21h38

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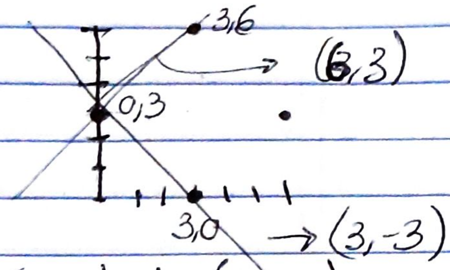
① $\iint_D (x-y)^2 \operatorname{sen}(x+y)\, dx\, dy$

D: $(\pi,0), (2\pi,\pi), (\pi,2\pi), (0,\pi)$

D: $(3,0), (6,3), (3,6), (0,3)$



x	y	u	v
0	π	0	0
π	0	π	0
π	2π	0	π
2π	π	π	π

$L_1: (0,\pi) + t\cdot(\pi,\pi)$

$L_2: (0,\pi) + t\cdot(\pi,-\pi)$

$(0 + t\pi,\ \pi - t\pi)$

$$\boxed{\begin{aligned} x &= \pi - u \\ y &= v + \pi \end{aligned}}$$

$\to x + y = 2\pi + v - u$

$x - y = -u - v$

$$\int_0^{\pi}\int_0^{\pi} (-u-v)^2 \operatorname{sen}(2\pi + v - u)\, du\, dv$$

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faltou Tempo

⑤ Faltou Tempo

18h20

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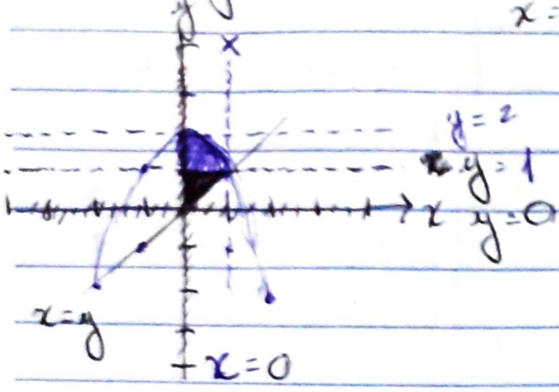
ⓔ ⓐ $I = \underbrace{\int_0^1 \int_0^y dx\,dy}_{A} + \underbrace{\int_1^2 \int_0^{\sqrt{2-y}} dx\,dy}_{B}$

ⓐ desenhe a região de integração e reescreva a integral na ordem dy dx



$x = \sqrt{2-y} \to x^2 = 2-y \to y = -x^2+2$

x	y
0	2
1	1
2	-2
-2	-2
-1	1

Tipo 2:

A	B
$0 \leq x \leq y$	$0 \leq x \leq \sqrt{2-y}$
$0 \leq y \leq 1$	$1 \leq y \leq 2$

Tipo 1:

A	B
$0 \leq x \leq 1$	$0 \leq x \leq 1$
$x \leq y \leq 1$	$1 \leq y \leq -x^2+2$

$$\int_0^1 \int_x^1 dy\,dx + \int_0^1 \int_1^{-x^2+2} dy\,dx$$

ⓑ $\left(\int_0^1 \int_0^y dx\,dy + \int_1^2 \int_0^{\sqrt{2-y}} dx\,dy\right) = I$

$\int_0^1 x\Big|_0^y dy + \int_1^2 x\Big|_0^{\sqrt{2-y}} dy = I$

$\int_0^1 y\,dy + \int_1^2 \sqrt{2-y}\,dy = I$

$\int \sqrt{2-y}\,dy \qquad 2-y = u$

$\int -\sqrt{u}\,du \qquad -1 = du$

$-\int u^{1/2}$

$-\left(\dfrac{u^{1/2+1}}{\frac{1}{2}+1}\right) = \dfrac{-u^{3/2}}{3/2} = \dfrac{-2u^{3/2}}{3}$

$-\dfrac{2\sqrt[3]{(2-y)^2}}{3}$

$\dfrac{y^2}{2}\Big|_0^1 - \dfrac{2\sqrt[3]{(2-y)^2}}{3}\Big|_1^2$

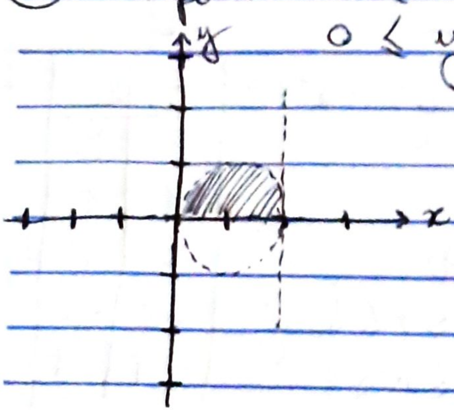
$\dfrac{1^2}{2} - \left(\dfrac{2\sqrt[3]{(2-2)^2}}{3} - \dfrac{2\sqrt[3]{(2-1)^2}}{3}\right)$

$\dfrac{1}{2} - \left(0 - \dfrac{2}{3}\right) = \dfrac{1}{2} + \dfrac{2}{3} = \dfrac{3}{6} + \dfrac{4}{6} = \boxed{\dfrac{7}{6}}$

③ $I = \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\; dy\, dx$

ⓐ Tipo 1: $0 \leq x \leq 2 \rightarrow 0 \leq x\,,\; x \leq 2$

$0 \leq y \leq \sqrt{2x-x^2} \rightarrow 0 \leq y\,,\; y \leq \sqrt{2x-x^2}$



$y^2 \leq 2x - x^2$

$y^2 + x^2 - 2x + 1 \leq 1$

$y^2 + (x-1)^2 \leq 1$

ⓑ Calcule a integral I usando uma conveniente mudança de coordenadas

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$$

$x^2 + y^2 = r^2$

$\sqrt{2x-x^2} = \sqrt{2r\cos\theta - r^2\cos^2\theta}$

$\sqrt{x^2+y^2} = \sqrt{r^2} = r$

$dy\,dx = r\, dr\, d\theta$

$$\int_0^{\pi/2}\int_0^{2} r^2\, dr\, d\theta = \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^2 d\theta = \int_0^{\pi/2} -0 + \frac{2^3}{3}\, d\theta = \frac{+8\theta}{3}\Big|_0^{\pi/2}$$

$$+\frac{8}{3}\cdot\frac{\pi}{2} - 0 = +\frac{4}{3}\pi = \boxed{\frac{4}{3}\pi} \approx 4{,}18\ldots$$

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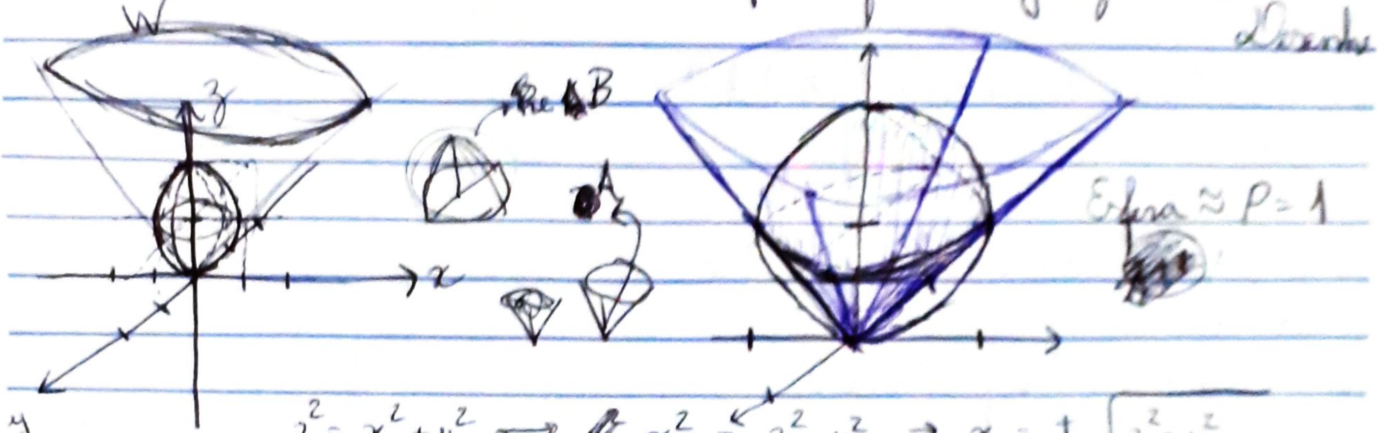
(4) $\iiint z^2 \, dx\, dy\, dz$ | sendo W o sólido interior ao cone $z = \sqrt{x^2+y^2}$ limitada pela esfera $x^2+y^2+(z-1)^2 = 1$.

Desenho W



$$z^2 = x^2 + y^2 \rightarrow x^2 = z^2 - y^2 \rightarrow x = \pm\sqrt{z^2 - y^2}$$
$$\rightarrow y^2 = z^2 - x^2 \qquad y = \pm\sqrt{z^2 - x^2}$$

A	B
$-\sqrt{z^2-y^2} \leq x \leq +\sqrt{z^2-y^2}$	$-\sqrt{(1-y^2-(z-1)^2)} \leq x \leq +\sqrt{(1-y^2-(z-1)^2)}$
$-\sqrt{z^2-x^2} \leq y \leq +\sqrt{z^2-x^2}$	$-\sqrt{(1-x^2-(z-1)^2)} \leq y \leq +\sqrt{(1-x^2-(z-1)^2)}$
$0 \leq z \leq 1$	$1 \leq z \leq 2$

$$x^2+y^2+(z-1)^2 = 1 \rightarrow x^2 = 1-y^2-(z-1)^2 \rightarrow x = \pm\sqrt{(1-y^2-(z-1)^2)}$$
$$\rightarrow y^2 = 1-x^2-(z-1)^2 \rightarrow y = \pm\sqrt{(1-x^2-(z-1)^2)}$$

Coordenadas esféricas

A Cone (A)	(B) Esfera
$0 \leq \rho \leq 1$	$0 \leq \rho \leq 1$
$0 \leq \theta \leq 2\pi$	$0 \leq \theta \leq 2\pi$
$0 \leq \phi \leq \pi/4$	$0 \leq \phi \leq \pi/2$

$$\iiint_W z^2 \, dx\, dy\, dz$$

$$\iiint_Q \rho^2\cos^2\phi \; \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

$$\iiint_Q \rho^4 \cos^2\phi \operatorname{sen}\phi$$

$$A: \int_0^{2\pi}\int_0^{\pi/4}\int_0^1 \rho^4 \cos^2\phi \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

④ $\iiint_Q \rho^4 \cos^2\phi \operatorname{sen}\phi$

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Cone

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{1} \rho^4 \cos^2\phi \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi/4} \frac{\rho^5}{5} \cos^2\phi \operatorname{sen}\phi \Big|_0^1 d\phi \, d\theta$$

$$\frac{1}{5}\int_0^{2\pi}\int_0^{\pi/4} \cos^2\phi \operatorname{sen}\phi \, d\phi \, d\theta$$

$$\frac{1}{5}\int_0^{2\pi} \frac{-1}{3} \cos^3\phi \Big|_0^{\pi/4} d\theta$$

$$\cancel{\frac{-1}{15}\int_0^{2\pi} (\cos(\pi/4))^3 \, d\theta}$$

$$\cancel{\frac{-1}{15}\cdot\left(\frac{\sqrt{2}}{2}\right)^3 \cdot \int_0^{2\pi} d\theta}$$

$$\cancel{\frac{-1}{15}\cdot\frac{2\sqrt{2}}{8}\cdot 2\pi = \boxed{\frac{-\pi\sqrt{2}}{60}}}$$

$$\frac{-1}{15}\left((\cos(\pi/4))^3 - (\cos(0))^3\right)\int_0^{2\pi} d\theta$$

$$\frac{-1}{15}\cdot\left(\left(\frac{\sqrt{2}}{2}\right)^3 - 1\right)\cdot 2\pi$$

$$\frac{-1\cdot(2\sqrt{2}-8)\cdot 2\pi}{15\cdot 8} = \boxed{\frac{-\pi\sqrt{2}+4\pi}{30}}$$

$$\frac{\pi(4-\sqrt{2})}{30} \qquad \frac{\pi(2\sqrt{2}-1)}{15\sqrt{2}}$$

Esfera:

$$B: \int_0^{2\pi}\int_0^{\pi/2}\int_0^{1} \rho^4 \cos^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$\frac{-1}{15}\int_0^{2\pi} \cos^3\phi \Big|_0^{\pi/2} d\theta$$

$$\frac{-1}{15}\cdot\left((\cos \pi/2)^3 - (\cos 0)^3\right)\cdot\int_0^{2\pi} d\theta$$

$$\frac{-1}{15}\cdot(0-1)\cdot 2\pi = \boxed{\frac{2\pi}{15}}$$

Esfera + Cone

$$\frac{2\pi}{15} + \frac{\pi(4-\sqrt{2})}{30} = \frac{4\pi + \pi(4-\sqrt{2})}{30}$$

$$\frac{\pi\cdot(4+4-\sqrt{2})}{30} = \boxed{\frac{\pi}{30}\cdot(8-\sqrt{2})}$$