of hindsight, this fear proved needless in most cases. We were not surprised that the group of assistants rated the fairness of both process and results less euphorically than the professors. This might partly be due to the fact that, according to German law, professors are not bound to any formal working hours, whereas assistants are. For this reason, professors were allowed to a

Table 5 Overall assessment (%)

	Professors			Assistants		
	Yes	No	n/a	Yes	No	n/a
Is the schedule sufficiently free of conflicts?	42.86	7.14	50.00	47.37	0.00	52.63
Should the system be used in the future?	64.29	7.14	28.57	68.42	0.00	31.58

priori rule out 2 days for their teaching, while full-time assistants as a general rule were considered to be always available.

We also asked whether the resulting plan was sufficiently free of conflicts. Table 5 indicates, that a small majority of professors and teachers felt unable to answer this question. The others reported their impression that the schedule was sufficiently free of conflicts. The last question was whether the tested system should be implemented permanently at our school. To our surprise only one professor answered that the system should not be used again. A clear majority of both professors and assistants advocated the introduction of the model-based approach to create timetables for the school. Our impression is that the quality of the created schedule compensated, to some extent, for the ungracious introduction of the process.

6 Conclusion and further work

Based on the gathered experience of this test, we concluded that a centralized planning approach, based on an integer programming model, is both organizationally and computationally feasible. We also found that it can indeed be used to create substantially better timetables than the school ever had. Faced with these results, the Dean decided to invest the necessary money to advance the previous web application for timetabling, in order to collect information on preferences and restrictions online via a web application. Future work on the model will take the link between the course scheduling and the examination scheduling problem into account. As explained in Sect. 2, in Hannover the course timetable determines the examination timetable, which is also important from the students' point of view. The model originally implemented in GAMS for the research purpose of testing the approach is currently being transferred to FlopC++, an open-source algebraic modeling language implemented as a C++ class (Hultberg 2003). FlopC++ can be used to together with the open-source solver CoinCbc (Forrest and Lougee-Heimer 2005) to solve the model. It is therefore possible to use open source software to effectively and efficiently solve the timetabling problem at the School of Economics and Management at Hannover University, with a solution quality that has been considered inconceivable for many years.

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