

# Embedded systems

## Pulse Width Modulation, PWM

The principle of *pulse width modulation (PWM)* has been growing in popularity when it comes to circuitry for power amplification, both for pure application and as drivers for among other things motors. The reasons for this are primarily the simplicity of the circuitry and the possibilities to create applications with low power dissipation.

We will start with an exposé of the commonly used types of amplification, discuss their merits and limitations and then move on to the role of PWM in this field.

## Amplifier classes

We will be discussing different types of amplifiers intended for power amplification, that is we are not only interested in raising the voltage level but also the current through the load. We will only look at transistor amplifiers.

### Class A

In the simplest form of *class A* amplifier we use one transistor as the amplification stage, *Figure 1*. This transistor is biased to a quiescent state where half the available current flow through the transistor and the voltage across the load is half the supply voltage. The input voltage will modulate the base voltage of the transistor and as a result the current through the transistor and the voltage across the load will vary around the quiescent values. Because of the variation around the quiescent state this amplifier can be made very linear but at a cost. Since the transistor is conducting even when there is no input signal the power consumption will be large and most of the power will be dissipated as losses, as heat. The efficiency of the class A amplifier will be less than 25 %, that is more than 75 % of the power will be lost in the form of heat that has to be transported away if the transistor is not to be destroyed.

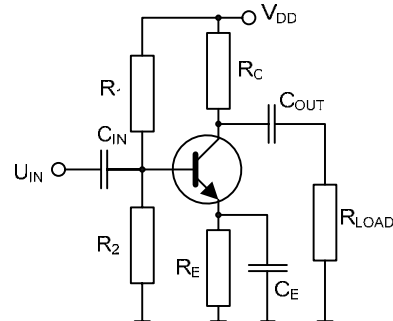


Figure 1 Class A amplifier



# Class B

In the *class B* amplifier we try to solve the problem of the big power consumption and the low efficacy. We have two transistors in a so called *push-pull configuration* and make sure that each of these devices are conducting during separate halfperiods of the insignal, *Figure 2*. When the signal is zero none of the transistors conduct. In this way there will be no power dissipation in rest. The configuration have some problems though. A transistor is not linear over the whole span from very small signals to large signals, we have a unlinear area for small signals. This means that the output signal will be distorted for small signals, that is in the area of small positive and negative output signals. We will get what is called *crossover distortion*, *Figure 3*. These amplifiers can theoretically give an efficiency of 78 % but will in reality give a lower efficiency.

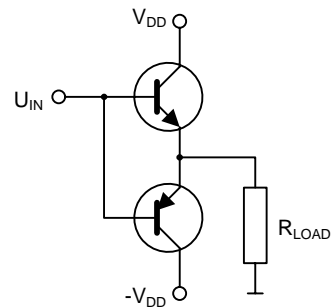


Figure 2 Simplified class C amplifier

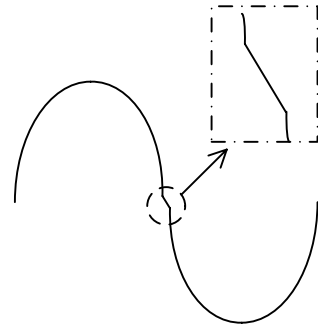


Figure 3 Crossover distortion

# Class AB

A *class AB* amplifier is a combination on the class A and class B theme. We start from the class B configuration but we make sure that the two transistors have a small bias voltage on their bases even in rest, *Figure 4*. In this way we make sure that the transistors work in their linear mode. The price we have to pay for this is that the bias voltages make the transistors conduct giving both voltage over the transistors and current trough them giving raise to power losses. This type of amplifier will seldom have an efficiency of more than 50 %.

The class AB type of power amplifier is by far the most frequently used amplifier type since it is quit easy to design with a reasonable efficiency.

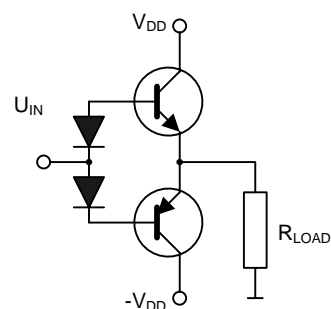


Figure 4 Simplified class AB amplifier

# Class C

The *class C* amplifier is a bit special and can only be used under special circumstances, *Figure 5*. The circuit can give large amplification but it is highly non-linear meaning that it gives different amplification for different frequencies in the input signal. The non-linearity can be of an acceptable level if we use the circuit over a small frequency range and that is something that is the case in radio transmitters and receivers (transceivers) where the signal is modulated on a carrier wave.

With the development of modern class AB amplifiers these have gradually started to replace class C amplifiers also in radio applications.

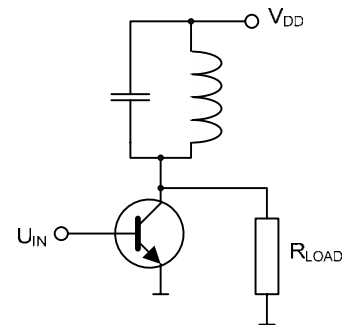


Figure 5 Simplified class C amplifier

# Class D

We have seen that a problem with the amplifier types mentioned is the power dissipation, the efficiency. Power is proportional to the voltage across the device and the current through it,  $P \propto U \cdot I$ , that is we could minimize this by making either of the two properties small and this is the idea behind the *class D* amplifier, *Figure 6*. We still have two transistors in a push-pull configuration but we switch the transistors between two states, non-conducting and saturated. In the non-conducting state the voltage across the transistor will be high and in the saturated state it will be low. When one of the transistors is non-conducting the other will be saturated.

Now how does this help us? No current will flow through a non-conducting transistor so there will be no power dissipation no matter the voltage across it. When the transistor is saturated the voltage through it will be small (around 0.2 Volts) and the power dissipation will be, not zero, but small although the current through the transistor might be high. This means that the power dissipation will be low and the efficiency will be high.

This might seem nice but if we think a bit further we realize that switching the transistors between two states, conducting and non-conducting can only give a faithful representation of the input signal if this is a square wave but our aim is to amplify analog signals, that is signals that can take on any voltage within their definition range. This is where the *pulse width modulation (PWM)* comes into play.

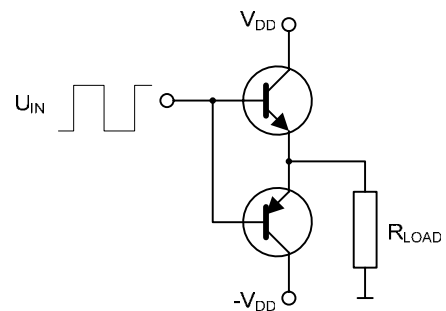


Figure 6 Simplified class D stage

## Pulse width modulation

We use a PWM modulator to change the *duty cycle* of the input pulse train in the class D amplifier in accordance with the analog voltage level. The duty cycle describes how large part of a period in the pulse signal that will be high (one) compared to the period time, the value is mostly given in percent. A duty cycle of zero (0) percent means that the signal is always low, a duty cycle of 50 percent means that the signal is high during

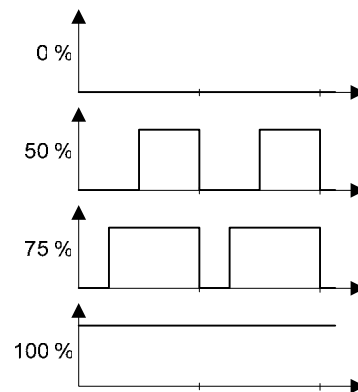
half the period and low during the other half, 75 percent duty cycle means that the signal is high during three fourths of the period time and a duty cycle of 100 percent means that the signal is always high, *Figure 7*.

Now the function of the PWM modulator is to create this pulse signal with varying duty cycle. The input signal to the modulator could be generated digitally or come from a A/D converter. Let's say we have the latter. We define the period of the generated pulse signal as a number of clock cycles. The easiest choice is to relate it to the resolution of the A/D converter. Let's say that the A/D converter works with  $n$  bits. The number of values we can have with this resolution is  $N = 2^n$  and we set the pulse period to  $N$  clock cycles and the reading from the A/D converter will simply give the number of clock cycles under a period where the signal should be high, that is it gives the duty cycle. We will get back to reasons for choosing other period times later.

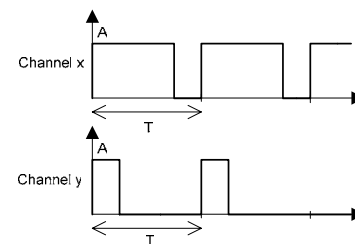
In most PWM-devices we have the option to decide if the PWM-period should start with a low or a high level. In some devices we can also decide if the signals should be *left* or *center aligned*. The difference could be easiest seen if we have two PWM-channels with the same frequency but different duty cycles. If the signals are left aligned it means that a new period of the signal starts at the same time on both channels, *Figure 8*. This means that if the two channels turn on some power hungry devices there will be a sudden current increase on both channels at the same time and this might cause some disturbance. If the PWM-signal are center aligned it is the center of the period that is at the same time on both channels, *Figure 9*. Now the devices won't turn on at the same time as long as the two channels doesn't have the same duty cycle. As we can see in the figures the period time of the center aligned signals are twice the period time of the left aligned signals.

If we look back our goal was to use PWM to generate an analog output signal but so far we have not reached that goal. Our output signal is still just a pulse train even if the power is increased. We need to do something about the pulse signal.

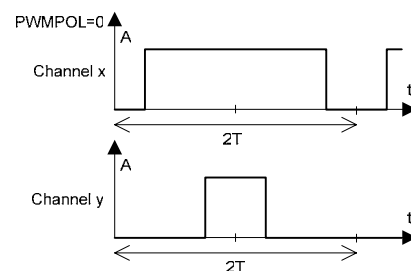
If we analyse the frequency content of the square wave (Fourier analysis) we will find that the signal can be described by the equation



*Figure 7 Example of duty cycles*



*Figure 8 Left aligned PWM-signals*



*Figure 9 Center aligned PWM-signals*

$$f(t) = A \cdot \frac{\tau}{T} + 4 \cdot A \cdot \frac{\tau}{T} \cdot \sum_{k=1}^{\infty} \frac{\sin\left(\frac{k \cdot \omega_0 \cdot \tau}{2}\right)}{k \cdot \omega_0 \cdot \tau} \cdot \cos(k \cdot \omega_0 \cdot t) \quad \omega_0 = 2 \cdot \pi \cdot f_0 = \frac{2 \cdot \pi}{T}$$

if the signal can be described by *Figure 10*.

From the equation we can see that the signal can be separated into a DC level  $A \cdot \frac{\tau}{T}$

and an infinite sum of frequency components. The frequency components have frequencies  $k \cdot f_0$  where  $k = 1, 2, 3, \dots$ , that is we have the base frequency  $f_0$  which is the

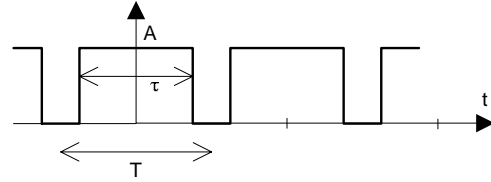


Figure 10 Pulse train

same as the frequency of the pulse train and the other frequencies are integer multipliers of this frequency. In reality the number of frequency components is not infinite. This would only happen if the signal transition from low to high and from high to low were infinitely fast and this can not be realized with real components.

The DC level will vary with the duty cycle of the square wave, that is when we vary the duty cycle of the PWM signal this level will give the analog output signal that we are looking for. To be precise this is not a DC level since it will vary with the duty cycle that we are constantly changing.

Now all we want to keep is this varying DC level and we have to remove the other frequencies from the signal. These frequencies will have higher frequencies than the frequency of the variation in the DC level and we can use a low pass filter to remove them.

An ideal low pass filter allows low frequencies to pass unaltered while higher frequencies are stopped. In a real implementation of the filter the higher frequencies will not be stopped but they will be more or less damped out. To get a filter with low losses we will use a LC-filter, *Figure 11*.

If we analyze the filter we get the transfer function

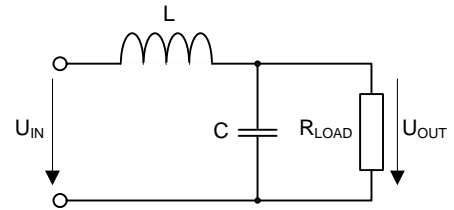


Figure 11 Output filter

$$H(\omega) = \frac{R \cdot \frac{1}{j \cdot \omega \cdot C}}{j \cdot \omega \cdot L + R \cdot \frac{1}{j \cdot \omega \cdot C}} = \frac{1}{1 - \omega^2 \cdot L \cdot C + j \cdot \omega \cdot \frac{L}{R}} = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j \cdot \frac{\omega}{Q \cdot \omega_0}}$$

Where the cut off frequency is

$$f_0 = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

And the Q value

$$Q = R \cdot \sqrt{\frac{C}{L}}$$

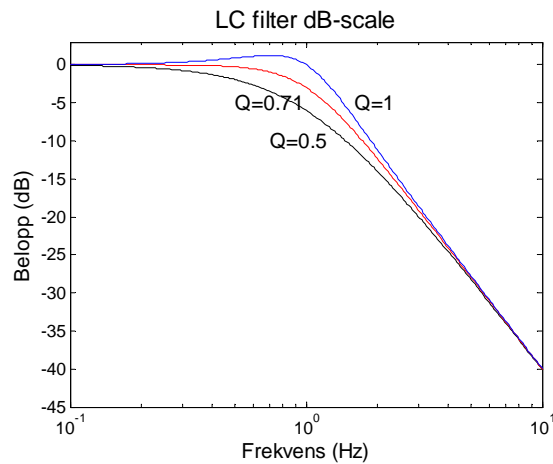
För  $Q$  values of 0.5,  $\frac{1}{\sqrt{2}}$  and 1 and

a cut off frequency of 1 Hz we get the frequency plots in *Figure 12*.

We can see that when the frequency gets higher than the cut off frequency  $f_0$  the signal gets more and more damped the higher the frequency is and to damp the higher frequency components of our PWM signal efficiently we need to make sure that these frequencies are much higher than the frequency of the varying DC level that we want to keep.

We can make a more efficient filter by increasing the order of the filter but then the filter would be more complicated.

One might think that we could replace this passive filter with an active one but we have to remember that the filter should be placed *after* the power amplifier and the operational amplifier in the active filter can not handle this power so we have to stay with the passive filter.



*Figure 12* Frequency plot of LC filter  $Q = 0.5, \frac{1}{\sqrt{2}}$  and 1 respectively,  $f = 1 \text{ Hz}$

## Resolution and filter damping

When we generate the PWM signal in our processor we set the period of the PWM signal as a number of clock periods and we set the duty cycle by controlling during how many of these clock periods the signal should be a logic one (1). The resolution of our PWM signal, that is the number of different duty cycles the signal can have is governed by the period of the signal, that is by the number of clock cycles that give this period. Now if the processor clock has a fixed frequency then the frequency of the PWM signal will get lower when we increase the resolution by increasing the number of clock periods in the period time. A lower frequency of the PWM signal means that it will get closer to the frequency of the desired signal, the variation in the DC level. This will give a lower damping of the unwanted frequency components in the LC filter if we don't increase the order of the filter. As a consequence we can see that the quality of the output signal is a trade off between the resolution of the PWM signal and the damping of the filter.

## PWM applications

Pulse width modulation has for a long time been used for the control of the motor speed in DC motors. There are several reasons for this. The accuracy of the speed, that is the resolution of the PWM signal is in many cases not that critical. The ability of the motor to react to duty cycle changes in the controlling PWM signal is pretty slow which means that the PWM frequency can be low. Actually the motor in itself

acts as a low pass filter which means that in many cases we can control the motor directly without any low pass filter. These things put together give a very simple implementation of the PWM control.

In recent years PWM control have grown in popularity in the design of audio amplifiers, that is class D amplifiers. This has followed on the constant development of processors that use higher and higher clock frequencies. This makes it possible to use higher PWM frequencies and therefore simplifies the development of more efficient low pass filters. We have also seen a healthy development of transistors that can handle high power at a high frequency.