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Information Retrieval

Language Models

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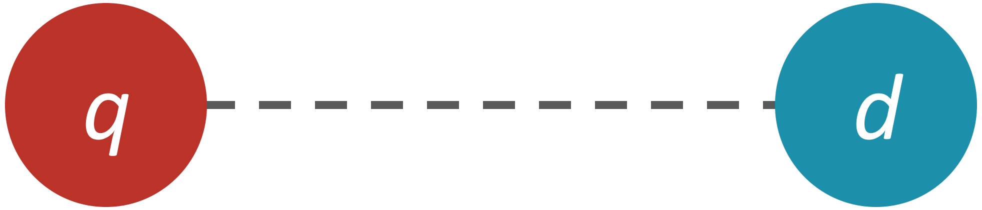
The ranking problem



$$f(q, d)$$

Ranking models recap

Boolean model

- Boolean query
- Set-based retrieval (no actual ranking)

Vector space models

- Query and documents as vectors
- Similarity-based ranking

Language modeling approach

Key intuition

- Users who try to think of a good query, think of words that are likely to appear in relevant documents
- A document is a good match to a query if it uses the same underlying ***language*** as the query

Statistical language model

A probability distribution over word sequences

- P(“Today is Wednesday”) ≈ 0.001
- P(“Today Wednesday is”) ≈ 0.0000000000001
- P(“The eigenvalue is positive”) ≈ 0.00001

Can also be regarded as a probabilistic mechanism for “generating” text, thus also called a “generative” model

Types of language models

Full dependence model

- $P(w_1 \dots w_k) = P(w_1)P(w_2|w_1) \dots P(w_k|w_1 \dots w_{k-1})$

Infeasible in practice

- Expensive computation
- Weak estimates (data sparsity)

Types of language models

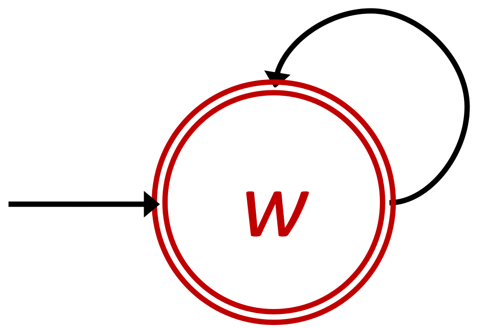
Tunable dependence via n-grams

- 3-gram ("trigram")
  $P(w_1 \dots w_k) = P(w_1)P(w_2|w_1) \dots P(w_k|w_{k-2}, w_{k-1})$
- 2-gram ("bigram")
  $P(w_1 \dots w_k) = P(w_1)P(w_2|w_1) \dots P(w_k|w_{k-1})$
- 1-gram ("unigram")
  $P(w_1 \dots w_k) = P(w_1)P(w_2) \dots P(w_k)$

Unigram language model

The simplest language model

- A one-state probabilistic finite automaton



state emission probabilities

the	0.20
a	0.10
frog	0.01
toad	0.01
said	0.03
likes	0.02
that	0.04
…	
STOP	0.20

P(”frog said that toad likes frog STOP”)

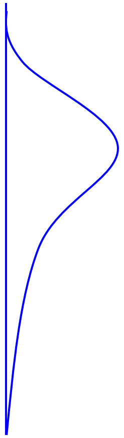
$= 0.01 \cdot 0.03 \cdot 0.04 \cdot 0.01 \cdot 0.02 \cdot 0.01 \cdot 0.02$

$= 0.0000000000048$

Example text generation

Model θ_1

…	
text	0.2
mining	0.1
association	0.01
clustering	0.02
…	
food	0.00001
…	

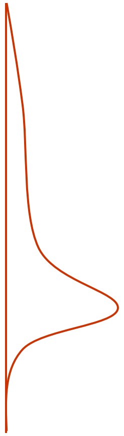


Sampling

Text
mining
paper

Model θ_2

…	
food	0.25
nutrition	0.1
healthy	0.05
diet	0.02
clustering	0.0001
…	



Sampling

Food
nutrition
paper

Evaluation of language models

Direct evaluation criterion

- Goodness of fit: data likelihood, perplexity, cross entropy, Kullback-Leibler divergence

Indirect evaluation criterion

- Task-dependent: we hope more “reasonable” LMs would achieve better task performance

Applications of language models

Language modeling offers a principled way to quantify the uncertainties associated with natural language

Speech recognition

- Given that we see “John” and “feels”, how likely will we see “happy” as opposed to “habit” next?

Applications of language models

Language modeling offers a principled way to quantify the uncertainties associated with natural language

Text categorization

- Given that we see “baseball” three times and “game” once in an article, how likely is it about “sports”?

Applications of language models

Language modeling offers a principled way to quantify the uncertainties associated with natural language

Document ranking

- Given that a user is interested in sports news, how likely would he or she use “baseball” in a query?

Query likelihood model

$$
\begin{aligned}
f(q,d) &\approx P(q,d) \\
&= P(q|d)P(d) \quad \text{Bayes' rule}
\end{aligned}
$$

Two core components

- $P(q|d)$: query likelihood
- $P(d)$: document prior

Computing $\boldsymbol{P(d)}$

Uninformative (uniform) prior

- $P(d) = 1/n$, for n documents in the corpus

Informative prior

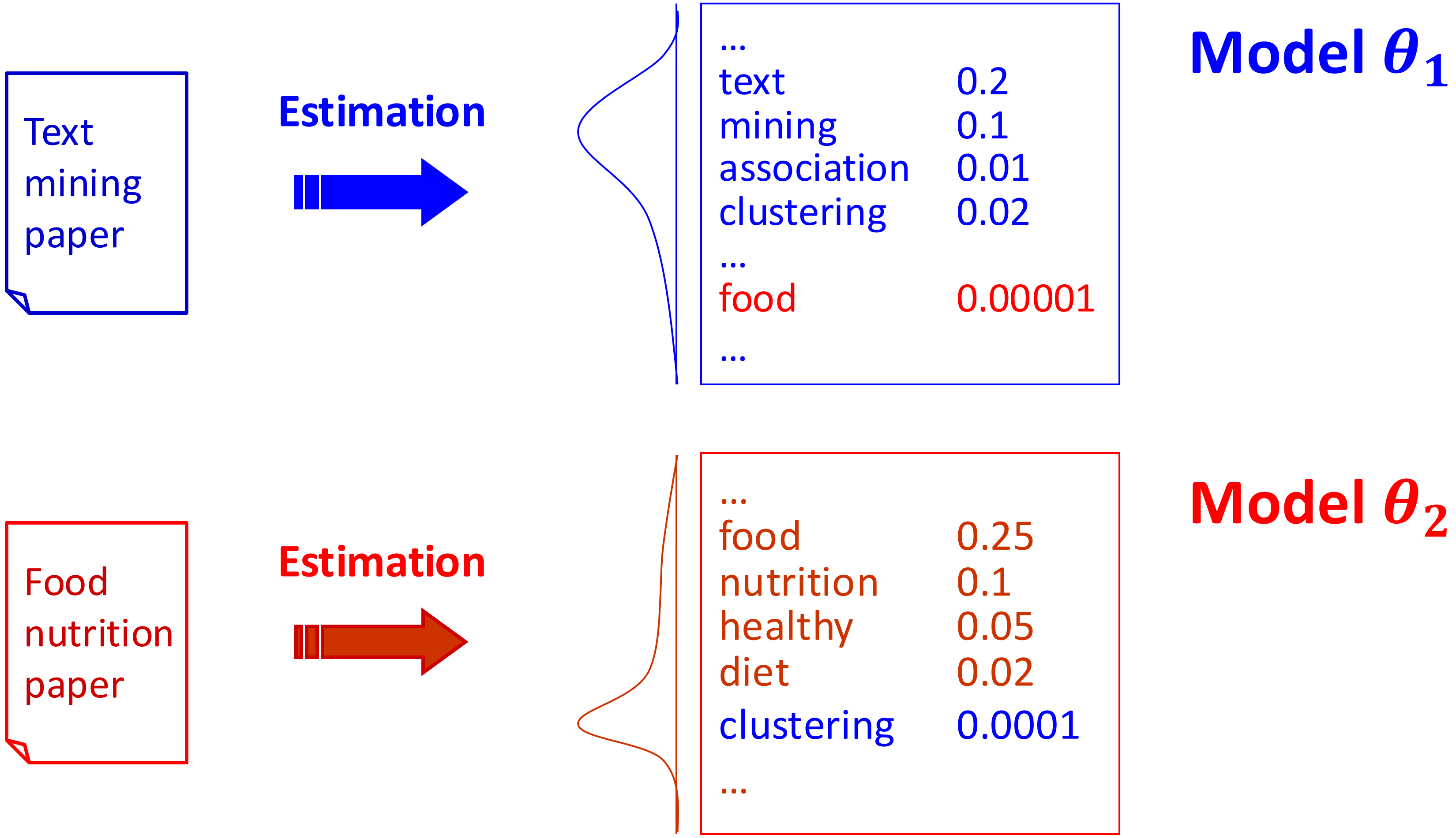
- Authoritativeness (e.g., PageRank)
- Accessibility (e.g., URL length)
- Readability (e.g., avg. sentence length)

Computing $P(q|d)$

Generative process

- Intuitively, the probability that a user who likes document d (modeled by θ_d) will pose query q

Ranking via query likelihood



Ranking via query likelihood

…	
text	0.2
mining	0.1
association	0.01
clustering	0.02
…	
food	0.00001
…	

Model $\boldsymbol{\theta_1}$

…	
food	0.25
nutrition	0.1
healthy	0.05
diet	0.02
clustering	0.0001
…	

Model $\boldsymbol{\theta_2}$

q = [data mining algorithms]

$$P(q|\theta_1)$$

vs.

$$P(q|\theta_2)$$

Computing $\boldsymbol{P(q|d)}$

Under a unigram (term independence) assumption

$$P(q|\theta_d) = \prod_{t \in q} P(t|\theta_d)^{\mathrm{tf}_{t,q}}$$

$$\propto \sum_{t \in q} \mathrm{tf}_{t,q} \log P(t|\theta_d)$$

How to estimate $P(t|\theta_d)$?

Maximum likelihood estimation (MLE)

Simply count observed occurrences

- $P_{\mathrm{MLE}}(t|\theta_d) = \frac{\mathrm{tf}_{t,d}}{|d|}$

Problems

- Observed terms will be scored too optimistically
- Unobserved terms will receive zero probability

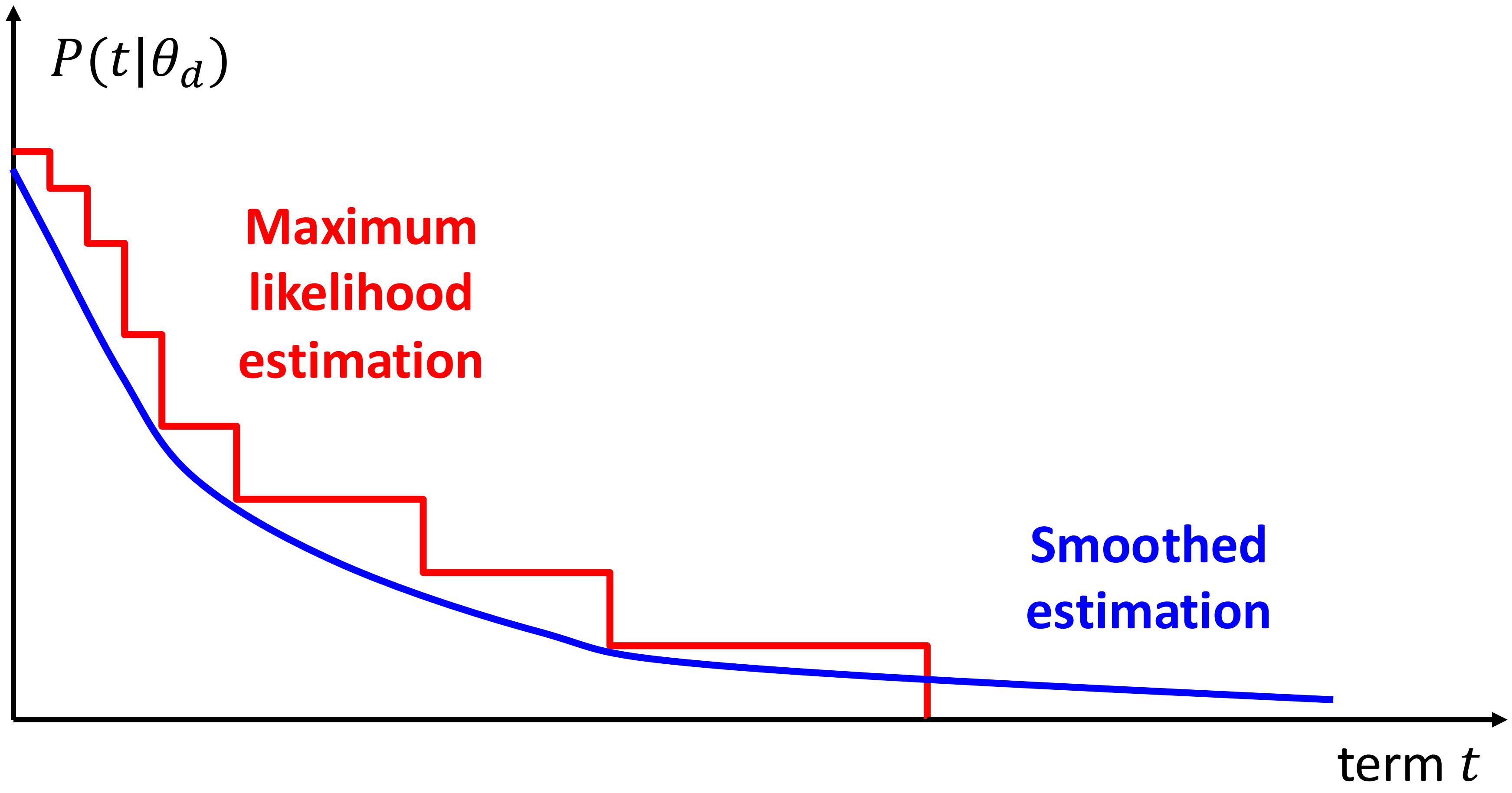
Smoothing probabilities

Smooth probabilities in language models

- Discount the probability of seen words
- Give some probability mass to unseen words

The probability of a non-occurring term should be close to its probability of occurrence in the corpus C

Smoothing probabilities



Smoothing probabilities

General form

- $P(t|\theta_d) = (1 - \alpha)\, P_{\mathrm{MLE}}(t|\theta_d) + \alpha\, P_{\mathrm{MLE}}(t|\theta_C)$

Smoothing controlled through parameter α

- Jelinek-Mercer: $\alpha = \lambda, 0 < \lambda < 1$

Jelinek-Mercer smoothed model

$$f(q,d) \propto \prod_{t \in q} P(t|\theta_d)^{\mathrm{tf}_{t,q}}$$

$$= \prod_{t \in q} \left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda \frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|} \right)^{\mathrm{tf}_{t,q}}$$

Jelinek-Mercer example ($\lambda = 1/2$)

q: [michael jackson]

- d_1: Jackson was a gifted entertainer
- d_2: Michael Jackson anointed himself King of Pop

$$P(q|\theta_1) = \left[\frac{0/5 + 1/12}{2}\right] \times \left[\frac{1/5 + 2/12}{2}\right] \approx 0.008$$

$$P(q|\theta_2) = \left[\frac{1/7 + 1/12}{2}\right] \times \left[\frac{1/7 + 2/12}{2}\right] \approx 0.018$$

<

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{t\in q} \mathrm{tf}_{t,q} \log \boxed{P(t|\theta_d)}$$ replace with JM formulation

$$= \sum_{\boxed{t\in q}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

run through terms in and not in d separately

$$= \sum_{\substack{t\in q\\ t\in d}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

= 0, based on summation indices

$$+ \sum_{\substack{t\in q\\ t\notin d}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\boxed{\mathrm{tf}_{t,d}}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{\substack{t \in q \\ t \in d}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda \frac{\mathrm{tf}_{t,C}}{|C|}\right) + \sum_{\substack{t \in q \\ \boxed{t \notin d}}} \mathrm{tf}_{t,q} \log\left(\lambda \frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

all terms, minus those in d

$$= \sum_{\substack{t \in q \\ t \in d}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda \frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

$$+ \boxed{\sum_{t \in q} \mathrm{tf}_{t,q} \log\left(\lambda \frac{\mathrm{tf}_{t,C}}{|C|}\right)} - \sum_{\substack{t \in q \\ t \in d}} \mathrm{tf}_{t,q} \log\left(\lambda \frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

document-independent
(doesn't affect ranking)

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{\boxed{\substack{t\in q\\ t\in d}}} \mathrm{tf}_{t,q} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right) - \sum_{\boxed{\substack{t\in q\\ t\in d}}} \mathrm{tf}_{t,q} \log\left(\lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)$$

same summation indexes

$$= \sum_{\substack{t\in q\\ t\in d}} \left(\boxed{\mathrm{tf}_{t,q}} \log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right) - \boxed{\mathrm{tf}_{t,q}} \log\left(\lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)\right)$$

common multiplier

$$= \sum_{\substack{t\in q\\ t\in d}} \mathrm{tf}_{t,q} \left(\boxed{\log\left((1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right) - \log\left(\lambda\frac{\mathrm{tf}_{t,C}}{|C|}\right)}\right)$$

subtraction of logs

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{t\in q} \mathrm{tf}_{t,q} \log\left(\boxed{\frac{(1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|} + \lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}}{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}}}\right) \quad \text{split fraction}$$

$$= \sum_{t\in q} \mathrm{tf}_{t,q} \log\left(\frac{(1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|}}{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}} + \boxed{\frac{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}}{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}}}\right) \quad = 1$$

$$= \sum_{t\in q} \mathrm{tf}_{t,q} \log\left(\frac{(1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|}}{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}} + 1\right)$$

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{t \in q} \mathrm{tf}_{t,q} \log\left(\frac{(1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|}}{\lambda \frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}} + 1 \right)$$

Where is tf-idf weighting?

$$f(q,d) \propto \sum_{t \in q} \mathrm{tf}_{t,q} \log\left(\frac{(1-\lambda)\frac{\mathrm{tf}_{t,d}}{|d|}}{\lambda\frac{\mathrm{tf}_{t,C}}{|\mathrm{C}|}} + 1\right)$$

- Proportional to term frequency: tf effect
- Inv. proportional to collection frequency: idf effect

How about document length?

General form

- $P(t|\theta_d) = (1-\alpha)\, P_{\mathrm{MLE}}(t|\theta_d) + \alpha\, P_{\mathrm{MLE}}(t|\theta_C)$

Smoothing controlled through parameter α

- Jelinek-Mercer: $\alpha = \lambda, 0 < \lambda < 1$
- Dirichlet: $\alpha = \frac{\mu}{|d|+\mu}$ $\quad |d| \to 0 \therefore \alpha \to 1$ $\quad |d| \to \infty \therefore \alpha \to 0$

How effective are these?

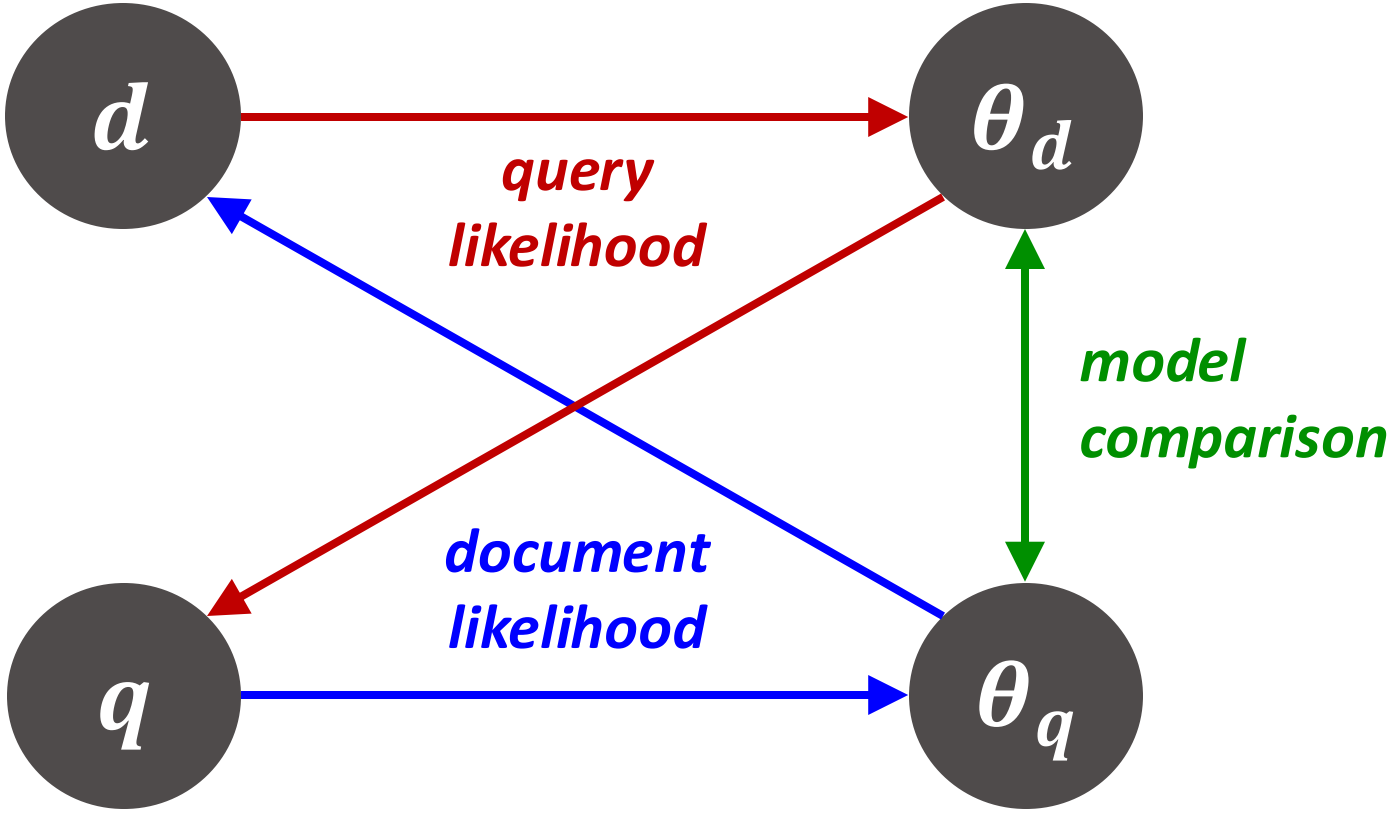
Dirichlet smoothing works the best

- Particularly effective for keyword queries

Length-adjusted smoothing

- Shorter documents get more smoothing
- Longer documents get less smoothing

Extended approaches



Document likelihood model

$$
\begin{aligned}
f(q,d) &\approx P(q,d) \\
&= P(d|q)P(q) \quad \text{Bayes' rule} \\
&\propto P(d|\theta_q)
\end{aligned}
$$

Problem: queries are short

- Poor estimation of θ_q

Solution: improve query model via feedback

Model comparison

Two steps

- Build both θ_q and θ_d
- Measure how much they ***diverge***

Divergence-based ranking

- $f(q,d) = -D(\theta_q || \theta_d)$

Model comparison

Kullback-Leibler (KL) divergence

- Asymmetric measure $[0,\infty]$

$$f(q,d) = -D_{\mathrm{KL}}(\theta_q || \theta_d)$$

$$= -\sum_t P(t|\theta_q) \log \frac{P(t|\theta_q)}{P(t|\theta_d)}$$

Summary

Principled ranking approach

- Statistical foundations (better parameter setting)

Effectiveness through smoothing

- Add key ranking components (idf, document length)

Flexible and extendable

- Relevance feedback, priors, other tasks

References

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Coming next…

Experimental Methods

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