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Ranking Models

Information-Theoretic Models

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Probabilistic ranking

A range of models
- Probabilistic relevance models
- Language models

Key distinguishing assumptions
- How to estimate the informativeness of a term
- How to regulate the influence of document length

Probabilistic relevance models

Term informativeness
- How much observing a term-document pair contributes to observing relevance
- $P(G|t,d)$, which boils down to tf-idf

Document length
- Either ignored (BIM) or heuristic (BM25)

Language models

Term informativeness
- How much observing the language of a document contributes to observing a query term
- $P(t|\theta_d)$, which boils down to smoothed tf

Document length
- Controlled via Bayesian smoothing

An information-theoretic look

The informativeness of a term occurrence is proportional to the amount of information it carries, with random occurrences being little informative
- Specialty terms: occur non-randomly
- Non-specialty terms: occur randomly

Can we measure random-ness?

Divergence from randomness (DFR)

> *The more the divergence of the frequency of a word t in a document d compared to its frequency in the collection, the more the information carried by t in d.*
>
> ∘ Amati and van Rijsbergen, 2002

Basic assumption #1

A term that carries little information is assumed to be randomly distributed over the whole collection C

- Given a term t, its probability distribution over the whole collection is referred to as $P_1(t|C)$
- The amount of information associated with this distribution is given by $-\log P_1(t|C)$

Basic assumption #2

An informative term is frequent in its **elite set** – the set of documents where the term occurs

- Given a term t, its probability distribution in an element d of the elite set is referred to as $P_2(t|d)$
- The less the term is expected, the higher is the amount of information gained: $1 - P_2(t|d)$

DFR scoring

General scoring

- $f(q, d) = \sum_{t \in q} w_{t,q} w_{t,d}$

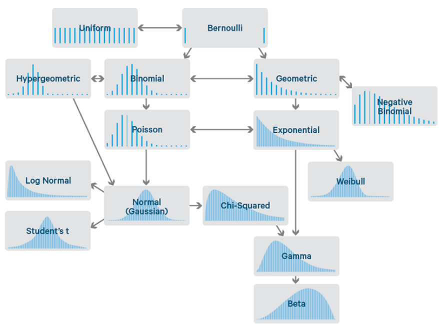
Where

- $w_{t,q} = \mathrm{tf}_{t,q} / \max_{t_i \in q} \mathrm{tf}_{t_i,q}$
- $w_{t,d} = \mathrm{inf}_1 \, \mathrm{inf}_2$
  $= -\log P_1(t|C) \times \left(1 - P_2(t|d)\right)$

Term weighting

Three steps

- Select a basic randomness model
- Apply the first normalization
- Normalize term frequencies



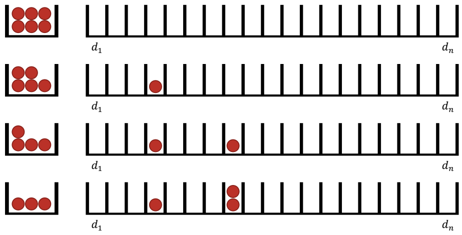
Credit: Sean Owen

Basic randomness model: $P_1(t|C)$

To compute the distribution of terms in the collection, distinct probability models can be considered

- Binomial (→ Poisson) distribution
- Bose-Einstein (→ Geometric) distribution
- Hypergeometric distribution

Binomial model (B)



Binomial model (B)

Basic event: occurrence of a single term in a document

- Bernoulli process with $p = 1/n$, for n documents

Example: $n = 1024$, $\mathrm{tf}_{t,C} = 10$, $\mathrm{tf}_{t,d} = 4$

- $P_1 = B(1024, 10, 4)$

$$= \binom{10}{4} p^4 (1-p)^6$$
$$= 0.00000000019$$

Binomial model (B)

General form

- $P_1 = B(n, \mathrm{tf}_{t,C}, \mathrm{tf}_{t,d})$

$$= \binom{\mathrm{tf}_{t,C}}{\mathrm{tf}_{t,d}} p^{\mathrm{tf}_{t,d}} (1-p)^{\mathrm{tf}_{t,C} - \mathrm{tf}_{t,d}}$$

Poisson approximation (P)

Let λ be the expected frequency of t in d

- $\lambda = \frac{\mathrm{tf}_{t,C}}{n}$ (constant)

For $n \to \infty$ ($p = 1/n \to 0$)

- $B(n, \mathrm{tf}_{t,C}, \mathrm{tf}_{t,d}) \approx Poiss(\lambda, \mathrm{tf}_{t,d})$

$$= \frac{e^{-\lambda} \lambda^{\mathrm{tf}_{t,d}}}{\mathrm{tf}_{t,d}!}$$

Poisson approximation (P)

$$\begin{aligned} \mathrm{inf}_1 &= -\log B(n, \mathrm{tf}_{t,C}, \mathrm{tf}_{t,d}) \\ &\approx -\log Poiss(\lambda, \mathrm{tf}_{t,d}) \\ &= -\log \frac{e^{-\lambda} \lambda^{\mathrm{tf}_{t,d}}}{\mathrm{tf}_{t,d}!} \\ &= -\mathrm{tf}_{t,d} \log \lambda + \lambda \log e + \log (\mathrm{tf}_{t,d}!) \end{aligned}$$

Poisson approximation (P)

Using Stirling's formula

- $n! = \sqrt{2\pi}\ n^{n+0.5}\ e^{-n}\ e^{(12n+1)^{-1}}$

$$\begin{aligned}\mathrm{inf}_1 \approx\ & \mathrm{tf}_{t,d} \log \frac{\mathrm{tf}_{t,d}}{\lambda} \\ & + \left(\lambda + \frac{1}{12\ \mathrm{tf}_{t,d} + 1} - \mathrm{tf}_{t,d}\right) \log e \\ & + 0.5 \log (2\pi\ \mathrm{tf}_{t,d})\end{aligned}$$

Bose-Einstein model (B_E)

Describes the number of particles with a certain energy

- In our setting, describes the probability that a document d contains $\mathrm{tf}_{t,d}$ occurrences of term t

Geometric model (G)

B-E can be approximated by a geometric distribution

- $P(t|C) = p(1-p)^{\mathrm{tf}_{t,d}}$, where where $p = 1/(1+\lambda)$

The amount of information associated with term t in the collection can then be computed as

$$\mathrm{inf}_1 \approx -\log\left(\frac{1}{1+\lambda}\right) - \mathrm{tf}_{t,d} \log\left(\frac{\lambda}{1+\lambda}\right)$$

First normalization: $P_2(t|d)$

Assumption: probability that the observed term contributes to select a relevant document is high, if the probability of encountering one more token of the same term in a relevant document is similarly high

Laplace's law of succession (L)

Useful when we have no advance knowledge of how many tokens of a term should occur in a relevant document of arbitrary large size

$$\begin{aligned}P_2 &= P(\mathrm{tf}_{t,d} + 1 | \mathrm{tf}_{t,d}, d) \\ &\approx \frac{\mathrm{tf}_{t,d} + 1}{\mathrm{tf}_{t,d} + 2} \approx \frac{\mathrm{tf}_{t,d}}{\mathrm{tf}_{t,d} + 1} \quad \text{replacing } \mathrm{tf}_{t,d} \text{ by } \mathrm{tf}_{t,d} - 1\end{aligned}$$

Laplace's law of succession (L)

$$\begin{aligned}\mathrm{inf}_2 &= 1 - P_2(t|d) \\ &\approx 1 - \frac{\mathrm{tf}_{t,d}}{\mathrm{tf}_{t,d} + 1} \\ &= \frac{1}{\mathrm{tf}_{t,d} + 1} \quad \text{tf saturation effect}\end{aligned}$$

First normalization: Bernoulli (B)

Add a new token to the collection: $\text{tf}_{t,C} \rightarrow \text{tf}_{t,C} + 1$

- Compute probability that additional token falls into the observed documents: $\text{tf}_{t,d} \rightarrow \text{tf}_{t,d} + 1$

Compare $B\big(n_t, \text{tf}_{t,C} + 1, \text{tf}_{t,d} + 1\big)$ vs $B\big(n_t, \text{tf}_{t,C}, \text{tf}_{t,d}\big)$ on the elite set only (n_t instead of n)

First normalization: Bernoulli (B)

$$P_2 = \frac{B\big(n_t, \text{tf}_{t,C} + 1, \text{tf}_{t,d} + 1\big)}{B\big(n_t, \text{tf}_{t,C}, \text{tf}_{t,d}\big)}$$

$$= \frac{\text{tf}_{t,C} + 1}{n_t(\text{tf}_{t,d} + 1)}$$

$$\text{inf}_2 = 1 - \frac{\text{tf}_{t,C} + 1}{n_t(\text{tf}_{t,d} + 1)}$$

tf saturation effect

idf effect

Second normalization: document length

Formulations thus far do not take into account the length of document d

- Solution: normalize term frequency $\text{tf}_{t,d}$

H1. $\text{tfn}_{t,d} = \text{tf}_{t,d} \frac{avl}{l_d}$

H2. $\text{tfn}_{t,d} = \text{tf}_{t,d} \log\left(1 + \gamma \frac{avl}{l_d}\right)$

Example model

PL2 [Amati, 2003]

- Randomness model: Poisson
- First normalization: Laplace
- Second normalization: H2

Many other effective models

Hypergeometric model (H)

Binomial distribution describes the probability of observing $\text{tf}_{t,d}$ after $\text{tf}_{t,C}$ independent draws

- $\text{tf}_{t,C}$ tokens are sampled ***with*** replacement

Hypergeometric distribution describes the probability of observing $\text{tf}_{t,d}$ after $\text{tf}_{t,C}$ non-independent draws

- $\text{tf}_{t,C}$ tokens are sampled ***without*** replacement

Hypergeometric model (H)

Because draws are not independent, the probability of observing a further token in a document is reduced

- In practice, no need for length normalization
- Also, no hyperparameter tuning

DPH [Amati et al. 2007]

- Very effective in web search tasks

Summary

Almost all variants of the model give very good results
- Poisson model slightly better than Binomial
- First normalization variants L and B give similar results
- Term frequency normalization H2 better than H1
- Hypergeometric model effective and parameter free

Divergence from independence (DFI)

Independence rather than randomness
- Non-specialty terms: occur at a more or less constant rate relative to other terms across documents

Independence quantification is distribution-free
- Non-parametric counterpart of DFR models
- Also very effective in practice

References

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References

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Amati and van Rijsbergen, ACM TOIS 2002

A nonparametric term weighting method for information retrieval based on measuring the divergence from independence
Kocabas, Dinçer, and Karaoglan, Inf. Retr. 2013

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Coming next…

Feedback Models

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