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Information Retrieval

# Vector Space Models

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# The ranking problem

Given

- Some evidence of the user's need

Produce

- A list of matching information items
- In decreasing order of relevance

# The ranking problem

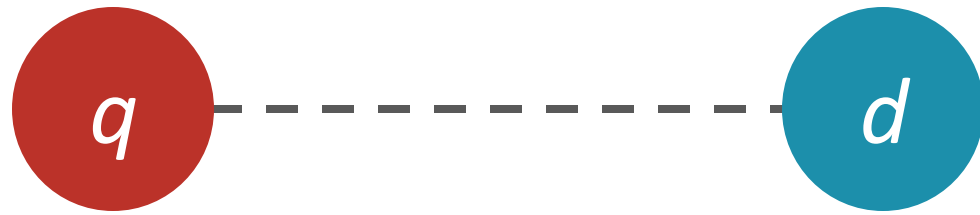
Given

- Some ~~evidence of the user's need~~ *query*

Produce

- A list of matching ~~information items~~ *documents*
- In decreasing order of relevance

# The ranking problem

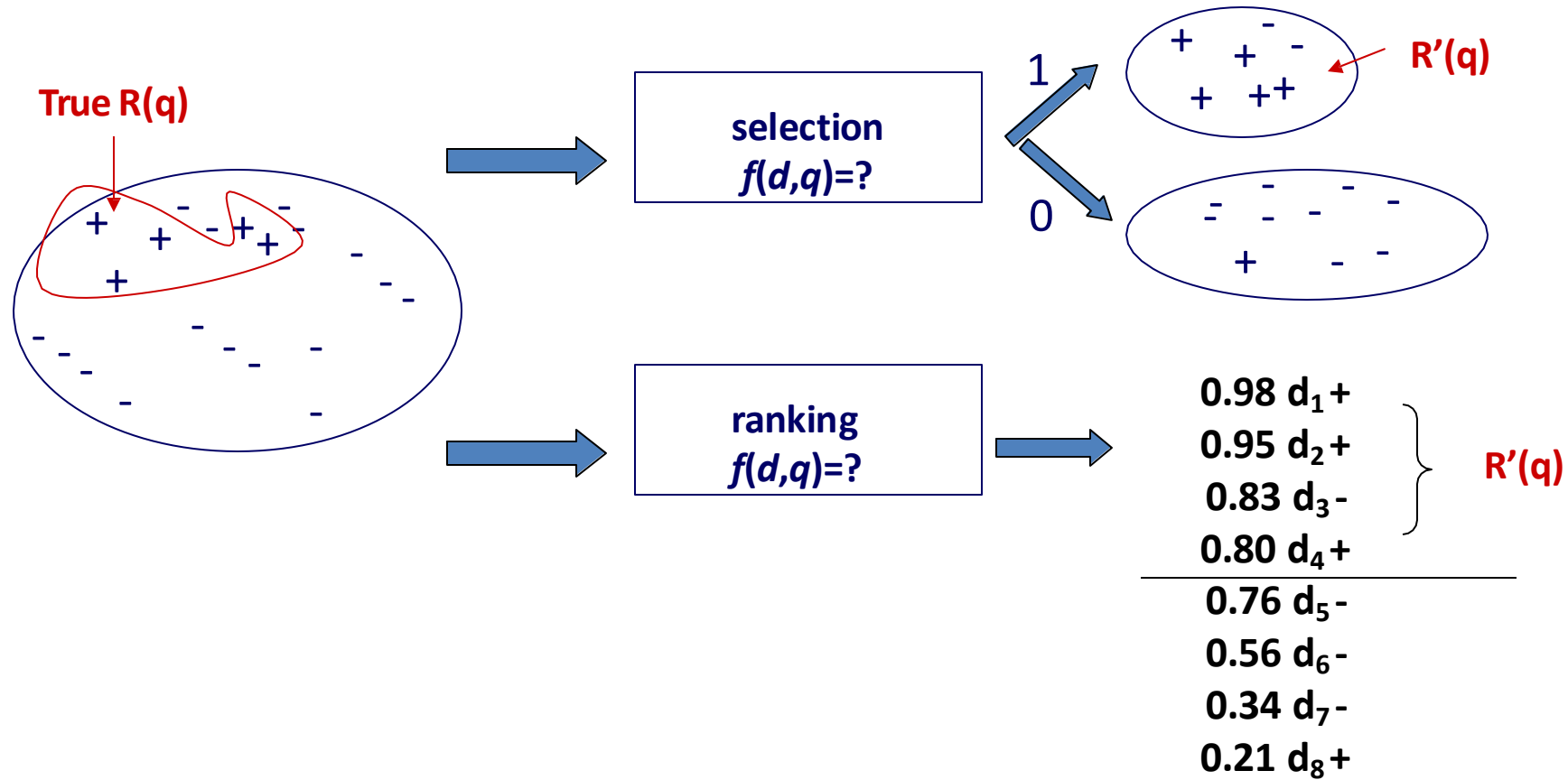


$$f(q, d)$$

# Why rank?

Couldn't  $f(q,d)$  be  
just an indicator  
function?

# Document selection vs. ranking



# Why not select?

The classifier is unlikely accurate

- Over-constrained: no relevants returned
- Under-constrained: too many relevants returned
- Hard to find an appropriate threshold

Not all relevant documents are equally relevant!

- Prioritization is needed

# Probability Ranking Principle (PRP)

“

*Ranking documents by decreasing probability of relevance results in optimal effectiveness, provided that probabilities are estimated (1) with certainty and (2) independently.*

◦ Robertson, 1977



# Ranking effectiveness

Effectiveness is about doing the right thing; it's about finding documents that are relevant to the user

Relevance is influenced by many factors

- Topical relevance vs. user relevance
- Task, context, novelty, style

Ranking models define ***a view of*** relevance

# Ranking models

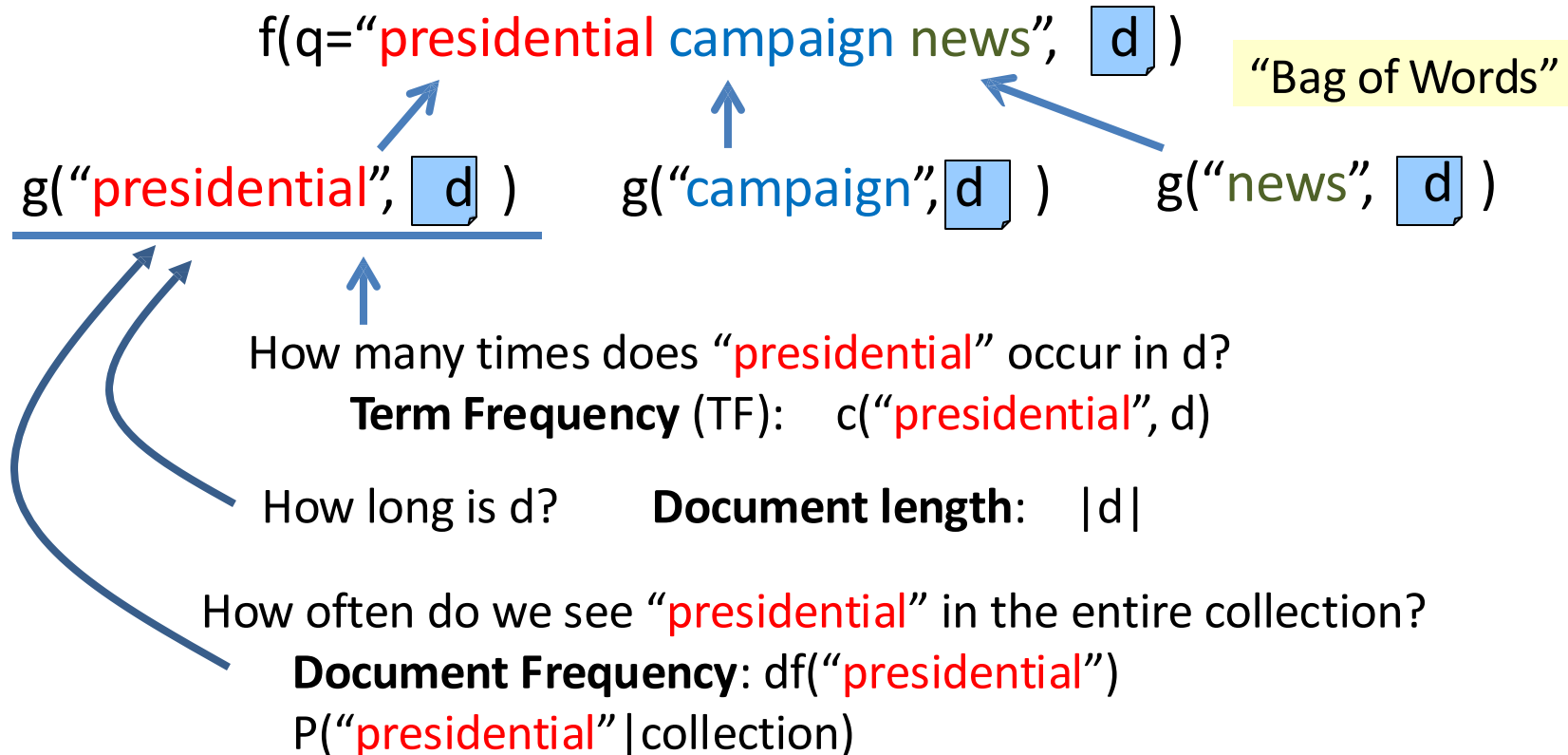
Provide a mathematical framework for ranking

- Each model builds upon different assumptions

Progress in ranking models has corresponded with improvements in effectiveness

- An effective model should score relevant documents higher than non-relevant documents

# Fundamental elements



# Many classical models

Similarity-based models:  $f(q, d) = \text{sim}(q, d)$

- Vector space models

Probabilistic models:  $f(d, q) = p(R = 1|d, q)$

- Classic probabilistic models
- Language models
- Information-theoretic models

# Many extended models

## Structural models

- Beyond bags-of-words

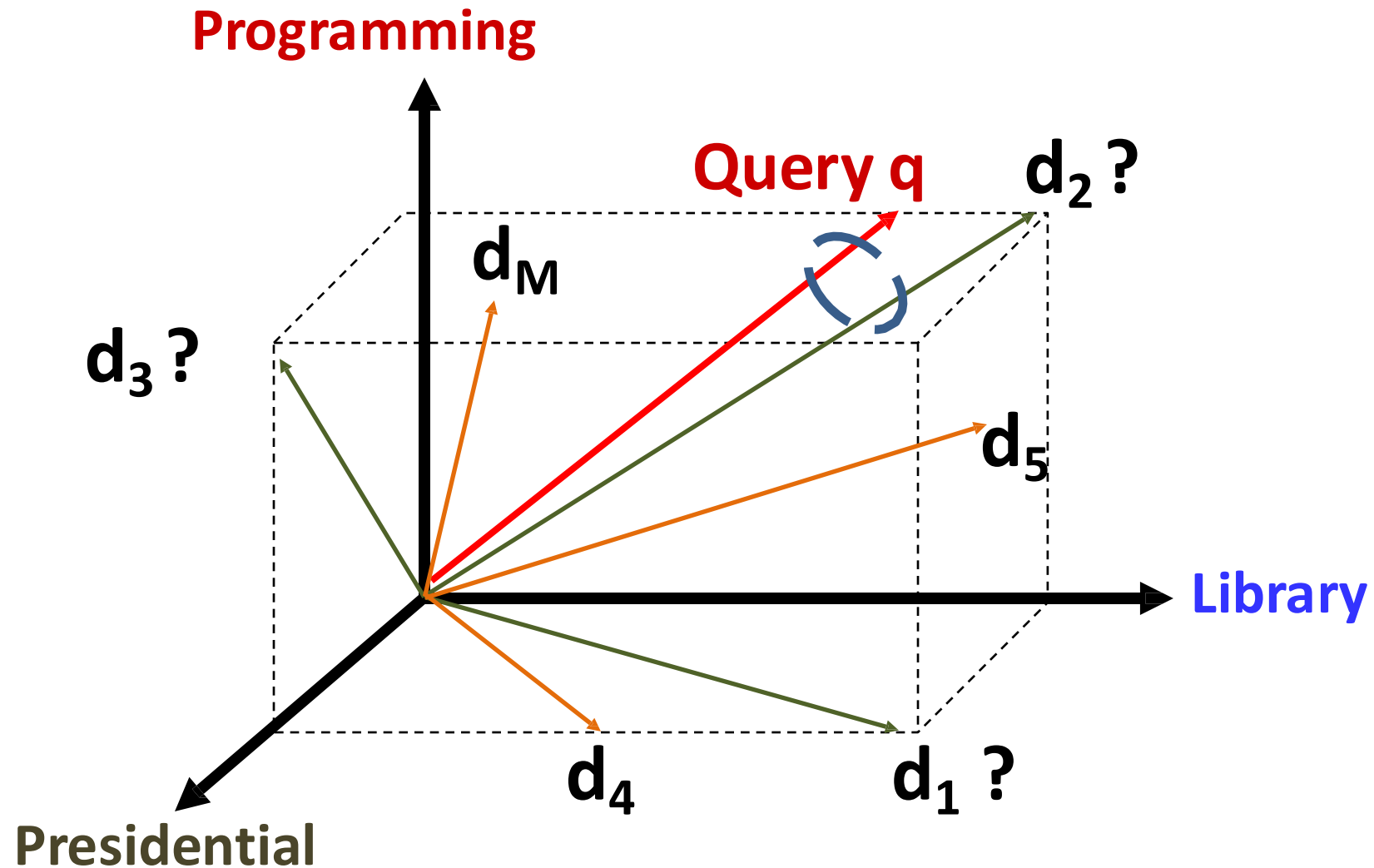
## Semantic models

- Beyond lexical matching

## Contextual models

- Beyond queries

# Vector Space Model (VSM)



# VSM is a framework

Queries and documents as term vectors

- Term as the basic concept (e.g., word or phrase)

A vocabulary  $V$  defines a  $|V|$ -dimensional space

- Vector components as real-valued term weights

Relevance estimated as  $f(q, d) = \text{sim}(q, d)$

- $q = (x_1, \dots, x_{|V|})$  and  $d = (y_1, \dots, y_{|V|})$

# What VSM doesn't say

How to define vector dimensions

- Concepts are assumed to be orthogonal

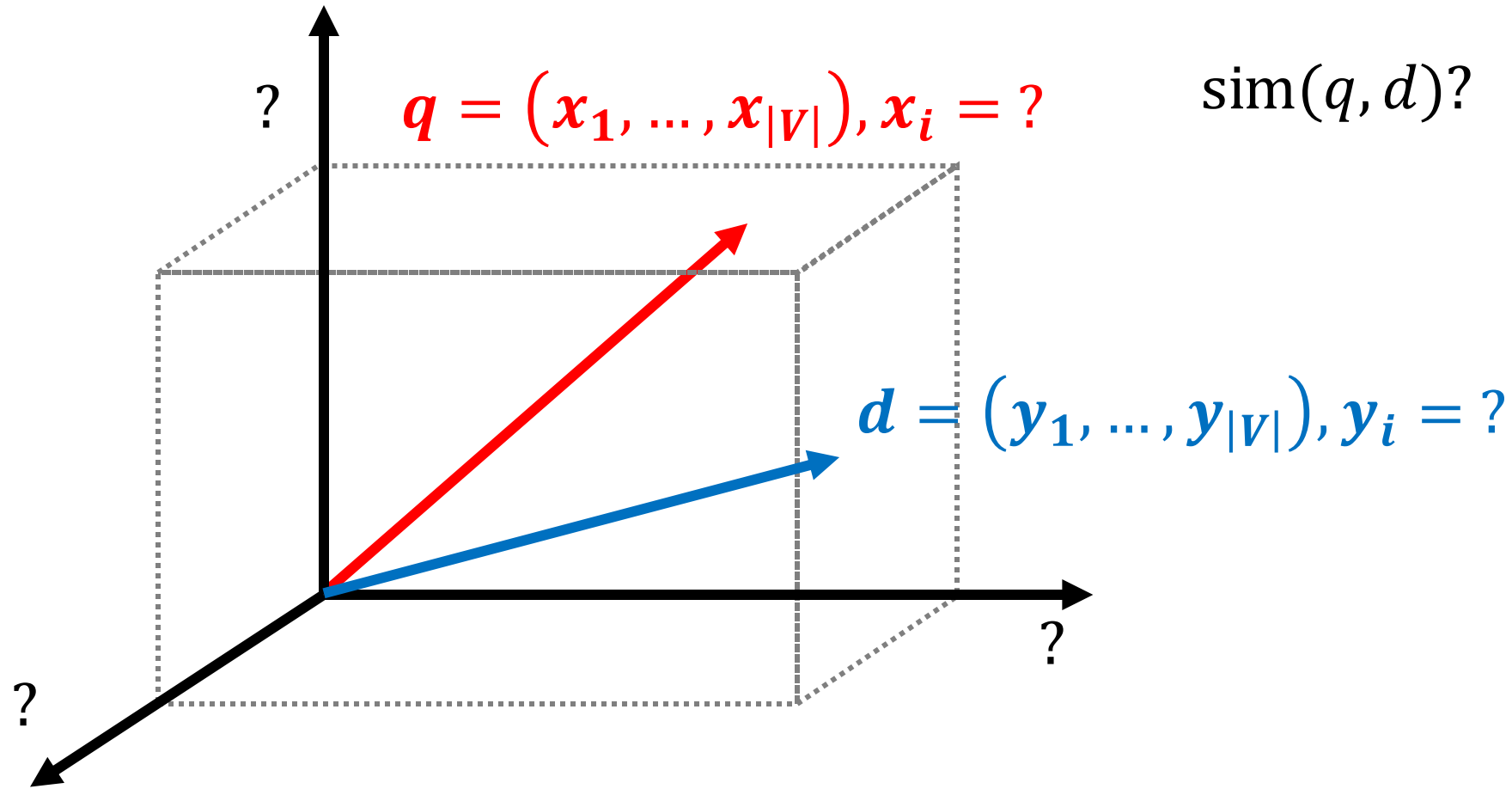
How to place vectors in the space

- Term weight in query indicates importance of term
- Term weight in document indicates topicality

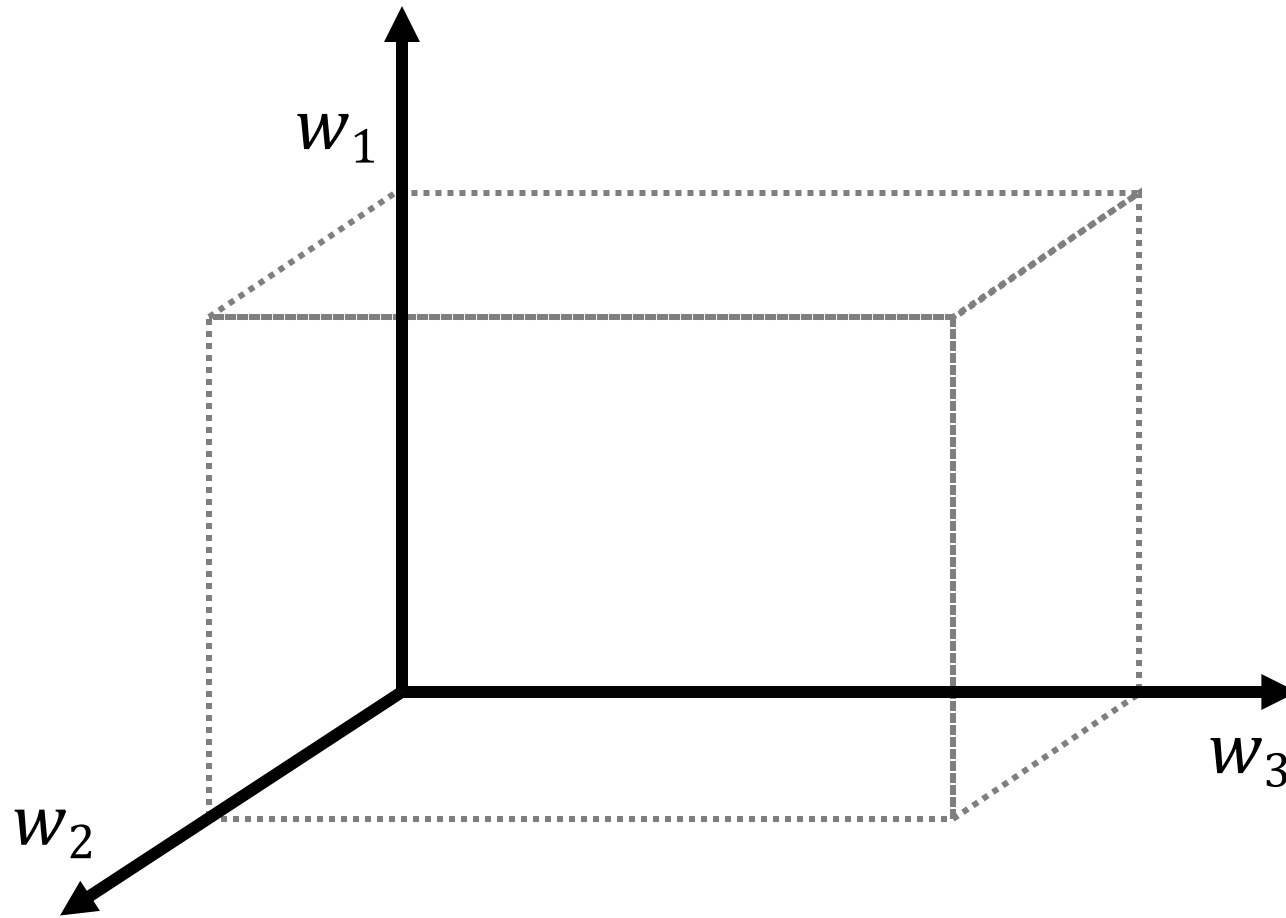
How to define the similarity measure



# What VSM doesn't say



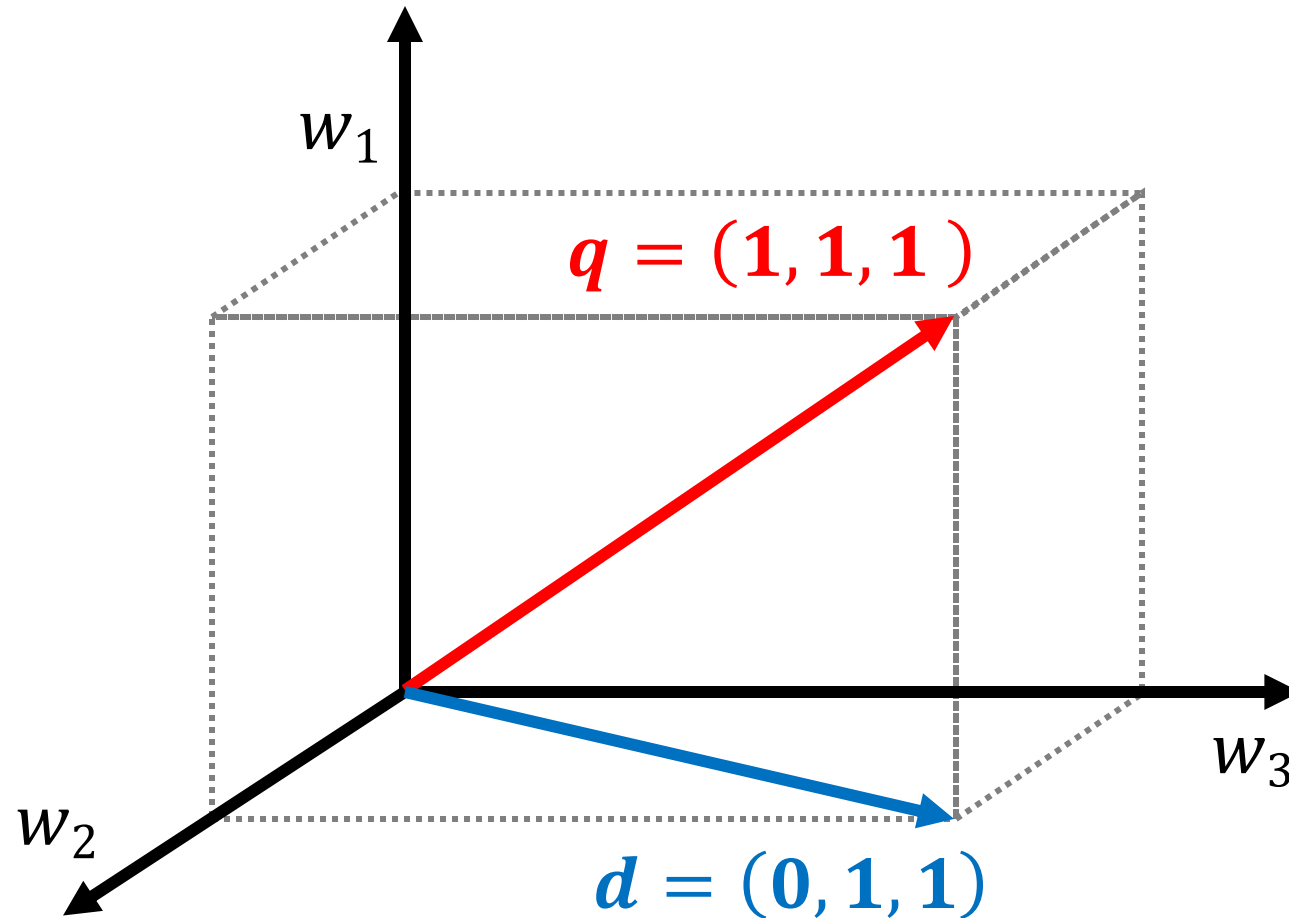
# Dimensions as a bag of words (BOW)



Vocabulary

$$V = (w_1, \dots, w_{|V|})$$

# Vectors placed as bit vectors

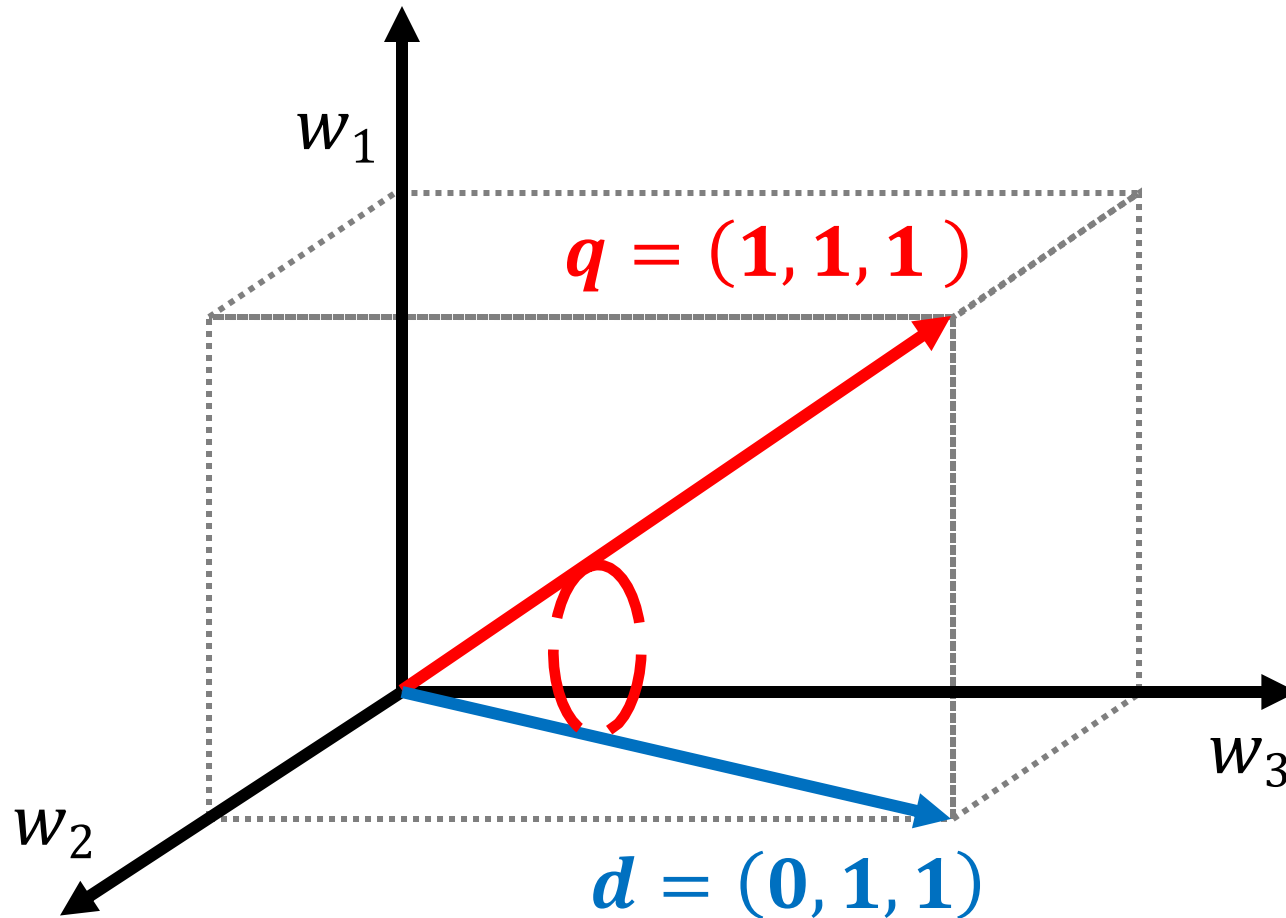


$$x_i, y_i \in \{0, 1\}$$

1: word  $w_i$  is present

0: word  $w_i$  is absent

# Similarity as dot product



$$\begin{aligned}\text{sim}(q, d) &= q \cdot d \\ &= x_1 y_1 + \dots + x_{|V|} y_{|V|} \\ &= \sum_{i=1}^{|V|} x_i y_i\end{aligned}$$

# Simplest VSM = BOW + bit vectors + dot

$$q = (x_1, \dots, x_{|V|})$$

$$d = (y_1, \dots, y_{|V|})$$

$$x_i, y_i \in \{0, 1\}$$

1: word  $w_i$  is present

0: word  $w_i$  is absent

$$\text{sim}(q, d)$$

$$= q \cdot d$$

$$= x_1 y_1 + \dots + x_{|V|} y_{|V|}$$

$$= \sum_{i=1}^{|V|} x_i y_i$$

*What does this ranking function intuitively capture?*

*Is this a good ranking function?*

# How would you rank these documents?

$q = [ \text{news about presidential campaign} ]$

ideal

|       |   |         |
|-------|---|---------|
| $d_1$ | ... news about ...  | $d_4 +$ |
| $d_2$ | ... news about organic food campaign...                                   | $d_3 +$ |
| $d_3$ | ... news of presidential campaign ...                                     | $d_1 -$ |
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ...   | $d_2 -$ |
| $d_5$ | ... news of organic food campaign...<br>campaign...campaign...campaign... | $d_5 -$ |

# Ranking using the simplest VSM

$q = [ \text{news about presidential campaign} ]$

|       |                                       |
|-------|---------------------------------------|
| $d_1$ | ... news about ...                    |
| $d_3$ | ... news of presidential campaign ... |

$V = \{ \text{news, about, presidential, campaign, food, ...} \}$

$$q = (1, 1, 1, 1, 0, \dots)$$

$$d_1 = (1, 1, 0, 0, 0, \dots) \quad \text{sim}(q, d_1) = 2$$

$$d_3 = (1, 0, 1, 1, 0, \dots) \quad \text{sim}(q, d_3) = 3$$

# Is it effective?

$q = [ \text{news about presidential campaign} ]$

|       |   | $f(q,d)$ | ranking | ideal   |
|-------|---|----------|---------|---------|
| $d_1$ | ... news about ...  | 2        | $d_2$   | $d_4 +$ |
| $d_2$ | ... news about organic food campaign...                                   | 3        | $d_3$   | $d_3 +$ |
| $d_3$ | ... news of presidential campaign ...                                     | 3        | $d_4$   | $d_1 -$ |
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ...   | 3        | $d_1$   | $d_2 -$ |
| $d_5$ | ... news of organic food campaign...<br>campaign...campaign...campaign... | 2        | $d_5$   | $d_5 -$ |



# What's wrong with it?

$q = [ \text{news about presidential campaign} ]$

|       |   |
|-------|---|
| $d_3$ | ... news of presidential campaign ...                                   |
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ... |

*Matching "presidential" more times deserves more credit!*

$f(q,d)$

ranking

ideal

$d_2$

$d_4 +$

$d_3$

$d_3 +$

$d_4$

$d_1 -$

3

$d_1$

$d_2 -$

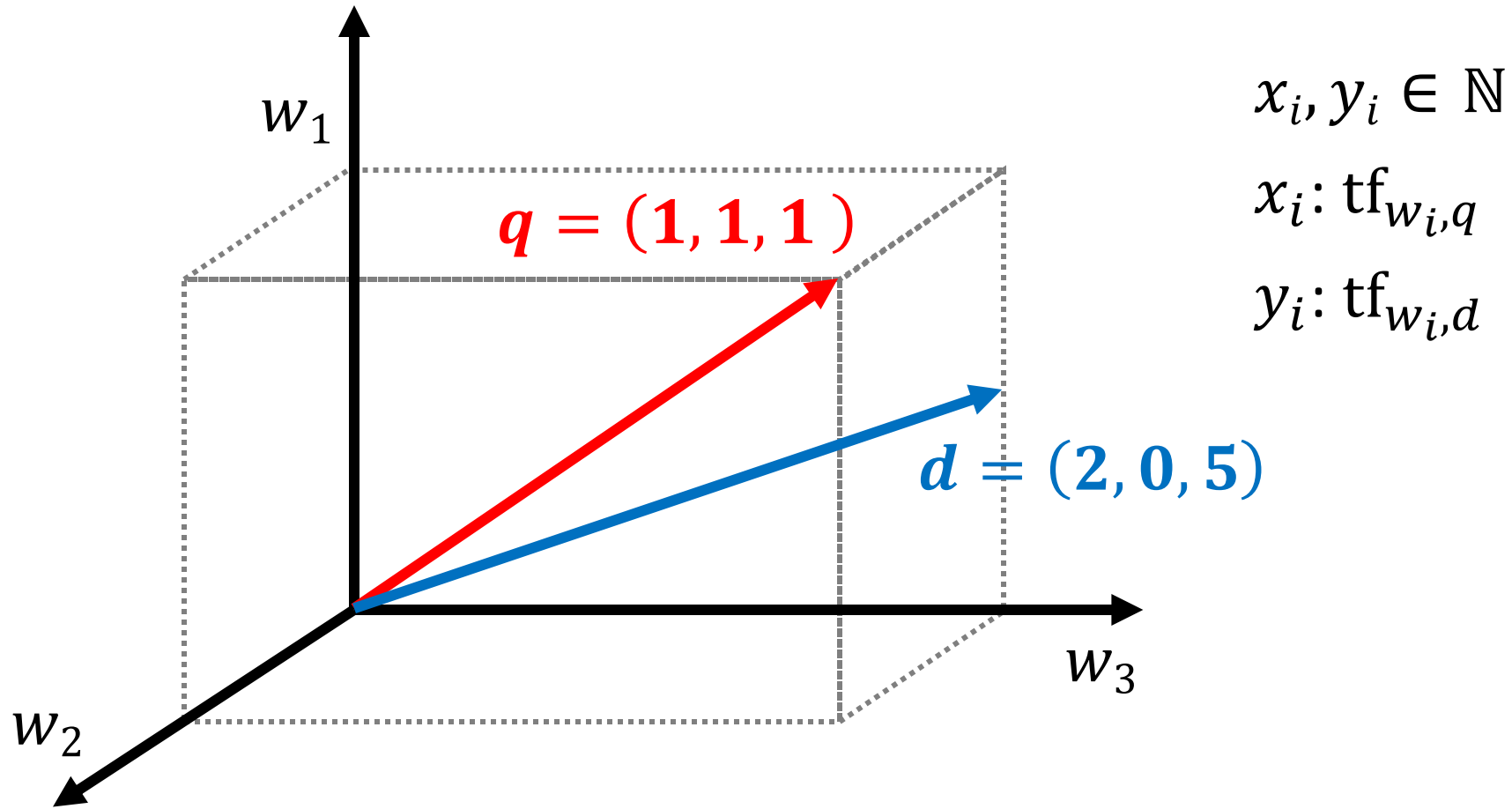
3

$d_5$

$d_5 -$



# Vectors placed as tf vectors



# Ranking using VSM with tf vectors

$q = [ \text{news about presidential campaign} ]$

|       |   |
|-------|---|
| $d_3$ | ... news of presidential campaign ...                                   |
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ... |

$V = \{ \text{news, about, presidential, campaign, food, ...} \}$

$$q = (1, 1, 1, 1, 0, \dots)$$

$$d_3 = (1, 0, 1, 1, 0, \dots) \quad \text{sim}(q, d_3) = 3$$

$$d_4 = (1, 0, 2, 1, 0, \dots) \quad \text{sim}(q, d_4) = 4$$

# What's wrong with it?

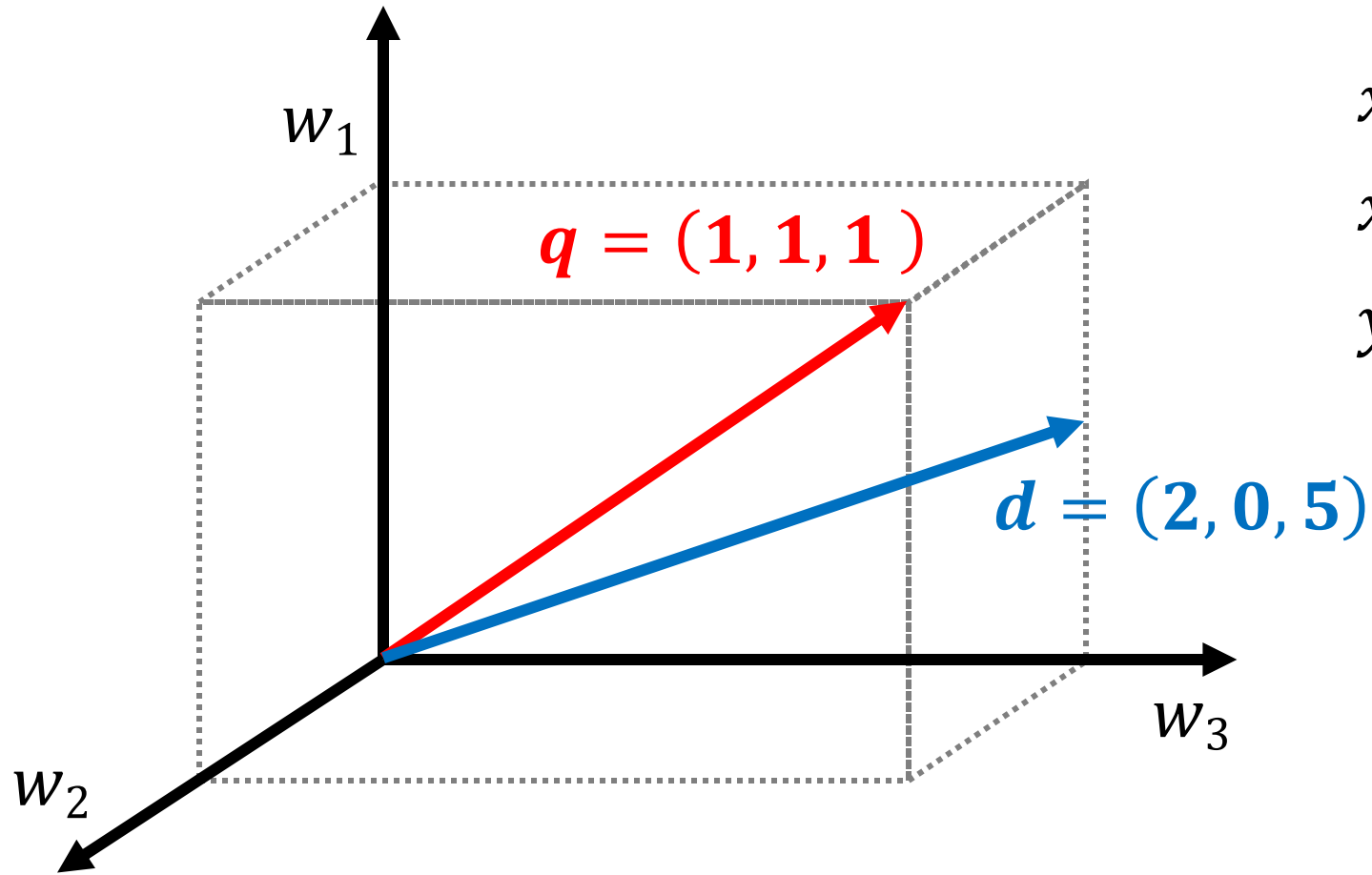
$q = [ \text{news about presidential campaign} ]$

|       |   |
|-------|---|
| $d_2$ | ... news about organic food campaign... |
| $d_3$ | ... news of presidential campaign ...   |

*Matching "presidential" is **more important** than matching "about"!*

| $f(q,d)$ | ranking | ideal   |
|----------|---------|---------|
|          | $d_2$   | $d_4 +$ |
| 3        | $d_3$   | $d_3 +$ |
| 3        | $d_4$   | $d_1 -$ |
|          | $d_1$   | $d_2 -$ |
|          | $d_5$   | $d_5 -$ |

# Vectors placed as tf-idf vectors



$$x_i, y_i \in \mathbb{R}$$

$$x_i: \text{tf}_{w_i, q} \text{ idf}_{w_i}$$

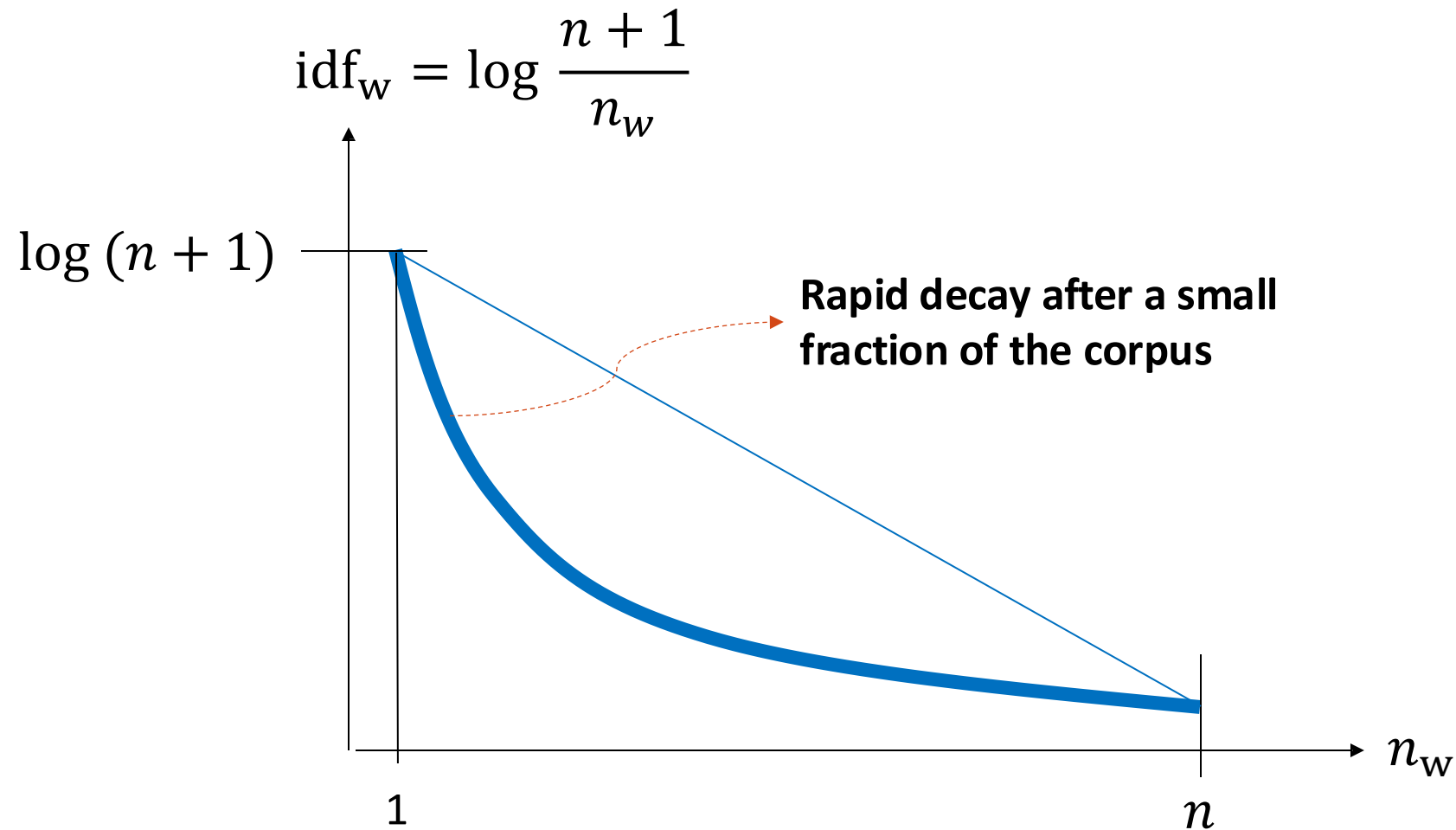
$$y_i: \text{tf}_{w_i, d} \text{ idf}_{w_i}$$

# Inverse document frequency (idf)

$$\text{idf}_w = \log \frac{n + 1}{n_w}$$

- $n$ : number of documents in the corpus
- $n_w$ : number of documents where  $w$  appears

# Why a log-based penalization?



# Ranking using VSM with tf-idf vectors

$q = [ \text{news about presidential campaign} ]$

|       |   |
|-------|---|
| $d_2$ | ... news about organic food campaign... |
|-------|---|

|       |                                       |
|-------|---------------------------------------|
| $d_3$ | ... news of presidential campaign ... |
|-------|---------------------------------------|

$V = \{ \text{news, about, presidential, campaign, food, ...} \}$

idf = (1.5, 1.0, 2.5, 3.1, 1.8, ...)

$q = (1, 1, 1, 1, 0, \dots)$

$d_2 = (1 * 1.5, 1 * 1.0, 0, 1 * 3.1, 0, \dots) \quad \text{sim}(q, d_2) = 5.6$

$d_3 = (1 * 1.5, 0, 1 * 2.5, 1 * 3.1, 0, \dots) \quad \text{sim}(q, d_3) = 7.1$



# Is it effective?

$q = [ \text{news about presidential campaign} ]$

|       |   | $f(q,d)$ | ranking | ideal   |
|-------|---|----------|---------|---------|
| $d_1$ | ... news about ...  | 2.5      | $d_5$   | $d_4 +$ |
| $d_2$ | ... news about organic food campaign...                                   | 5.6      | $d_4$   | $d_3 +$ |
| $d_3$ | ... news of presidential campaign ...                                     | 7.1      | $d_3$   | $d_1 -$ |
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ...   | 9.6      | $d_2$   | $d_2 -$ |
| $d_5$ | ... news of organic food campaign...<br>campaign...campaign...campaign... | 13.9     | $d_1$   | $d_5 -$ |

# Is it effective?

$q = [ \text{news about presidential campaign} ]$

|       |  |
|-------|--|
| $d_4$ | ... <b>news</b> of <b>presidential campaign</b> ...<br>... <b>presidential</b> candidate ...                       |
| $d_5$ | ... <b>news</b> of organic food <b>campaign</b> ...<br><b>campaign</b> ... <b>campaign</b> ... <b>campaign</b> ... |

| $f(q,d)$ | ranking | ideal   |
|----------|---------|---------|
| 2.5      | $d_5$   | $d_4 +$ |
| 5.6      | $d_4$   | $d_3 +$ |
| 7.1      | $d_3$   | $d_1 -$ |
| 9.6      | $d_2$   | $d_2 -$ |
| 13.9     | $d_1$   | $d_5 -$ |

# Ranking using VSM with tf-idf vectors

$q = [ \text{news about presidential campaign} ]$

|       |   |
|-------|---|
| $d_4$ | ... news of presidential campaign ...<br>... presidential candidate ... |
|-------|---|

|       |   |
|-------|---|
| $d_5$ | ... news of organic food campaign...<br>campaign...campaign...campaign... |
|-------|---|

$V = \{ \text{news, about, presidential, campaign, food, ...} \}$

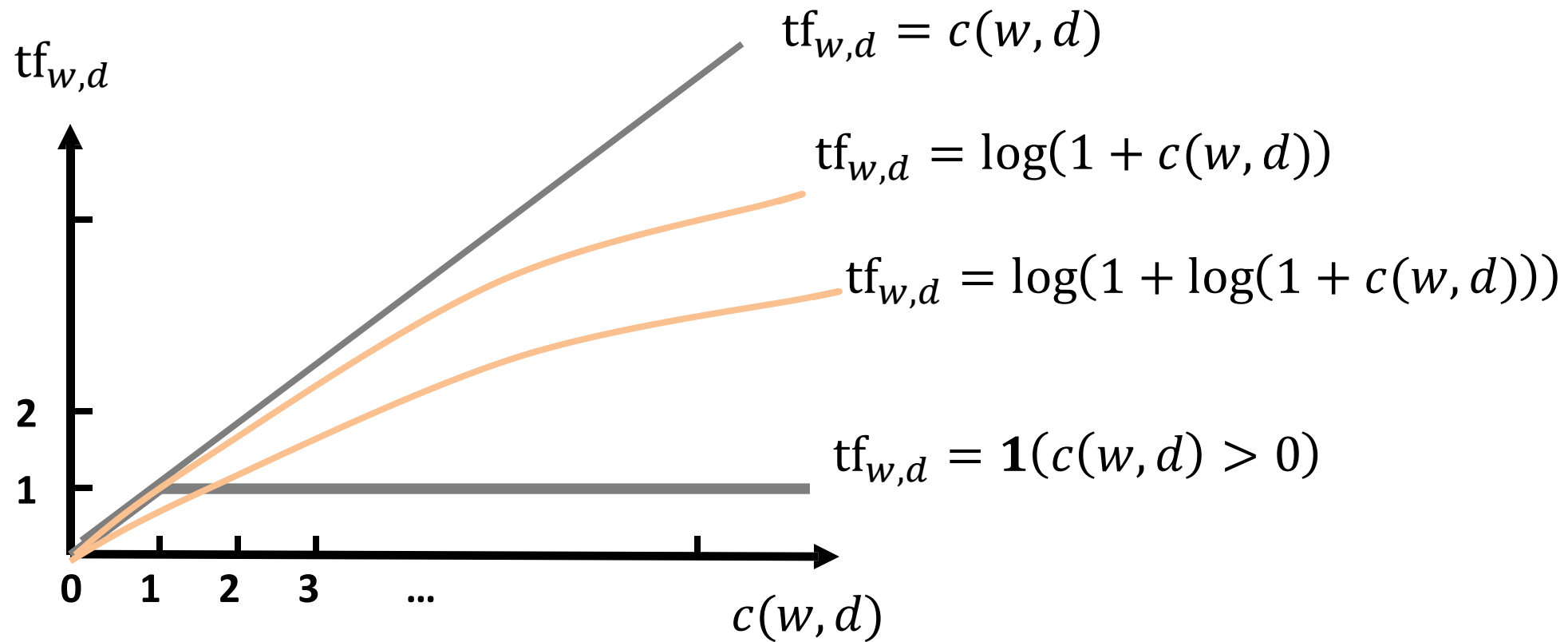
idf = (1.5, 1.0, 2.5, 3.1, 1.8, ...)

$q = (1, 1, 1, 1, 0, \dots)$

$d_4 = (1 * 1.5, 0, 2 * 2.5, 1 * 3.1, 0, \dots) \quad \text{sim}(q, d_4) = 9.6$

$d_5 = (1 * 1.5, 0, 0, 4 * 3.1, 1 * 1.8, \dots) \quad \text{sim}(q, d_5) = 13.9$

# Transforming tf



# What about document length?

$q = [ \text{news about presidential campaign} ]$

|       |  |           |
|-------|--|-----------|
| $d_4$ | ... <b>news</b> of <b>presidential campaign</b> ...<br>... <b>presidential</b> candidate ... | 100 words |
|-------|--|-----------|

|       |  |            |
|-------|--|------------|
| $d_6$ | ... <b>campaign</b> ..... <b>campaign</b> .....<br>.....<br>..... <b>news</b> .....<br>.....<br>..... <b>news</b> .....<br>.....<br>.....<br>..... <b>presidential</b> ..... <b>presidential</b> ..... | 5000 words |
|-------|--|------------|

$f(q, d_6) > f(q, d_4)?$

# Document length normalization

Penalize long documents

- Avoid matching by chance
- Must also avoid over-penalization

A document is long because

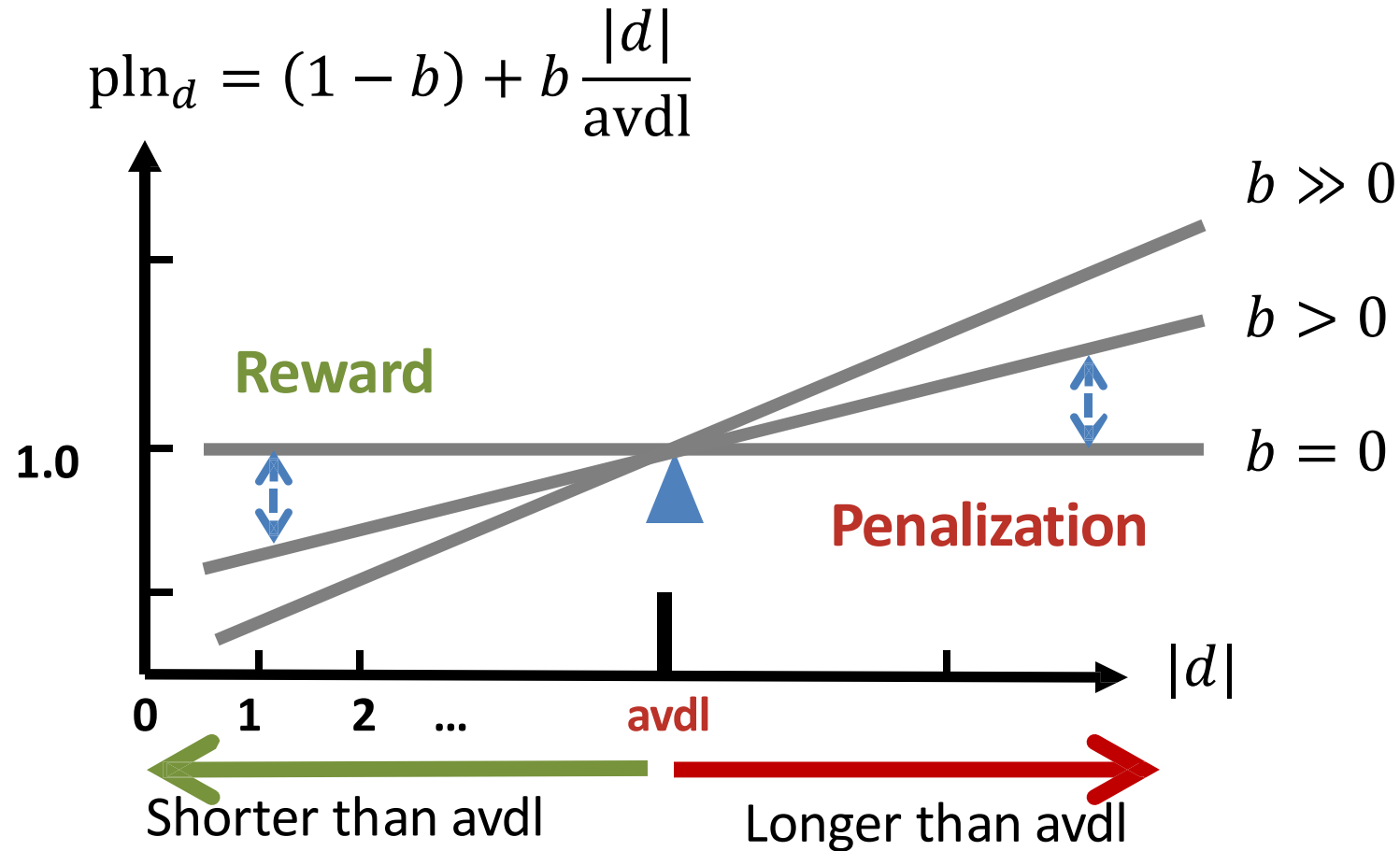
- It uses more words → more penalization
- It has more content → less penalization

# Pivoted length normalization (pln)

$$\text{pln}_d = (1 - b) + b \frac{|d|}{\text{avdl}}$$

- $|d|$ : document length in tokens
- $\text{avdl}$ : average document length in the corpus
- $b \in [0,1]$ : parameter

# Pivoted length normalization (pln)





# State-of-the-art VSM ranking

Pivoted length normalization VSM [Singhal et al. 1996]

$$\circ f(q, d) = \sum_{w \in q} c(w, q) \frac{\ln(1 + \ln(1 + c(w, d)))}{(1-b) + b \frac{|d|}{avdl}} \log \frac{n+1}{n_w}$$

Okapi/BM25 [Robertson and Walker, 1994]

$$\circ f(q, d) = \sum_{w \in q} c(w, q) \frac{(k_1+1) c(w, d)}{c(w, d) + k_1 \left( (1-b) + b \frac{|d|}{avdl} \right)} \log \frac{n+1}{n_w}$$

# Summary

Fundamental ranking components

- Term and document frequency
- Document length

VSM is a framework

- Components as term and document weights
- Relevance as query-document similarity

# Summary

Lack of theoretical justification

- Axiomatic approaches, probabilistic approaches

Room for further improvement

- Structure, semantics, feedback, context
- Feature-based models

# References

[Text Data Management: A Practical Introduction to Information Retrieval and Text Mining](#), Ch. 6

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[Search Engines: Information Retrieval in Practice](#), Ch. 7

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[The probability ranking principle in IR](#)

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Coming next...

# Language Models

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