Primeira Prova - 2020/1

1. a. $\int_0^2 c\cdot e^{-2x}\,dx = 1 \Rightarrow c\cdot\int_0^2 e^{-2x}\,dx = 1$ $\quad u = -2x$

$c\cdot\int_a^b -\frac{1}{2} e^{u}\,du \to c\cdot -\frac{1}{2} e^{u}\Big|_a^b \to c\cdot -\frac{1}{2} e^{-2x}\Big|_0^2 = 1$

$\left[-\frac{1}{2} c\cdot e^{-4}\right] - \left[-\frac{1}{2} c\cdot e^{0}\right] = 1$

$-\frac{c\cdot e^{-4}}{2} + \frac{c}{2} = 1 \to c - e^{-4}c = 2$

$c = 2/(1-e^{-4})$

b. $\int_{0.5}^1 c\cdot e^{-2x}\,dx \Rightarrow c\cdot -\frac{1}{2} e^{-2x}\Big|_{0.5}^1$

$\left[c\cdot -\frac{1}{2}\cdot e^{-2}\right] - \left[c\cdot -\frac{1}{2} e^{-1}\right] \to -\frac{ce^{-2}}{2} + \frac{ce^{-1}}{2}$

$c\left(\frac{e^{-1}-e^{-2}}{2}\right) \to \frac{\cancel{2}}{1-e^{-4}}\left(\frac{e^{-1}-e^{-2}}{\cancel{2}}\right) = \frac{e^{-1}-e^{-2}}{1-e^{-4}}$

c. $F(x) = P(X \leq x) \to P(X \in (0,x)) = \int_0^x ce^{-2x}\,dx$

$c\cdot -\frac{1}{2} e^{-2x}\Big|_0^x \to \left[-\frac{1}{2} c\cdot e^{-2x}\right] - \left[c\cdot -\frac{1}{2}\cancel{e^0}\right] = \frac{c}{2} - \frac{ce^{-2x}}{2}$

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$$F(x) = c\left(\frac{1-e^{-2x}}{2}\right) \rightarrow \frac{\cancel{2}}{1-e^{-4}}\left(\frac{1-e^{-2x}}{\cancel{2}}\right)$$

$$F(x) = \frac{1-e^{-2x}}{1-e^{-4}} \qquad F(x) = \begin{cases} 0, & x < 0 \\ \left(\frac{1-e^{-2x}}{1-e^{-4}}\right), & x \in (0,2) \\ 1 & \text{caso contrário} \end{cases}$$

d. $E(X) = \int x\, f(x)\, dx$

$$\int x\, c \cdot e^{-2x}\, dx \rightarrow c \cdot \int \frac{e^{v} \cdot v}{4}\, dv$$

$u = v \quad v' = e^{v}$

$$\frac{c}{4}\int v \cdot e^{v}\, dv \rightarrow \frac{c}{4}\left(e^{v} v - \int e^{v}\, dv\right)$$

$$\frac{c}{4}\left(e^{v} \cdot v - e^{v}\right) \rightarrow \frac{c}{4}\left(e^{-2x}(-2x) - e^{-2x}\right)$$

$$\frac{c}{4}\left(-2x\, e^{-2x} - e^{-2x}\right) \rightarrow E(X) = \frac{c \cdot e^{-2x}}{4}\left(-2x-1\right)$$

$$E(X) = \frac{\cancel{2}}{1-e^{-4}} \cdot \frac{e^{-2x}}{\cancel{4}\,2} \cdot (-2x-1) = \left.\frac{e^{-2x}(-2x-1)}{2(1-e^{-4})}\right|_{0}^{2}$$

$$E(x) = \frac{e^{-4}(-2 \cdot 2 - 1)}{2 - e^{-4}} - \frac{e^{0}(-2 \cdot 0 - 1)}{2(1-e^{-4})} \cong 0,463$$

2. Resultado Teste

Defeito	$T+$	$T-$	
$V+$	$P(T+ \mid V+)$	$P(T- \mid V+)$	$=1$
$V-$	$P(T+ \mid V-)$	$P(T- \mid V-)$	$=1$

$P(V_+) = 1\%$
$P(V_-) = 99\%$

$P(T_+ \mid V_+) = 80\%$ $\quad P(T- \mid V_+) = 20\%$
$P(T_+ \mid V_-) = 5\%$ $\quad P(T- \mid V_-) = 95\%$

$$P(V- \mid T+) = \frac{P(T+ \mid V-)\,P(V-)}{P(T+)} = \frac{0{,}05 \cdot 0{,}99}{P(T+)}$$

$$\begin{aligned}
P(T+) &= (T+ \cap V+) \cup (T+ \cap V-) \\
&= P(T+ \cap (V+ \cup V-)) \\
&= P(T+ \mid V+)\,P(V+) + P(T+ \mid V-)\,P(V-) \\
&= 0{,}8 \cdot 0{,}01 + 0{,}05 \cdot 0{,}99 \\
&= 0{,}008 + 0{,}0495 \\
&= 0{,}0575
\end{aligned}$$

$$P(V- \mid T+) = \frac{0{,}05 \cdot 0{,}99}{0{,}0575} = \frac{0{,}0495}{0{,}0575} = 0{,}86 \rightarrow 86\%$$

3. $P(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ $\qquad E[x] = \sum_{x_i=0}^{\infty} x\,P(X = x_i)$

$$E[X] = \sum_{x_i=0}^{\infty} x_i \left[\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right] \rightarrow \underbrace{0 \cdot \left[\frac{e^{-\lambda}\lambda^{0}}{0!}\right]} + \sum_{x_i=1} x_i \left[\frac{e^{-\lambda}\lambda^{x_i}}{x_i!}\right]$$

$$\sum_{x_i=1} \lambda \left[\frac{e^{-\lambda}\lambda^{x_i-1}}{(x_i-1)!}\right] \rightarrow \lambda e^{-\lambda} \sum_{x_i=0} \left[\frac{\lambda^{x_i}}{(x_i)!}\right] \rightarrow \lambda e^{-\lambda} \cdot e^{\lambda}$$

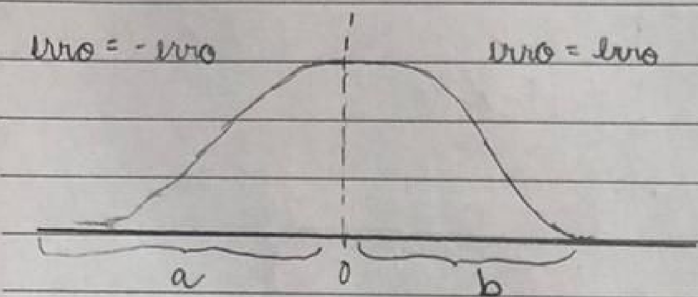
somatório converge

$$E[x] = \lambda e^{-\lambda} e^{\lambda} \rightarrow E[x] = \lambda$$

4. erro em $x \to Z = N(0,2)$ $\sigma = \sqrt{2}$

erro $= |Z|$

erro $= \left| \dfrac{X - \mu}{\sigma} \right| = \left| \dfrac{X}{\sigma} \right|$



$$P(W) = P(Z \geq 0) + P(Z < 0)$$

$$P(W) = 2\left(\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-0)^2}{2\sigma^2} \right) \right)$$

$$= 2\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\left(x^2/2\sigma^2\right)} \right) \to 2\left(\frac{e^{-x^2/4}}{\sqrt{2}\sqrt{2}\sqrt{\pi}} \right) = \cancel{2}\left(\frac{e^{x^2/4}}{\cancel{2}\sqrt{\pi}} \right) = \frac{e^{-x^2/4}}{\sqrt{\pi}}$$

$$E(x) = \int_0^{\infty} x \frac{e^{-x^2/4}}{\sqrt{\pi}} = \frac{1}{\sqrt{\pi}} \int x e^{-x^2/4} = \frac{1}{\sqrt{\pi}} \int_a^b -2e^u \, du = \frac{-2}{\sqrt{\pi}} \left[e^u \right]_a^b$$

$$= \frac{-2}{\sqrt{\pi}} \left[e^{-x^2/4} \right]_0^{\infty} = \frac{-2}{\sqrt{\pi}} e^{\cancel{-(\infty^2/4)}\,-\infty} + \frac{2}{\sqrt{\pi}} \cdot e^{-0^2/4}$$

$$= \cancel{\frac{-2}{\sqrt{\pi}} e^{-\infty}} + \frac{2}{\sqrt{\pi}} e^0 = 2/\sqrt{\pi}$$

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5. a. X é {1, 2, 3 . . .}

b. P(de dar dupla) = 1/6   ~ $P(\bar{D}) = 5/6$

K → nº da jogada em que dá certo
↳ se K = 1 foram 0 fracassos
Y = K − 1
↳ podemos usar uma geométrica tal que fracassos = (K − 1)

$P(K=1) = P(Y=0) = \frac{1}{6} = \theta$

$P(K=2) = P(Y=1) = \frac{5}{6} \cdot \frac{1}{6} = (1-\theta)\,\theta$

$P(Y=2) = (1-\theta)^2 \cdot \theta$

$P(Y=3) = (1-\theta)^3 \cdot \theta$

$P(X=k) = (1-\theta)^{k-1} \cdot \theta$

$P(X=k) = \left(\frac{5}{6}\right)^{k-1} \cdot \frac{1}{6}$

c. $E(X) = \sum_{k=1} (1-\theta)^{k-1} \cdot \theta \cdot k$

$= \theta \left(\sum_{k=1} (1-\theta)^{k-1} \cdot k \right)$

$= \frac{\theta}{1-\theta} \sum_{k=1} (1-\theta)^{k} \cdot k$

$= \frac{\cancel{\theta}}{\cancel{1-\theta}} \cdot \frac{\cancel{1-\theta}}{\theta^2} = 1/\theta$

$E(x) = 1/\theta = 1/(1/6) = 6$