

# Homework

- Sort the following functions in the ascending order of Big O notation:

$2^{10}$ ,  $2^{\log n}$ ,  $3n + 100 \cdot \log n$ ,  $4n$ ,  $n \cdot \log n$ ,  $4n \cdot \log n + 2n$ ,  $n^2 + 10n$ ,  $n^3$ ,  $2^n$

```

51 // Tính thời gian chạy (đơn vị mili giây)
52 duration<double, milli> duration = end - start;
53 return duration.count(); // Trả về thời gian chạy
54 }
55
56 // *****
57 /* Ass1
58   Biến đổi:
59   4nLog(n)+2n = O(nLogn)
60   2^10 = O(1)
61   2^logn = n^log2 = n = O(n)
62   3n+100Logn = O(n)
63   4n = O(n)
64   2^n = O(2^n)
65   n^2+10n = O(n^2)
66   n^3 = O(n^3)
67   nLogn = O(nLogn)
68
69   Sắp xếp:
70   STT Big Onotation Functions
71   1 O(1) 2^10
72   2 O(Logn)
73   3 O(n) 2^logn; 3n+100Logn; 4n;
74   4 O(nLogn) 4nLog(n)+2n; nLogn
75   5 O(n^2) n^2+10n,
76   6 O(n^3) n^3
77   7 O(2^n) 2^n
78   8 O(n!)
79 */
80
81 // *****
82 // Ass2
83 double powerOfTwo_Iterative (int n){
84   /*
85   ExponentialFunc
86   Hàm thực hiện nh
87   Đầu vào:
88   int n: số mũ của hàm

```

- Given an integer number  $n$ , your task is to write two different algorithms in pseudo-codes to calculate  $2^n$ , and evaluate the complexity of the algorithms.

```

1 long long pow_2_n (int n):
2     long long res = 1;
3     for int i = 0 -> n-1:
4         res *= 2;
5     return res;

```

$$T_{12} = 2 = O(1)$$

$$T_3 = n - 1 = O(n)$$

$$T_4 = 1 = O(1)$$

$$T_5 = 1 = O(1)$$

$$T_{12345} = T_{12} + T_3 * T_4 + T_5 = O(n)$$

```

1 long long binpow(long long a, long long b) {
2     if (b == 0)
3         return 1;
4     long long res = binpow(a, b / 2);
5     if (b % 2)
6         return res * res * a;
7     else
8         return res * res;
9 }

```

- $T_1 = O(1)$
- $T(b == 0) = O(1)$
- $T_4 = 1 + T(\text{binPow})$
- $T_{5678} = O(1)$
- $T(\text{binpow}) = O(\log b)$

3. Your task is to write operations of queue data structure in pseudo-codes using an array, then evaluate the complexities of the operations.

```

1 void deleteFirstEleInArr (int* A, int& n)
2   for (int i = 0; i < n - 1; i++) {
3       A[i] = A[i + 1];
4   }
5   n--;
6}

class Queue {
    int* values;
    int countValue;
public:

    // Enqueue & dequeue
1 void enqueue(int value) {
2     values[countValue] = value;
3     countValue++;
4 }
5 void dequeue() {
6     deleteFirstEleInArr(values, countV
7 }
};

```

- deleteFirstEleInArr
  - $T_1 = 1 = O(1)$
  - $T_2 = n - 2 = O(n)$
  - $T_3 = 3 = O(1)$
  - $T_5 = 1 = O(1)$
  - $T(\text{deleteFirstEleInArr}) = T_1 + T_2 + T_3 + T_5 = O(n)$
- enqueue
  - $T_1 = 1 = O(1)$
  - $T_2 = 2 = O(1)$
  - $T_3 = 1 = O(1)$
  - $T(\text{enqueue}) = T_1 + T_2 + T_3 = O(1)$
- dequeue
  - $T_6 = T(\text{deleteFirstEleInArr}) = O(n)$

4. Your task is to write operations of queue data structure in pseudo-codes using a linked list, then evaluate the complexities of the operations.

```

33 // Enqueue & dequeue
34 void enqueue(int value) {
35     if (head == NULL) {
36         head = new Node (value, NULL);
37         return;
38     }
39     Node* p = head;
40     while (p->next != NULL) {
41         p = p->next;
42     }
43     p->next = new Node (value, NULL);
44 }
45 void dequeue() {
46     if (head == NULL) {
47         return;
48     }
49     Node* tmp = head;
50     head = head->next;
51     delete tmp;
52 }
53 };

```

- enqueue
  - $T_{34} = 1 = O(1)$
  - $T_{35} = 1 = O(1)$
  - $head == NULL$ 
    - $T_{36} = 3 = O(1)$
    - $T(head == NULL) = O(1)$
  - $head \neq NULL$ 
    - $T_{39} = 1 = O(1)$
    - $T_{40} = n - 1 = O(n)$  (coi n là số lượng phần tử trong linked list)
    - $T_{41} = 1$
    - $T_{43} = 3 = O(1)$
    - $T(head \neq NULL) = O(n)$
  - $T(enqueue) = \max(T(head == NULL), T(head \neq NULL)) = O(n)$
- dequeue
  - $T_{46} = 1 = O(1)$
  - $head == NULL$ 
    - $T(head == NULL) = 0 = O(1)$
  - $head \neq NULL$ 
    - $T_{49} = 1 = O(1)$
    - $T_{50} = 1 = O(1)$
    - $T_{51} = 1 = O(1)$
    - $T(head \neq NULL) = O(1)$

- $T(\text{dequeue}) = \max(T(\text{head} == \text{NULL}), T(\text{head} \neq \text{NULL})) = O(1)$

5. Your task is to write operations of stack data structure in pseudo-codes using an array, then evaluate the complexities of the operations.

```

23 // Push & pop
24 void push(int value) {
25     if (countValue == 100) {
26         return;
27     }
28     values[countValue] = value;
29     countValue++;
30 }
31 void pop() {
32     if (countValue != 0) {
33         countValue--;
34     }
35 }

```

- push
  - $T_{24} = 1 = O(1)$
  - $T_{25} = 1 = O(1)$
  - $T(\text{countValue} == 100) = 1 = O(1)$
  - $\text{countValue} \neq 100$ 
    - $T_{28} = 2 = O(1)$
    - $T_{29} = 1 = O(1)$
    - $T(\text{countValue} \neq 100) = O(1)$
  - $T(\text{push}) = \max(T(\text{countValue} == 100), T(\text{countValue} \neq 100)) = O(1)$
- pop
  - $T_{32} = 1 = O(1)$
  - $T(\text{countValue} == 0) = 0 = O(1)$
  - $\text{countValue} \neq 0$ 
    - $T_{33} = 1 = O(1)$
    - $T(\text{countValue} \neq 0) = 1 = O(1)$
  - $T(\text{pop}) = \max(T(\text{countValue} == 0), T(\text{countValue} \neq 0)) = O(1)$

$O(1)$

6. Your task is to write operations of stack data structure in pseudo-codes using a linked list, then evaluate the complexities of the operations.

```
38 // push & pop
39 void push(int value) {
40     if (head == NULL) {
41         head = new Node(value, NULL);
42         return;
43     }
44     Node* p = head;
45     while (p->next != NULL) {
46         p = p->next;
47     }
48     p->next = new Node(value);
49 }
50 void pop() {
51     if (head == NULL) {
52         return;
53     } else if (head->next == NULL) {
54         delete head->next;
55         head = NULL;
56     }
57     Node* p = head;
58     while (p->next->next != NULL) {
59         p = p->next;
60     }
61     delete p->next;
62     p->next = NULL;
63 }
64 };
```

- push
  - $T_{39} = 1 = O(1)$
  - $T_{40} = 1 = O(1)$
  - $head == NULL$ 
    - $T_{41} = 3 = O(1)$
    - $T(head == NULL) = O(1)$
  - $head \neq NULL$ 
    - $T_{44} = 1 = O(1)$
    - $T_{45.46} = n - 1 = O(n)$  ( $n \sim$  là số lượng phần tử trong stack)
    - $T_{48} = 3 = O(1)$
    - $T(head \neq NULL) = O(n)$
  - $T(push) = \max(T(head == NULL), T(head \neq NULL)) = O(n)$
- pop
  - $T_{51} = 1 = O(1)$
  - $T(head == NULL) = 1 = O(1)$
  - $head \neq NULL$ 
    - $T_{53} = 1 = O(1)$
    - $head \rightarrow next == NULL$ 
      - $T_{54} = 1 = O(1)$
      - $T_{55} = 1 = O(1)$

- $T(\text{head} \rightarrow \text{next}) = O(1)$
- $\text{head} \rightarrow \text{next} \neq \text{NULL}$ 
  - $T_{57} = 1 = O(1)$
  - $T_{58.59} = n - 2 = O(n)$
  - $T_{61} = 1 = O(1)$
  - $T_{62} = 1 = O(1)$
  - $T(\text{head} \rightarrow \text{next} \neq \text{NULL}) = O(n)$
- $T(\text{head} \neq \text{NULL}) = \max(T(\text{head} \rightarrow \text{next} == \text{NULL}), T(\text{head} \rightarrow \text{next} \neq \text{NULL})) = O(n)$
- $T(\text{pop}) = \max(T(\text{head} \neq \text{NULL}), T(\text{head} \rightarrow \text{next} \neq \text{NULL})) = O(n)$