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Information Retrieval

Learning to Rank: Pairwise and Listwise

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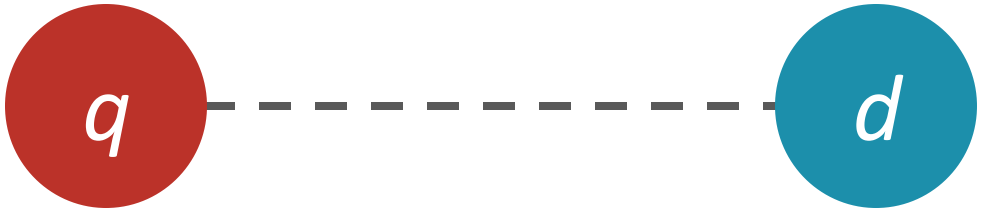
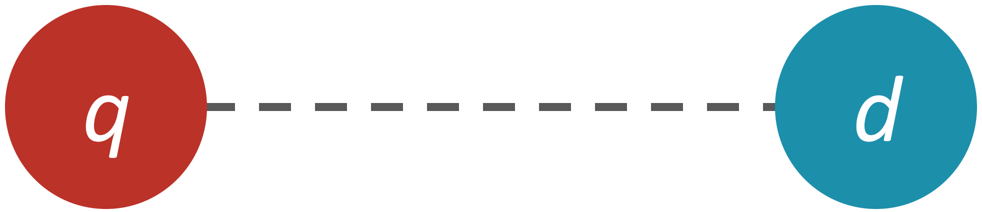
The ranking problem



$$f(q, d)$$

Learning to rank



$$\boldsymbol{f}(\mathbf{x})$$

Learning to rank

Feature-based representation

- Individual models as ranking ”features”

Discriminative learning

- Effective models learned from data
- Aka machine-learned ranking

Pointwise approach

Several approaches

- Regression-based
- Classification-based
- Ordinal regression-based

$$\{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\}$$

Limitations of the pointwise approach

Ranking requires getting relative scores right

- Pointwise approaches learn absolute scores

Higher positions should matter more than lower ones

- Pointwise loss functions are agnostic to positions

Queries should be equally important

- Queries with many relevant documents dominate

Pairwise approach

Pairwise classification-based

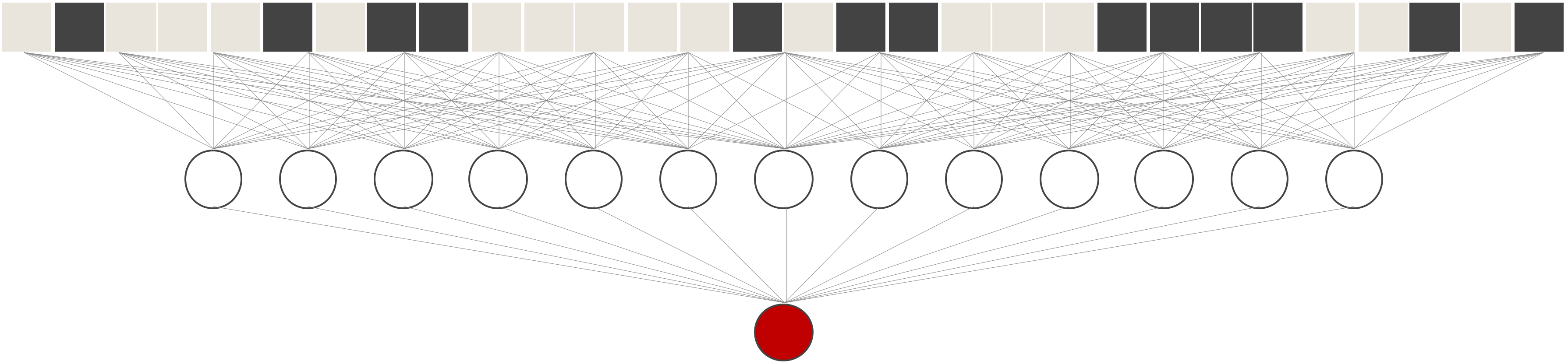
- RankNet
- RankBoost
- Ranking SVM
- IR-SVM

$$\{(x_1, x_2, 1), (x_2, x_1, 0),$$
$$(x_1, x_3, 1), (x_3, x_1, 0),$$
$$\dots$$
$$(x_{n-1}, x_n, 1), (x_n, x_{n-1}, 0)\}$$

RankNet
(Burges et al., ICML 2005)

Shallow (2-layer) neural network model

- Sigmoid activations



Gradient descent optimizer

RankNet
(Burges et al., ICML 2005)

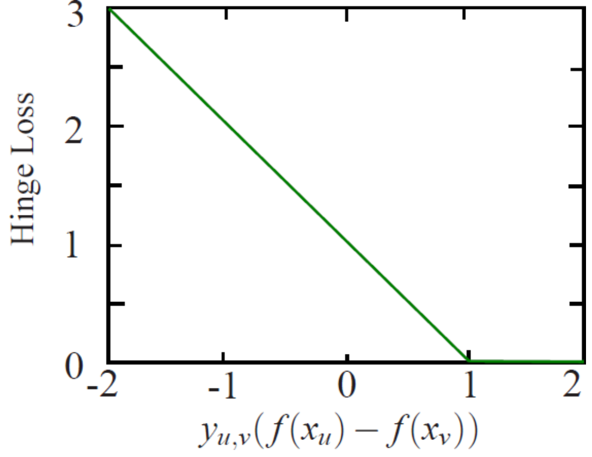
Pairwise prediction converted to probability

- $\hat{y}_{uv} = f(x_u, x_v) = \frac{\exp(f(x_u) - f(x_v))}{1 + \exp(f(x_u) - f(x_v))}$ ***(logistic function)***

Cross entropy loss

- $$\mathcal{L}(f; x_u, x_v, y_{uv}) = -\overbrace{y_{uv}}^{\text{relevant}} \log \overbrace{\hat{y}_{uv}}^{\text{predicted relevant}} - \underbrace{(1 - y_{uv})}_{\text{non-relevant}} \log \underbrace{(1 - \hat{y}_{uv})}_{\text{predicted non-relevant}}$$

Ranking SVM
(Herbrich et al., ALMC 2000; Joachims, KDD 2002)



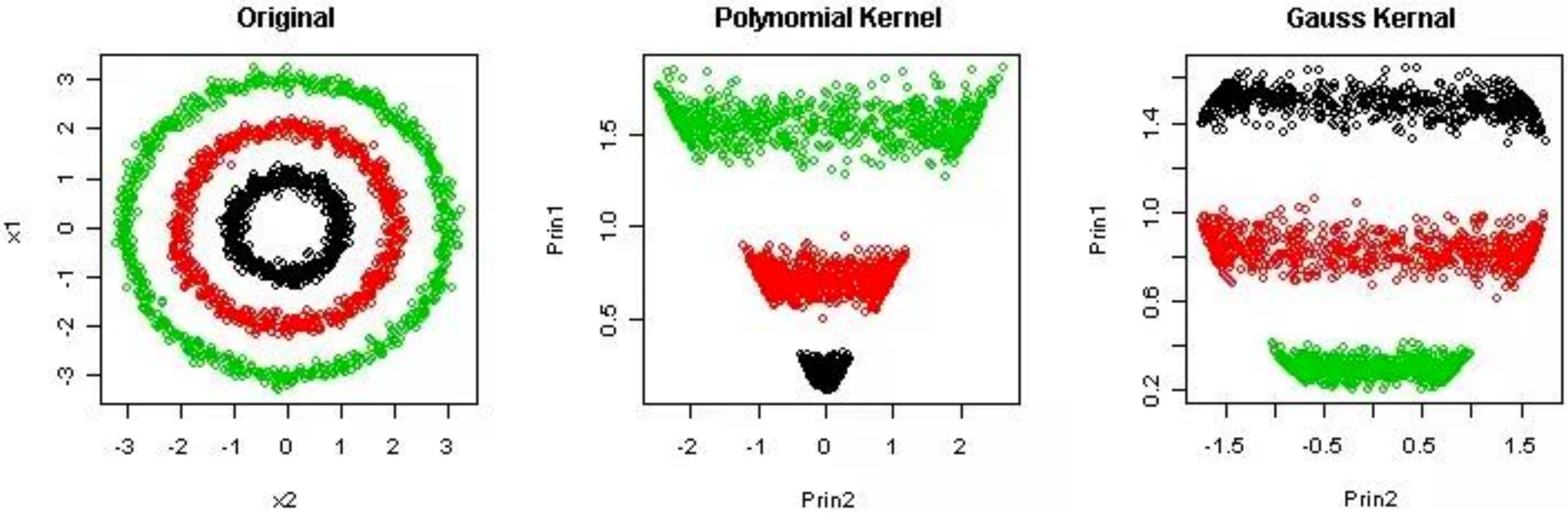
Hinge loss

- $\mathcal{L}(f; x_u, x_v, y_{uv}) = \max(0, 1 - y_{uv}(f(x_u) - f(x_v)))$

Nice properties inherited from standard SVM

- Good generalization via margin maximization
- Non-linear models via the kernel trick

Kernel trick



Limitations of the pairwise approach

Pairwise labels ignore graded relevance

- $(x_1, x_2, 1)$, regardless of the grades of x_1 and x_2

Query dominance exacerbated

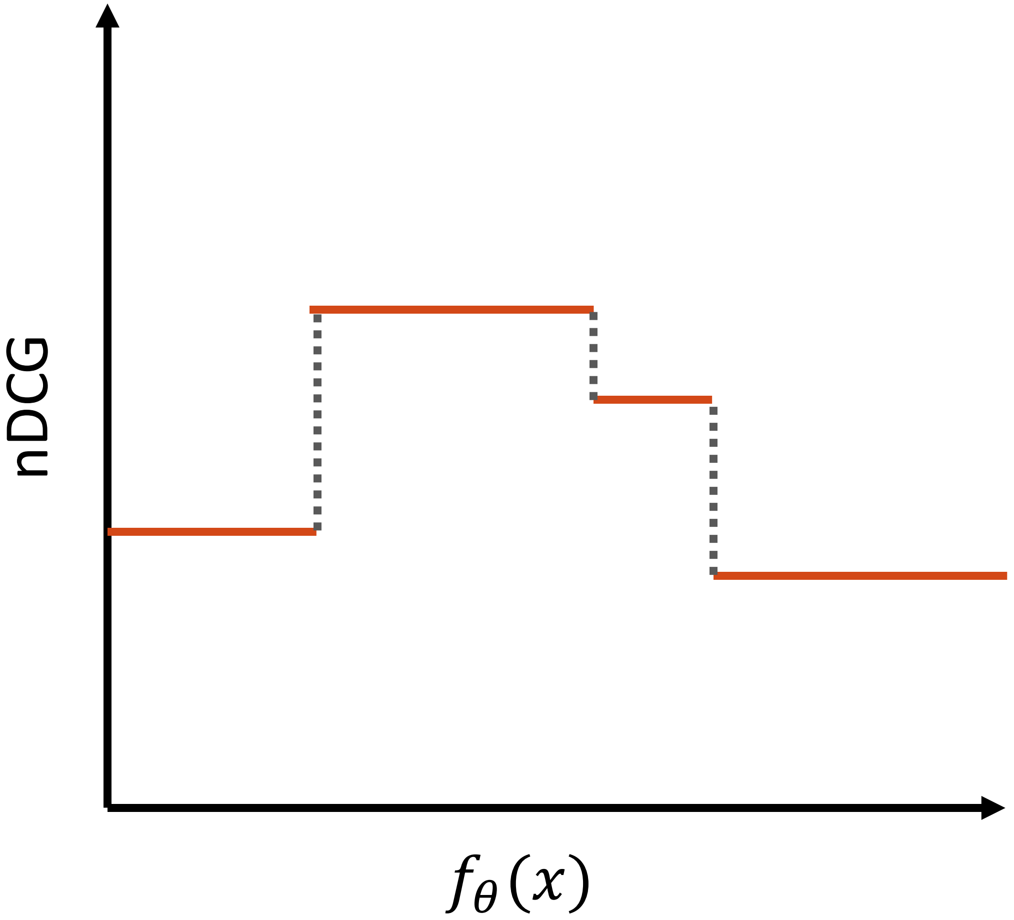
- A query with more docs will have way more pairs

Ranking positions still not taken into account

- Swaps at the top more important than at the bottom

Can we optimize ranking metrics directly?

Ranking metrics generally non-differentiable



Piecewise constant functions

- **Flat:** zero derivative
- **Discontinuous:** undef derivative

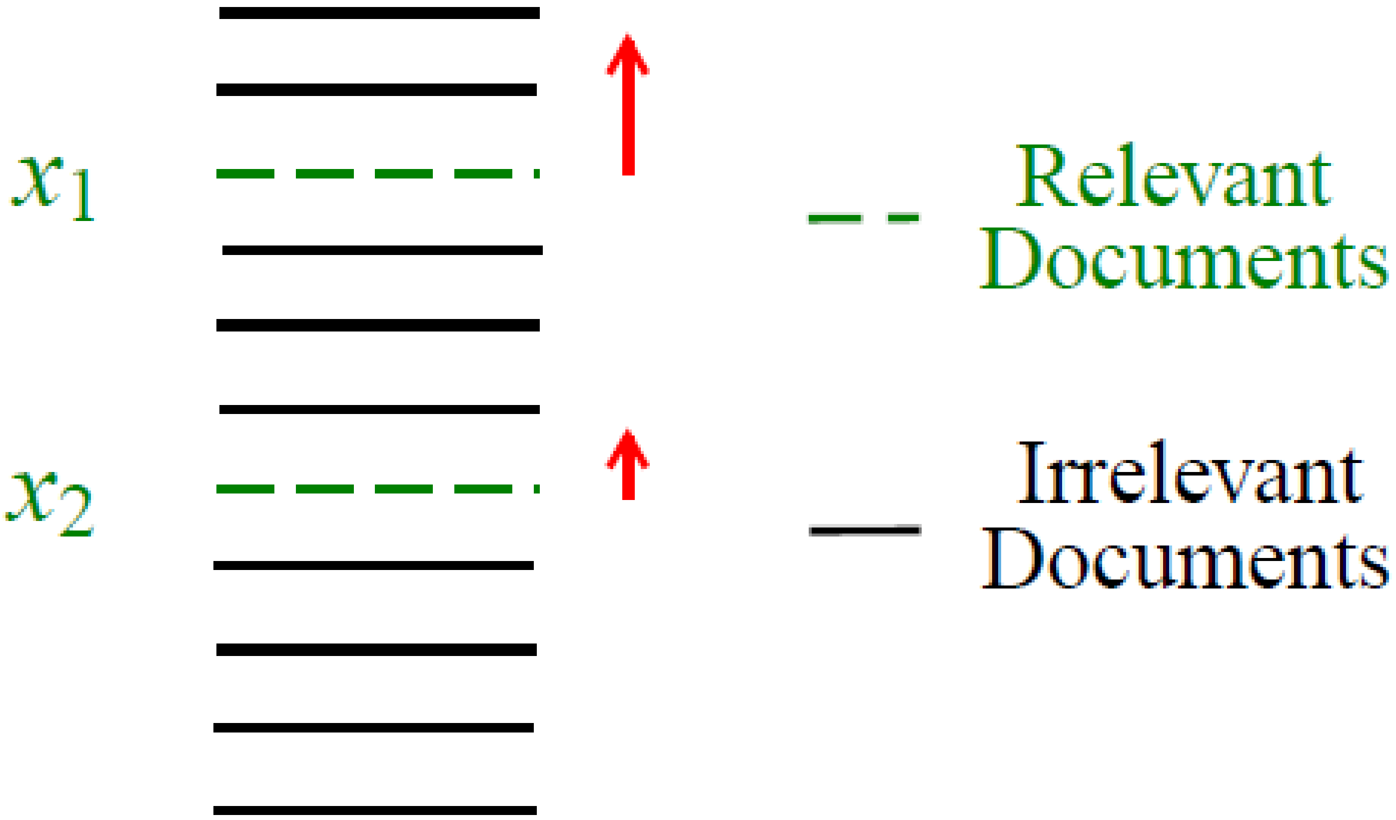
LambdaRank
(Burges, NIPS 2006)

An extension of RankNet

- Ranking evaluation metrics (which are position-based) are directly used to define the gradient with respect to each document pair in the training process

Why is it feasible to directly define the gradient?

LambdaRank
(Burges, NIPS 2006)



$$\frac{\partial L}{\partial s_1} > \frac{\partial L}{\partial s_2}$$

LambdaRank

(Burges, NIPS 2006)

Gradient determines magnitude of updates

- $w = w - \alpha\, \nabla \mathcal{L}(w)$

Vector $\nabla \mathcal{L}(w) = \left(\frac{\partial \mathcal{L}(w)}{\partial w_1}, \frac{\partial \mathcal{L}(w)}{\partial w_2}, \ldots, \frac{\partial \mathcal{L}(w)}{\partial w_d} \right)$

- $\frac{\partial}{\partial w_k} \mathcal{L}(w) = \sum_{\langle u,v \rangle} \underline{\lambda_{uv}}\, \underline{x_k^{(i)}}$

gradient magnitude (prediction error)   *k^{th} feature score*

LambdaRank
(Burges, NIPS 2006)

Lambda function

- An arbitrary surrogate for the gradient magnitude, assuming no particular loss function

$$\lambda_{uv} \equiv \frac{2^{y_u} - 2^{y_v}}{1 + \exp\left(f(x_u) - f(x_v)\right)} \left|\Delta \mathrm{nDCG}(x_u, x_v)\right|$$

LambdaMART
(Wu et al., Tech. Report 2008)

MART = Multiple Additive Regression Trees

- Commercial name for gradient boosted trees

Boosted tree version of LambdaRank

- Lambda functions guide the construction of weak learners (regression trees) via boosting
- $h_t^* = \text{argmin}_{h_t} \sum_{(x,y)} (h_t(x) - (-\alpha \, \nabla \mathcal{L}(f_t))^2$

Listwise approach

Metric-specific loss

- Optimize evaluation metrics

Non-metric-specific loss

- Optimize other listwise functions

$$\{(\mathbf{x}, \mathbf{y})\}$$

Metric-specific listwise ranking

It is natural to directly optimize what is used to evaluate the ranking results, but not trivial

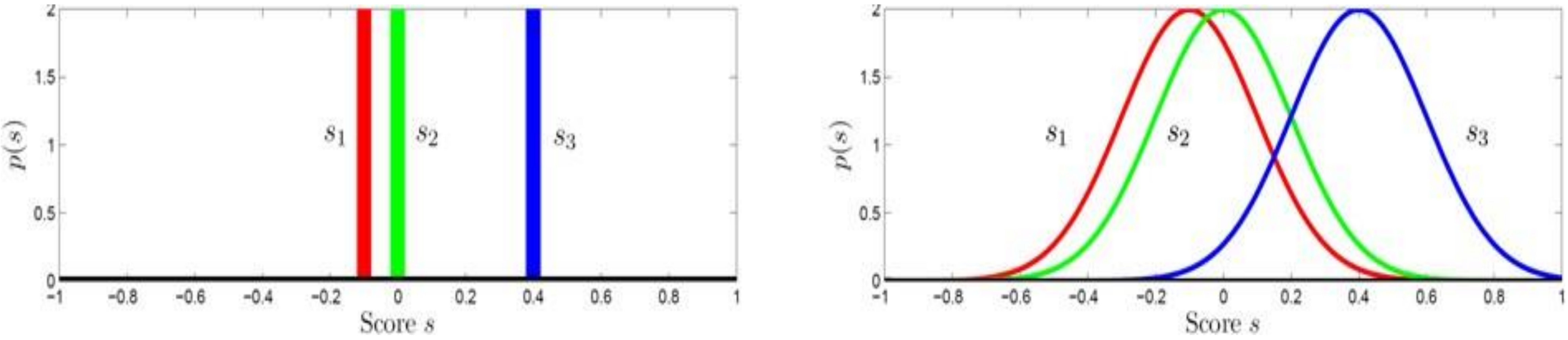
- Evaluation metrics such as nDCG and MAP are non-continuous and non-differentiable
- Most optimization techniques were developed to handle continuous and differentiable cases

SoftRank
(Taylor et al. LR4IR 2007/WSDM 2008)

Key idea: “soften” the evaluation metric

Score s_i as a random variable

- $P(s_i) = N(s_i | f(x_i), \sigma_s^2)$

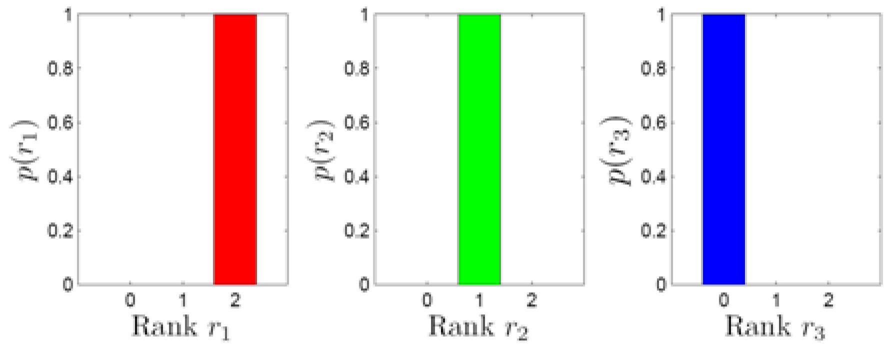


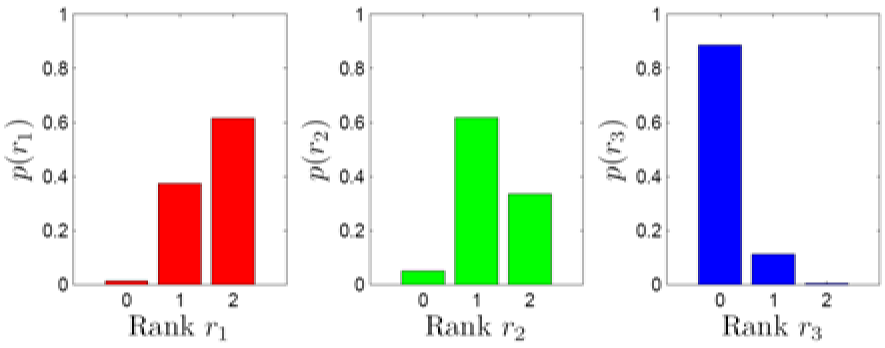
SoftRank
(Taylor et al. LR4IR 2007/WSDM 2008)

We've constructed a score distribution per rank

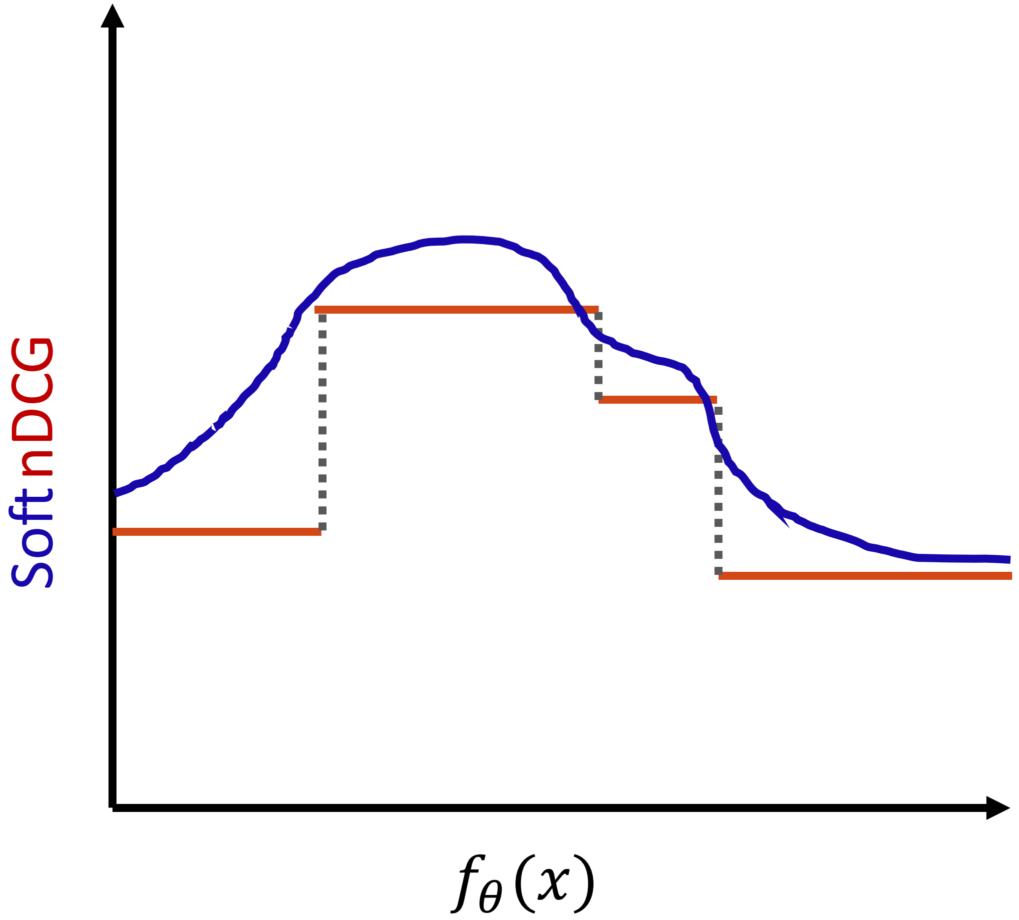
- Non-deterministic scores make it possible to find a document in any ranking position

Map score distribution to rank distribution





SoftRank
(Taylor et al. LR4IR 2007/WSDM 2008)



Now each document has a non-deterministic position

- Each draw results in a permut.
- Each permut. has a given nDCG

Compute SoftNDCG as the expected nDCG over all possible permutations (smooth and differentiable)

Optimize via gradient descent

Metric-specific listwise ranking

Workarounds

- Soften or upper-bound non-smooth objective

Optimize the non-smooth objective directly

- Use genetic programming (RankGP)
- Use ranking evaluation metric to update the training data distribution via boosting (AdaRank)

AdaRank

(Xu and Li, SIGIR 2007)

Algorithm 2 Learning Algorithms for AdaRank

Input: document group for each query

Given: initial distribution $\mathcal{D}_1$ on input queries

training set biased by query difficulty

For $t = 1, \cdots, T$

Train weak ranker $f_t(\cdot)$ based on distribution $\mathcal{D}_t$.

single-feature weak ranker

Choose $\alpha_t = \frac{1}{2} \log \frac{\sum_{i=1}^{n} \mathcal{D}_t(i)(1+M(f_t,\mathbf{x}^{(i)},\mathbf{y}^{(i)}))}{\sum_{i=1}^{n} \mathcal{D}_t(i)(1-M(f_t,\mathbf{x}^{(i)},\mathbf{y}^{(i)}))}$

weighted accuracy of f_t

Update $\mathcal{D}_{t+1}(i) = \frac{\exp(-M(\sum_{s=1}^{t} \alpha_s f_s,\mathbf{x}^{(i)},\mathbf{y}^{(i)}))}{\sum_{j=1}^{n} \exp(-M(\sum_{s=1}^{t} \alpha_s f_s,\mathbf{x}^{(j)},\mathbf{y}^{(j)}))}$,

update difficulty estimates

Output: $\sum_t \alpha_t f_t(\cdot)$.

Non-metric-specific listwise ranking

Defining listwise loss functions based on the understanding of unique properties of ranking

Representative algorithms

- ListNet
- ListMLE
- BoltzRank

Ranking loss is non-trivial!

An example

- Function f: $f(A) = 4, f(B) = 2, f(C) = 3 \quad (ACB)$
- Function h: $h(A) = 4, h(B) = 6, h(C) = 3 \quad (BAC)$
- Ground-truth y: $y(A) = 6, y(B) = 4, y(C) = 3 \quad (ABC)$

Which function (f or h) is closer to the ground-truth?

- According to nDCG, f should be closer to y!

ListNet
(Cao et al., ICML 2007)

Given f: $f(A) = 4, f(B) = 2, f(C) = 3$

- Deterministically: $P(ACB) = 1$

What if other permutations have a non-zero prob?

- For a k-permutation π^k under function f

$$P_f\left(\pi^k\right) = \frac{f(\pi_1)}{\sum_{i=1}^{k} f(\pi_i)} \times \frac{f(\pi_2)}{\sum_{i=2}^{k} f(\pi_i)} \times \cdots \times \frac{f(\pi_{k-1})}{\sum_{i=k-1}^{k} f(\pi_i)}$$

ListNet
(Cao et al., ICML 2007)

Given f: $f(A) = 4, f(B) = 2, f(C) = 3$

- Deterministically: $P(ACB) = 1$

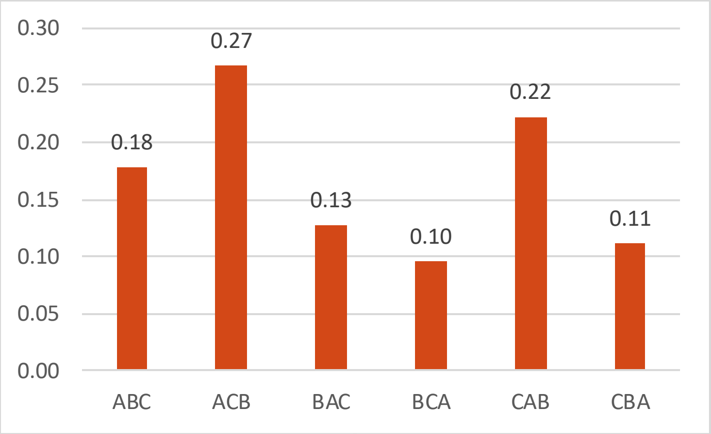
What if other permutations have a non-zero prob?

- Probabilistically: $P(ACB) = \frac{f(A)}{f(A)+f(C)+f(B)} \times \frac{f(C)}{f(C)+f(B)}$

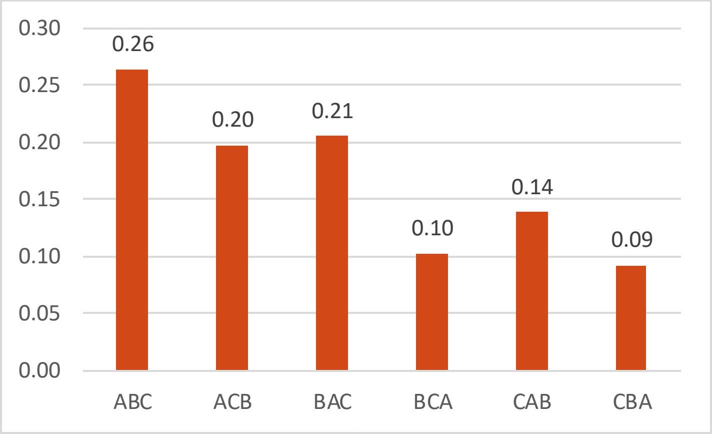
$$= \frac{4}{4+2+3} \times \frac{2}{2+3} = 0.27$$

Distance between ranked lists

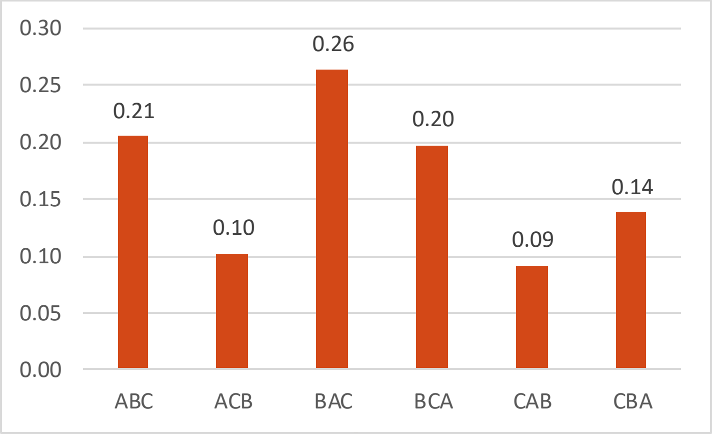
$f\colon f(A) = 4, f(B) = 2, f(C) = 3$



$y\colon y(A) = 6, y(B) = 4, y(C) = 3$



$h\colon h(A) = 4, h(B) = 6, h(C) = 3$



$D(P_y||P_f) = 0.10$

$D(P_y||P_h) = 0.14$

f is closer!

KL divergence loss

Loss function = KL-divergence between two permutation probability distributions ($\varphi = \exp$)

- $\mathcal{L}(f; \mathbf{x}, y) = D(\underbrace{P_{\varphi(y)}}_{\text{ground-truth probability distro}} || \underbrace{P_{\varphi(f(\mathbf{x}))}}_{\text{predicted probability distro}})$

Neural net model, gradient descent optimizer

Summary

Listwise approaches more closely model ranking

- Take all the documents associated with the same query as one learning instance
- Ranking position is visible to the loss function

State-of-the-art on standard benchmarks (LETOR)

- Particularly effective for top-heavy evaluation

RankLib tutorial

RankLib v2.18

```
0: MART (gradient boosted regression tree)
1: RankNet
2: RankBoost
3: AdaRank
4: Coordinate Ascent
6: LambdaMART
7: ListNet
8: Random Forests
```

RankLib tutorial

https://sourceforge.net/p/lemur/wiki/RankLib%20How%20to%20use/

```
> java -jar bin/RankLib-2.18.jar \
    -train MQ2008/Fold1/train.txt \
    -validate MQ2008/Fold1/vali.txt \
    -test MQ2008/Fold1/test.txt \
    -ranker 6 -metric2t NDCG@10 -metric2T ERR@10 \
    -save mymodel.txt
```

References

Learning to rank for information retrieval
Liu, 2011

Learning to rank for information retrieval and natural language processing
Li, 2014

Learning for Web rankings
Grangier and Paiement, 2011

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Coming next...

Neural Models

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