

Does the chromatic Mach bands effect exist?

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The achromatic Mach bands effect is a well-known visual illusion, discovered over a hundred years ago. This effect has been investigated thoroughly, mainly for its brightness aspect. The existence of chromatic Mach bands, however, has been disputed. In recent years it has been reported that chromatic Mach bands are not perceived under controlled iso-luminance conditions. However, here we show that a variety of chromatic Mach bands, consisting of chromatic and achromatic regions, separated by a saturation ramp, can be clearly perceived under iso-luminance and iso-brightness conditions. In this study, observers' eye movements were recorded under iso-brightness conditions. Several observers were tested for their ability to perceive the chromatic Mach bands effect and its magnitude, across different cardinal and non-cardinal chromatic Mach bands stimuli. A computational model of color adaptation, which predicted color induction and color constancy, successfully predicts this variation of chromatic Mach bands. This has been tested by measuring the distance of the data points from the "achromatic point" and by calculating the shift of the data points from predicted complementary lines. The results suggest that the chromatic Mach bands effect is a specific chromatic induction effect.

Keywords: chromatic Mach bands, chromatic adaptation, computational model

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Introduction

The classic Mach bands effect (named after physicist Ernest Mach) is a visual illusion noted by illusory bright and dark bands seen where a luminance plateau meets a ramp. This visual illusion became a classic case of distinguishing between physical and perceptual aspects of sensation (Pease, 1978; Pessoa, 1996). Although this illusion of brightness, as well as other illusions, such as simultaneous contrast, has generated much interest, no acceptable explanation has been formulated for this phenomenon.

The existence of chromatic Mach bands has been disputed for decades (Daw, 1964; Ercoles-Guzzoni & Fiorentini, 1958; Gur & Syrkin, 1993; Jacobson & MacKinnon, 1969; Koffka & Harrower, 1931; Matthews, 1967). Several studies reported the existence of the chromatic effect, but none provided a comprehensive report about the specific perceived colors. In addition, the effect has not been tested for a variety of chromatic Mach bands stimuli, nor did any of the studies cite a general trend in the nature of the perceived colors (Daw, 1964; Jacobson & MacKinnon, 1969; Koffka & Harrower, 1931). The earliest evidence of the existence of chromatic Mach bands, deriving from a saturation difference, refers

only to the color red (Koffka & Harrower, 1931). Later, Matthews (1967) reported on the existence of three chromatic Mach bands consisting of different chromatic regions. Jacobson and MacKinnon (1969) also reported two cases where chromatic Mach bands exist, where the stimulus regions were of different hue and saturation, and the color wheel was of the same hue and luminance but saturated differently. Daw (1964) reported that three out of six observers perceived the chromatic bands in Mach bands stimulus, but the nature of the bands was not described.

There is no general consensus regarding the appearance of colored Mach bands (Boff, Kaufman, & Thomas, 1986; Pease, 1978; Pessoa, 1996; Valberg, 2005; Wyzecki & Stiles, 1982). Recent reports have claimed that chromatic Mach bands are not perceived under iso-luminance or iso-brightness conditions (Gur & Syrkin, 1993; Pessoa, 1996).

Most of the previous investigations of chromatic Mach bands employed stimuli composed of two chromatically homogeneous regions with a chromatic ramp (Pease, 1978; Pessoa, 1996). It is questionable, however, whether this stimulus composition is appropriate for identifying the chromatic Mach bands effect. The problematic stimulus composition can be scrutinized on a specific stimulus paradigm. For example, a Mach bands stimulus composed of homogeneous red and green regions with a

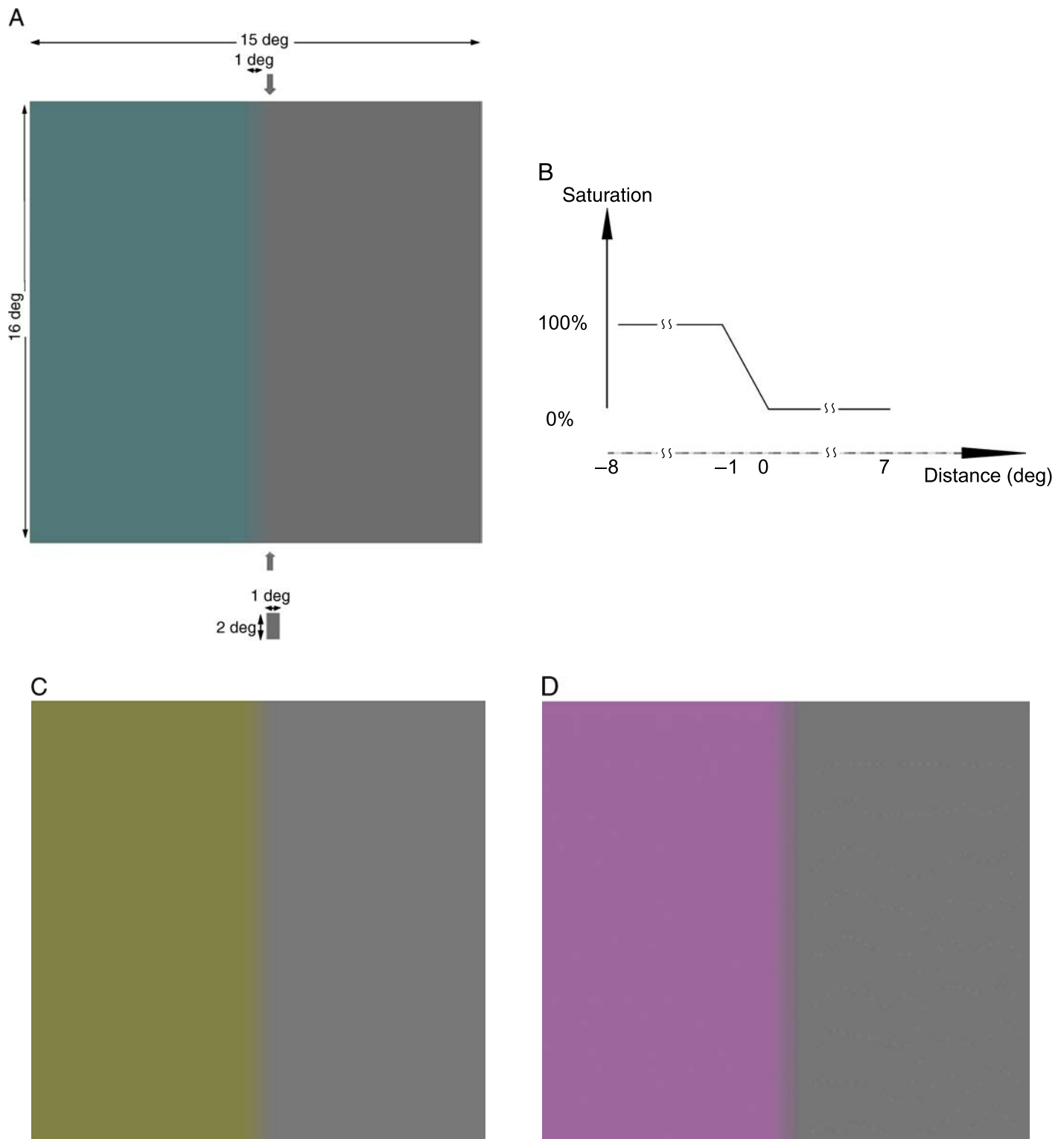


Figure 1. Three examples of the experimental chromatic Mach bands stimuli: (A) The chromatic Mach bands with the stimulus procedure, which includes the matching color patch (below) and gray arrows indicating the location of the chromatic band. Note that some of the observers perceived the chromatic Mach bands effect after several seconds. (B) Saturation profile of the chromatic Mach bands along x-axis of each Mach bands stimulus's region. These stimulus regions were under iso-luminance conditions (or iso-brightness conditions in Experiment 2). The width of the ramp region was 1° . (C) Additional examples of the chromatic (hue) Mach band stimuli (yellow) and magenta (D).

chromatic saturation ramp between them will cause a cyan band illusion on the green side of the Mach bands stimulus. (Note that red and green colors are not complementary colors.) This perceived cyan color (yielded from the red chromatic region) will probably not be perceived as such since this expected perceived color will interfere with the physically green color of the stimulus's region. (In other words, the “induced” color is mixed with the real color). Thus, the perceived “cyanish” color is hardly observed. Such color interaction probably occurred in the previously reported Mach bands paradigms, which used chromatic regions (which are not complementary) for their Mach bands stimuli (Green & Fast, 1971; Gur & Syrkin, 1993; van der Horst & Bouman, 1967; Ware & Cowan, 1987).

The above doubt concerning the previous stimulus composition also resulted from the predictions of the adaptation and color constancy models (Spitzer & Barkan, 2005; Spitzer & Rosenbluth, 2002; Spitzer & Semo, 2002), which also predicted the color induction effect (Spitzer & Barkan, 2005). We speculated whether of the first order adaptation mechanism (Spitzer & Barkan, 2005), which causes an enhancement of the differences between the stimulus (its color or intensity) and its surrounding area, is presumably the mechanism that produces both the induction and Mach bands effects. These models led us to the idea that the chromatic Mach bands are manifested as a specific case of chromatic induction. Consequently, we chose to test this assumption by using a chromatic region as an “inducer” and an achromatic region as the “induced” region.

In our paradigm, the effect of the color induction can be expressed on the achromatic region near the ramp zone, avoiding interaction of the two chromatic sources in the same location (that of the perceived chromatic Mach band and the physical chromatic region of Mach band stimulus) (Figure 1A).

In addition to the experimental aspects, the potential of the adaptation model to predict the chromatic Mach bands effect has been tested and specific model predictions have been compared to the psychophysical results.

Experiment 1: Iso-luminance

We chose to use iso-luminance and iso-brightness stimuli since one of the main goals of these experiments was to test the possibility of obtaining complementary colors from the induction model predictions, i.e., the predictions of the first order adaptation model (see Introduction and Methods; Spitzer & Barkan, 2005; Spitzer & Rosenbluth, 2002; Spitzer & Semo, 2002). Accordingly, we chose to present the results on the CIE 1931 (Y , x , y) chromatic space, allowing for comprehensible portrayal of complementary colors.

Methods

Observers

Three men and three women (all under the age of 40) were tested for the quantitative test in iso-luminance conditions. Five of the observers were naïve and one (co-author AT) was experienced. All the observers had normal (or corrected to normal) vision and were tested for normal color vision with the “Ishihara test” for color blindness (1989).

Apparatus

The stimuli were generated and the experiments controlled by the computerized VSG2/5 stimulus generator (Cambridge Research Systems, Cambridge, England). A 37-inch computer screen (Mitsubishi MegaView Pro 37) was used to display the stimuli.

In order to correctly present the different colors, we measured the hues of the stimuli on the screen (using a Minolta CS-100 Colorimeter) while the screen was set to “white” ($R = G = B = 255$) and adjusted to match the CIE equal-energy-white (E : $x = y = 0.333$).

The VSG automatic gamma correction procedure was performed on the monitor display (in a dark room), for each phosphor separately and for all of the phosphors together. (The inputs to the monitor were separated through BNC connectors for each of the three guns R, G, and B. In addition to these BNC connectors, two BNC connectors were used for H and V inputs.) The gamma correction procedure used the OptiCAL (Cambridge Research Systems, Cambridge, England). The OptiCAL has a guaranteed photopic error (F'_1) of less than 3%, typically less than 2% (i.e. $F'_1 < 3\%$, typically $< 2\%$). F'_1 is an indication of the accuracy of how well the sensor matches the CIE photopic curve.

Spatial inhomogeneities of the display were reduced using the VSG2/5 (or the Visage in Experiments 2 and 3). Spatial Gamma corrections were also performed according to the OptiCAL's local luminance measurements at different locations on the display.

The “color viewer” (option within the Cambridge Research Systems VSG software) was used to test the validity of the luminance calibration. The equal luminosity and the stimulus's chromatic properties were measured from the screen (Minolta CS-100). The VSG generates a patch (according to the user's choice) using the same area over which the calibration was previously performed to display the selected point in the “color space.” This test patch was also measured with a Minolta colorimeter (CS-100) to verify the validity of the automatic calibration.

According to these measurements, we changed the Gamma correction of the screen to achieve approximate linearity. A look-up table adjusted any remaining non-linearities. This adjustment relied on a previous study (Bohnsack, Diller, Yeh, Jenness, & Troy, 1997), which showed that the chroma and the intensity of the screen are

a reflection (of at least 99.4%) of the summed values of the three phosphors.

This procedure was also performed in order to calibrate the 20-inch monitor (Mitsubishi Diamond Pro 2070^{SB}) used in Experiments 2 and 3.

Stimuli

In order to test the existence of the chromatic effect, we built a paradigm that maximized conditions enabling manifestation of the effect: All the chromatic Mach bands stimuli (in Experiment 1) were constructed with maximum chromatic saturation. Each Mach bands stimulus was constructed so that the chromatic and achromatic regions (Table 1) were built to comprise iso-luminance (OptiCAL Cambridge Research Systems and colorimeter Minolta CS-100).

Twelve chromatic Mach bands stimuli were formed: six comprised the cardinal colors red, green, and blue and their complementary colors (cyan, magenta, and yellow), and six consisted of the intermediate non-cardinal colors (i.e., colors that are neither cardinal nor complementary to cardinal colors; Figure 1, Table 1). (Complementary colors are colors that can be additively mixed to produce an achromatic color.) The stimuli were presented on a black background screen while the room was completely darkened, 2–3 cd/m². The stimulus was presented on only about half of the area of the monitor and was centered, thus revealing less spatial in-homogeneity. Pilot experiments on much smaller stimuli, 3° by 3.2° were also performed.

In order to include all the Mach bands stimuli with the same levels of luminance, we had to choose the luminance value with the highest common denominator among all the stimulus colors. The main problem arose from the

“blue” region. The luminance of saturated blue color is low. Other stimulus colors cannot manifest the appearance of high chromaticity with the same luminance appropriate for blue color—they will appear too dark. If iso-luminance conditions were fulfilled for all stimuli, i.e., application of all chromatic stimuli in blue color luminance, the perceived color effect from other chromatic stimuli would yield a smaller chromatic Mach bands effect, or non-observable effect. Thus, existence of the chromatic effect might not be manifested. Accordingly, each Mach bands stimulus was composed of a chromatic and achromatic pair of iso-luminance regions, linked by a chromatic ramp. Consequently, we divided the different chromatic Mach bands stimuli into three iso-luminance groups (Table 1). This enabled the stimuli to express a significant effect, since they were nearing their maximal possible chromaticity. These new stimulus conditions enabled us to test whether a perceived chromatic Mach bands effect exists under iso-luminance conditions (for each Mach bands stimulus and across groups of Mach bands stimuli).

The size of the stimulus was 15° by 16° (Figure 1A). The spatial width of the saturation ramp between each pair of regions in all the Mach bands stimuli was 1°. The saturation ramp constituted the linear saturation decrease from that point presented on the chromatic region to zero saturation, as presented on the achromatic region (Figure 1B). The test patch size was 1° by 2° (Figure 1A).

The chromaticity of the ramp was calculated according to the Euclidian distance of the specific chromaticity point (on the CIE 1931 (x, y)-chromaticity diagram) from the achromatic point. Each chromatic Mach bands stimulus was built under iso-luminance conditions (the chromatic region, the achromatic region, and the ramp). The actual intensities for three iso-luminance luminance groups of stimuli are detailed in Table 1. (Note that the color gamut of our CRT is larger than the color gamut of common CRTs.)

Procedure

The observers’ task was to adjust the chromaticity of the test patch to match the color of the Mach bands while altering their gaze between the chromatic Mach bands and the test patch (Figure 1A). The observers were positioned with their eyes 120 cm from the 37-inch computer screen (Mitsubishi MegaView Pro 37).

The observers were instructed to first look at the relevant region (Figure 1A, arrows) until the Mach bands appeared and only then to start matching the patch color to the color of the perceived band (Figure 1A, without arrows). The observers were required to match the perceived chromatic Mach bands (on the achromatic region) by adjusting the chromaticity of a small rectangular test patch to match the perceived chromatic band (Figure 1A). Matching was performed by using six keyboard keys, three to increase the RGB values and three

Color	Chromatic side			Achromatic side		
	x	y	Y (cd/m ²)	x	y	Y (cd/m ²)
Orange	0.523	0.460	38.5	0.33	0.33	38.5
Turquoise	0.201	0.241	38.5			
Olive-green	0.379	0.587	38.5			
Purple	0.290	0.068	38.5			
Pink	0.466	0.148	38.5			
Aquamarine	0.237	0.463	38.5			
Green	0.274	0.677	38.5			
Magenta	0.369	0.104	38.5			
Yellow	0.445	0.530	38.5			
Blue	0.167	0.009	16.6	0.33	0.33	16.6
Cyan	0.219	0.353	35.5	0.33	0.33	35.5
Red	0.735	0.265	35.5			

Table 1. The CIE 1931 (Y, x, y) values of the chromatic Mach bands stimuli.

to decrease the RGB values. This was accomplished by using pre-program software that kept the chosen colors under iso-luminance conditions. Unlimited time was allowed for matching. Color values were chosen from a table of iso-luminance values (Minolta CS-100), which had been prepared previously off-line. The matched color in the patch (Figure 1A), which was determined by the observer, was measured with a colorimeter (Minolta CS-100).

Each of the observers performed six blocks of trials (on different days), which consisted of twelve types of chromatic Mach bands (Table 1). Each block of trials was presented four times, while the order of the specific Mach bands was randomized. A black screen was presented between trials for 1 sec.

Analysis

Chroma

The chroma was calculated as follows (Wyszecki & Stiles, 1982, p. 168):

$$\text{Chroma} = \sqrt{(u^*)^2 + (v^*)^2}, \quad (1)$$

where u^* equals u' and v^* denotes $1.5 \cdot v'$.

In the event that the data point did not “fall” on the complementary relevant line, we calculated its projected point on the relevant complementary line. The distance measured between the projected point on the straight line and the achromatic point (calculated in the CIE 1976 (L, u', v')-color space) was taken as the “chroma.” This complementary color “chroma” was assessed and expressed in chroma values (Equation 1).

All analysis calculations were performed via the CIE 1976 (L, u', v')-color space and then presented in the CIE 1931 (x, y)-chromaticity diagram for the iso-luminance experiments (Experiment 1). This color space was used due to the need to perform statistics on the different measures, such as the chroma, and the shift from the complementary line (Figures 2–7) to establish the data points’ significance. Such a task can be done on a uniform or close to uniform color space (at short color distances). This also enabled us to compare the results obtained under iso-luminance and iso-brightness conditions. The logic of this data analysis has also been applied to the iso-brightness experiments (Experiment 2) that are presented in the CIE 1976 (L, u', v')-chromaticity diagram.

Estimation of the perceived complementary color-shift measure

It seems that most of the data points obtained appear on the complementary relevant line (or are in proximity to this line). We calculated the amount of the “shift” through the Euclidian distance of the data point from the relevant complementary line (Equation 1). The logic of this data analysis has also been applied to the iso-brightness experiments (Experiment 2).

Calculation of relative chroma of the perceived complementary color

In order to determine the perceived Mach bands complementary colors’ magnitudes of significance, relative to the chromatic region of the Mach bands, we calculated the perceived chroma of the intersection point. For this, we divided the chroma of the perceived Mach band (chroma of the “induced”) with the chroma of the chromatic region (chroma of the “inducer”).

Results

The chromatic Mach bands effect was determined by six observers. The observers reported seeing the chromatic Mach bands on the achromatic region adjacent to the ramp. Some of the naive observers who saw these chromatic Mach bands (Figure 1) for the first time reported seeing the effect after a maximum of 10 sec. All observers reported seeing instability in the width of the chromatic Mach bands, which periodically shrank and expanded during observation. (Note that the observers gazed at the location indicated by arrows. The arrows appeared before the matching procedure and defined the region of the effect; Methods.) An additional “band” was observed on the chromatic side of the ramp (Figure 1). The scope of the study limited us to test the quantitative effect of the perceived chromatic band that appears on the achromatic side.

Figures 2–7 demonstrate the hue of chromatic Mach bands perceived by the six observers from the variety of chromatic Mach bands stimuli (Table 1). The results only refer to those chromatic bands perceived in the achromatic region (Figure 1). For each observer the results are presented in three plots according to the three different luminance groups (38.5, 35.5, and 16.6 cd/m^2 ; Table 1). All the plots are presented in the CIE 1931 (x, y)-chromatic diagram (Table 1; Methods). The perceived chromatic Mach bands’ data points are presented as colored circles (Figures 2–7). The data points (colored circles) have been obtained by averaging the results of single trials through the CIE 1976 (L, u', v')-chromaticity coordinates. These averaged values (data points) have been calculated for presentation in the CIE 1931 (Y, x, y)-chromaticity coordinates. The specific colors of the circles (“induced”) were represented by the same color as the specific chromatic Mach bands region (“inducer,” diamonds in Figures 2–7) that yielded the perceived color. The gray lines connect the “inducer,” its complementary color, and the standard achromatic point (empty circle).

We tested the statistical significance of the obtained data points’ real perceived chromatic effect. This was performed by calculating the shift in the perceived color from the achromatic point in the CIE 1976 (L, u', v')-color space, which is a measure of the degree of perceived saturation. The resulting data points, for all of the Mach bands stimuli obtained across all six observers, demonstrated a signifi-

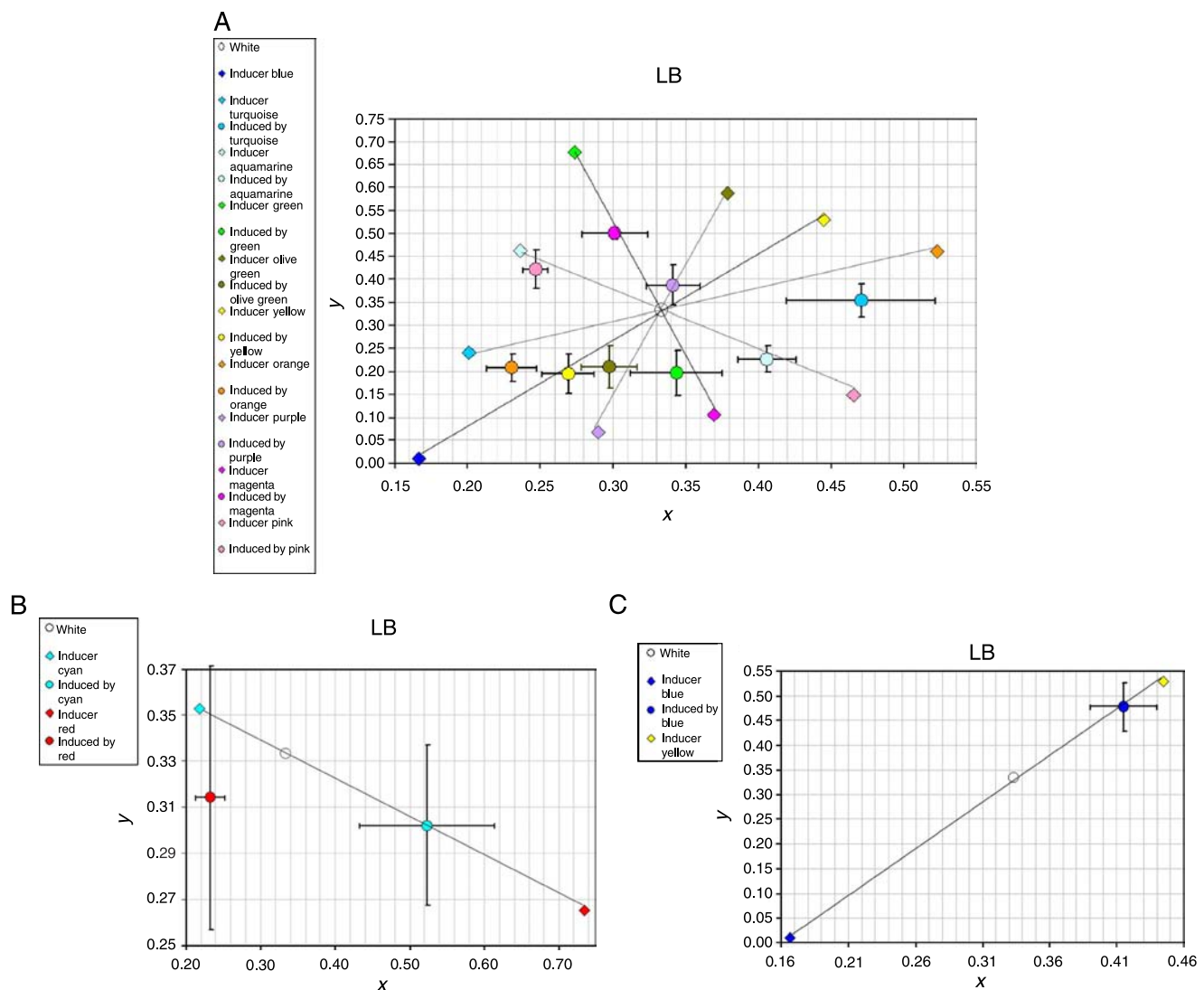


Figure 2. Significant perceived chromatic Mach bands from one of the six observers. Observer LB's perceived chromatic Mach bands data points (colored circles), across the three levels of luminance, have been averaged through the CIE 1976 (u' , v')-chromaticity coordinates and are presented in the CIE 1931 (x , y)-chromaticity coordinates, 38.5, 35.5, 16.6 cd/m^2 (Table 1), in panels A, B, and C, correspondingly. Each of the specific colored diamonds indicates the specific chromatic regions of the Mach bands stimuli (Table 1). We presented the perceived “induced” band's color (circles) with the specific color of the chromatic Mach bands region (“inducer,” diamonds). The gray lines connect the “inducer,” its complementary color, and the standard achromatic point (empty circle). Figures 2–7 present the same format of results, with relevant results obtained by different observers (LB, VT, AP, OS, SC, and AT).

cant perceived saturation (multivariate χ^2 test (Richard & Dean, 1988), $p < 0.025$). Thus, the results indicate the existence of perceived chromatic Mach bands effect with significant saturation. However, the perceived saturation varied across the different chromatic Mach bands stimuli which shared the same luminance (Figures 2–6, 7A–7C).

Figure 8 demonstrates the magnitudes of the chromatic Mach bands' perceived saturations across all the observers (colored bars). The results show that even though all observers showed significant saturation across chromatic Mach bands (Table 1), magnitudes varied across the observers and also across the specific chromatic region of the Mach bands stimulus. The variation across the stimuli is referenced below (Figure 8).

An additional related question regarding the experimental results is whether there is a general trend in the perceived colors, in the cardinal and non-cardinal chromatic regions (Krauskopf, Zaidi, & Mandler, 1986; Webster & Mollon, 1994). The question of the existence of two groups has been tested across the above two features, namely the saturation and the shift of the data points from the relevant complementary line.

Figure 8 shows a significant chroma of complementary color obtained from all chromatic Mach bands stimuli, both cardinal and non-cardinal colors, across all the six observers. The saturation values of the complementary colors are fairly diverse across the chromatic stimuli (even though the comparison was carried out separately for each

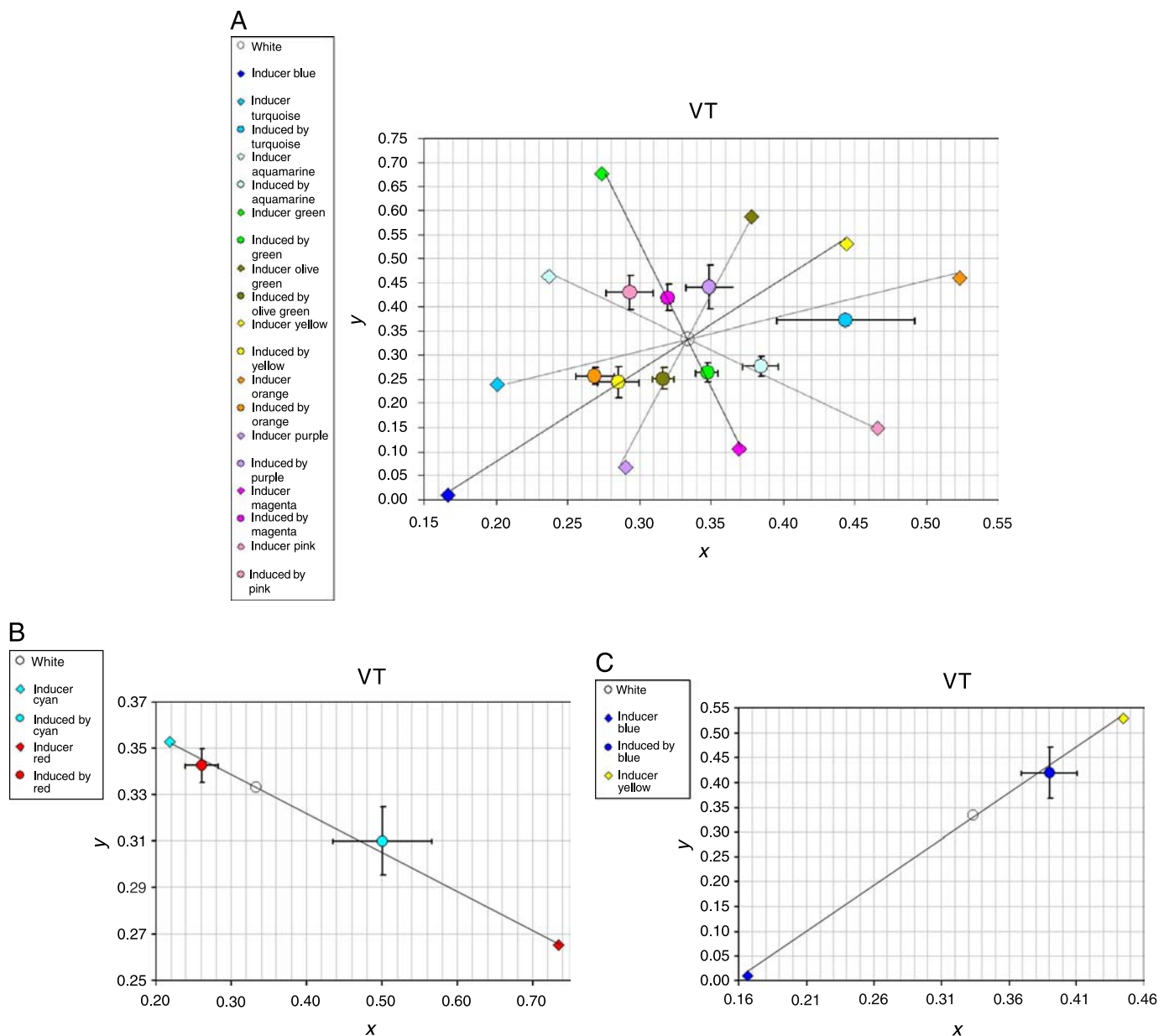


Figure 3. The perceived chromatic Mach bands for observer VT (with the same format as in Figure 2).

luminance group), with no significant differences recorded between the cardinal and the non-cardinal colors' Mach bands (Wilcoxon test, $p > 0.6$).

Shift

Most of the data points obtained were on the relevant complementary line (or in proximity to this line). We tested this assumption statistically by calculating the amount of “shift” (the Euclidian distance) of the data point from the relevant complementary line (Methods). In order to test the statistical significance of the ascription of each data point to the relevant complementary line, we performed two statistical tests: a two-tailed t test and a multivariate χ^2 test (Richard & Dean, 1988). The null

hypothesis in both tests assumed that each data point was attributed to the “appropriate” complementary line. Two alternative hypotheses attribute the data points (the perceived colors) to one of the two adjacent complementary lines, which were not relevant to the specific chromatic Mach bands stimulus. The t test aimed to examine whether the distance of data points belongs to the population of zero distance from the relevant complementary line. The statistical test for all observers across all cardinal colors data points was very significant and showed a relevant chromatic complementary line where $p < 0.005$. All of the observers' data points also showed the same tendency of high statistical significance, for the non-cardinal colors, but only for 4 or 5 chromatic Mach bands. (In this case 28 out of 36 data points showed very

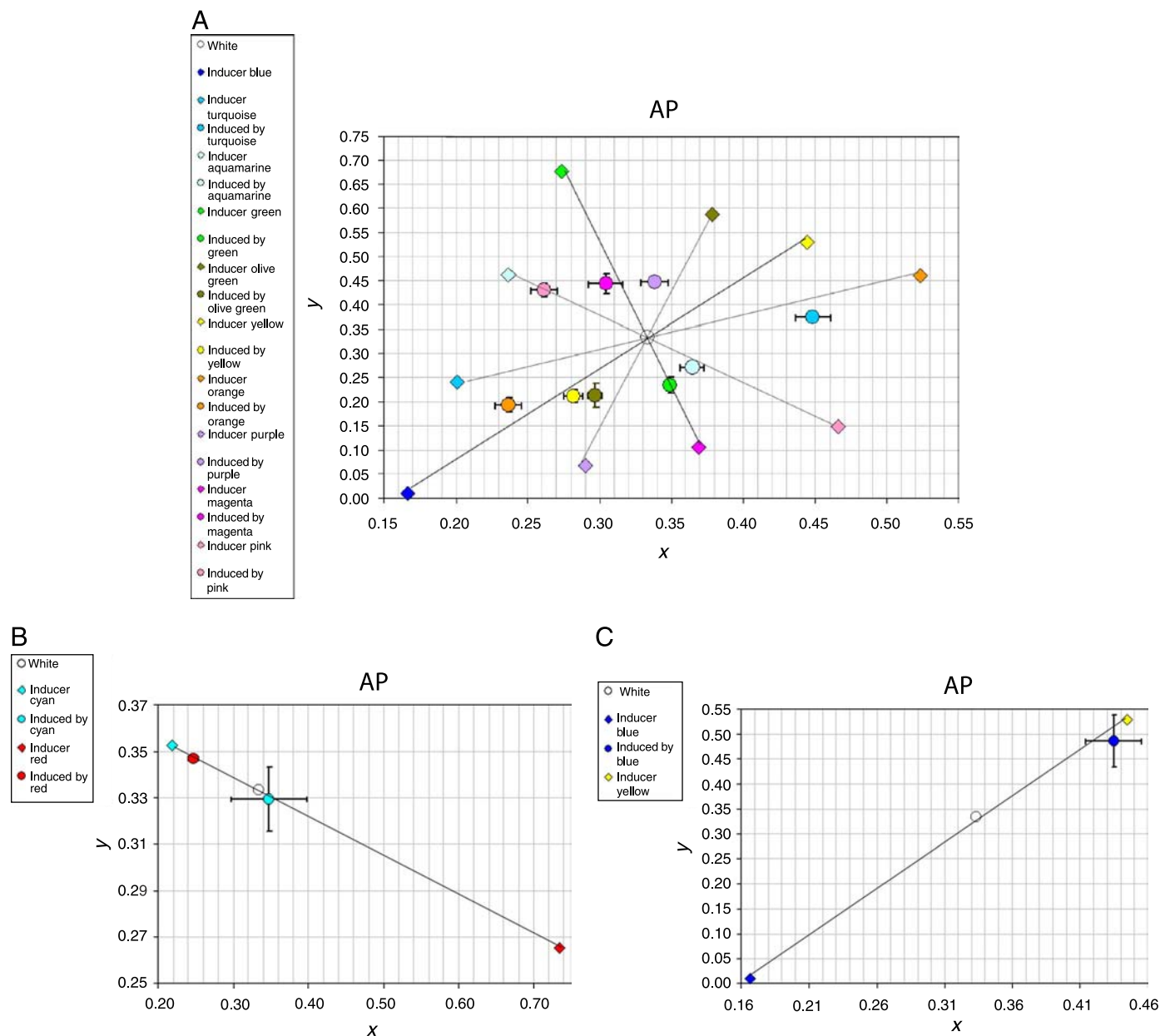


Figure 4. The perceived chromatic Mach bands for observer AP (with the same format as in Figure 2).

significant results, $p < 0.005$, and the other data points were statistically significant, $p < 0.025$.)

We added the multivariate χ^2 test (Richard & Dean, 1988) in order to examine the two components of color, i.e., the x and the y of the CIE 1931 (x, y)-chromaticity diagram. The data points showed a significant trend of adjustment in the direction of perceived relevant complementary colors, across all observers of all six cardinal chromatic Mach bands, $p < 0.001$. (This very significant result indicates that we cannot reject the null hypothesis that the data points are attributed to the population of the relevant complementary color line; Figures 2–7.) All of the observers' cardinal color stimuli data points showed very significant results in relation to the perceived data points that were obtained from the

chromatic Mach bands' non-cardinal colors stimuli (even though they were less significant than the results of the cardinal colors' data points) according to the non-parametric test, $p < 0.002$.

Figure 9 presents the “shift” of the data points from the complementary lines, in bar histograms, across the different chromatic Mach bands and across the different observers. The predicted “shift” responses of the computational model are also presented (dark blue bar) and will be discussed below. The results are presented separately for the three different stimuli luminances. Figure 9 shows a tendency of higher “shift” from the relevant complementary color line of the non-cardinal colors' Mach bands (Table 1).

Statistical testing has been performed in order to examine whether Mach bands stimuli with the cardinal

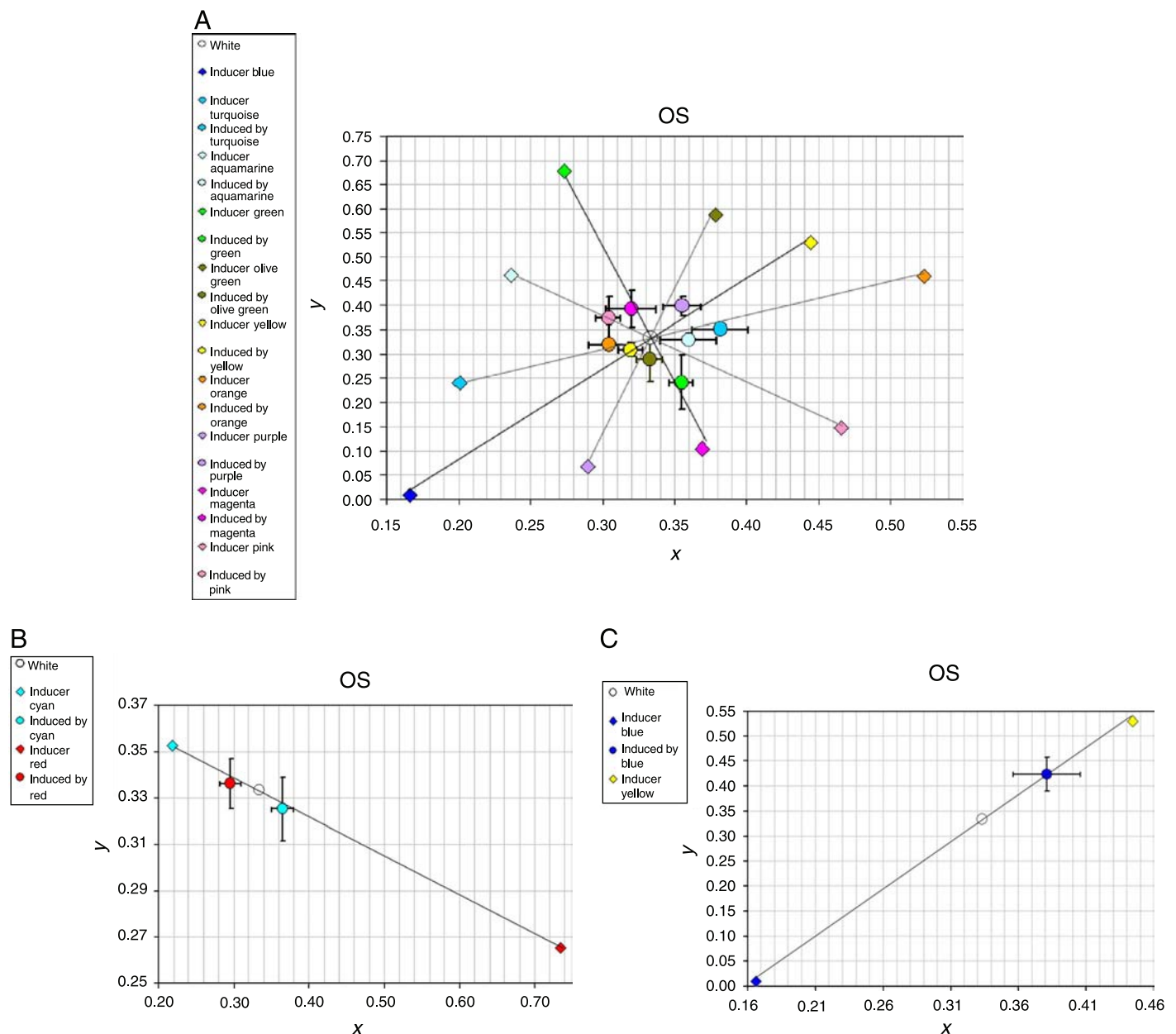


Figure 5. The perceived chromatic Mach bands for observer OS (with the same format as in Figure 2).

and the non-cardinal chromatic regions yielded different magnitudes of shift. The results showed a significantly larger (though still minor) shift of the predicted data points of non-cardinal colors stimuli vs. the cardinal colors' data points (Wilcoxon test, $p < 0.01$).

Model predictions

In the obtained model predictions (Appendix A), chromatic Mach bands stimuli were the inputs' images (Spitzer & Barkan, 2005; Spitzer & Semo, 2002). (We applied the first order chromatic adaptation algorithm that has also been used as the color constancy algorithm. Accordingly, Mach bands stimuli were presented as input images to the algorithm in the very same way as real images;

the appropriate luminance presented in the experimental luminance was 38.5 cd/m^2 , 35 cd/m^2 , and 16.6 cd/m^2 . In addition, this model was designed with the same parameters as those applied to real images.) The model yielded the predicted "perceived" Mach bands image, which includes the obtained chromatic bands on the achromatic region, adjacent to the ramp (Figures 10A–10C).

In order to evaluate the model predictions, we measured the color of the output Mach bands image obtained at the "perceived" Mach bands location (colorimeter Minolta CS-100). These predicted chromatic values are presented in THE CIE 1931 (x , y)-chromaticity diagram in Figures 10A–10C. Figure 10 presents the model predictions for the "perceived" colors (with the relevant colored circles) and presents the color of the Mach bands stimuli

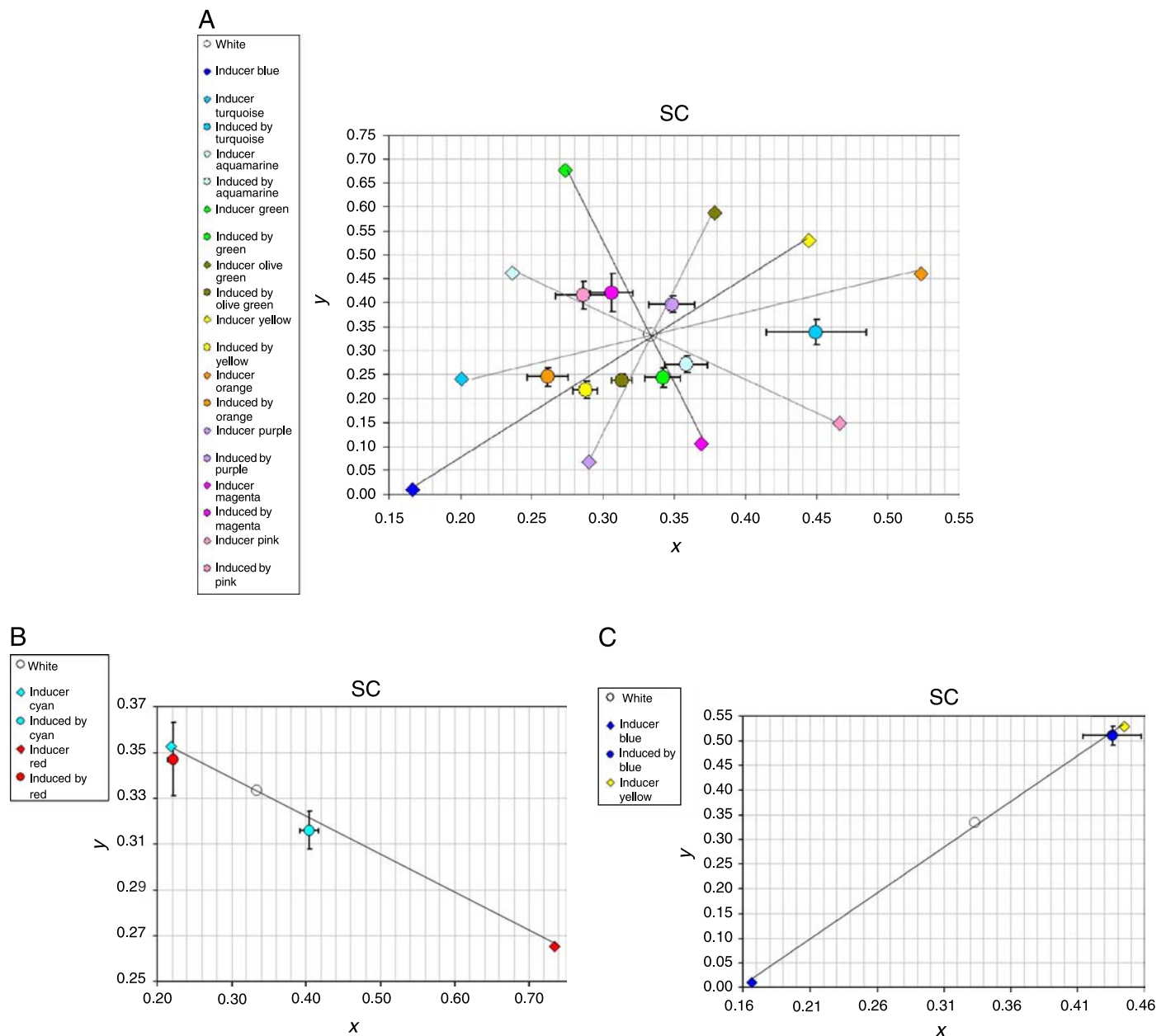


Figure 6. The perceived chromatic Mach bands for observer SC (with the same format as in Figure 2).

chromatic regions, the “inducers” (colored diamonds). (Note that this figure has the same structure as Figures 2–7.) All predicted data points yielded a chromatic appearance. Most of the predicted data points shown are located on or near the relevant complementary line. Note that the predicted results were calculated in three degrees of luminance as the experimental stimuli (Figures 10D–10F).

A similar question arose regarding the experimental perceived chroma, and the shift of the perceived complementary color from the relevant complementary color line, was tested for the model’s predictions (dark blue bar in Figures 8 and 9).

The predicted chroma (the dark blue bar; Figure 8) was obtained with large variability as a function of the specific

chromatic region of the Mach bands stimulus (“inducer”). These results raised the question of whether the variation in the magnitude of the predicted perceived chroma is due to the variation in the magnitude of the “inducer.” To test this, we calculated the perceived relative saturation of both the experimental and predicted results. (We did not present this here, because of the limited scope of paper.) The relative results show a small tendency of smaller variability in the experimental and predicted perceived saturations. It appears that the model’s results are compatible with the experimental results in the variability aspect. (These results led us to use iso-chroma stimuli in Experiment 2.)

The actual values of the “data points” of the predicted shift from the relevant complementary line, which are not

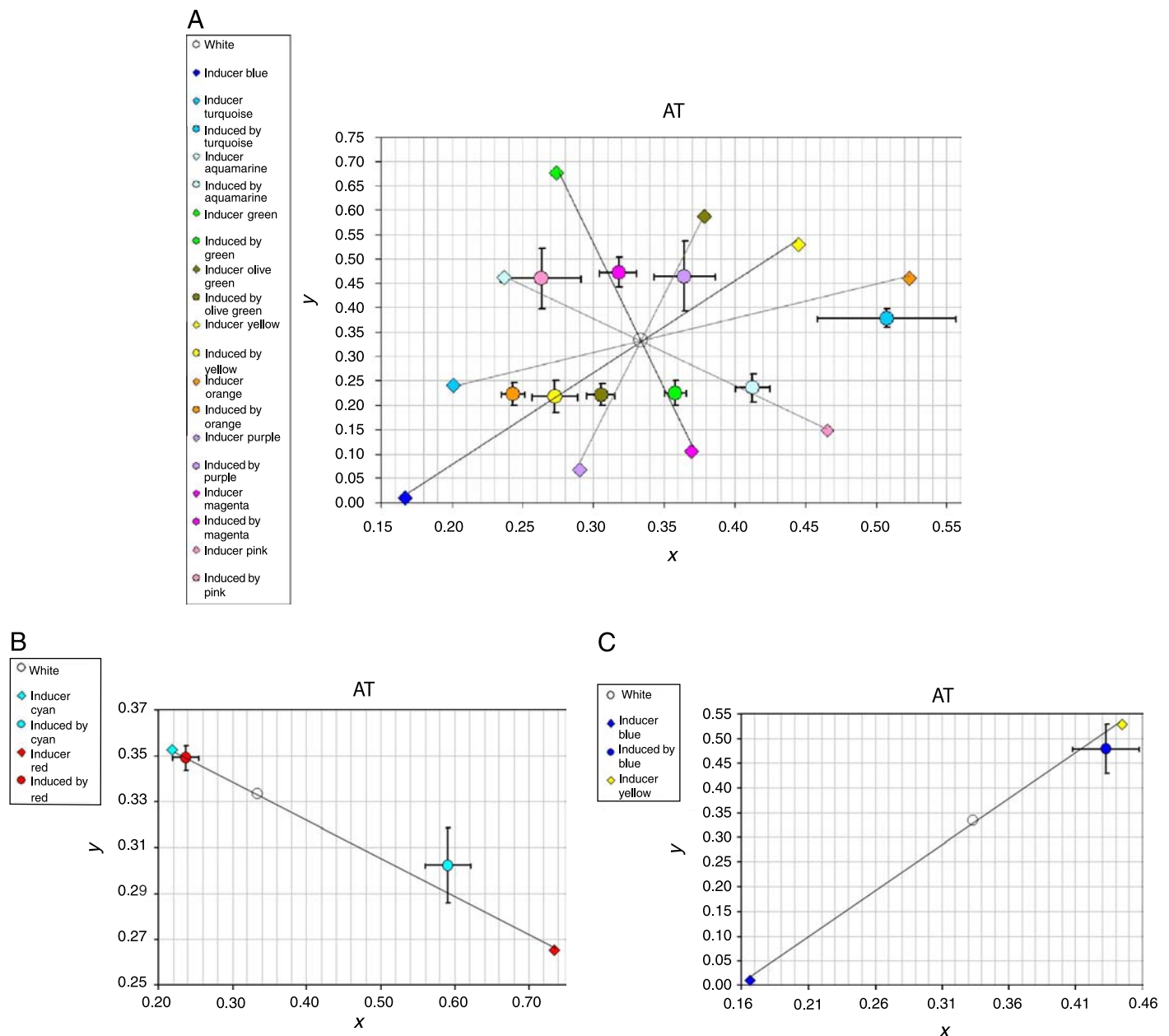


Figure 7. The perceived chromatic Mach bands for observer AT (with the same format as in Figure 2).

zero, are also varied as a function of the chromatic region of the mach band stimulus (Figure 9). These peculiar results strengthen the shift in the experimental results (Figure 9). We tested whether the predicted shift values are larger for the intermediate chromatic regions (non-cardinal) than those found for the experimental results. We found that indeed the predicted shift values yielded from the stimuli with non-cardinal colors are larger (Wilcoxon test, $p < 0.01$). Surprisingly, the model yielded a similar trend of shifting and shift predictions for the stimuli with the non-cardinal colors, especially for stimuli which yielded consistent shift across observers (for example, stimuli with turquoise and with orange colors). The obtained predicted and experimental results referring to the shift values raised the

question of whether the predicted shift would obtain zero values while the stimuli are presented under iso-brightness conditions (Experiment 2) rather than iso-luminance conditions (as presented in Figures 2–9).

Experiment 2: Iso-brightness

To rule out the possibility that the obtained chromatic Mach band effect derived from differences of brightness mainly within each of the Mach band stimuli, we

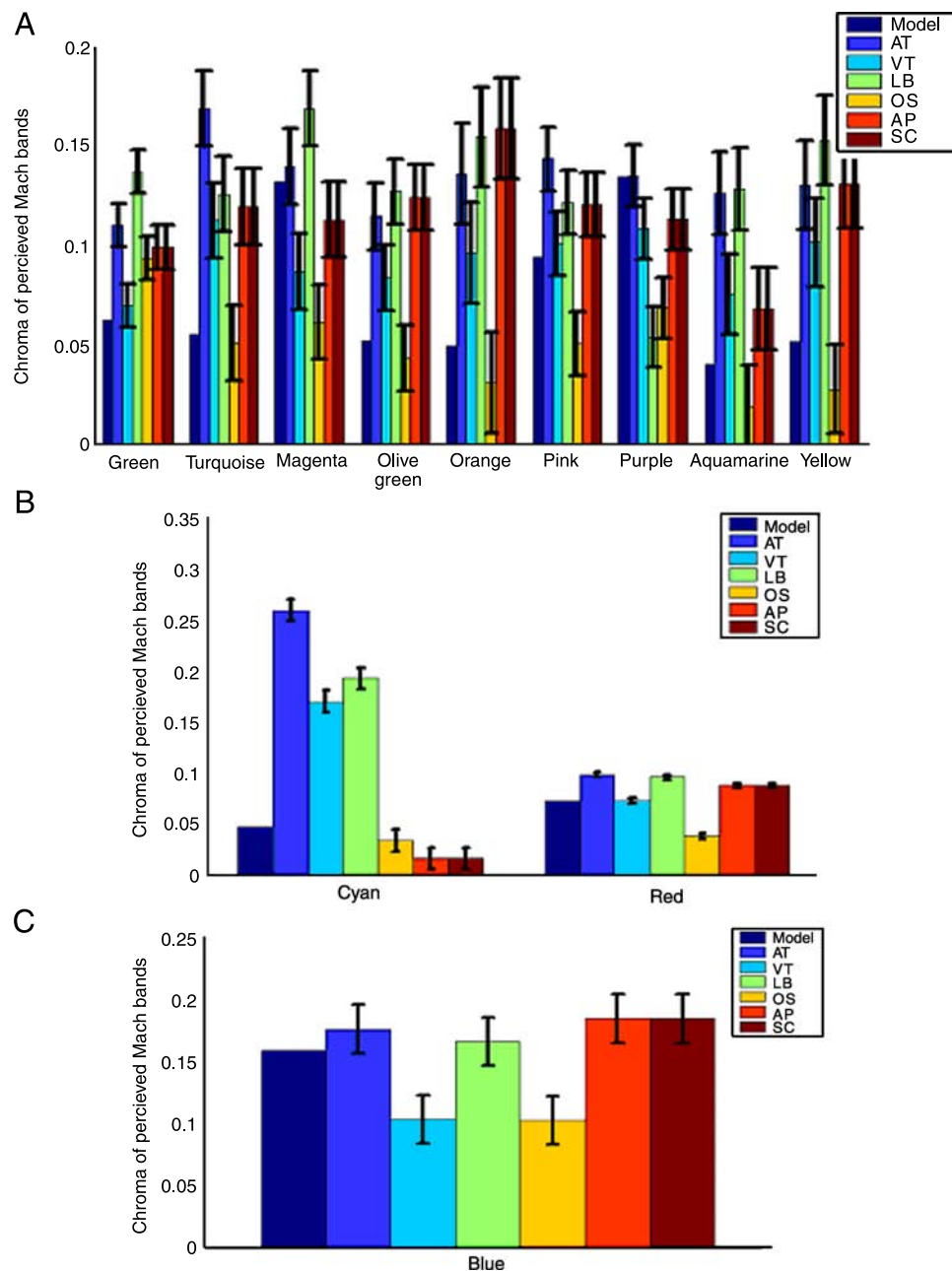


Figure 8. The perceived color "chroma" across the different Mach bands stimuli (x-axis) and across the six observers (different colors), separately for each stimulus luminance (38.5 cd/m^2 , 35.5 cd/m^2 , and 16.6 cd/m^2 at Figures 8A–8C, respectively). The dark blue bar presents the model prediction. The "chroma" term refers to the distance between the data point of perceived color (colored circle in Figures 2–7), on the complementary line and the achromatic point in the CIE 1976 (L , u' , v')-color space. In the case that a data point does not "fall" on the complementary relevant line, we calculated the projected point of this data point on the relevant complementary line, and we referred to the distance from the projected point on the strait complementary line to the achromatic point.

performed the above experiment under controlled iso-brightness conditions.

Methods

Observers

In order to test the chromatic Mach bands under iso-brightness conditions, two men and two women (under

the age of 40), with normal color vision, were tested for the perception of chromatic Mach bands (similar to the stimuli in Experiment 1; Figure 1). Three of the observers were naïve observers and one (co-author AT) had experience with iso-brightness experiments. All the observers had normal (or corrected to normal) vision and were tested for normal color vision with the "Ishihara test" for color blindness (Ishihara, 1989).

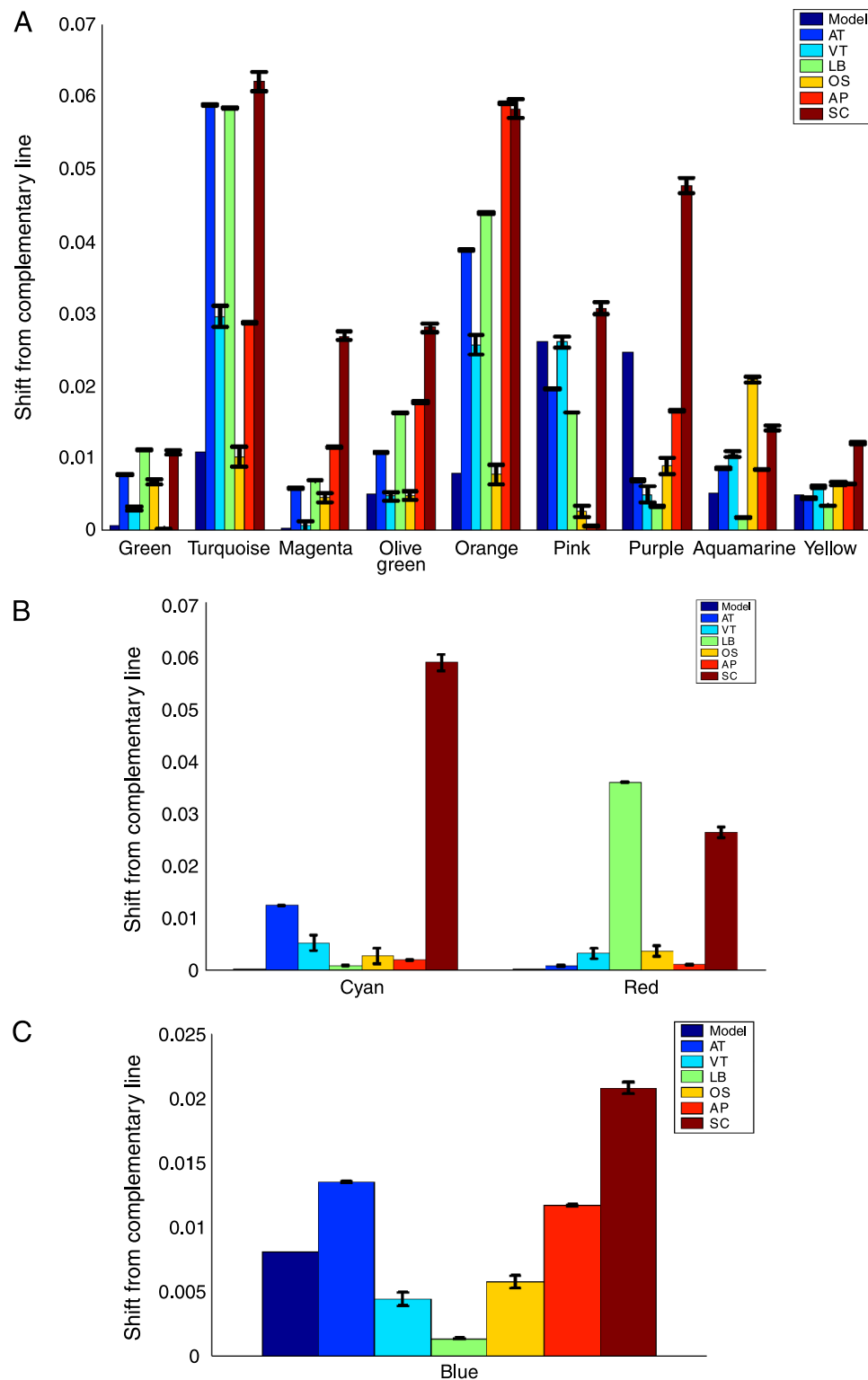


Figure 9. The “shift” of the data points from the relevant complementary lines, across the different Mach bands stimuli (x-axis) and across the 6 observers, presented separately for each stimulus luminance (38.5 cd/m², 35 cd/m², and 16.6 cd/m² at Figures 9A–9C, correspondingly). These “shifts” were calculated as the Euclidian distances (in the CIE 1976 (L , u' , v')-color space) of each data point from the relevant complementary line (gray lines in Figures 2–7). The model predicted “shift” responses of the computational model (Figures 10D–10F) for each of the Mach bands stimuli at the relevant luminance (A, B, and C), also presented (dark blue bar). The figure shows a general trend of higher “shift” from the relevant non-cardinal colors’ Mach band stimuli complementary color line than from relevant cardinal colors’ Mach bands stimuli complementary color line (Table 1).

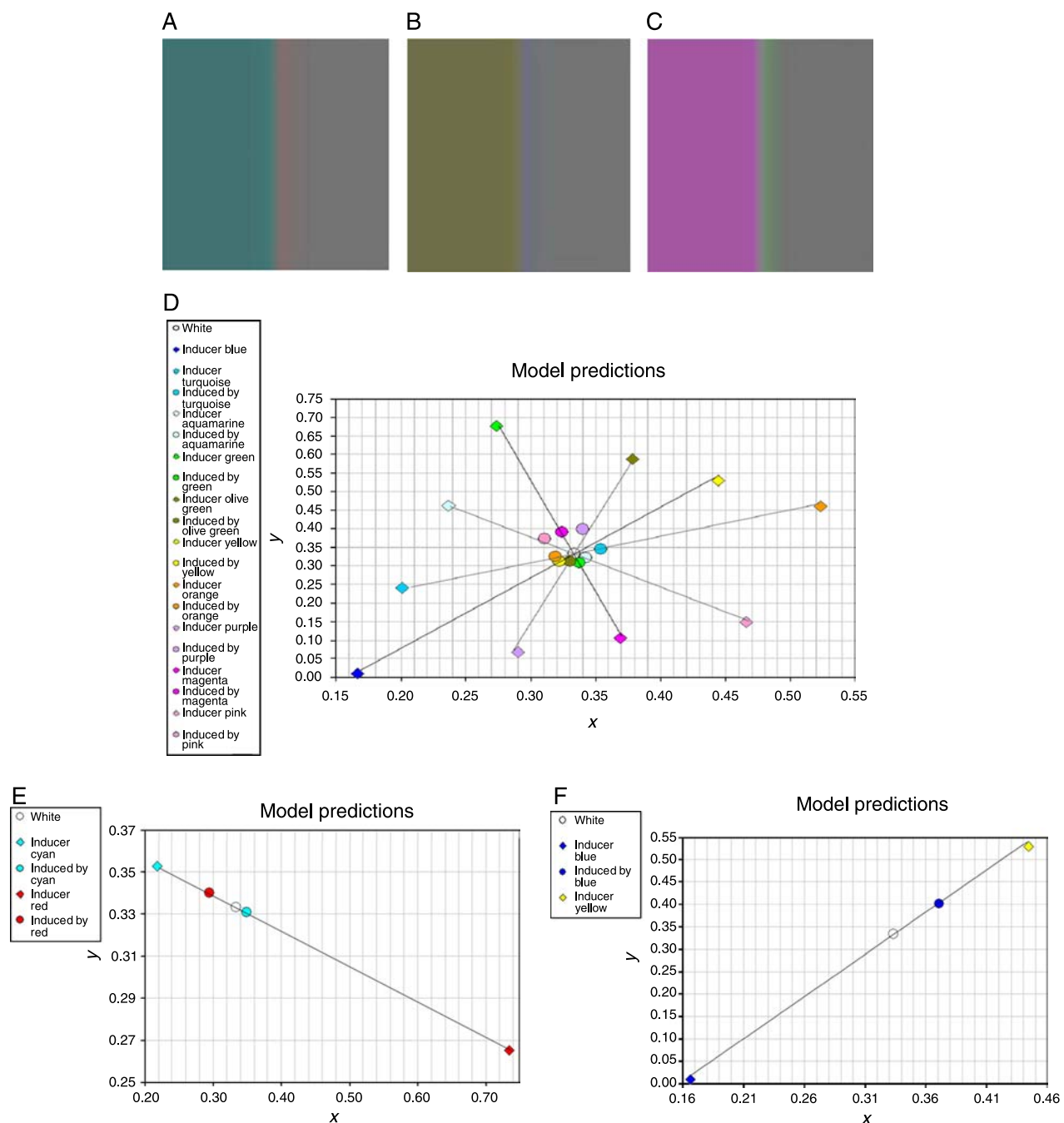


Figure 10. Demonstration of the model's predictions of the existence of chromatic Mach bands effect. (A–C) Three examples of predictions of the perceived chromatic Mach bands (the original stimuli are presented in Figures 1A–1D). (D–F) Predicted “perceived” colors (colored circles) yielded from the whole set of chromatic Mach bands stimuli (Table 1), presented in the CIE 1931 (Y, x, y)-chromaticity coordinates. The different colored diamonds in Figures 10D–10F indicate the specific chromatic regions of the Mach bands stimuli (Figures 1A–1D). The specific color of the “perceived” color is presented similarly to Figures 2–7.

Each observer did a calibration of brightness across 3 cardinal colors and their complementary colors (red, green, blue, cyan, yellow, magenta), along 12 saturation levels of each color (each level was kept in iso-chroma; Equation 1) using the “minimum motion” method (Experiment 2 in Anstis & Cavanagh, 1983). The chromatic/achromatic grating cycling rate was 15 Hz,

and their spatial frequency was 0.625 cycles per degree. (This spatial frequency has been chosen in order to be as close as possible to the spatial frequency of our stimulus.) The size of the grating was the same size as the Mach bands stimulus, i.e., 7° by 8°.

In this experiment each observer was requested to adjust the luminance of the chromatic stimulus until it appeared

steady with “no motion.” The perceived brightness of each chromatic stimulus region is equal to the perceived brightness of the same achromatic stimulus region (with luminance of 40 cd/m²).

The results of this calibration were referred to as the set of stimuli and matching patches organized as a look-up table for the rest of the experimental procedure. Each observer’s individual look-up table was built out of 73 iso-brightness hues (12 saturation levels for each of the 6 colors and one achromatic level). Following the “minimum motion” experiment for each observer, a look-up table was constructed for each observer separately.

Stimuli and matching procedure

Observers were positioned with their eyes 100 cm from the 20-inch monitor (Mitsubishi Diamond Pro 2070^{SB}). The stimuli were generated and the experiments controlled by the computerized Visage stimulus generator (Cambridge Research Systems, Cambridge, England). The Gamma correction and look-up table adjustments were done using the same procedure as in Experiment 1. The size of the stimulus was 7° by 8° (similar to Figure 1A). The spatial width of the saturation ramp between each pair of regions in all the Mach bands stimuli was 1°. The stimuli were presented on a black background screen, while the room was completely darkened, 2–3 cd/m².

The saturation ramp constituted the linear saturation decrease from that point, presented on the chromatic region to zero saturation, as was presented on the achromatic region (similar to Figure 1B). We wanted to keep all the chromatic Mach bands under iso-chroma conditions, therefore, except for cyan, none of the chromatic Mach bands’ regions were 100% saturated. The chromaticity distance was calculated according to the Euclidian distance of the specific chromaticity point from the achromatic point (on the CIE 1976 (u' , v')-chromaticity diagram). Each chromatic Mach band stimulus was built under iso-brightness conditions (using “minimum motion” results supplied in the look-up table). In this experiment we used two test patches (instead of one as

in Experiment 1) below and above the achromatic region located adjacent to the ramp. At this location, the chromatic Mach band is expected to be perceived (arrows locations in Figure 1A; Figure 3 in Lu & Sperling, 1996). The observers were instructed to only look at the achromatic region between the two matching patches.

Matching was performed by using a response box (CB6, Cambridge Research Systems). The observer was required to perform the matching using two keys in the response box to choose the appropriate hue and two keys to set the appropriate saturation of the chosen hue. The entire repertoire of possible chosen matching colors was taken from the preset of 73 hues, which was individually built for each observer from hues with iso-brightness and iso-chroma. The repertoire of possible chosen matching colors included 12 saturation levels for each of the 6 colors (cardinal colors and their complementary colors) as well as one achromatic level.

Each of the observers performed six blocks of trials (on different days), which consisted of six types of chromatic Mach bands (Table 2). Each block of trials was presented four times, while the order of the specific Mach bands was randomized. A black screen was presented between trials for 1 sec.

Results

The existence of chromatic Mach bands effect under iso-brightness conditions was determined by four observers. Figure 11 demonstrates a significant perceived saturation. The statistical significance of the results showed significant shift of the perceived color from the achromatic point, across all four observers and all the Mach bands stimuli (colored circles) ($p < 0.001$, multivariate χ^2 test; Richard & Dean, 1988). The structure of figures that presents the perceived results is similar to the corresponding figures in Experiment 1. However, since the stimuli have been presented in iso-brightness conditions, the data points are presented on the CIE 1976 (u' , v')-chromaticity diagram. Figure 12 demonstrates the variability in perceived chroma across the different chromatic Mach bands stimuli, but the results also showed significant variability among the different observers.

We analyzed the results in the same way as in Experiment 1, i.e., perceived chroma magnitude (Figure 12) and the shift from the relevant complementary line (Figure 13). The statistical tests showed very significant perceived chroma ($p < 0.001$ multivariate χ^2 test), across all the observers and all chromatic Mach bands. There was large variability in the perceived saturation results among the observers and also across the specific chromatic Mach bands stimulus (“inducer”).

Figure 13 shows a very small shift of the data from the relevant complementary line; note that the ordinate has a scale of 10^{-10} , which is very significant. The small shift

Color	Chromatic side		Achromatic side	
	u'	v'	u'	v'
Green	0.152	0.532	0.210	0.474
Magenta	0.269	0.416		
Yellow	0.205	0.556		
Blue	0.216	0.391		
Cyan	0.129	0.467		
Red	0.293	0.480		

Table 2. The CIE 1976 (u' , v') values of the iso-brightness and iso-chroma of the chromatic Mach bands stimuli. (L , the brightness was pre-calibrated by each observer, individually).

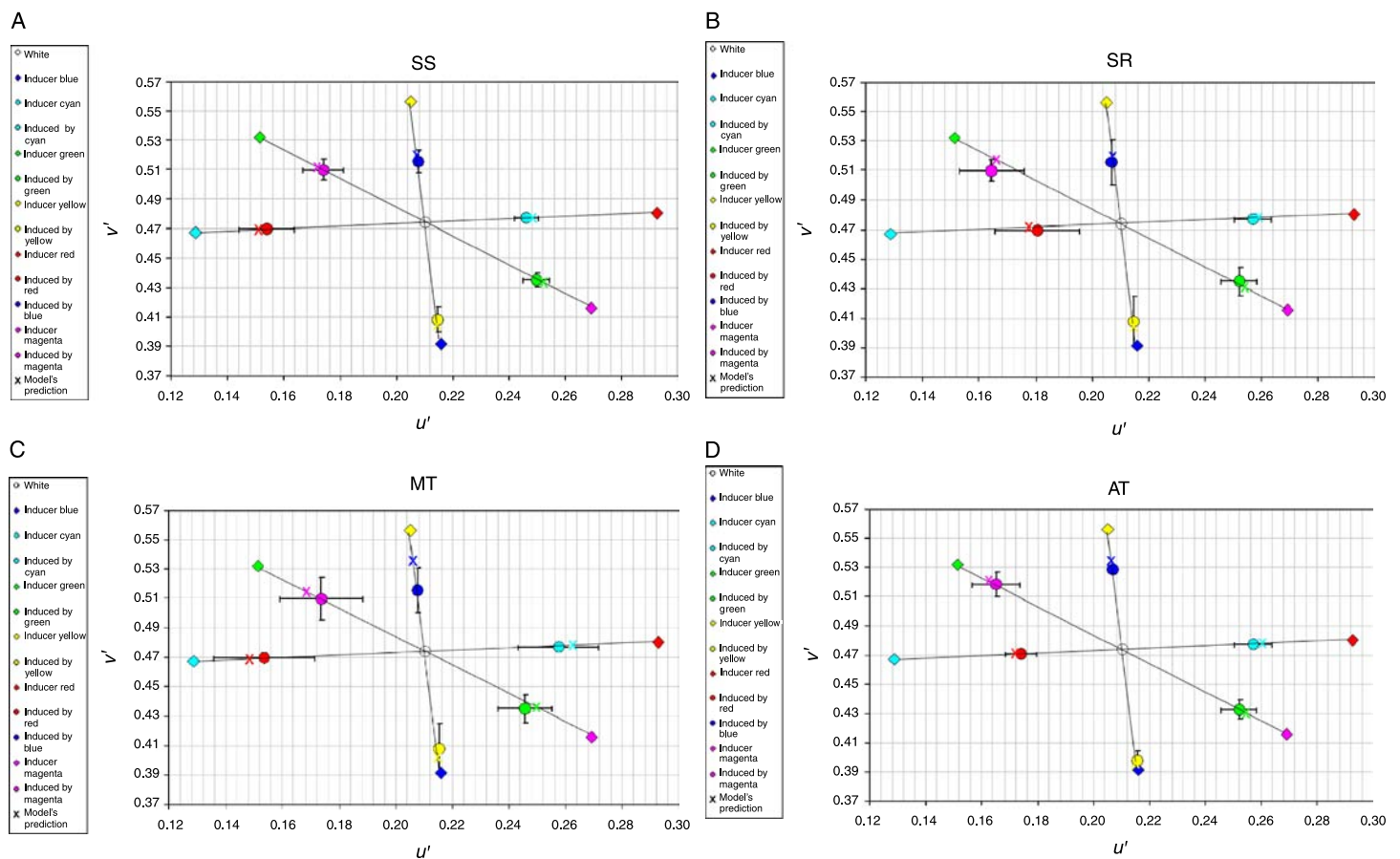


Figure 11. The perceived chromatic Mach bands, colored circles (in the CIE 1976 (u' , v') chromaticity coordinates) under iso-brightness conditions, across 4 observers. (A–D) Results obtained from different observers: SS, SR, MT, and AT, correspondingly. The chromatic regions (“inducers”) are represented by colored diamonds. The chromatic “x”-s represent the predicted chromatic Mach bands (Appendix A).

values obtained indicate that we cannot reject the null hypothesis that the data points are attributed to the population of the relevant complementary color line, $p < 0.001$, multivariate χ^2 test (Richard & Dean, 1988). Since we aimed to model the perceived Mach bands under iso-brightness conditions, we were obliged to calculate the model’s predictions, separately, for each chromatic Mach band and for each observer (narrow yellow bars in Figures 12 and 13).

Experiment 3: Control experiment—Recording eye movements

We repeated the above experiment, under iso-brightness conditions, while recording the eye movements of three observers with an eye tracker. This was necessary in order to rule out the possibility that the perceived chromatic Mach bands effect, Experiments 1 and 2, was caused by

eye movements, i.e., resulted from after effect due to exposure to chromatic region.

Methods

Observers

Three women (under the age of 35) with normal (or corrected to normal) vision were tested for normal color vision with the “Ishihara test” for color blindness (Ishihara, 1989). Two of the observers (MG and NG) were naive to the iso-brightness experiment but had experience with eye tracking experiments, and one observer was co-author (AT).

Apparatus and experimental procedure

The observers were positioned with their eyes 150 cm from the 20-inch monitor (Mitsubishi Diamond Pro 2070^{SB}) using the SensoMotoric Instruments (SMI) High-Speed (iView) eye tracking system.

The iView system is a video eye tracker based on infrared light; thus, it enables recordings in a condition of

complete darkness. The sampling frequency is 240 Hz, which gives the temporal resolution of 4.2 milliseconds (enabling reliable detection of saccades and micro-saccades). For each frame in the video, the software of the system calculates the center positions of both the pupil and of the corneal-reflection. The system calculates the gaze position for each frame by subtracting both and using calibration data (a process which is performed prior to each recording when gaze position is needed). The stated accuracy of tracking resolution is 0.01° . The gaze position accuracy is 0.25° – 0.5° . (Note that the ramp width is 1° .)

The eye-tracking column was fixed to a heavy marble table which absorbed most of the vibrations caused by the participants' body movements. The observers were restrained by a chin rest, a forehead rest, and a bite bar. Significant contribution to noise reduction was provided during long sessions, where the participants performed trials that involved hand movements to register their responses. During these long sessions, the bite bar reduces movements of the head (primarily due to neck and back muscle fatigue). The sessions in our experiments usually lasted no more than 12 minutes. Each observer performed two recording sessions (with a break in between each session of at least 10 minutes). In one session the participants' left eye was covered with a patch, while in the other session the participants viewed the stimuli with both eyes. In all the recordings, only the right eye was tracked.

The computerized Visage stimulus generator (Cambridge Research Systems, Cambridge, England) generated stimuli and controlled the experiments. The Gamma correction and look-up table adjustments were carried out using the same procedure as in [Experiment 2](#) but were collected manually through the iView reflecting glass (to eliminate their chromatic aberrations) with a Colorimeter (Minolta CS-100).

Each observer performed the “minimum motion” procedure through the iView in order to build her own iso-brightness look-up table ([Experiment 2](#)).

Prior to every recording session, the observer was required to perform a calibration procedure in order to obtain eye-tracking data about the gaze position.

The recording session was done during performance of the matching procedure ([Experiment 2](#)), i.e., during the observer's determination of perceived chromatic Mach bands. Each session consisted of experimental trials: two repetitions of each of the six chromatic Mach bands ([Table 2](#)) in randomized order.

Before each trail in the above session, the observer performed a calibration procedure across 10 key points ([Figure 14](#)). The numbers in [Figure 14](#) indicate the order of the points' appearance on the monitor. The size of each point in [Figure 14](#) is $0.5 \times 0.5^\circ$. The locations of the points are the stimulus corners, the centers of the matching patches, and the corners of the chromatic ramp. Each point was presented for 42 msec (10 samples of the iView recording). This calibration was followed by

420 msec of a black screen in order to eliminate adaptation to the points.

The above procedure was carried out in order to perform registration between the two systems (i.e., the iView and the Visage systems). In addition, this procedure also enabled synchronization between the iView and the Visage systems. This requirement for synchronization is due to the unlimited time allowed for the matching procedure ([Experiment 2](#)).

In this experiment, as well as in [Experiment 2](#), the observers were directly instructed not to look at the chromatic region and chromatic ramp, instead they were instructed to stare at the region located between the matching patches ([Figure 1A](#); Lu & Sperling, 1996). Due to previous studies relating to achromatic Mach bands and eye movements (Riggs, Ratliff, Cornsweet, & Cornsweet, 1953; Riggs, Ratliff, & Keeseey, 1961), we performed the above specific procedure to examine the chromatic Mach bands. It has been found that testing the achromatic Mach bands while viewing a specific location without eye movement can cause fading of the Mach bands' appearance. Therefore, the requirement to focus the gaze on the ramp's edge region seemed to be a good compromise.

Note that as in [Experiment 2](#) each trial was followed by 420 msec of a black screen before starting over with the 10 key points calibration and the next trail.

Results

To approve or to postpone the possibility of obtaining chromatic Mach bands, we tested the accumulated eye movements data across three observers, while performing the chromatic Mach bands test under iso-brightness conditions. This control experiment was performed with fewer repetitions ([Methods](#) of [Experiment 2](#)).

In order to scrutinize the observers' results, we presented the data points of the eye movements, recorded by the eye tracker, in reference to their spatial location on the chromatic Mach bands stimulus ([Figure 15](#)). [Figure 15](#) presents the gaze positions of observer MG while performing the matching procedure of the chromatic Mach band of cyan stimulus. It can be seen that almost all of the data points are distributed within the achromatic region ([Figure 15](#)). More specifically, most of the data points are located around the spatial key points no. 3, 4, 7, and 8 ([Figure 14](#)).

To obtain statistical knowledge about the eye movements, we accumulated the data points across the eye movements during the six chromatic Mach bands stimuli experiments (all the 24 trails), separately for each of the three observers ([Figure 16](#)). These accumulated sets of results are presented with histograms. Note these histograms represent only the distributions of data points across the x-axis ([Figure 15](#)). The spatial location of these histograms is in reference to the value of zero degrees of

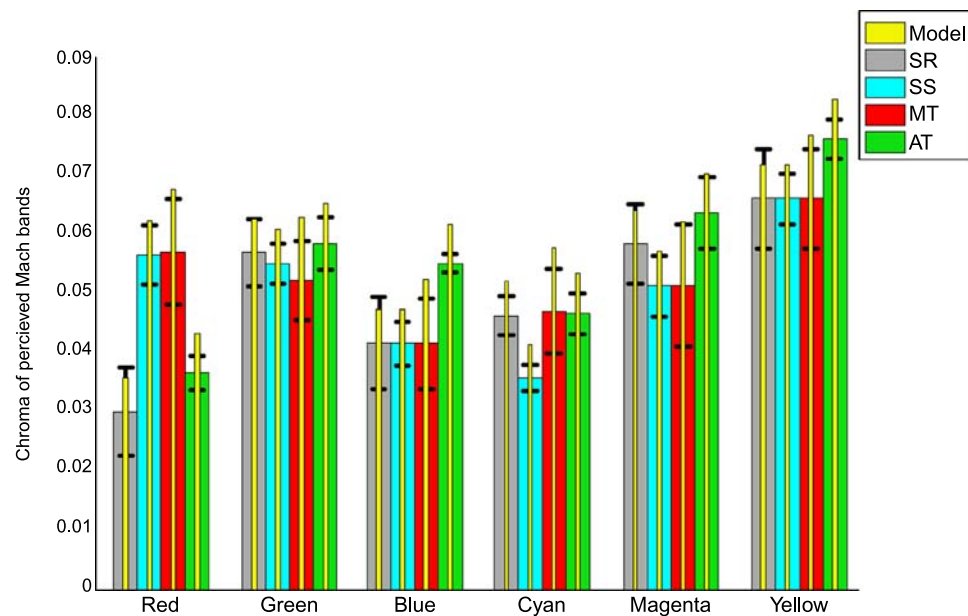


Figure 12. The perceived Mach bands' chroma, across the different Mach bands stimuli (x-axis), and across the four observers.

the x -axis in Figure 16, which presents the edge of the chromatic ramp and the achromatic region (Figure 1A).

The eye movements' results show that a significant percentage of the distribution of data points is significantly biased towards the achromatic region, across all three observers. The results show that in each of the three accumulated histograms less than 0.5% of the data points are located in the chromatic region. Analysis of the eye

movements on the y -axis shows that more than 80% of the above 0.5% of the data points converge around the matching patches (i.e., points no. 4 and 7 in Figure 14). The x -axis data points of all three observers in the gaze position histogram show a Poisson distribution with λ ranging between 1.04 and 1.10, for an α of 0.001.

Unlimited time was given to perform the color matching task, however, when analyzing the results, it was

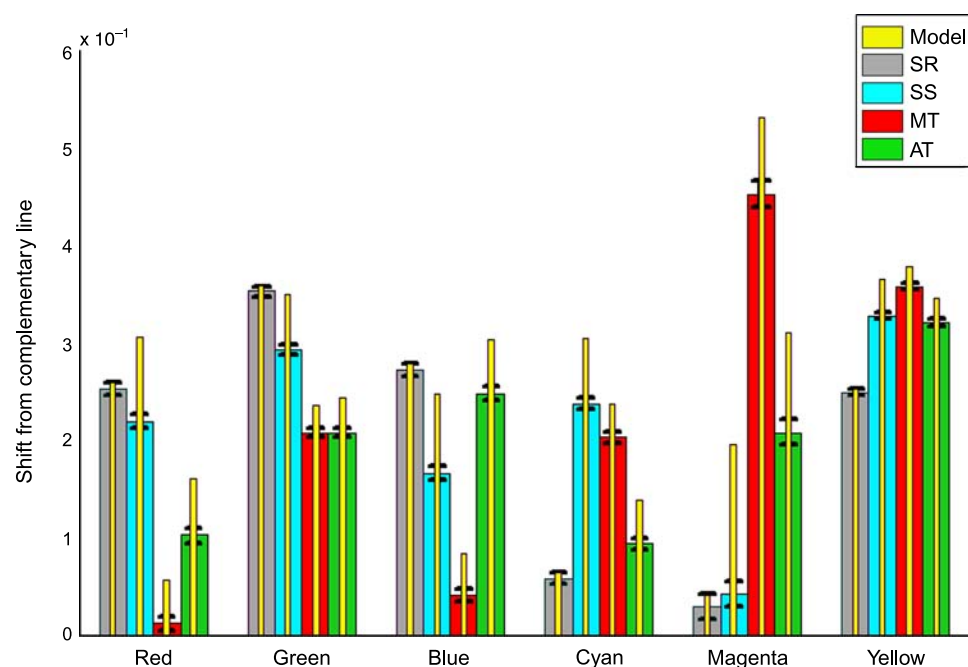


Figure 13. The results of the "shift" of data points from the complementary line, across the different Mach bands stimuli (x-axis), and across the four observers.

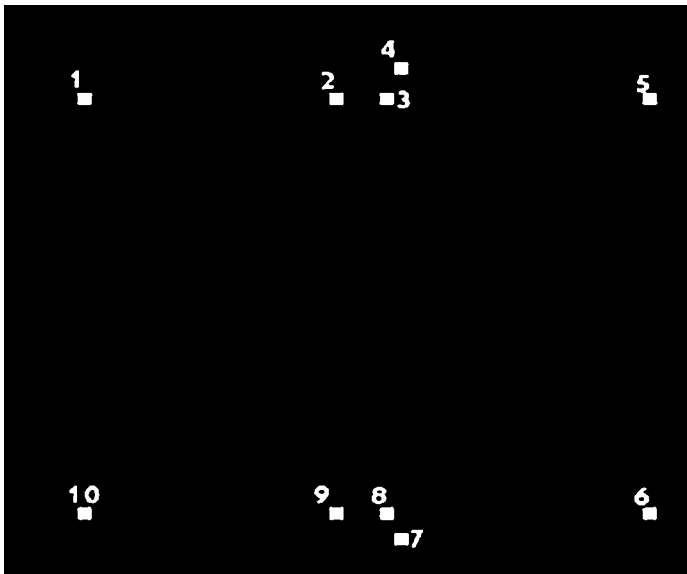


Figure 14. Ten key points calibration procedure. The numbers represent the order of appearance of the key points. The locations of the points are the stimulus corners, the centers of the matching patches, and the corners of the chromatic ramp. Each point is presented for 42 msec.

found that the average time to perform the task was about 300–500 msec. This time included both eye and hand movements (using the response box).

The observers reported that the observed side effect of instability regarding the width of the chromatic Mach bands, which periodically shrank and expanded during “free” observation, vanished during the eye movements experiment, and instead a constant Mach bands width was seen.

Since this experiment is only a control experiment to verify the trend of results obtained during the restricted eye movements condition, the number of repetitions trails is small. The chromatic Mach bands effect under iso-brightness conditions was determined by three observers while recording their eye movements. Figure 17 demonstrates significant perceived saturation across the six stimulus conditions ($p < 0.001$, t test), even though the number of repetitions is small. This is expressed by significant shifts in the perceived color (colored circles) from the achromatic point (empty circle), in both the CIE 1976 u' and v' axes, across all three observers ($p < 0.001$, multivariate χ^2 test; Richard & Dean, 1988).

In addition, we calculated the shift from the relevant complementary line. It can be seen that there are significant small shifts of the data points from the relevant complimentary line, less than 10^{-8} in the CIE 1976 (u' , v')-chromaticity coordinates (Figure 17). These results show a trend similar to the results yielded from Experiment 2 (Figures 11 and 13). The obtained small shift values indicate that we cannot reject the null hypothesis that the data points are attributed to the population of the

relevant complementary color line, $p < 0.001$ multivariate χ^2 test (Richard & Dean, 1988).

Even though significant chromatic Mach bands have been perceived during this control experiment, the magnitude of chroma obtained is somewhat less but not significantly less than the magnitude of chroma in Experiment 2 (Wilcoxon test $p > 0.7$; Figure 18). However, the overall differences between the predicted and the actual perceived chromatic Mach bands have not been demonstrated. Note that the results of Experiments 2 and 3 can only be compared qualitatively, since the observers in Experiment 3 viewed the stimuli through the iView reflecting glass and since the additional observers did not participate in Experiment 2.

The model predictions that were calculated for each chromatic Mach bands stimulus for each observer were more saturated than the observers' results (Figure 15).

Discussion

The existence of chromatic Mach bands under iso-luminance and under iso-brightness conditions has been demonstrated here. Furthermore, the nature and the magnitude of the perceived Mach bands' colors as well as the characterization of these Mach bands in such specific stimulus configuration have been studied for the first time. This effect has also been shown to be perceived while eye movements are controlled, demonstrating that eye movements occur mainly in the achromatic zone. Thus, we can rule out the possibility that the chromatic

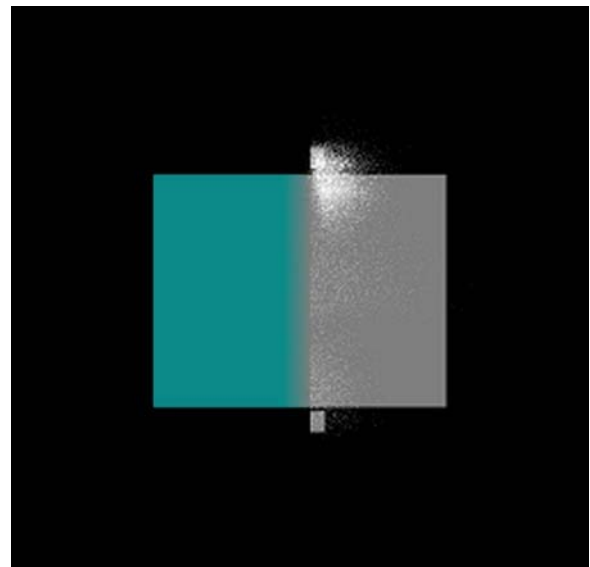


Figure 15. A demonstration of the distribution of gaze positions which present the eye movements (white points) data points, located in reference to the Mach bands stimulus, for observer MG, across all the repetitions of this specific stimulus. The gaze positions of observer MG while matching the chromatic Mach band of cyan stimulus (for all 4 repetitions).

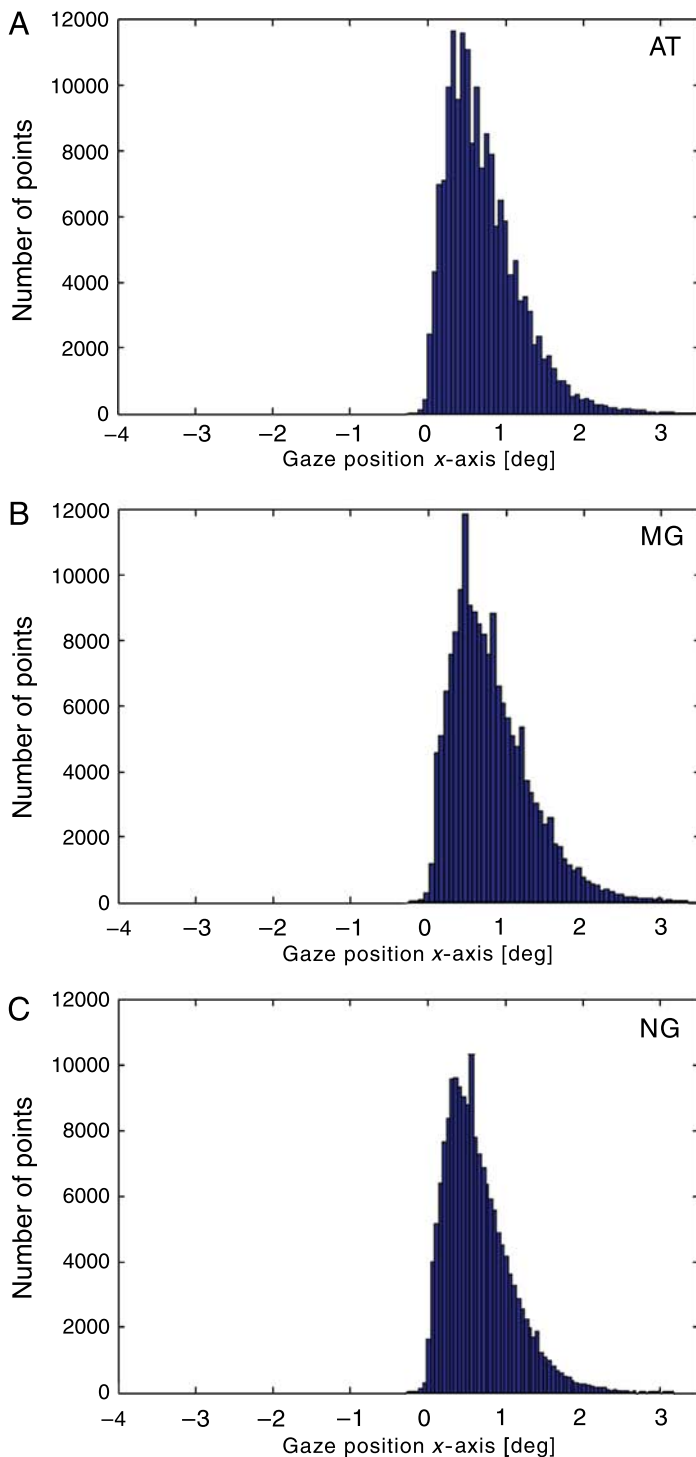


Figure 16. Histogram of the gaze position across all repetitions, for each observer, separately. The 0° in the x-axis presents the edge of the chromatic ramp and the achromatic region, as indicated at x-axis of [Figure 1B](#).

Mach bands effect occurs due to eye movements or that it is a sort of simple after image effect.

The specific stimulus paradigm, consisting of chromatic and achromatic regions, enabled us to produce the large

repertoire of perceived chromatic Mach bands. The results showed that the perceived Mach bands have been significantly saturated, across all the stimuli and for all the observers.

The results showed a systematic rule that the specific perceived color is always the complementary color of the Mach band's chromatic region, the “inducer.” These results led us to test whether the chromatic Mach bands effect is a specific case of the induction effect (Hurvich, 1981; Spitzer & Barkan, 2005). This was tested by performing simulations of the Mach bands stimuli as input to an induction model that also performs color constancy (Spitzer & Semo, 2002). We performed simulations of both sets of Mach bands stimuli under iso-luminance and iso-brightness conditions. Significant perceived chroma was found across all the stimuli and observers. Significant correspondence to the relevant complementary line of all data points, mainly from cardinal colors Mach bands at both iso-luminance and iso-brightness conditions, was also found. This significant perceived chroma has also been found when eye movements were tracked ([Experiment 3](#)).

In order to validate the definite existence of the chromatic effect as part of chromatic induction in our experiments, two critical conditions were created: The first one was that the experiments were established under controlled iso-luminance and iso-brightness conditions. The second one was the structure of Mach bands stimuli, built from chromatic and achromatic regions, used as “inducer” and “induced” regions. The results of experiments under iso-luminance conditions showed significant perceived chromatic bands from all the various chromatic Mach bands. The experiments under iso-brightness conditions proved that the effect occurs under all the required and common controls. [Experiment 3](#) supports the obtained data and showed that it has not been derived mainly from eye movements. We believe that since the results of [Experiment 3](#) yielded very similar results to those of [Experiment 2](#), it is not reasonable to assume that similar results occurred simply (as a necessary condition) due to eye movements. This statement does not rule out the possible additional contribution of eye movements to the chromatic Mach bands effect. The eye movements contribute, for example, to the observed periodic shrinking and expansion of Mach bands, as seen in the observers' report ([Results](#) of [Experiment 1](#)).

The consequent use of a “proper” set of chromatic Mach bands stimuli, each consisting of chromatic and achromatic iso-luminance (and iso-brightness) regions, experimentally supported the view that chromatic Mach bands exist. This characterizes, for the first time, the actual perceived colors on a variety of chromatic stimuli ([Tables 1](#) and [2](#)). The color of the perceived chromatic band was always complementary or nearly complementary to the chromatic stimulus's region ([Figures 2–7](#)). However, the shift of the data points is larger under iso-luminance conditions than under iso-brightness conditions. This

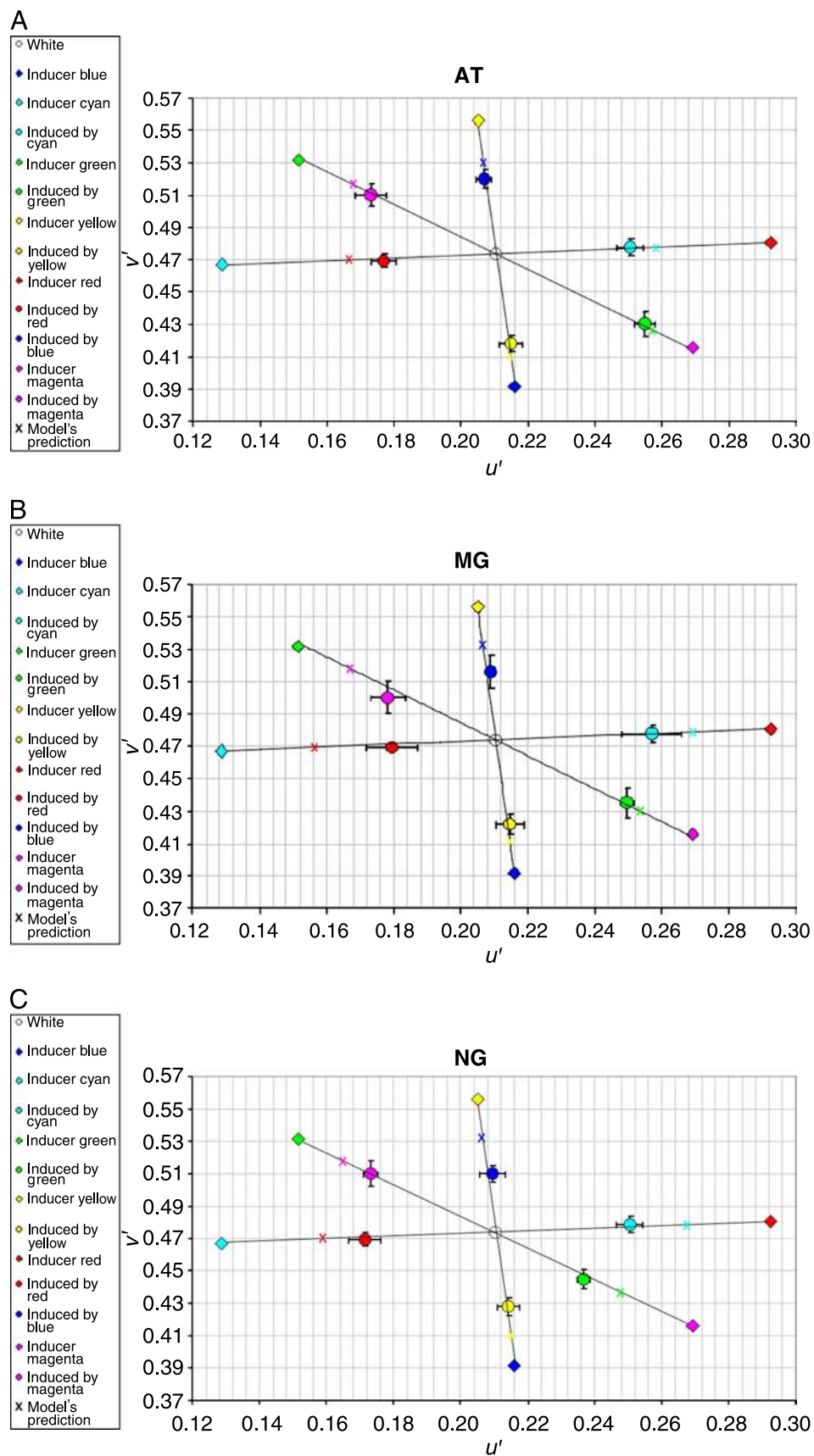


Figure 17. Perceived results of chromatic Mach bands under iso-brightness and during eye movements tracking. The structure of this figure is similar to Figure 11.

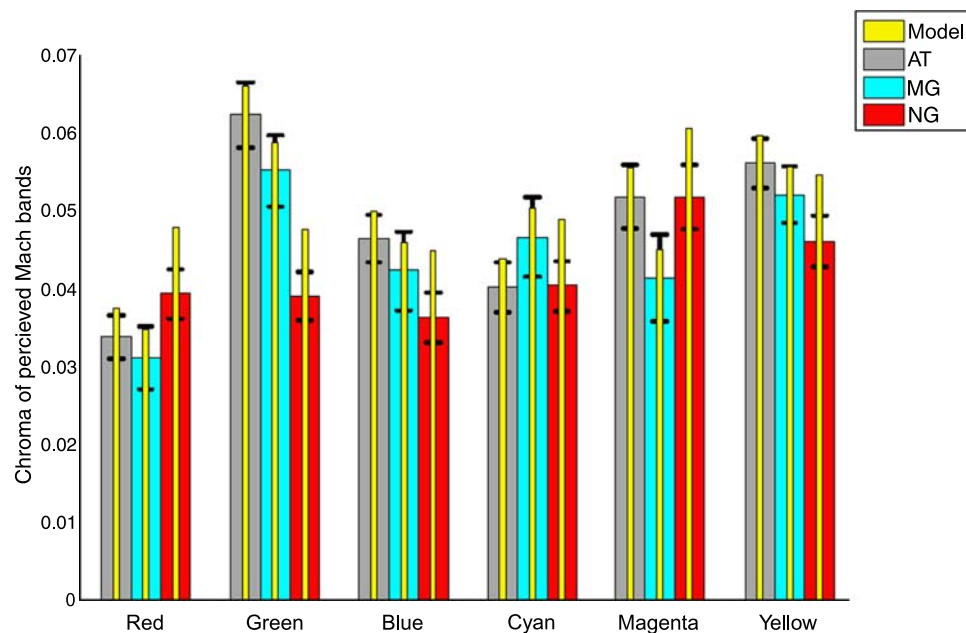


Figure 18. The chroma of perceived Mach bands, across the different Mach bands stimuli (x-axis), and across the three observers during eye movement tracking.

might simply be due to the fact that the cardinal color “inducer” causes a stronger effect, and the stimuli under iso-brightness were only from the cardinal color group. (The two technical procedures in [Experiments 1](#) and [2](#) might have also contributed to these differences; see Methods of [Experiments 1](#) and [2](#).) The complementary nature has also been found when eye movements were controlled ([Figure 17](#)).

We have shown that the computational model, which describes the first order adaptation mechanism and which predicts both the induction and the color constancy effects, can also predict a variety of perceived Mach bands colors as well as the complementary color nature effect ([Figures 10, 11, and 17](#)). The computational results, which show that all of the above effects are reasonable, derive from the basic effect of an enhancement of the differences between the stimulus (its color or intensity) and its surrounding area. This specific basic effect is part of the general effect or adaptation, which is performed through the curve-shifting mechanism (Dahari & Spitzer, 1996; Sakmann & Creutzfeldt, 1969; Shapley & Enroth-Cugell, 1984; [Appendix A](#)). The purpose of adaptation is to maintain a high level of response sensitivity under different illuminations (or contrast) and stimulus conditions (contexts). An adaptation mechanism thus enables the system to maintain a high gain for a large stimulus range (Barkan, Spitzer, & Einav, 2008; Dahari & Spitzer, 1996; Spitzer & Barkan, 2005; Spitzer & Semo, 2002).

The model results under iso-brightness conditions yielded better predictions than the model’s prediction in [Experiment 1](#) (under iso-luminance conditions). This is a result of the different input to the model in [Experiment 2](#),

which was adjusted for each observer separately, due to his “minimum motion” calibration. The input to the model in [Experiment 1](#), however, was a general one, across all the observers. This different model application could lead to the difference in the model prediction of the observers’ results across the two luminance conditions.

The model succeeded in predicting the existence of the chromatic Mach bands effect and perceived color across different stimuli, and under different luminance, brightness, and eye movements conditions. It has to be noted that both experimental and model predictions showed significant chromatic Mach bands effect, especially in [Experiment 3](#), where the observer was instructed to try to avoid gazing at the chromatic regions. These results support the model approach that the remote region (beyond the classical receptive field; [Equations A3–A5](#)) plays a major role in the expression of the induction effect and consequently the Mach bands effect.

Additional prediction is required and a related question raised is whether the same type of model, in reference to the luminance channel, has the ability to predict the classical Mach bands. A second question relates to chromatic Mach bands stimuli, which consist of two complementary regions. The rationale for using this model is also that it performs chromatic enhancement, as predicted for the Mach bands stimuli. This prediction derives simply from our mechanism proposal that the perceived Mach bands effect is a specific case of induction effect and therefore the complementary color is expected. Note that the Mach bands effect, as part of the induction effect, resulting from the adaptation mechanism (Dahari & Spitzer, 1996; Spitzer & Barkan, 2005), is expected to be affected by both temporal and

spatial domains. Thus, the temporal aspects of the chromatic Mach bands should still be tested.

We tested the two critical conditions using our chromatic induction model (the current model) and the achromatic model (Spitzer, Karasik, & Einav, 2003), referred to below. Additional model simulations were performed on the chromatic model, while taking one chromatic example of complementary colors in the two Mach band's regions (chromatic–chromatic Mach bands stimuli), as input to the model (Figure 19C). The figure demonstrates the expression of each chromatic Mach band, separately from each chromatic region (Figures 19A and 19B).

Additional input has been applied for prediction of Mach bands stimulus with opponent colors (Figure 20C). The effect of each of the chromatic Mach bands separately can be seen in Figures 20A and 20B, while in the superposition (Figure 20C), the effect is an illusive one. The model's prediction supports the perceived color from our chromatic Mach bands (Figures 10D, 19D, 19E, 20D, and 20E). Note that the model predictions in all the examples have been made using the very same set of parameters as those applied in Experiments 1–3.

The model's results show that chromatic enhancement is more prominent in the stimulus with complementary colors (Figure 19F) than in the stimulus with opponent colors (Figure 20F). The predictions of chromatic–chromatic Mach bands stimuli (Figures 19C and 20C) are consistent with the perceived Mach bands stimuli, even though these were not tested here systematically. Although the predictions of chromatic–chromatic Mach bands stimuli (Figures 19F and 20F) were modeled directly from these stimuli (Figures 19C and 20C), the very same results are also obtained from the summation chromatic Mach bands (Figures 19D, 19E, 20D, and 20E). The model's prediction of the red-green Mach bands stimulus is that little or no chromatic effect (Figure 20F) occurs. This is not in complete disagreement with previously reported effects (Gur & Syrkin, 1993; Valberg, 2005).

The current computational model, which successfully predicts the chromatic Mach bands with its various appearances, does not predict the traditional Mach bands, since it was not built for this computationally. (We chose to “normalize” the luminance effects in the current simulation and concentrate on the chromatic aspects of this specific model due to computational adjustments.) Consequently, we tested the achromatic stimuli on a variation of the current model by taking into account only the luminance component and relating to the on-type opponent receptive field (Dahari & Spitzer, 1996; Spitzer et al., 2003; Valberg, 2005). Modeling the traditional Mach bands stimulus with this model variation is not in disagreement with Valberg (2005) “...Mach bands to achromatic stimuli may well arise from activity of both PC and MC cells.” The prediction of this model variation in relation to the achromatic Mach bands can be seen in

Figure 21B. The model predicted both the bright and the dark bands in the Mach bands stimulus.

Besides the similarity of the chromatic and achromatic Mach bands effect, the chromatic Mach bands still showed a somewhat slower effect. The reason for this might be the different temporal parameters of the chromatic and achromatic adaptation mechanisms that should still be tested.

The mechanism that causes the Mach bands effect is regarded as yet unknown in the literature. Even though explanations have been suggested by several models, there is no existing accepted model that explains the effect (Pessoa, 1996; Ross, Diamond, & Badcock, 2003). All the previous suggested models related to achromatic Mach bands stimuli. The common and early explanation and models for the Mach bands reported center-surround antagonistic interactions at the retina (or expressed by terms of lateral inhibitory mechanism) (Mach, 1865; Morrone, Ross, Burr, & Owens, 1986; Ratliff, 1965; Ross, Morrone, & Burr, 1989). These models are insufficient for explanation of the chromatic Mach bands effect (see review, Pessoa, 1996); however, they are critical for the achromatic effect (Ross, Holt, & Johnstone, 1981). Luiz Pessoa (1996) divided the Mach bands models into three classes: (a) feature based, based on operators such as even-symmetry and odd-symmetry mechanisms (Tolhurst, 1972). According to this model the optimal edge detector response is in the middle of the ramp, while the optimal bar detector is at the inflection points. This approach was further supported by Ratliff's (1984) results. (b) Rule-based models (convolution responses) that were developed, whose specific rules, however, have not all been justified (Kingdom & Moulden, 1992; Watt & Morgan, 1985). (c) Filling-in model which suggests that edges and lines are basic primitives of early vision (Pessoa, 1996). Additional more recent explanations and models of the achromatic Mach bands effect have been suggested by Purves, Williams, Nundy, and Lotto (2004).

The chromatic and saturation aspects have not been addressed in any of these models, and we suspect that they cannot predict the complementary nature of chromatic effects, namely that their models would transfer to predict the achromatic effect. The only suggested model which relates to the chromatic effect is Valberg's (2005) model, which included a simple demonstration of response of the color coding opponent receptive field (limulus eye of Hartline, 1940), to chromatic Mach bands, which consisted of adjacent chromatic regions (Valberg, 2005), concluded that “chromatic Mach band seem not to exist.”

Opponent receptive fields, retinal or cortical color-coding opponent cells, are sensitive to color. Opponent receptive fields such as the receptive fields of R/G or G/R cells are not necessarily composed of complementary colors. Thus, the main dilemma over and above accounting for the color-coding cells is their inability to simply and basically predict the perceived complementary colors (for

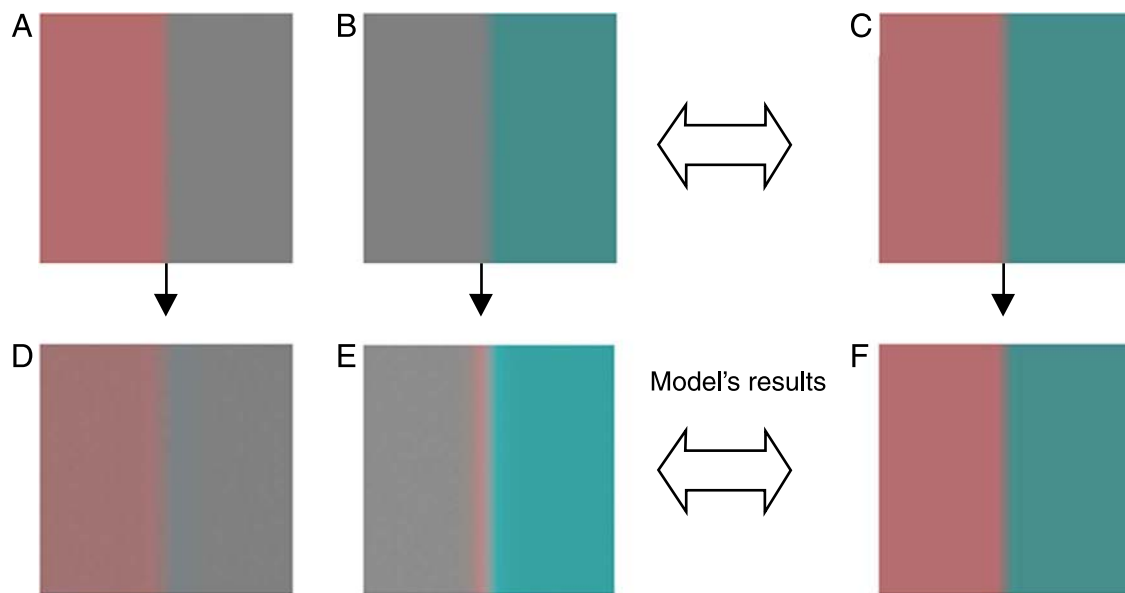


Figure 19. The chromatic–chromatic Mach bands stimulus consisting of complementary colors and its model's predictions. (A, B) present the chromatic regions separately, while they are located adjacent to achromatic regions, in order to explain the effect appearing from the super positioned stimulus (C). The model's predictions for each stimulus component separately (D, E) and for the chromatic–chromatic stimulus (F).

example, the reddish region will cause perception of a cyan Mach band). Moreover, based on our current observations, we doubt whether the application of current models to test Mach bands, for chromatic Mach bands effect, will predict a significant perceived complementary colors effect using both cardinal and non-cardinal Mach bands stimuli.

Henning, Millar, and Hill (2000) suggested an adaptation model which is a modification of two previous adaptation models, that of Cornsweet and Yellott (1985, 1986) and that of Kortum and Geisler (1995). The authors reported that the models succeeded in predicting several effects including the achromatic Mach bands effect, even

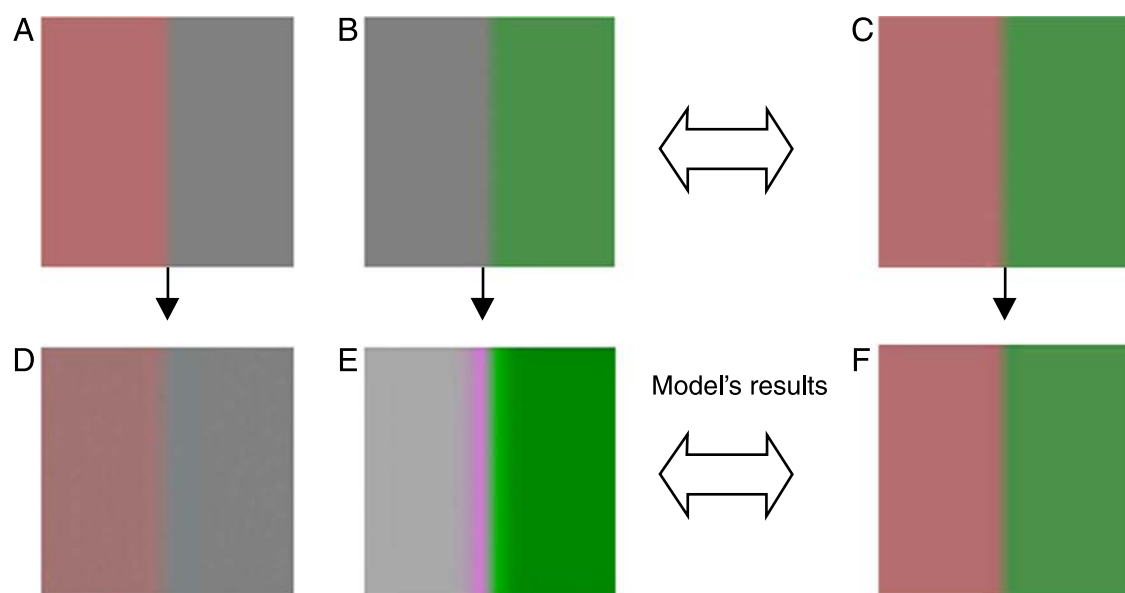


Figure 20. The chromatic–chromatic Mach bands stimulus consisting of opponent colors and its model's predictions. (A, B) The chromatic regions separately, while they are located adjacent to achromatic regions, in order to explain the effect appearing from the super positioned opponent color stimulus (C). The model's predictions for each stimulus component separately (D, E) and for the illusive chromatic–chromatic stimulus appearance (F).

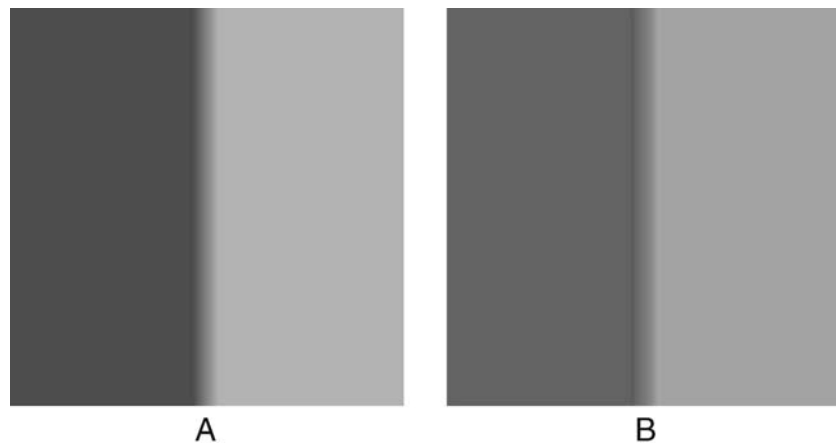


Figure 21. Traditional achromatic Mach bands stimulus (A) and its model predictions (B), see text.

though this has not been demonstrated. Their models contain several elements similar to our current and previous adaptation models (Dahari & Spitzer, 1996; Spitzer & Semo, 2002), such as the basic Naka–Rushton equation and curve shifting mechanism, but the spatial and temporal considerations differ. Note also that the Henning et al. (2000) model related only to models and predictions of the achromatic effects.

Although the model predicts significant perceived complementary colors from both types of stimuli, the model also predicts the small but significantly larger shift of the perceived colors observed from the complementary line (Figures 2–7, and 9) to the non-cardinal colors stimuli. The effect of perceived complementary colors on the induction effect from both the cardinal and non-cardinal colors has also been described in psychophysical experiments (Krauskopf et al., 1986; Webster & Mollon, 1994; Zaidi, 1999). Webster and Mollon's (1994) results showed greater selectivity following adaptation to the cardinal color channels, L–M or S axes. These findings show that even the small shift that has been predicted by the adaptation model (Appendix A) has been manifested in both chromatic Mach bands effect and suprathreshold color induction (Webster & Mollon, 1994).

Over recent years, consensus has been reached regarding the nature of the perceived complementary colors from chromatic induction, but a longstanding debate continues concerning the neuronal locus of color induction (Mizokami, Paras, & Webster, 2004; Webster, Malkoc, Bilson, & Webster, 2002). The debate also relates to the Mach bands effect. Several research groups have suggested that the debate about the induction effect was resolved by the Krauskopf and colleagues (1986) study. They claimed that chromatic induction occurs at the cortical level, resulting more from the interaction within the cortical level of chromatic mechanisms than from the interaction within the retinal opponent mechanisms (Krauskopf et al., 1986; Zaidi, 1999). This approach was based on the assumption that retinal opponent color-coding

cells' mechanism cannot account for induction appearance of complementary colors (and that the complementary color appearance cannot be built from the opponent color cells).

Webster and Mollon (1994, 1995) suggested that color appearance depends on channels that can be selectively tuned to any color-luminance direction and suggested several cortical possible mechanisms for their results.

Contrary to the above approaches, regarding the cortical locus of the mechanism of the color induction due to color adaptation (Krauskopf et al., 1986; Mizokami et al., 2004; Webster & Mollon, 1994, 1995; Zaidi, 1999), the current model's prediction is based on retinal opponent color coding receptive fields that predict the complementary color of the induction effect (Spitzer & Barkan, 2005; Spitzer & Semo, 2002) and the Mach bands effect (Figure 10). Thus, the model predictions contradict the assumptions of previous studies that the complementary colors effect cannot be derived from an opponent receptive field type (Brown & MacLeod, 1997; Krauskopf et al., 1986; Zaidi, 1999).

The fact that the same chromatic adaptation model, which related to different chromatic appearance effects, can predict several effects such as the Mach bands effect, simultaneous chromatic contrast (chromatic induction) and color constancy, manifests its generic uses. Such a generic model's mechanism is utilized as an advantage for a computational model.

In conclusion, it has been shown here that the chromatic Mach bands effect does exist. Here we showed that a variety of chromatic Mach bands, from a novel Mach bands paradigm, consisting of chromatic and achromatic regions, have been clearly perceived under both iso-luminance and iso-brightness conditions. This is also supported by the eye movements control experiment. It has been shown that the perceived Mach bands color is always the complementary color. The computational model for induction predicted the chromatic Mach bands. Both experimental and computational model results support the suggestion that the chromatic Mach bands effect is a specific chromatic induction effect.

Appendix A

A short summary of the model

The current model for the adaptation of the first order is represented here briefly (for the self-content of this paper) in order to predict the chromatic Mach bands effect (Spitzer & Barkan, 2005; Spitzer & Semo, 2002).

The quantum catch of each of the three cone-types, L_{cone} , M_{cone} , and S_{cone} , is expressed by an inner product of the cone pigment sensitivities, the spectral composition of the illumination and the reflectance properties of the surface (Wyszecki & Stiles, 1982). In the case of the synthetic images (such as Mach bands stimuli; Figure 1) and the real images, the RGB values were transformed to L_{pigment} , M_{pigment} , and S_{pigment} values through the CIE 1931 (Y, x, y) -color space (Wyszecki & Stiles, 1982).

The “center” signals for the three spectral regions, L_{cen} , M_{cen} , and S_{cen} , that feed the retinal level are defined as an integral of the cones quantum catches, over the center subregion, with a Gaussian decaying spatial weight function.

$$\begin{aligned} L_{\text{cen}}(x_0, y_0) &= \iint_{\text{cen-area}} L_{\text{cone}}(x, y) f_c(x - x_0, y - y_0) dx dy \\ M_{\text{cen}}(x_0, y_0) &= \iint_{\text{cen-area}} M_{\text{cone}}(x, y) f_c(x - x_0, y - y_0) dx dy \\ S_{\text{cen}}(x_0, y_0) &= \iint_{\text{cen-area}} S_{\text{cone}}(x, y) f_c(x - x_0, y - y_0) dx dy, \end{aligned} \quad (\text{A1})$$

where the variables L_{cen} , M_{cen} , and S_{cen} at locations x_0, y_0 represent the response of the center area of the receptive field of each cell type which is centered at location x_0, y_0 . The following equations are similarly expressed, but in order to simplify x_0, y_0 will be substituted as $x_0 = y_0 = 0$.

Where f_c is defined by:

$$f_c(x, y) = \frac{\exp[-(x^2 + y^2)/\rho_{\text{cen}}^2]}{\pi \rho_{\text{cen}}^2}; x, y \in \text{center area}, \quad (\text{A2})$$

where ρ represents the radius of the center region of the receptive field of the color-coding cells. The “center” can be stimulated by as little as a single cone, as frequently occurs in the fovea (the center of the gaze). The “surround” signals L_{srd} , M_{srd} , and $(L + M)_{\text{srd}}$ are similarly defined, with a spatial weight function three times larger in diameter than that of the “center” (Shapley & Enroth-Cugell, 1984).

The “remote” signal represents the peripheral area that extends far beyond the borders of the classic RF of the

P-RGC (Creutzfeldt, Crook, Kastner, Li, & Pei, 1991; Creutzfeldt, Kastner, Pei, & Valberg, 1991). The “remote” area has the shape of an annulus, concentric to that of the “center” and of the “surround.” The four “remote” signals, $L_{(\text{remote})}$, $M_{(\text{remote})}$, $S_{(\text{remote})}$, and $(L + M)_{(\text{remote})}$ that feed the P-RGCs’ level are defined as the inner product of each cone output with a remote spatial weight function f_r :

$$\begin{aligned} L_{(\text{remote})} &= \iint_{\text{remote area}} L_{\text{cone}}(x, y) f_r(x, y) dx dy \\ M_{(\text{remote})} &= \iint_{\text{remote area}} M_{\text{cone}}(x, y) f_r(x, y) dx dy \\ S_{(\text{remote})} &= \iint_{\text{remote area}} S_{\text{cone}}(x, y) f_r(x, y) dx dy \\ (L + M)_{(\text{remote})} &= \iint_{\text{remote area}} (L + M)_{\text{cone}}(x, y) f_r(x, y) dx dy. \end{aligned} \quad (\text{A3})$$

The adaptation process for each subregion is performed according to its content σ_1 and its remote area, σ_r (Equation A5). The history of the stimulation of the relevant region and its remote area and the change in σ ($\sigma = \sigma_{\text{local}} + \sigma_{\text{remote}}$) that produces a gain control effect is equivalent to the curve shift of the “response vs. log luminance (or color)” curve also reflected and has been shown experimentally (Dahari & Spitzer, 1996; Sakmann & Creutzfeldt, 1969; Shapley & Enroth-Cugell, 1984; Spitzer & Semo, 2002). For on-center cells, the response is expressed as $R_{\text{op}(L+M-)}$, $R_{\text{op}(M+L-)}$, and $R_{\text{op}(S+(L+M)-)}$. The response of $R_{\text{op}(L+M-)}$ is presented:

$$\begin{aligned} R_{\text{op}(L+M-)}(t) &= \frac{L_{\text{cen}}}{L_{\text{cen}} + \sigma_{L+M- \text{cen}}(t)} - \frac{M_{\text{srd}}}{M_{\text{srd}} + \sigma_{L+M- \text{srd}}(t)} \\ R_{\text{op}(M+L-)}(t) &= \frac{M_{\text{cen}}(t)}{M_{\text{cen}} + \sigma_{M+L- \text{cen}}(t)} - \frac{L_{\text{srd}}}{L_{\text{srd}} + \sigma_{M+L- \text{srd}}(t)} \\ R_{\text{op}(S+(L+M)-)}(t) &= \frac{S_{\text{cen}}}{S_{\text{cen}} + \sigma_{S+(L+M)- \text{cen}}(t)} - \frac{(L+M)_{\text{srd}}}{(L+M)_{\text{srd}} + \sigma_{S+(L+M)- \text{srd}}(t)}. \end{aligned} \quad (\text{A4})$$

When dealing with single images (such the Mach bands stimulus), we referred to the final stage of adaptation, constant in time, and already reaching a steady state ($t = \infty$) in the adaptation process. In current usage of the model, only this stage of adaptation has been analyzed. The

adaptation factor σ of “local” and “remote” components for the L^+M^- signal, for example, is given by:

$$\sigma_{L^+M^- - \text{cen}}(t = \infty) = \sigma_{L^+M^- - \text{cen} - \text{local}} + \sigma_{L^+M^- - \text{cen} - \text{remote}},$$

where

$$\begin{aligned}\sigma_{L^+M^- - \text{cen}}(t = \infty) &= a_{\text{cen}}L_{\text{cen}} + b_{\text{cen}} + c_{\text{cen}}L_{\text{remote}} \\ \sigma_{L^+M^- - \text{srdn}}(t = \infty) &= a_{\text{srdn}}M_{\text{srdn}} + b_{\text{srdn}} + c_{\text{srdn}}M_{\text{remote}}.\end{aligned}\tag{A5}$$

The adaptation factors for surround subregions are similarly defined. a , b , and c are constants but obtain different values for the “center” and for “surround” (Dahari & Spitzer, 1996; Shapley & Enroth-Cugell, 1984).

In order to quantitatively estimate the model’s performance and perform the algorithm on real images, the activation levels of the simulated responses have been transformed into a perceived color through an inverse function with few assumptions. The first and main assumption is that the remote area in the inverse function is achromatic, with the same degree of luminance as in the direct function.

The induction effects (including the Mach bands effect) appear as part of the visual system processing to enhance the perceived chromatic or luminance differences between objects and surfaces, also under different chromatic illuminations, as occurs in the color constancy phenomenon sharing the same mechanism (Fairchild, 1998; Spitzer & Barkan, 2005; Spitzer & Semo, 2002). We believe that if the chromatic Mach bands were built from complementary colors, the Mach bands would be observed simply as chromatic enhancement, similar to how they are observed in classical achromatic stimuli. This might be a subject for future research.

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