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Information Retrieval

Learning to Rank: Pointwise

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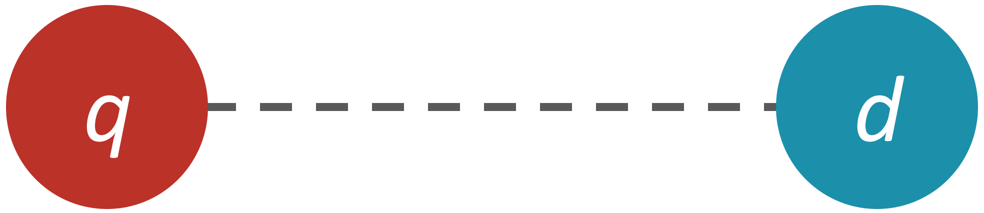
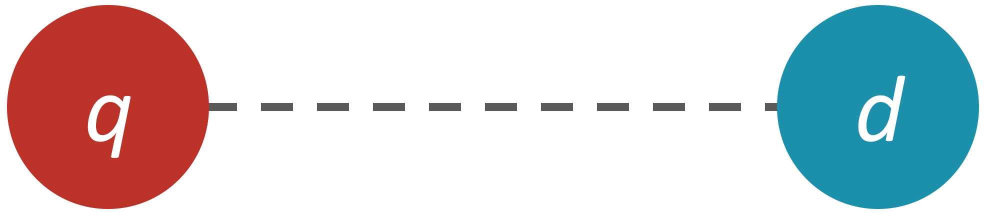
The ranking problem



$$f(q, d)$$

Learning to rank



$$\boldsymbol{f}(\mathbf{x})$$

Learning to rank

Feature-based representation

- Individual models as ranking ”features”

Discriminative learning

- Effective models learned from data
- Aka machine-learned ranking

Building blocks

Goal is to learn a ranking model

- $f : \mathcal{X} \rightarrow \mathcal{Y}$

That minimizes some loss function

- $\mathcal{L} : f(\mathcal{X}) \times \mathcal{Y} \rightarrow \mathcal{R}$

Ideally, we would like a low test error

- We settle for a good training error / capacity trade-off

What is a learning algorithm?

Given

- A family of functions $\mathcal{F}$ (e.g., linear, trees, neural nets)
- A measure of loss $\mathcal{L}$ (error + capacity)

Learning can be cast as an optimization problem

- $$\begin{aligned} f^* &= \operatorname{argmin}_{f \in \mathcal{F}} \mathcal{L}(f, \text{train}) \\ &= \operatorname{argmin}_{f \in \mathcal{F}} \operatorname{Err}(f, \text{train}) + \lambda \operatorname{Reg}(f) \end{aligned}$$

Classical algorithms

Linear learning algorithms

- Regression (LASSO, Ridge reg.)
- Classification (logistic reg., linear SVM, AdaBoost)

Non-linear learning algorithms

- Neural networks
- (Boosted) regression trees

Linear learning algorithms

$\mathcal{F}$ is the set of linear functions

- $f_w(x) = w^T x$, where $x \in \mathcal{R}^d$, $w \in \mathcal{R}^d$
  - x: d-dimens. feature vector describing an example
  - w: weight vector defining the function f_w
  - $f_w(x)$ is the prediction of f_w for example x

Linear learning algorithms

Most typical regularizers

- L2 regularization, $||w||_2^2 = \sum_j w_j^2$
  (prefer ”flat” weights)
- L1 regularization, $||w||_1 = \sum_j |w_j|$
  (prefer ”sparse” weights)

Classical algorithms

Linear learning algorithms

- **Regression (LASSO, Ridge reg.)**
- Classification (logistic reg., linear SVM, AdaBoost)

Non-linear learning algorithms

- Neural networks
- (Boosted) regression trees

Linear learning for regression

Ordinary least squares

- $\mathcal{L}(w) = \frac{1}{2}\sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$

Ridge regression

- Add L2 regularization ($+\lambda \, \|w\|_2^2$)

LASSO

- Add L1 regularization ($+\lambda \, \|w\|_1$)

Linear learning for regression

Ordinary least squares

- $\mathcal{L}(w) = \frac{1}{2}\sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$

Typical optimization strategy

- Compute the gradient of $\mathcal{L}(\cdot)$ as $\nabla\mathcal{L}(\cdot)$
- Take a step (iterate) in the opposite direction
  $w = w - \alpha\, \nabla\mathcal{L}(w)$, where α is the learning rate

Computing gradients

Gradient as a d-dimens. vector of partial derivatives

- $\nabla\mathcal{L}(w) = \left(\frac{\partial\mathcal{L}(w)}{\partial w_1}, \frac{\partial\mathcal{L}(w)}{\partial w_2}, \dots, \frac{\partial\mathcal{L}(w)}{\partial w_d}\right)$

How to compute each partial derivative?

- Manual derivation… d may be very large!
- Automatically

Computing gradients

Ordinary least squares

- $\mathcal{L}(w) = \frac{1}{2}\sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$

For a given dimension k

- $\frac{\partial}{\partial w_k}\mathcal{L}(w) = \frac{\partial}{\partial w_k}\frac{1}{2}\sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$

Computing gradients

$$\frac{\partial}{\partial w_k}\mathcal{L}(w) = \frac{\partial}{\partial w_k}\frac{1}{2}\sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$$

$$= \sum_{i=1}^{m}\frac{\partial}{\partial w_k}\frac{1}{2}\left(f_w(x^{(i)}) - y^{(i)}\right)^2$$

$$= \sum_{i=1}^{m} 2\,\frac{1}{2}\left(f_w(x^{(i)}) - y^{(i)}\right)\frac{\partial}{\partial w_k}\left(f_w(x^{(i)}) - y^{(i)}\right)$$

$$= \sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)\frac{\partial}{\partial w_k}f_w(x^{(i)})$$

Computing gradients

$$
\begin{aligned}
\frac{\partial}{\partial w_k} f_w\left(x^{(i)}\right) &= \frac{\partial}{\partial w_k} \mathrm{w}^T x^{(i)} \\
&= \frac{\partial}{\partial w_k} \sum_{j=1}^{d} w_j x_j^{(i)} \\
&= \frac{\partial}{\partial w_k} \left(w_k x_k^{(i)} + \sum_{j \neq k} w_j x_j^{(i)} \right) \\
&= x_k^{(i)}
\end{aligned}
$$

Computing gradients

$$\frac{\partial}{\partial w_k}\mathcal{L}(w) = \sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)\frac{\partial}{\partial w_k}f_w(x^{(i)})$$

$$= \sum_{i=1}^{m}\underbrace{\left(f_w(x^{(i)}) - y^{(i)}\right)}_{\textit{prediction error}}\underbrace{x_k^{(i)}}_{k^{th}\textit{ feature score}}$$

Computing gradients

$$\frac{\partial}{\partial w_k}\mathcal{L}(w) = \sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right)\frac{\partial}{\partial w_k} f_w(x^{(i)})$$

$$= \sum_{i=1}^{m}\left(f_w(x^{(i)}) - y^{(i)}\right) x_k^{(i)}$$

- $m = |D|$: batch gradient descent
- $m = c \ll |D|$: mini-batch gradient descent
- $m = 1$: stochastic gradient descent

Classical algorithms

Linear learning algorithms

- Regression (LASSO, Ridge reg.)
- **<u>Classification (logistic reg., linear SVM, AdaBoost)</u>**

Non-linear learning algorithms

- Neural networks
- (Boosted) regression trees

Linear learning for classification

Prediction

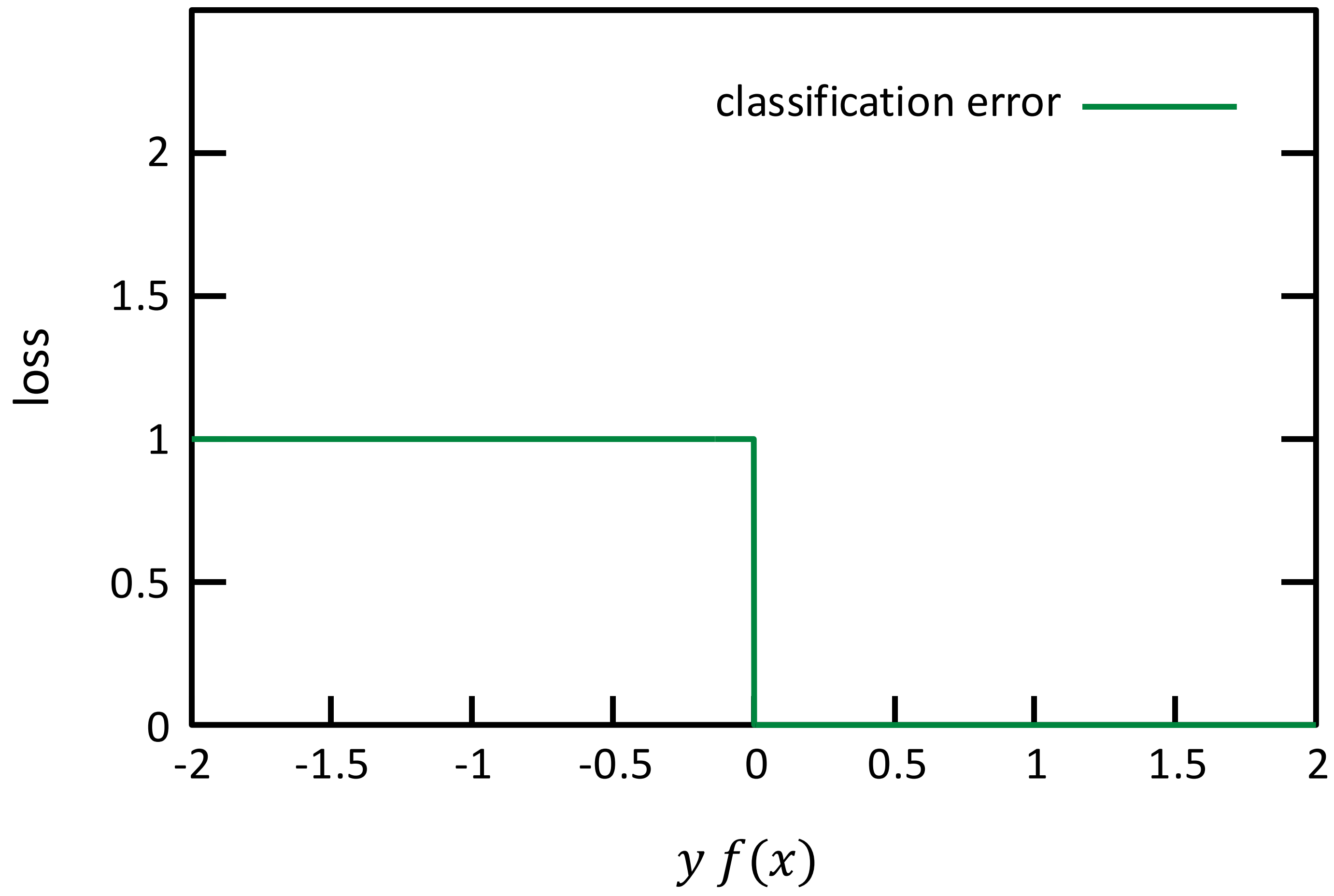
- $f_w(x) = w^T x$ $\quad$ > 0 predicts $+1$

  ≤ 0 predicts -1

Classification error

- $\mathcal{L}(w) = \frac{1}{m} \sum_{i=1}^{m} \mathbf{1}\left(y^{(i)} f_w\left(x^{(i)}\right) < 0\right)$
- **Problem:** non-differentiable!

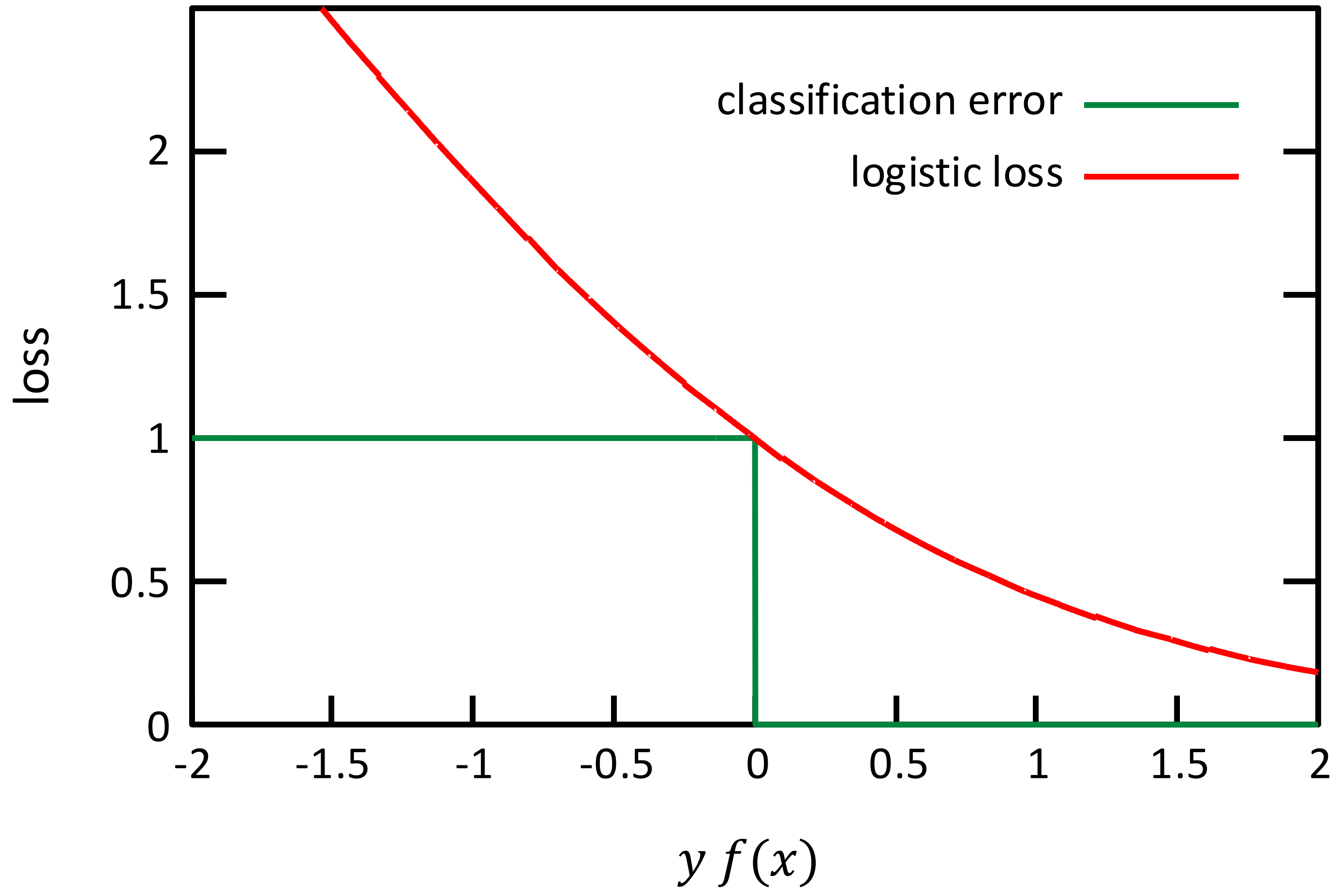
Upper-bounding classification error



Classification error

$\mathbf{1}(y f_w(x) < 0)$

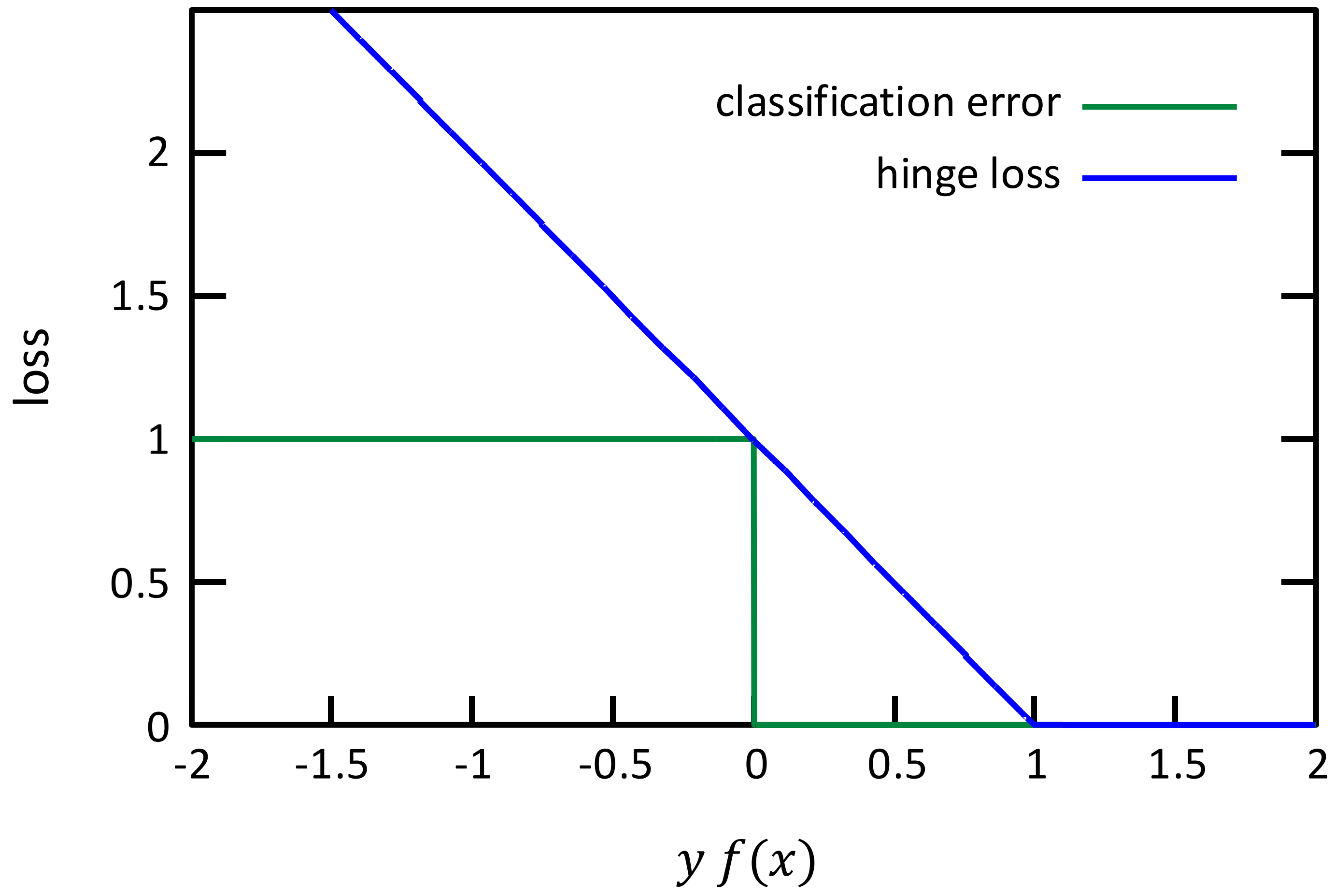
Upper-bounding classification error



Logistic loss

$$\log\left(1 + \exp(-y f_w(x))\right)$$

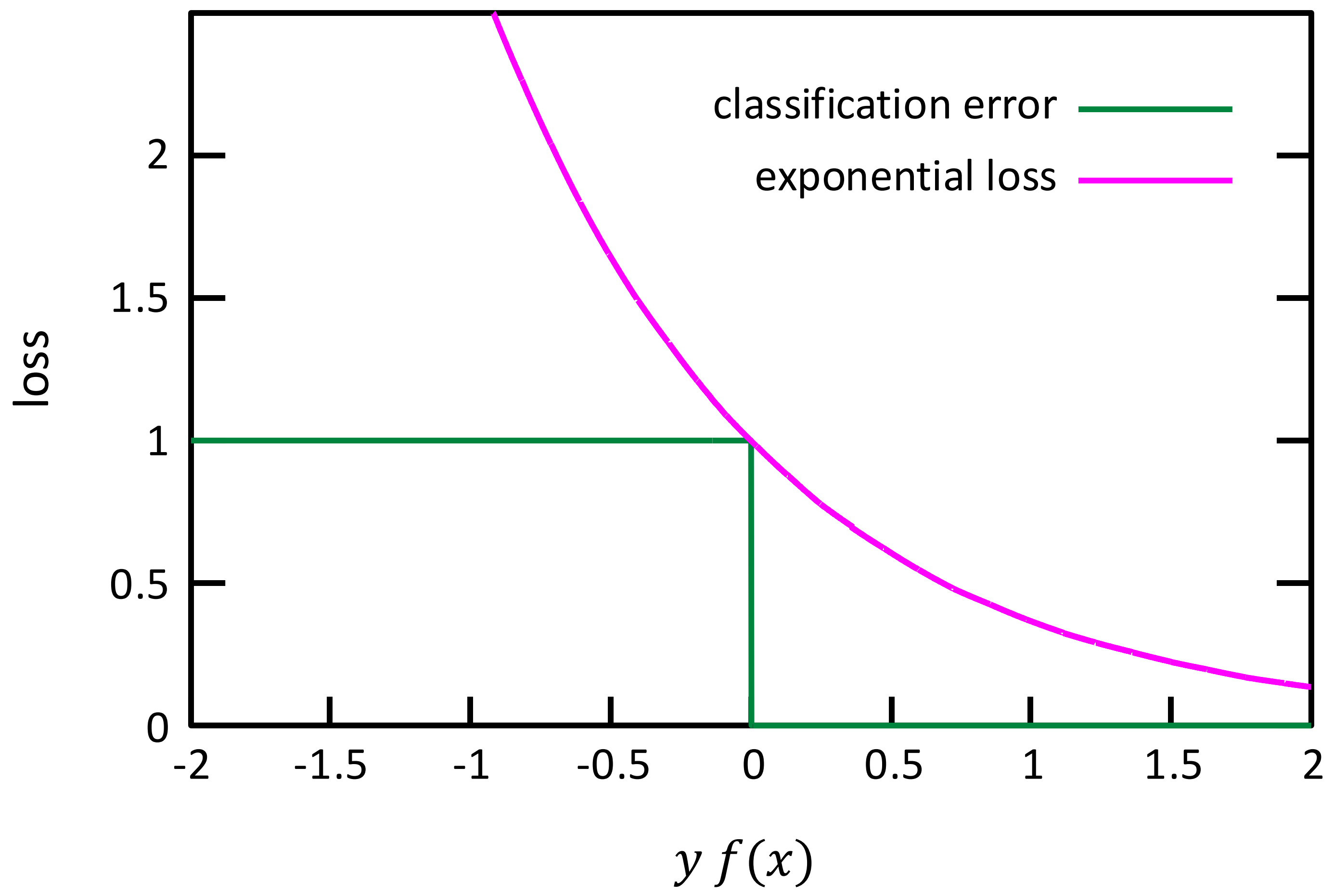
Upper-bounding classification error



Hinge loss

$$\max\left(0, 1 - y f_w(x)\right)$$

Upper-bounding classification error



Exponential loss

$\exp(-y f_w(x))$

Linear learning for classification

Logistic regression

- Logistic loss + L1 or L2 regularizer

Support Vector Machines (SVM)

- Hinge loss + L2 regularizer

AdaBoost

- Exponential loss + L1 regularizer

Classical algorithms

Linear learning algorithms

- Regression (LASSO, Ridge reg.)
- Classification (logistic reg., linear SVM, AdaBoost)

Non-linear learning algorithms

- **Neural networks**
- (Boosted) regression trees

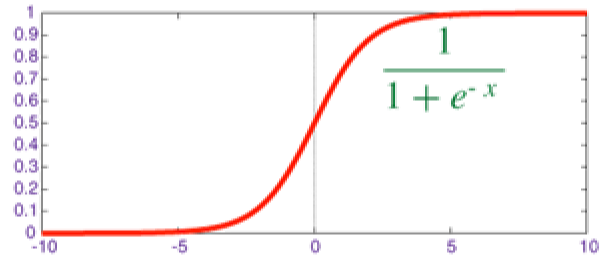
Neural networks

Composable functions through several layers

- Inspired by biological neural networks (your brain)

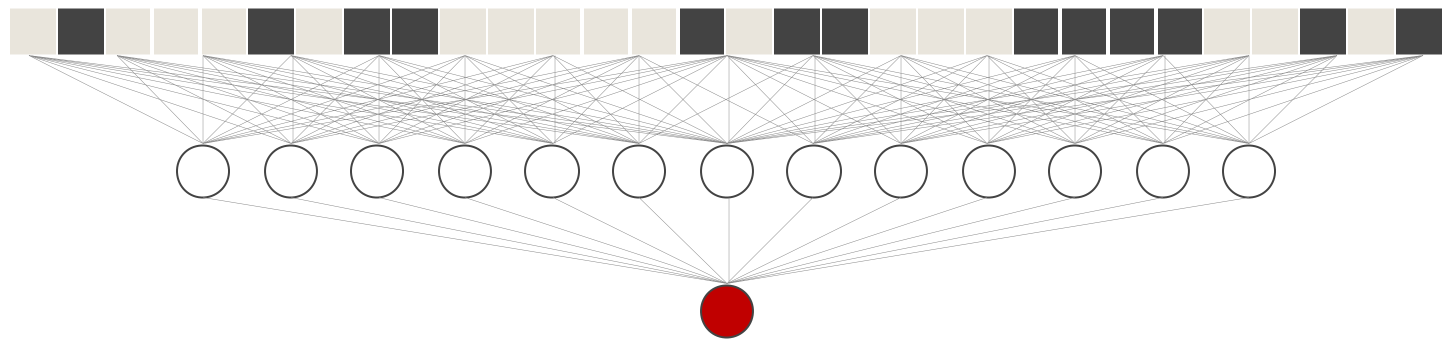
Each layer performs a linear operation potentially followed by a non-linear activation

- $\ell_0 = x$ and $\ell_i = \underset{\textit{activation}}{\underline{\sigma_i}}(\underset{\textit{weights}}{\underline{W_i}}\ell_{i-1} + \underset{\textit{biases}}{\underline{b_i}})$



sigmoid function

Example: shallow (2-layer) network



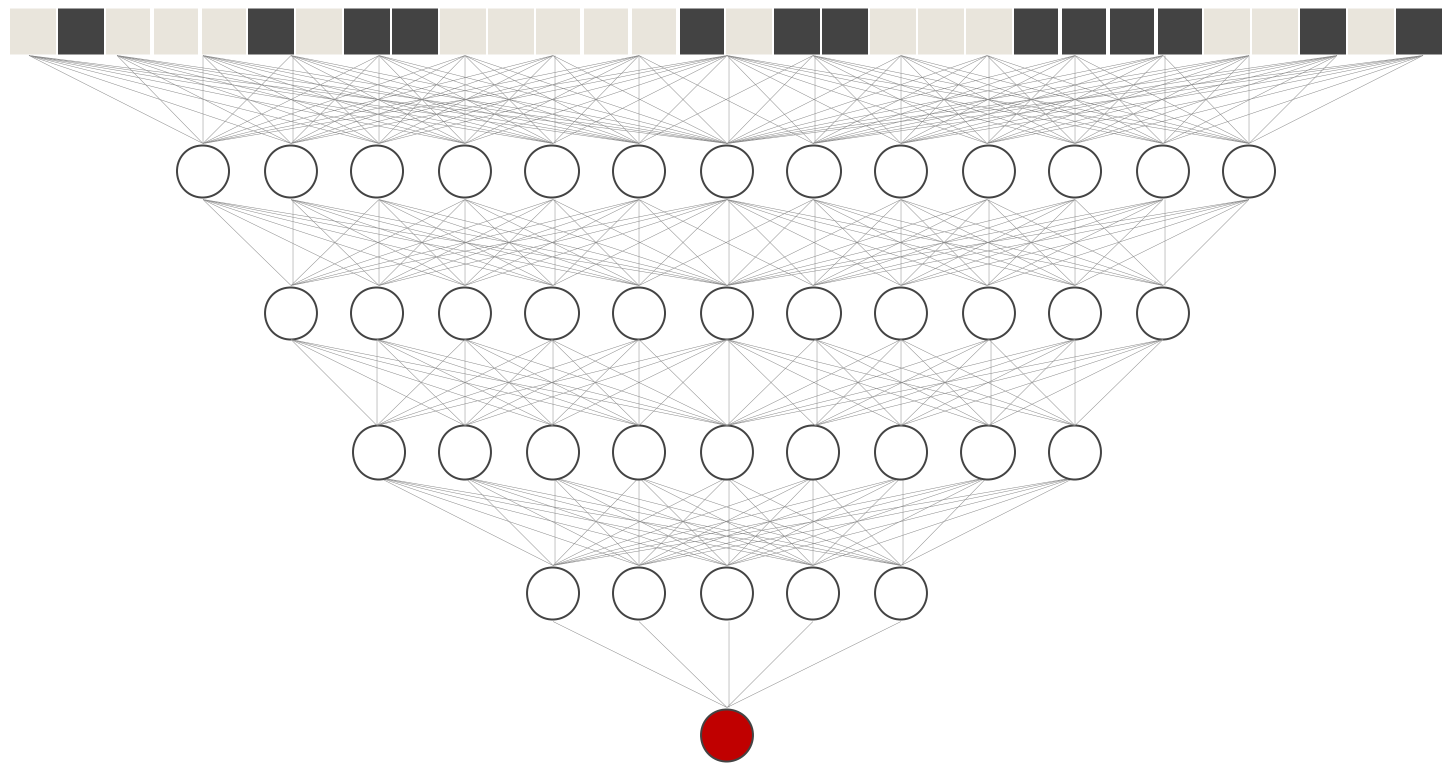
$f_w(x) = \ell_2$

$$\underset{[d \times 1]}{\ell_0} = x$$

$$\underset{[h_1 \times 1]}{\ell_1} = \sigma_1(\underset{[h_1 \times d]}{W_1}\underset{[d \times 1]}{\ell_0} + \underset{[h_1 \times 1]}{b_1})$$

$$\underset{[1 \times 1]}{\ell_2} = \sigma_2(\underset{[1 \times h_1]}{W_2}\underset{[h_1 \times 1]}{\ell_1} + \underset{[1 \times 1]}{b_2})$$

Example: deep (5-layer) network



$$\underset{[d\times 1]}{\ell_0} = x$$

$$\underset{[h_1\times 1]}{\ell_1} = \sigma_1(\underset{[h_1\times d]}{W_1}\underset{[d\times 1]}{\ell_0} + \underset{[h_1\times 1]}{b_1})$$

$$\underset{[h_2\times 1]}{\ell_2} = \sigma_2(\underset{[h_2\times h_1]}{W_2}\underset{[h_1\times 1]}{\ell_1} + \underset{[h_2\times 1]}{b_2})$$

$$\underset{[h_3\times 1]}{\ell_3} = \sigma_3(\underset{[h_3\times h_2]}{W_3}\underset{[h_2\times 1]}{\ell_2} + \underset{[h_3\times 1]}{b_3})$$

$$\underset{[h_4\times 1]}{\ell_4} = \sigma_4(\underset{[h_4\times h_3]}{W_4}\underset{[h_3\times 1]}{\ell_3} + \underset{[h_4\times 1]}{b_4})$$

$$\underset{[1\times 1]}{\ell_5} = \sigma_5(\underset{[1\times h_4]}{W_5}\underset{[h_4\times 1]}{\ell_4} + \underset{[1\times 1]}{b_5})$$

Neural network learning

Parameters (W_i's and b_i's) learned via gradient descent

- Find $\{W_i, b_i\}$ which minimize loss

Works with any differentiable loss

- Cross-entropy commonly used
- L2 regularization on $\{W_i, b_i\}$

Architecture selected using validation data

Classical algorithms

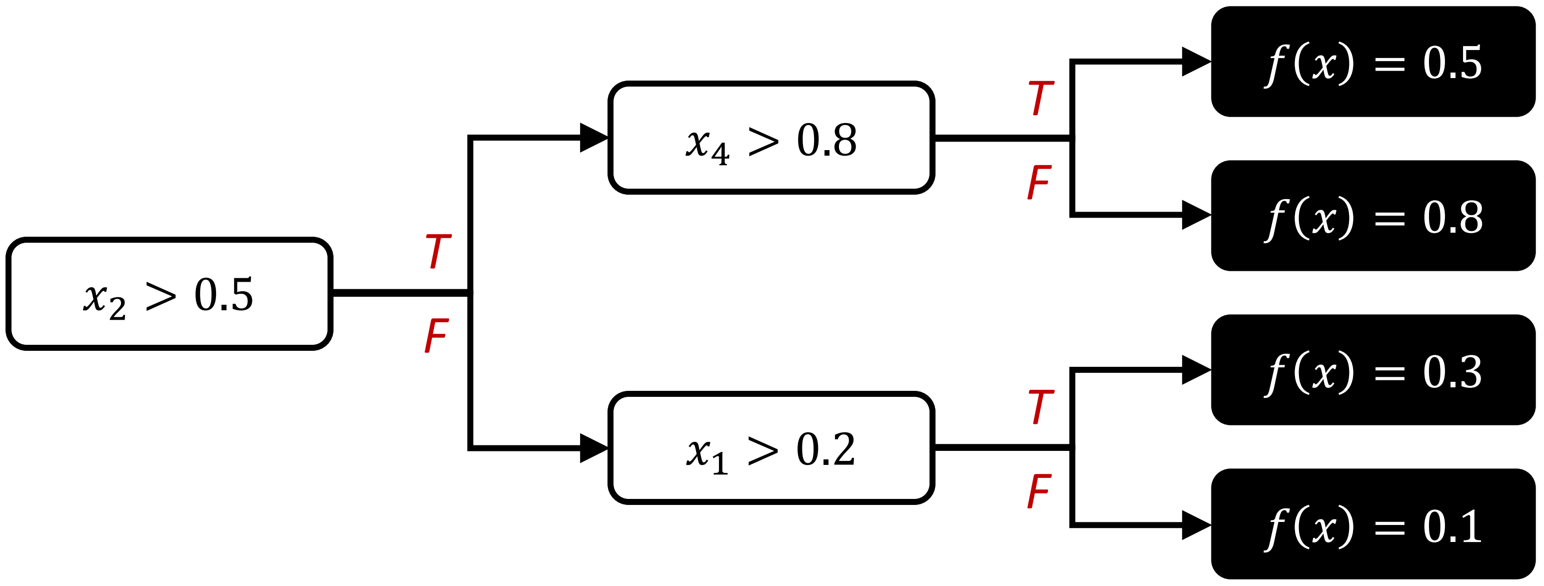
Linear learning algorithms

- Regression (LASSO, Ridge reg.)
- Classification (logistic reg., linear SVM, AdaBoost)

Non-linear learning algorithms

- Neural networks
- **(Boosted) regression trees**

Regression trees



$x = (0.3, 0.6, 0.2, 0.5)$

$x_2 > 0.5$? ***T***

$x_4 > 0.8$? ***F***

$\therefore f(x) = 0.8$

Examples travel the tree from the root to one leaf

Each node performs a test on the input x

Each leaf corresponds to a prediction $f(x)$

Greedy learning

1. Start with a tree containing only the root
2. Splitting: for each node with depth < max depth
   - Find best test and create two leaves from the node
   - Find best prediction for these two leaves
3. Rep. #2 until no more nodes with depth < max depth

Greedy learning

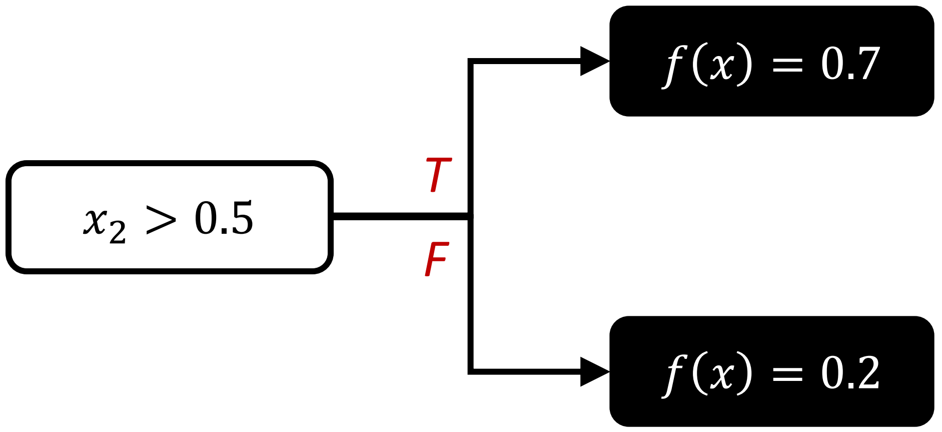
$\boxed{f(x) = 0.5}$

Best split: $x_2 > 0.5$

Best predictions: $f(x) = 0.7 \text{ if } x_2 > 0.5$

$f(x) = 0.2 \text{ otherwise}$

Greedy learning

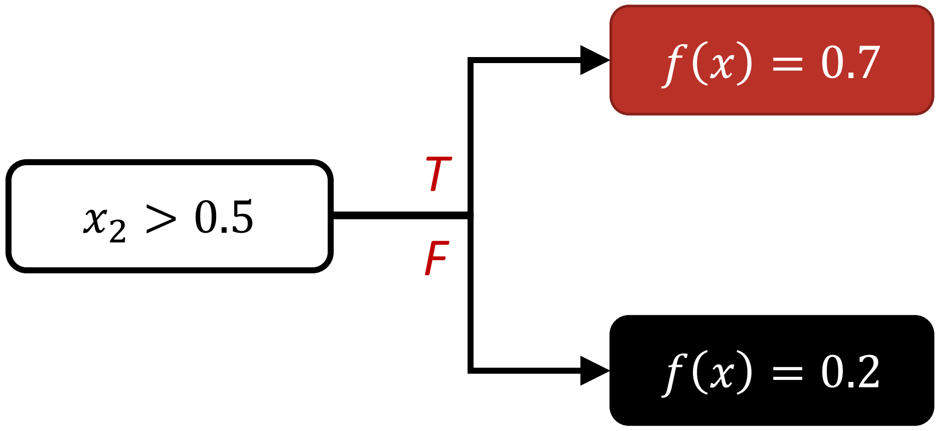


Best split: $x_2 > 0.5$

Best predictions: $f(x) = 0.7$ if $x_2 > 0.5$

$f(x) = 0.2$ otherwise

Greedy learning

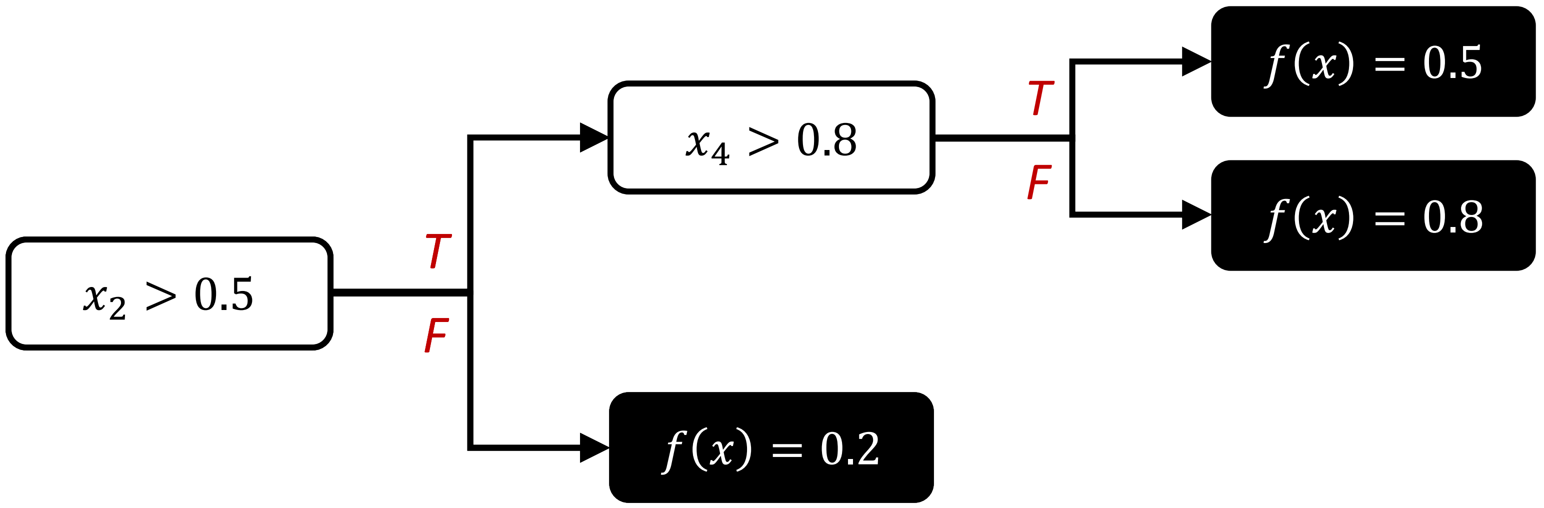


Best split: $x_4 > 0.8$

Best predictions: $f(x) = 0.5 \text{ if } x_4 > 0.8$

$f(x) = 0.8 \text{ otherwise}$

Greedy learning

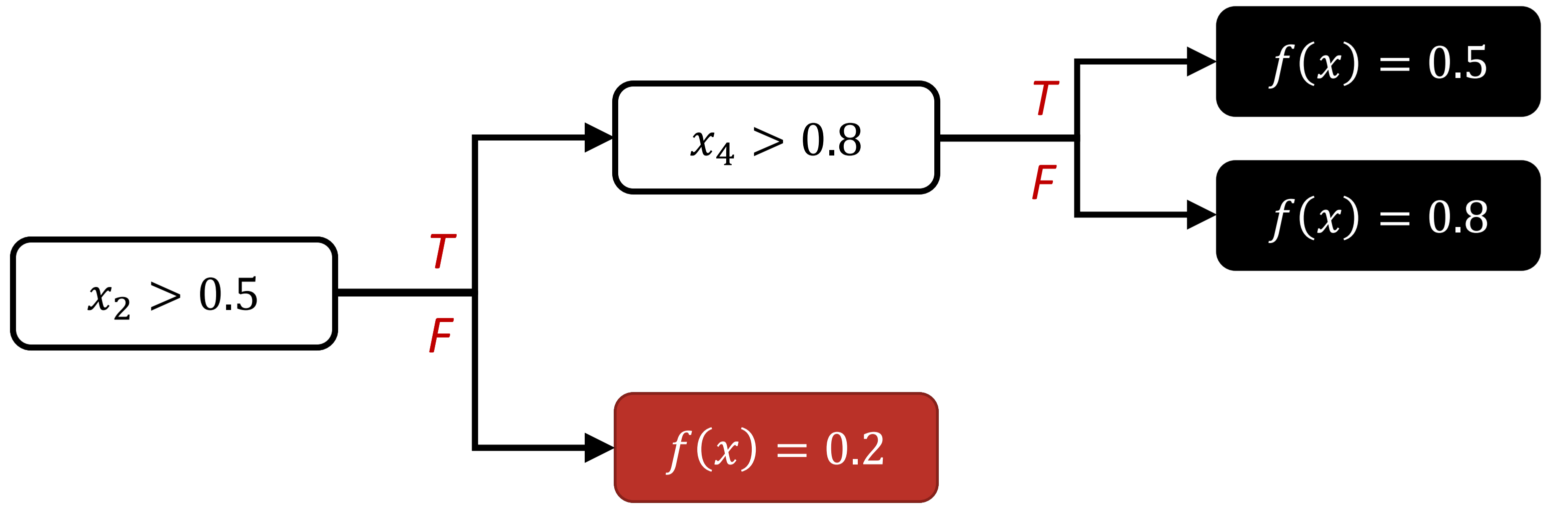


Best split: $x_4 > 0.8$

Best predictions: $f(x) = 0.5$ if $x_4 > 0.8$

$f(x) = 0.8$ otherwise

Greedy learning

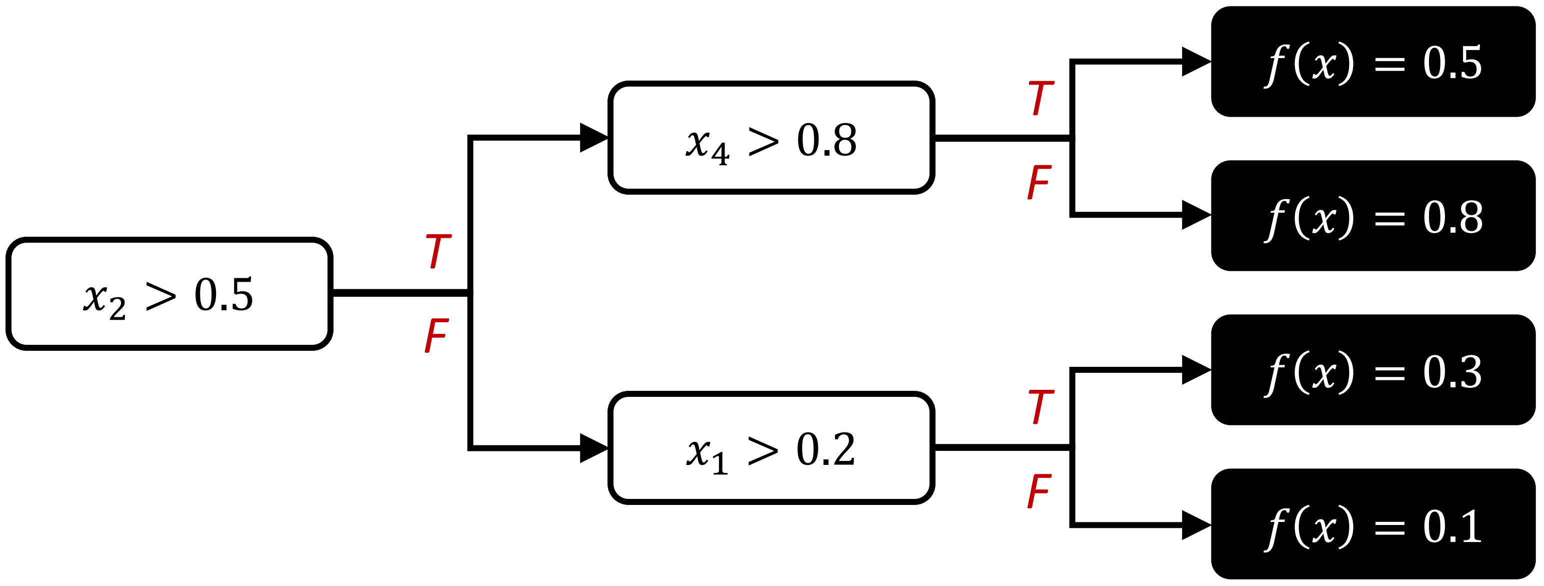


Best split: $x_1 > 0.2$

Best predictions: $f(x) = 0.3$ if $x_1 > 0.2$

$f(x) = 0.1$ otherwise

Greedy learning



Best split: $x_1 > 0.2$

Best predictions: $f(x) = 0.3$ if $x_1 > 0.2$

$f(x) = 0.1$ otherwise

Greedy learning a regression tree

1. Start with a tree containing only the root

2. Splitting: for each node with depth < max depth
   - Find best test and create two leaves from the node
   - Find best prediction for these two leaves

3. Rep. #2 until no more nodes with depth < max depth

Splitting a node

Given all S_k examples reaching node k, find:

- i, τ: defining the test $x_i > \tau$

Let $S_k^l \equiv S_k : x_i \leq \tau$ and $S_k^r \equiv S_k : x_i > \tau$

Minimize: $\mathcal{L} = \sum_{(x,y)\in S_k^l} \left(\bar{y}^l - y\right)^2 + \sum_{(x,y)\in S_{k^r}} (\bar{y}^r - y)^2$

- $\underbrace{\bar{y}^l \equiv \frac{1}{|S_k^l|} \sum_{(x,y)\in S_k^l} y}_{f(x) \text{ in the left branch}}$ and $\underbrace{\bar{y}^r \equiv \frac{1}{|S_k^r|} \sum_{(x,y)\in S_k^r} y}_{f(x) \text{ in the right branch}}$

Regression trees

Advantages

- Low computation cost for prediction
- Easy to interpret

Disadvantages

- Only models piecewise constant functions
- Learning deep trees requires lots of examples

Gradient boosted regression trees (GBRTs)

A GBRT is an ensemble of regression trees

- $f(x) = \sum_{t=1}^{T} h_t(x)$, where h_t is a regression tree

Advantages

- Models more complex functions than a single tree
- Learning many shallow trees requires less training data than learning one deep tree

Gradient boosting

For $t = 1, \dots, T$

- Current model: $f_t = \sum_{i=1}^{t-1} h_i$
- Learn h_t^* to "correct" f_t
  - $f_{t+1} = f_t + h_t^* \approx y$

    $\therefore h_t^* \approx \underline{y - f_t \qquad \textit{residual}}$
  - $h_t^* = \operatorname{argmin}_{h_t} \sum_{(x,y)} (h_t(x) - (y - f_t(x)))^2$

    $= \operatorname{argmin}_{h_t} \sum_{(x,y)} (h_t(x) - (-\alpha \nabla \mathcal{L}(f_t)))^2$

Summary

Pointwise approaches borrowed from other tasks

- Regression, classification, ordinal classification

Straightforward, yet not quite suited for ranking

- Ranking requires getting relative scores right
- Higher positions matter more than lower positions
- All queries are equally important

References

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Coming next...

Learning to Rank: Pairwise and Listwise

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