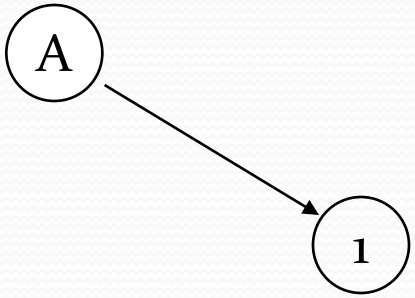
Codificação em Redes (Network Coding)

Prof. Luiz Filipe M. Vieira
DCC/UFMG

lfvieira@dcc.ufmg.br

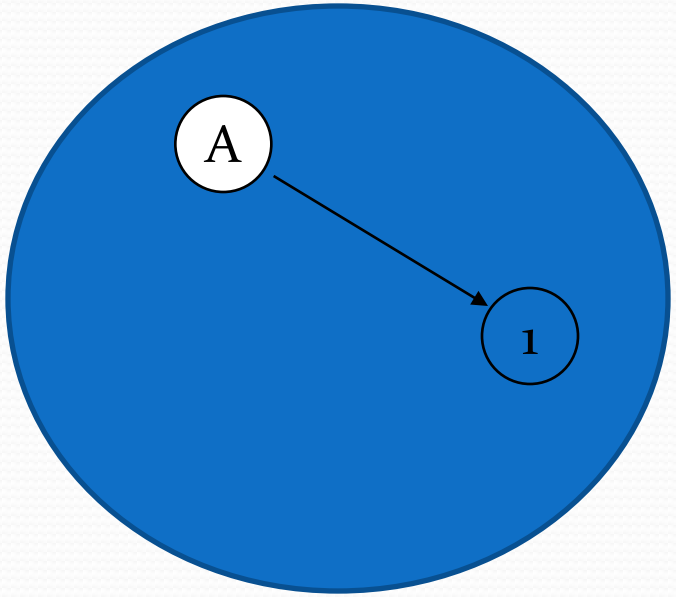
Modeling Networks

- Use Graphs (V,E)
- Vertex
- Edge
- Edges connect vertices that communicate



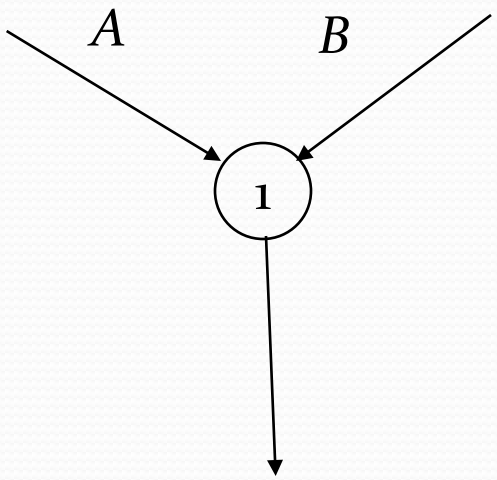
Modeling Wireless Networks

- Use Graphs (V,E)
- Vertex
- Edge
- Edges connect vertices that communicate



What is Network Coding?

- Nodes inside the network can perform coding
- Coding: any type of operation involving the messages
- It has many potentials gains



Operation on A and B

Operation

- Truth Table

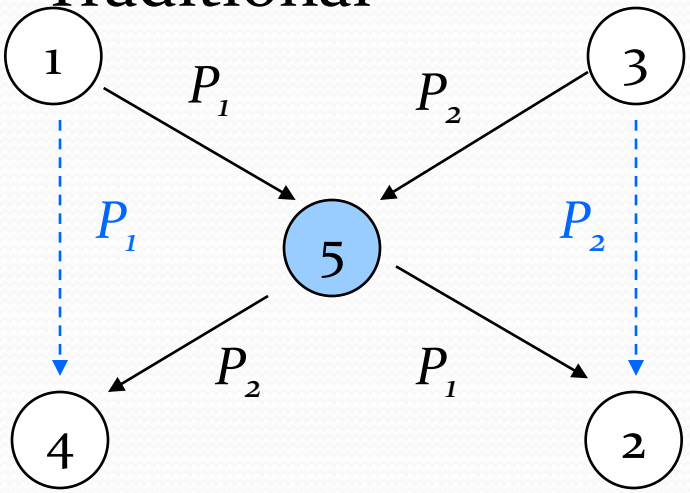
A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

What is A XOR A?

What is A XOR A XOR B?

Wireless Network

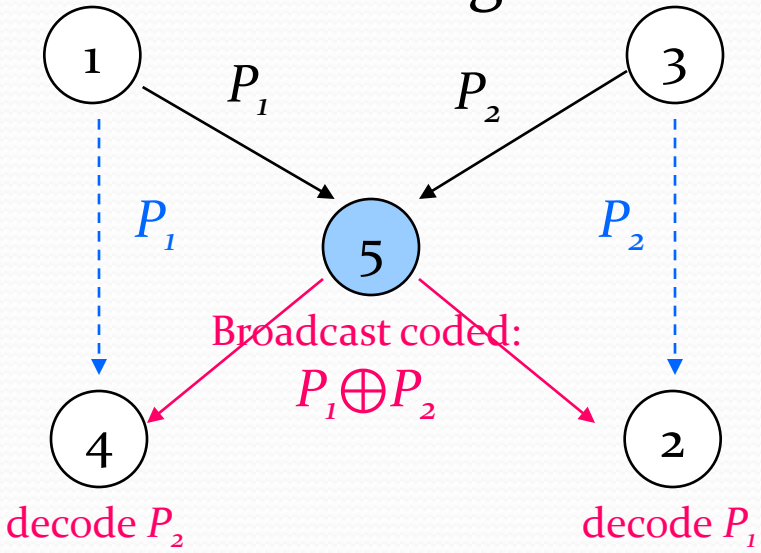
- Traditional



Two flows:1→5→2

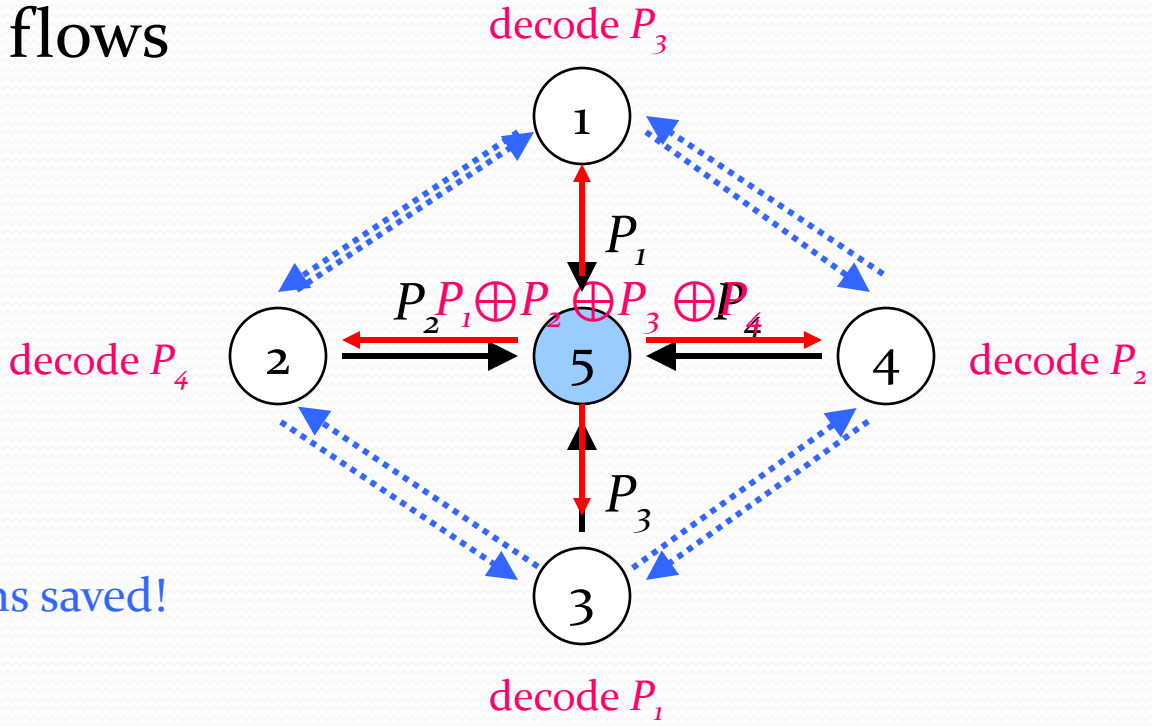
3→5→4

- Network Coding



Network Coding

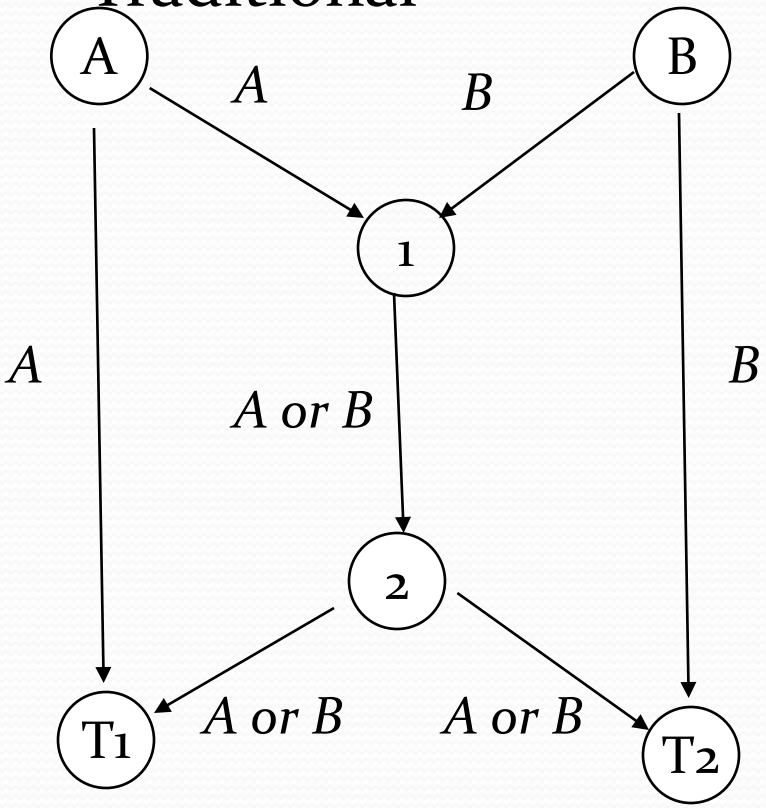
- More complicated scenario…
- 4 bi-directional flows



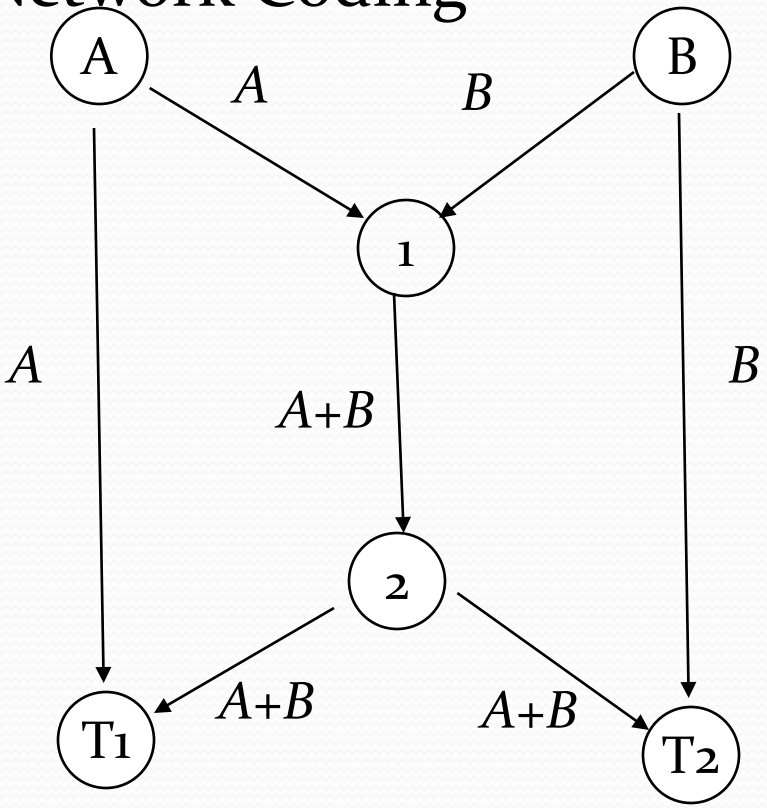
Three transmissions saved!

Butterfly Network

- Traditional

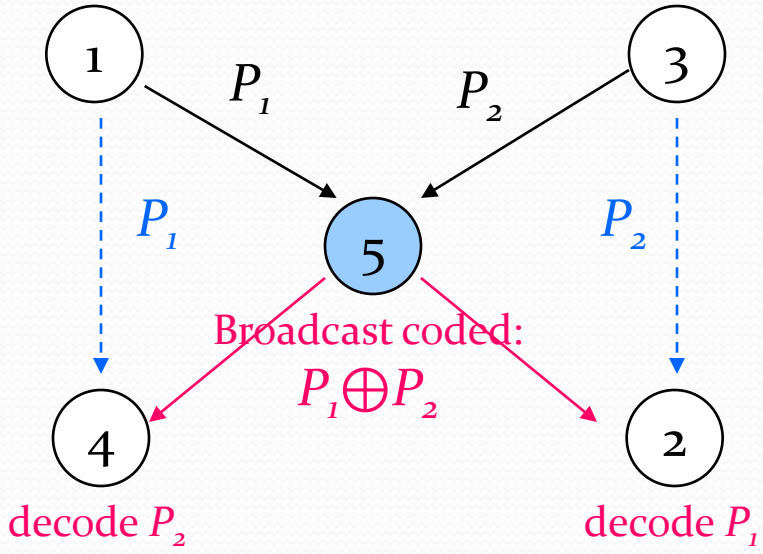


- Network Coding



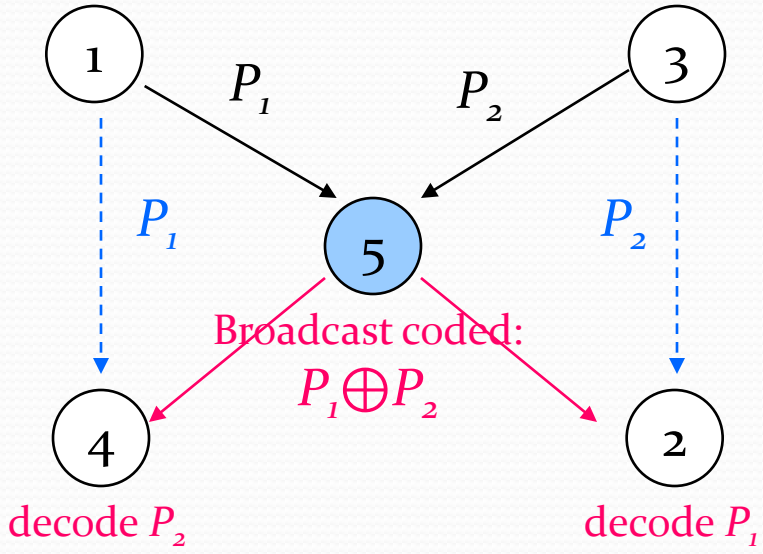
Advantages of Network Coding

- Increases throughput:
  - More information can be sent over the network in a given period of time.



Advantages of Network Coding

- Decreases energy consumption:
  - ➡Less transmissions

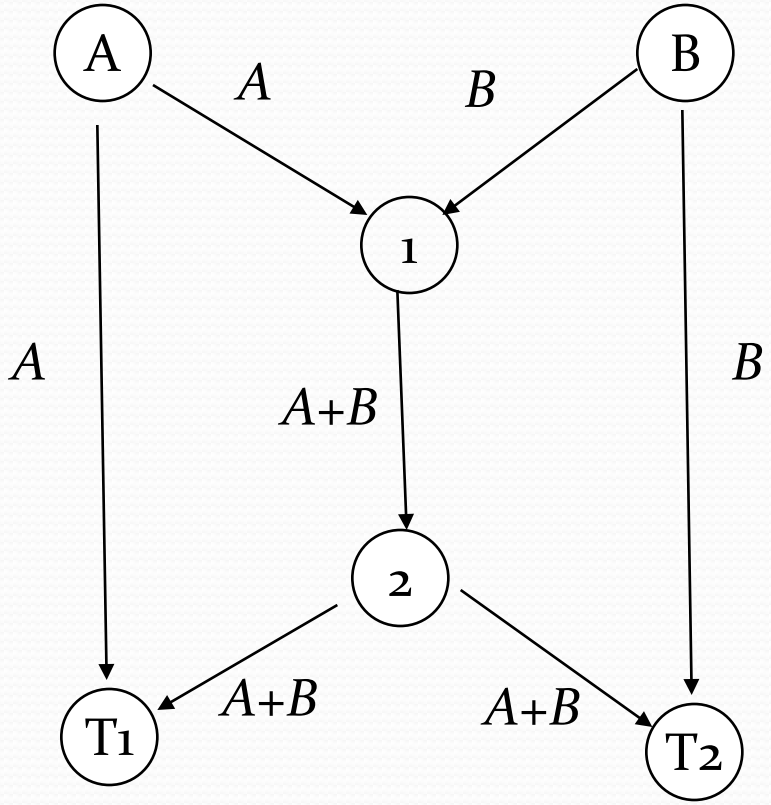


Advantages of Network Coding

- Robustness:
  - Messages may be lost
  - By appropriately combining packets, information may be recovered

Applicability - Wired Protocols

- Same idea as the butterfly network

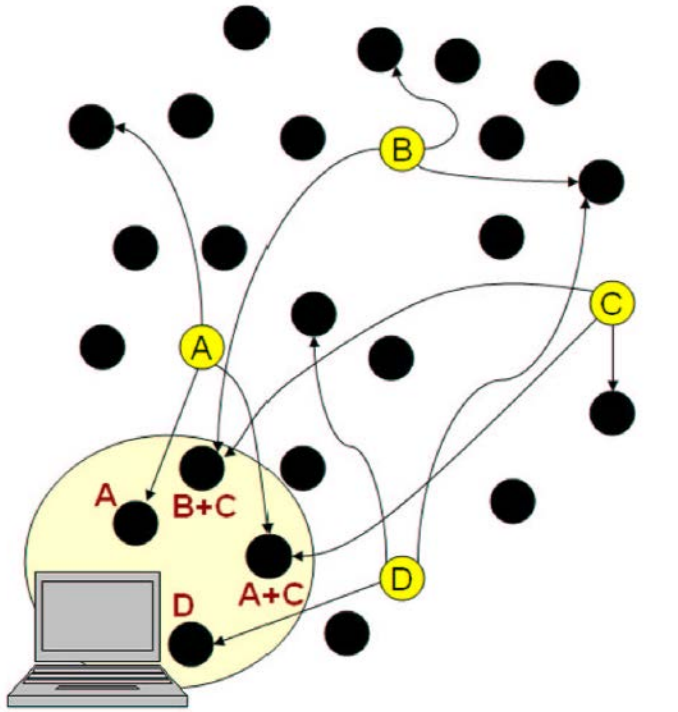


Applicability - Wireless Protocols

- Packet loss occurs
- Shared medium
- How to take advantage?
- Focus on wireless protocols

Applicability - Distributed Storage

- K data nodes
- N storage nodes
- Data is diffuse to storage nodes
- Randomly select storage nodes
- Data collector can recover it by accessing k storage nodes

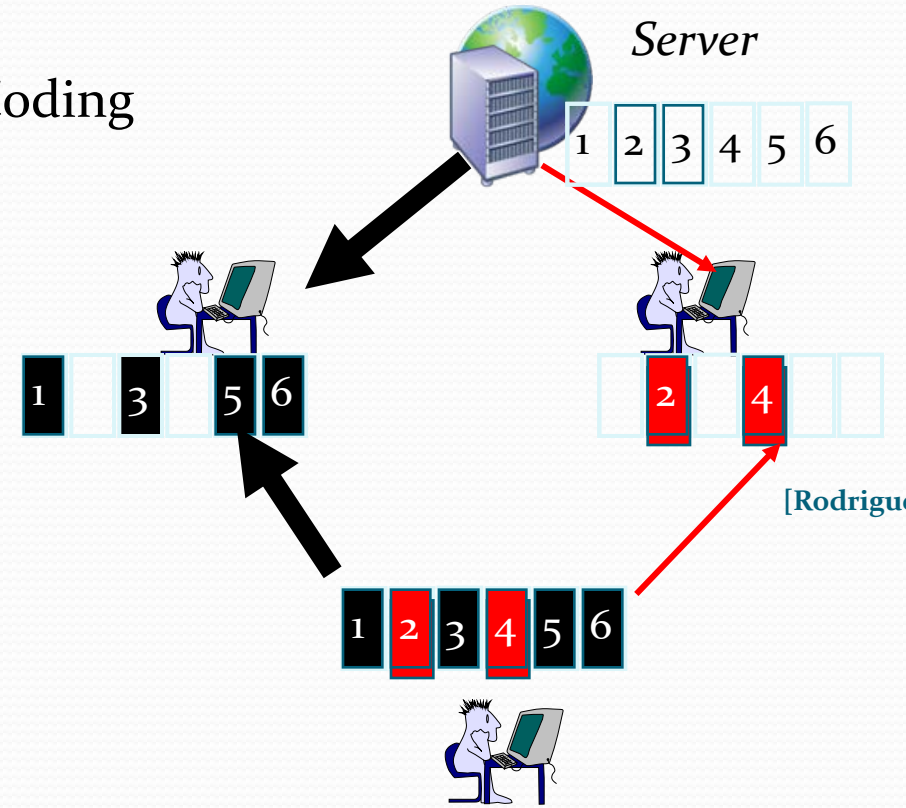


- K= 4 measuring information
- N=23 storage nodes

A. G. Dimakis et al.,Decentralized Erasure codes for distributed networked storage, TON 2007

Network Coding in P2P Swarming

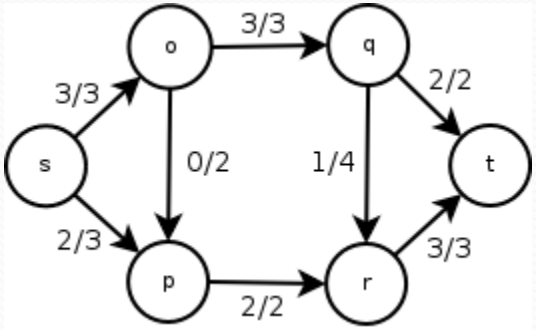
- P2P Swarming
  - File is divided into many small pieces for distribution
  - Clients request different pieces from the server/other peers
  - When all pieces are downloaded, clients can re-construct the whole file
  - Rare piece problem
- P2P using Network Coding
  - Avalanche



[Rodriguez, Biersack, Infocom'oo]

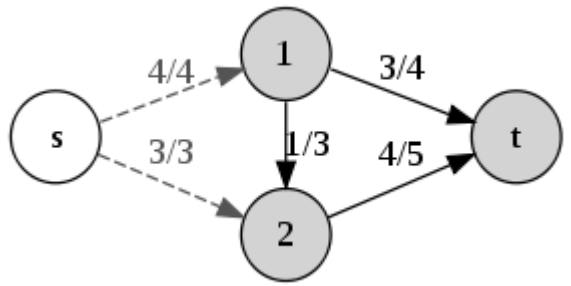
Max-Flow

- Given:
  - a network G(N,E), and
  - edges capacities
- Find the maximum flow from a sink to a terminal



Max-Flow Min-Cut

- The maximum amount of flow passing from the source to the sink is equal to the minimum capacity that needs to be removed from the network so that no flow can pass from the source to the sink.



Max Flow – LP Formulation

Max-Flow LP:

maximize f_{RS}

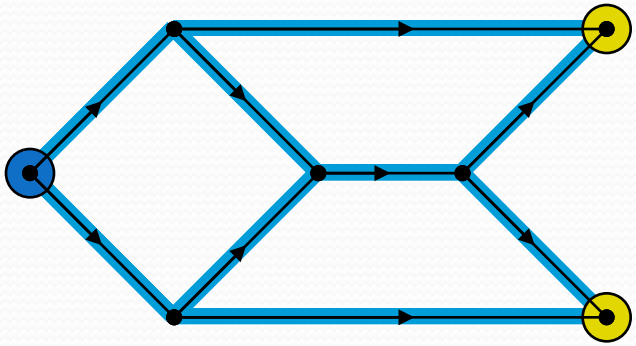
subject to

$$\sum_{(v,u)\in E} f_{vu} = \sum_{(u,w)\in E} f_{uw}, \quad \forall\, u \in V \quad \text{(flow conservation)}$$

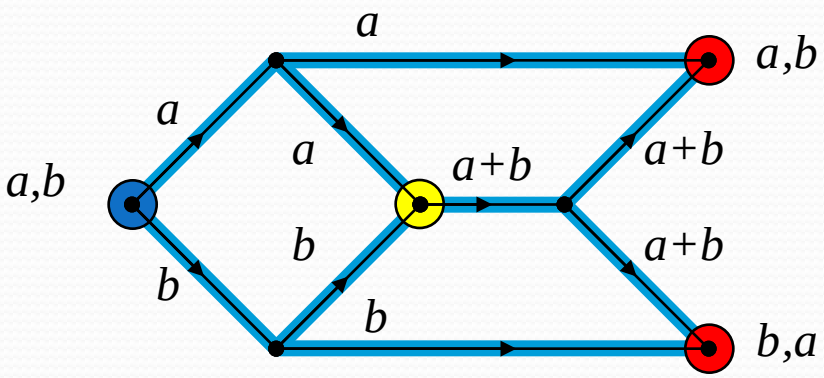
$$f_{vu} \le c_{vu}, \quad \forall\, (v,u) \in E \quad \text{(capacity constraints)}$$

$$f_{vu} \ge 0, \quad \forall\, (v,u) \in E$$

NC achieves multicast capacity



optimal routing
throughput = 1



network coding
throughput = 2

- Alswede, Cai, Li, Yeung (2000):
  - $\min_{t \in T} \text{MinCut}(s,t)$ is always achievable by network coding
  - $h = \min_{t \in T} \text{MinCut}(s,t)$ is "multicast capacity"

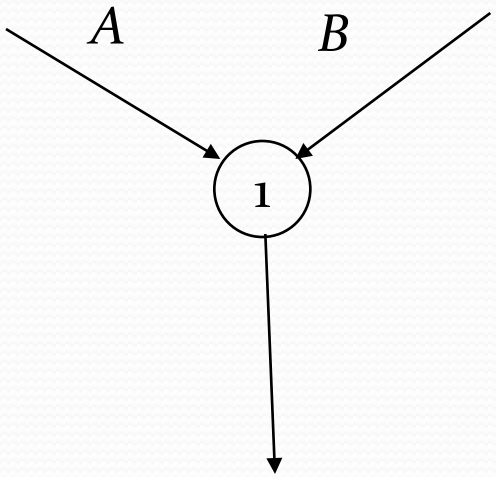
sender

receiver

coding node

Linear Coding

- Each node generates a new packet, which is a linear combination of the earlier received packets on the link, by coefficients in a finite field.



Linear Operation on A and B

Linear Coding

- A message generated so Xk is related to the received messages Mi by the relation:

$$X_k = \sum_{i=1}^{s} g_k^i \cdot M_i$$

- Each node forwards the computed value Xk along with all the coefficients used in the kth level

- The values are the coefficients from the Galois field GF(2s)

- Makes encoding and decoding easy to implement in practice

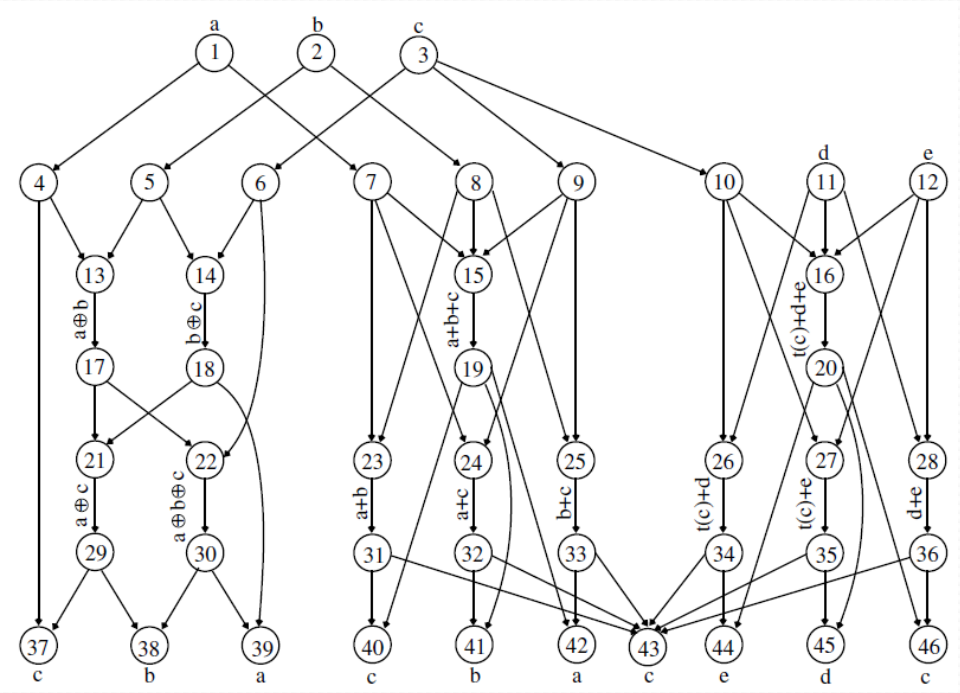
Linear Coding – Theoretical Results

- For one source Li et. al gave an explicit construction of a code for multicast in a network that achieves the max-flow
- Linear Coding can achieve max-flow

Linear Network Coding, Li et al., IEEE Transactions on Information Theory, 2003

Insufficiency of Linear Coding

- In 2005, it was shown that the linear coding is not sufficient in general (multisource, multisink with arbitrary demands)



Insufficiency of Linear Coding, Dougherty et al., IEEE Transactions on Information Theory, 2003.

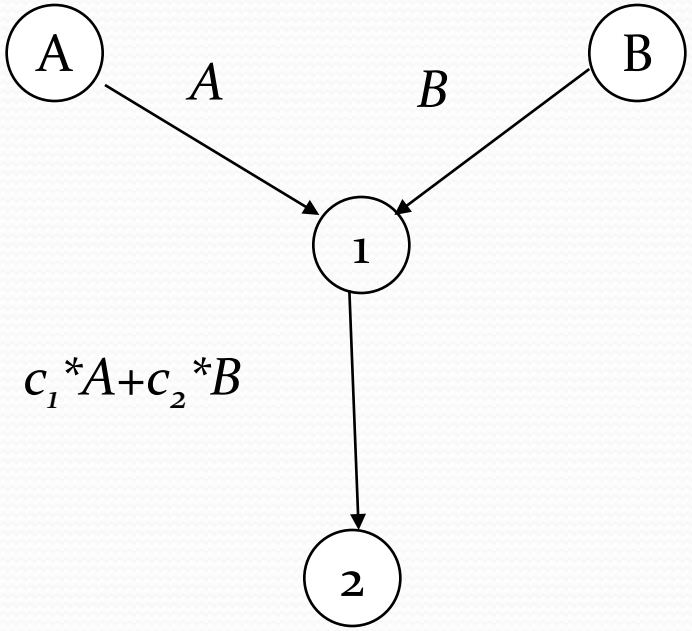
Random Linear Coding

- Randomized coding approach
- For robust, distributed transmission and compression of information in network
- Interior nodes perform encoding
- Destiny performs decoding

The Benefits of Coding over Routing in a Randomized Setting, Ho et al., IEEE International Symposium on Information Theory Theory, 2003

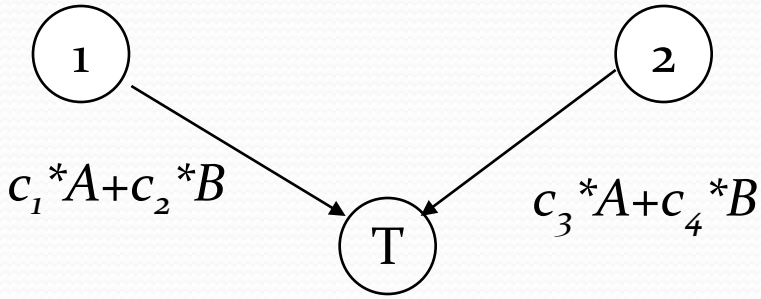
Random Linear Coding - Encoding

- Interior nodes independently choose random linear coefficients
- Output is a linear combination of inputs multiplied by random coefficients

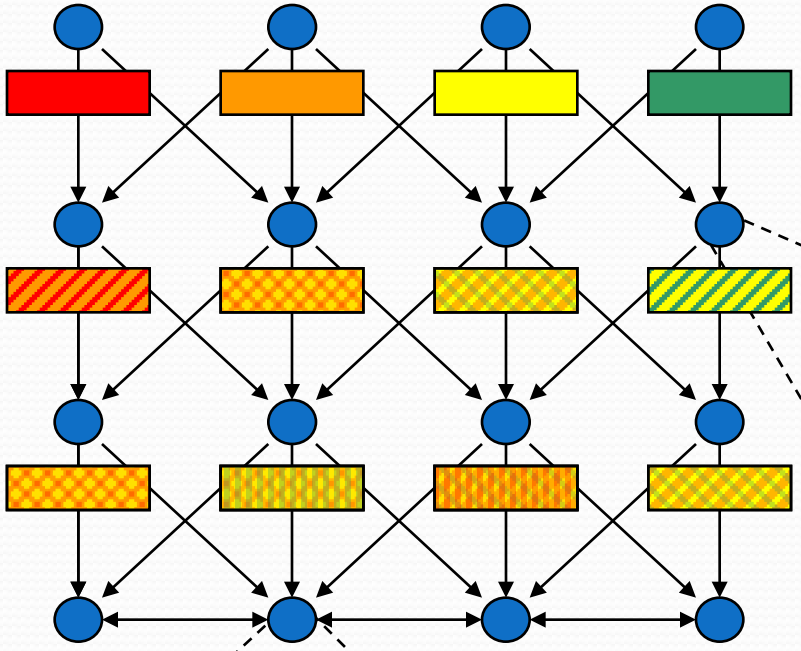


Random Linear Coding - Decoding

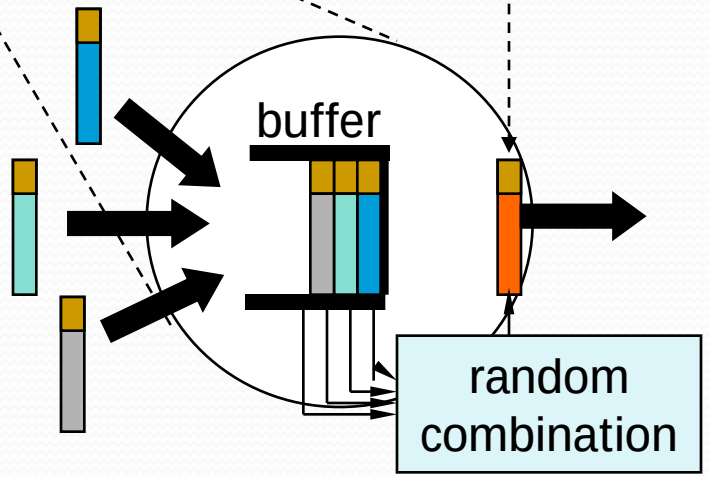
- Receiver nodes can decode if they receive as many independent linear combinations as the number of source packets
- Coefficient are stored into a matrix T
- Compute the inverse of T

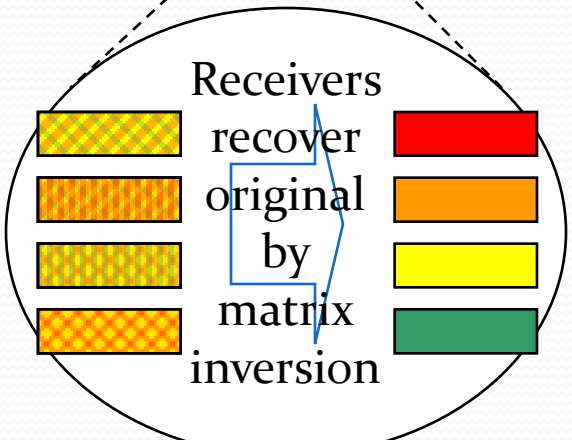


Random network coding



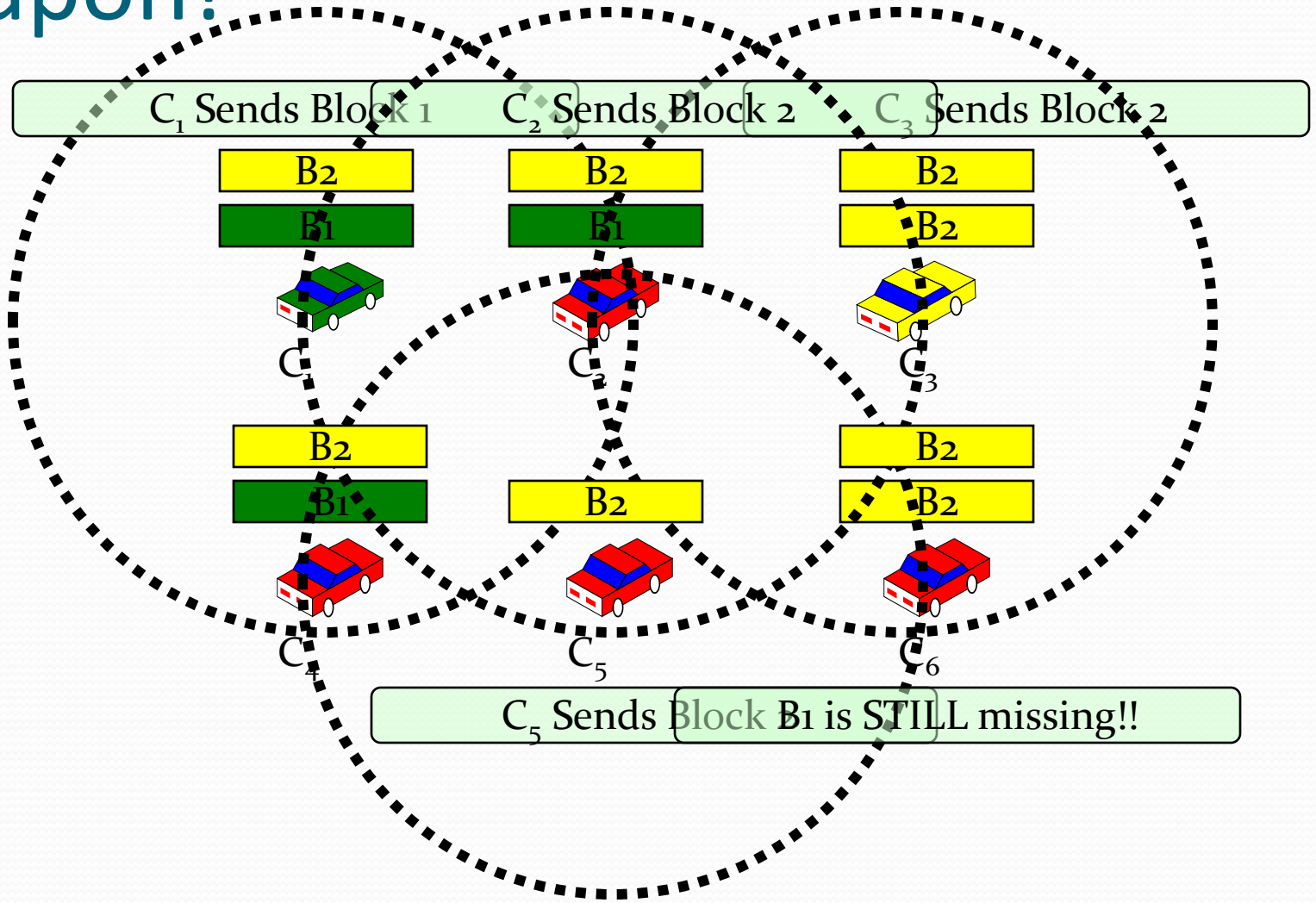
Every packet **p** carries **e** = [e_1 e_2 e_3] encoding vector prefix indicating how it is constructed (e.g., coded packet $\mathbf{p} = \sum e_i \boldsymbol{x}_i$ where $\boldsymbol{x}_i$ is original packet)





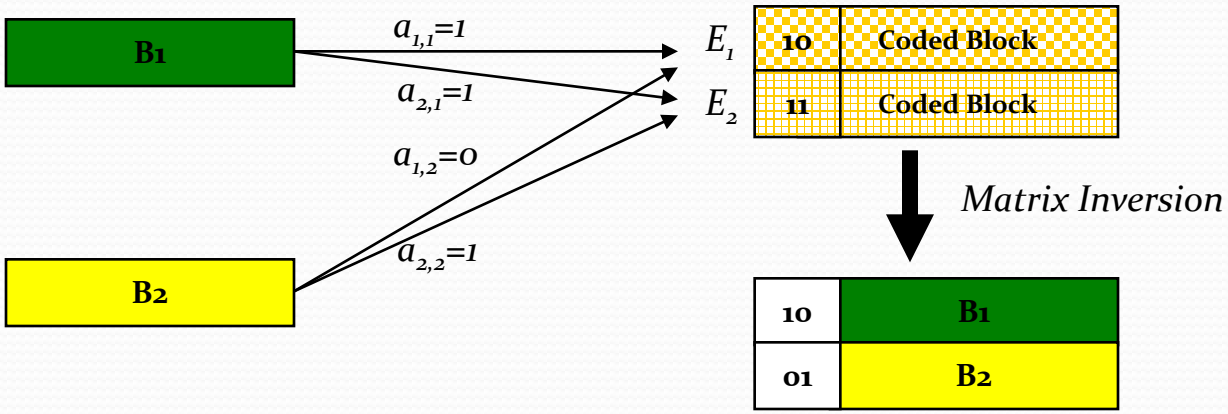
Intermediate nodes randomly mix incoming packets to generate outgoing packets

Swarming limitation: Missing coupon!



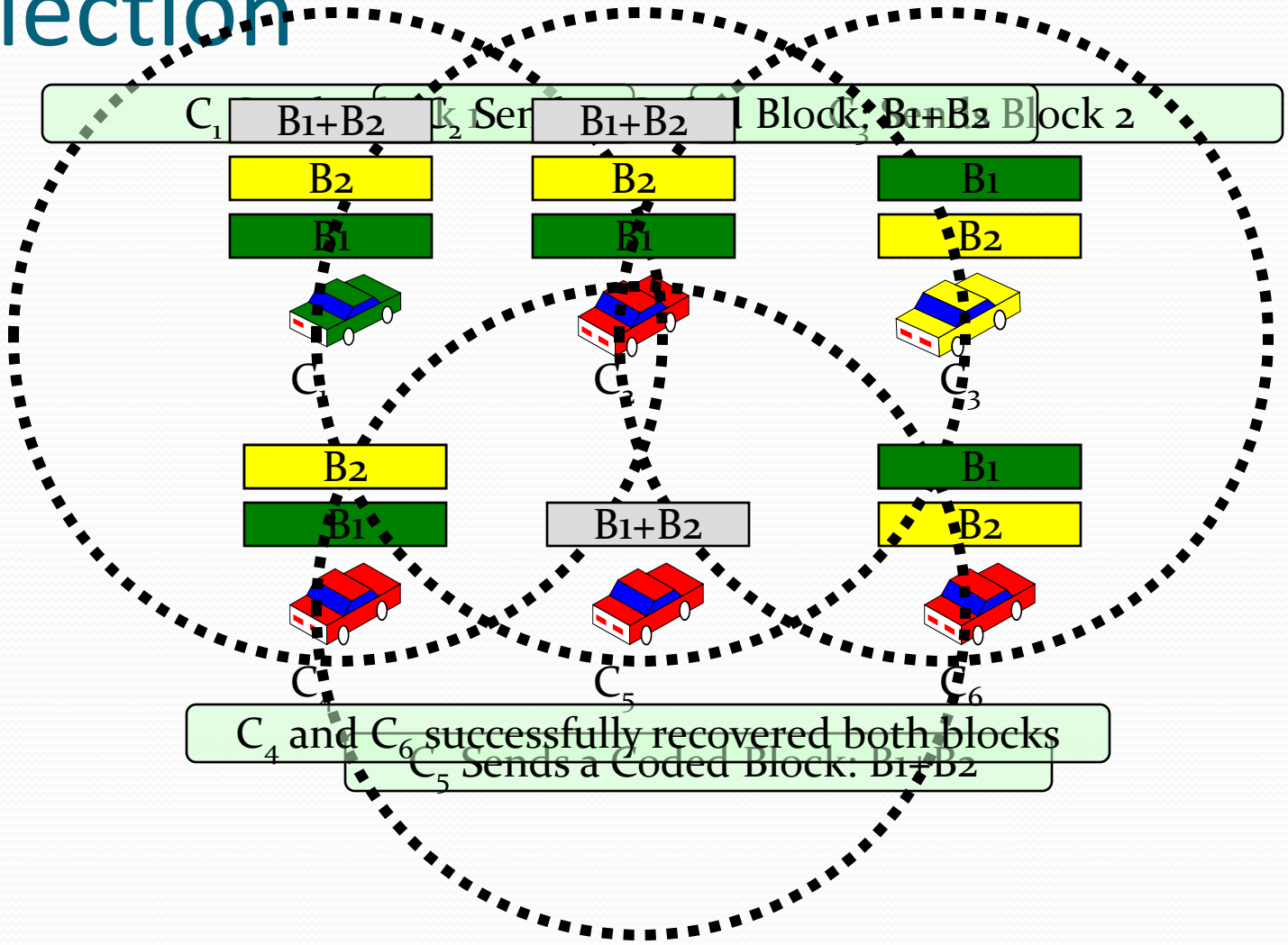
Network Coding

- Let a file has *k* blocks: $[B_1\ B_2\ \ldots\ B_k]$
- Encoded block E_i is generated by
  - $E_i = a_{i,1}*B_1 + a_{i,2}*B_2 + \ldots + a_{i,k}*B_k$
  - $a_{i,x}$: randomly chosen over the finite field
- Any "k" linearly independent coded blocks can recover $[B_1\ B_2\ \ldots\ B_k]$ by matrix inversion
- Network coding maximizes throughput and minimizes delay



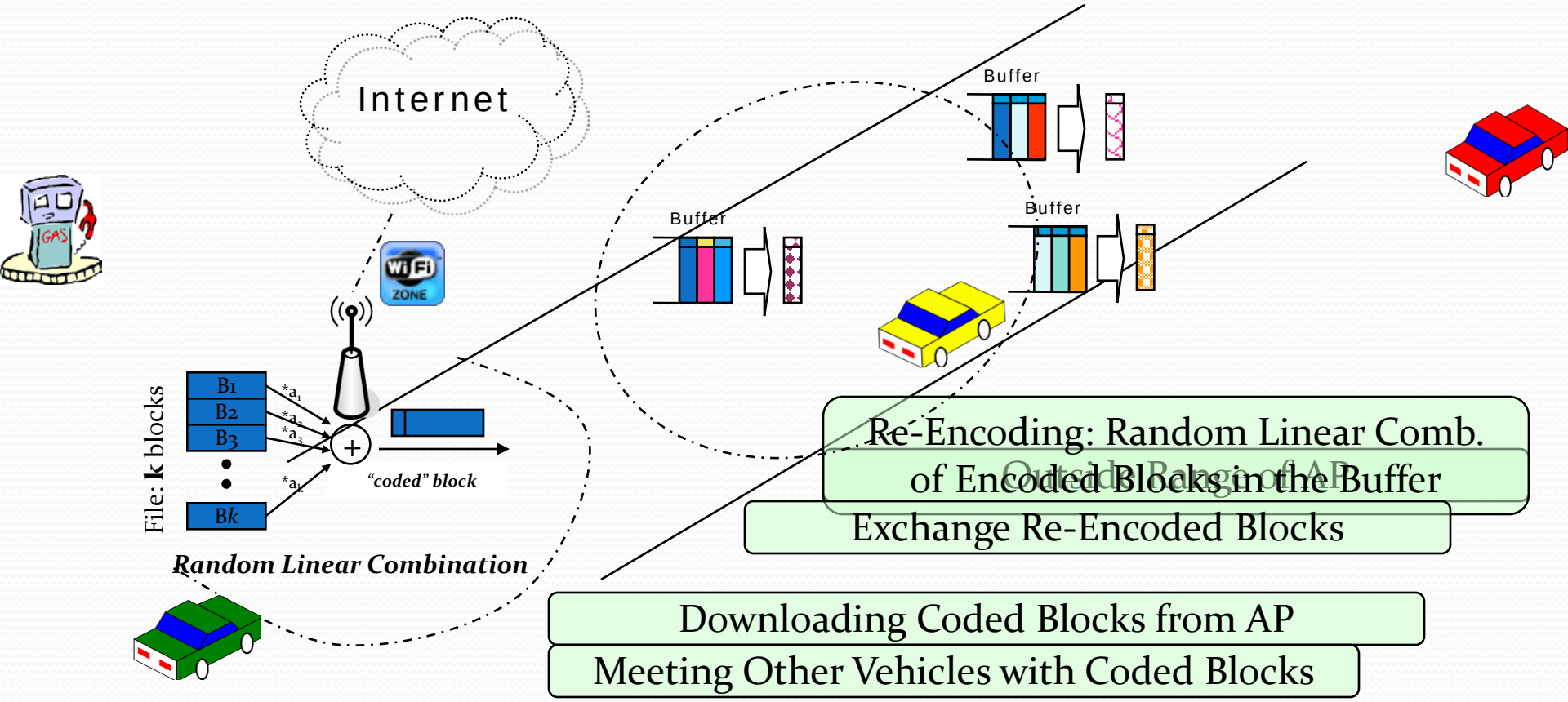
Network coding over the finite field GF(2)={0,1}

Network coding helps coupon collection



Protocols

- **Single-hop pulling**



Conclusion

- NC achieves multicast capacity
- Reduces complexity
- It is Practical
- Many Applications

- Unicast, Multicast
- MANETS, VANET

32