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Recommender Systems

Similarity-based Recommendation

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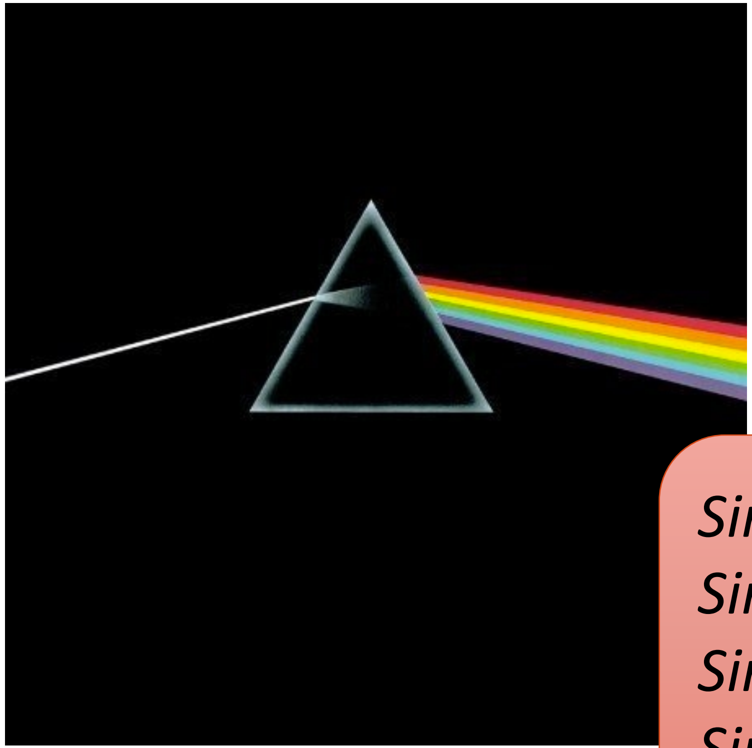
Content-based recommendation

You bought



You may like

Similar artist: Pink Floyd
Similar origin: England
Similar genre: Rock
Similar period: 1970s

Content-based recommendation

Collaborative filtering

- Leverages item ratings
- Agnostic to item content

Content-based filtering

- Leverages item content
- Agnostic to item ratings

*Applicable **to any kind of item** (e.g., text, audio, video, food)*

*Applicable even in **extreme cold-start** scenarios*

Vector representation

Each item is a vector

- One component for each term
- High dimensionality

Each user is a vector

- Some combination of item vectors

How to weight term occurrences?

Occurrence-based weighting

Each item is a vector in $\{0, 1\}^{|V|}$

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTHONY	1	1	0	0	0	1
BRUTUS	1	1	0	1	0	0
CAESAR	1	1	0	1	1	1
CALPURNIA	0	1	0	0	0	0
CLEOPATRA	1	0	0	0	0	0
MERCY	1	0	1	1	1	1
WORSER	1	0	1	1	1	0
. . .						

How representative is a term?

Count-based weighting

Each item is a vector in $\mathbb{N}^{|V|}$

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTHONY	157	73	0	0	0	1
BRUTUS	4	157	0	2	0	0
CAESAR	232	227	0	2	1	0
CALPURNIA	0	10	0	0	0	0
CLEOPATRA	57	0	0	0	0	0
MERCY	2	0	3	8	5	8
WORSER	2	0	1	1	1	5
. . .						

How discriminative is a term?

TF-IDF

Given a term t and an item i

- tf_{ti}: *term frequency* of term t in item i
- idf_t: *inverse document frequency* of term t

  $$\mathrm{idf}_t = \log \frac{n}{n_t}$$

  - n: number of items in the collection
  - n_t: number of items where t appears

*TF*IDF*: weight given to each term

TF-IDF-based weighting

Each item is a vector in $\mathbb{R}^{|V|}$

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
ANTHONY	5.25	3.18	0.0	0.0	0.0	0.35
BRUTUS	1.21	6.10	0.0	1.0	0.0	0.0
CAESAR	8.59	2.54	0.0	1.51	0.25	0.0
CALPURNIA	0.0	1.54	0.0	0.0	0.0	0.0
CLEOPATRA	2.85	0.0	0.0	0.0	0.0	0.0
MERCY	1.51	0.0	1.90	0.12	5.25	0.88
WORSER	1.37	0.0	0.11	4.15	0.25	1.95
. . .						

Vector representation

Each item is a vector

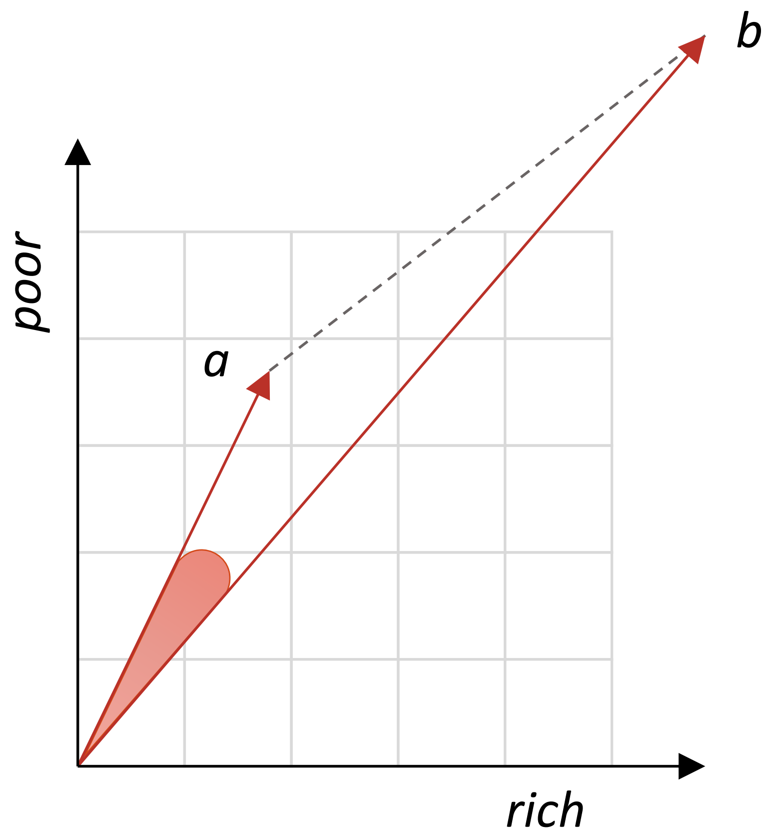
- One component for each term
- High dimensionality

Each user is a vector

- Some combination of item vectors

How to compute similarity?

Computing similarities



a: "social inequality raises"
b: "rich-poor gap grows"

Euclidean distance

- Distance between vectors' endpoints

Cosine

- Angular distance between vectors

Which similarity?

Angle vs. distance

Thought experiment

- Take an item vector i_1
- Append i_1 to itself, forming i_2 (i.e., $i_2 = 2i_1$)

"Semantically", i_1 *and* i_2 are equivalent

- The cosine between the two vectors is maximal
- … the Euclidean distance can be quite large

Cosine similarity

$$\mathrm{sim}(u,i) = \cos(\vec{u},\vec{i}) = \frac{\vec{u}\cdot\vec{i}}{\|\vec{u}\|\|\vec{i}\|} = \frac{\sum_{t=1}^{|V|} u_t i_t}{\sqrt{\sum_{t=1}^{|V|} u_t^2}\sqrt{\sum_{t=1}^{|V|} i_t^2}}$$

$\vec{u}$ and $\vec{i}$ are term-weight vectors

- u_t is the TF-IDF weight of term t in user u
- i_t is the TF-IDF weight of term t in item i
- $\|\vec{u}\|$ and $\|\vec{i}\|$ are the norms of $\vec{u}$ and $\vec{i}$

Quick recap

We know how to represent items

- Each item is a vector over terms

We know how to compute vector similarities

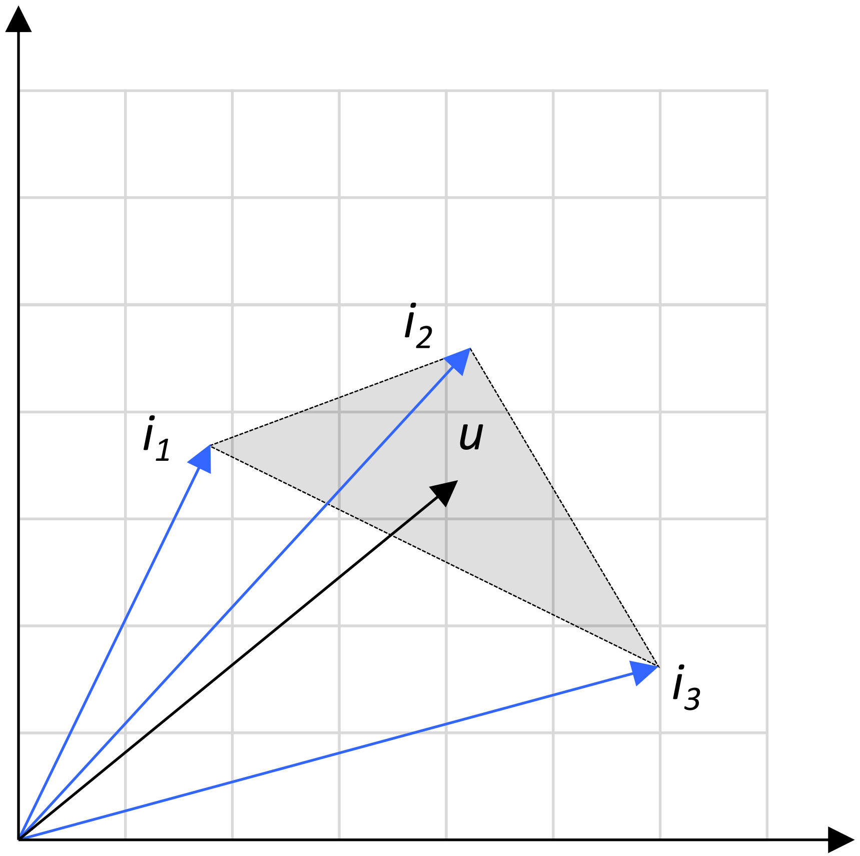
- Cosine of the angle between the vectors

We can now produce recommendations

- Rank items by their similarity to the user

How to represent the user?

Representing the user



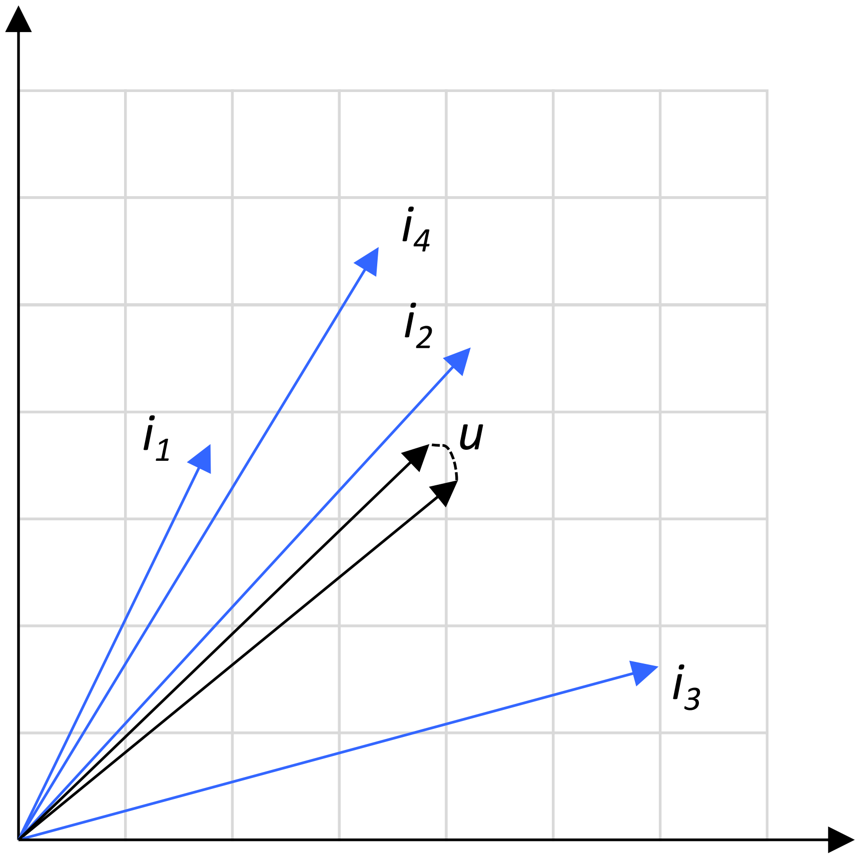
User has rated

- i_1: ★★★
- i_2: ★★★★★
- i_3: ★★★

User prototype

$$\vec{u} = 3\vec{i}_1 + 5\vec{i}_2 + 3\vec{i}_3$$

Incremental updates



User has rated

- i_1: ★★★
- i_2: ★★★★★
- i_3: ★★★
- i_4: ★★★★★

User prototype

$$\vec{u} = 3\vec{i}_1 + 5\vec{i}_2 + 3\vec{i}_3 + 5\vec{i}_4$$

Rocchio recommendation

Each item is a vector $\vec{\imath}$

- One component for each term

Each user is a vector $\vec{u}$

- $\vec{u} = \frac{1}{|I_u|} \sum_{j \in I_u} r_{uj} \, \vec{\jmath}$
  - I_u: items rated by user u
  - r_{uj}: rating of user u to item j

Prediction score

- $\text{sim}(\vec{u}, \vec{\imath}) = \cos(\vec{u}, \vec{\imath})$

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

(input) utility matrix

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

(input) item feature matrix

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

Computing the user feature matrix

$$u_{11} = \frac{1\times 2 + 4\times 1.5 + 5\times 1 + 2\times 0 + 5\times 1}{5} = 3.6$$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	u_{11}				
t_2					
t_3					
t_4					

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

Computing the user feature matrix

$$u_{12} = \frac{1\times1+4\times10+5\times0+2\times20+5\times5}{5} = 21.2$$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6				
t_2	u_{12}				
t_3					
t_4					

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

Computing the user feature matrix

$$u_{13} = \frac{1\times0+4\times1+5\times3+2\times1+5\times3}{5} = 7.2$$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6				
t_2	21.2				
t_3	u_{13}				
t_4					

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

Computing the user feature matrix

$$u_{14} = \frac{1\times4+4\times0+5\times1+2\times1+5\times0}{5} = 2.2$$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6				
t_2	21.2				
t_3	7.2				
t_4	u_{14}				

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1		4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

How to recommend to user u_1?

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6	2.4	3.5	4.9	8.1
t_2	21.2	32.6	18.5	9.0	31.0
t_3	7.2	5.6	5.3	4.3	8.4
t_4	2.2	2.8	5.8	3.2	3.8

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1	r_{12}	4		5		2			5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

Recommending to u_1

$$\hat{r}_{12} = cos(\vec{u}_1, \vec{i}_2)$$

$$\hat{r}_{12} = \frac{3.6\times0+21.2\times2+7.2\times2+2.2\times1}{\sqrt{3.6^2+21.2^2+7.2^2+2.2^2}\sqrt{0^2+2^2+2^2+1^2}}$$

$$\hat{r}_{12} = 0.86$$

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6	2.4	3.5	4.9	8.1
t_2	21.2	32.6	18.5	9.0	31.0
t_3	7.2	5.6	5.3	4.3	8.4
t_4	2.2	2.8	5.8	3.2	3.8

Rocchio example

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
u_1	1	.86	4	.61	5	.99	2	.25	.96	5
u_2		2		3		3	5		1	
u_3	4				4		3			2
u_4		2	1	5			1	3		3
u_5	3		4		3	5			4	

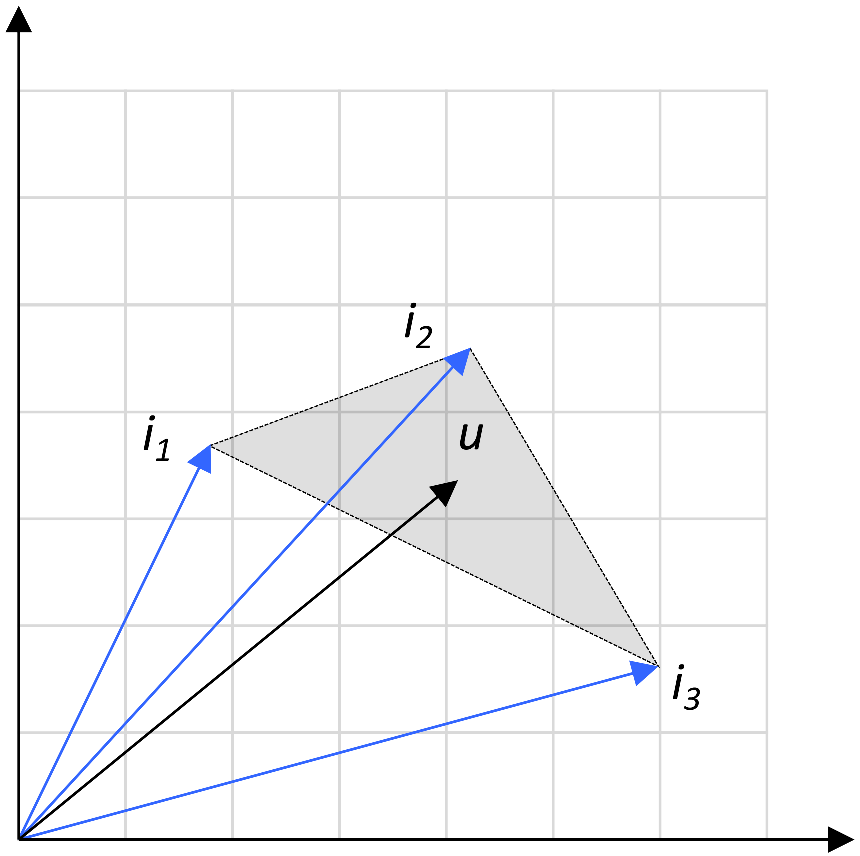
Recommending to u_1

- i_6 : 0.99
- i_9 : 0.96
- i_2 : 0.86
- i_4 : 0.61
- i_8 : 0.25

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}
t_1	2	0	1.5	0.5	1	2.5	0	7.5	3.2	1
t_2	1	2	10	1	0	16	20	0	8	5
t_3	0	2	1	1	3	5	1	2	1	3
t_4	4	1	0	2	1	0	1	2	1	0

	u_1	u_2	u_3	u_4	u_5
t_1	3.6	2.4	3.5	4.9	8.1
t_2	21.2	32.6	18.5	9.0	31.0
t_3	7.2	5.6	5.3	4.3	8.4
t_4	2.2	2.8	5.8	3.2	3.8

Representing the user



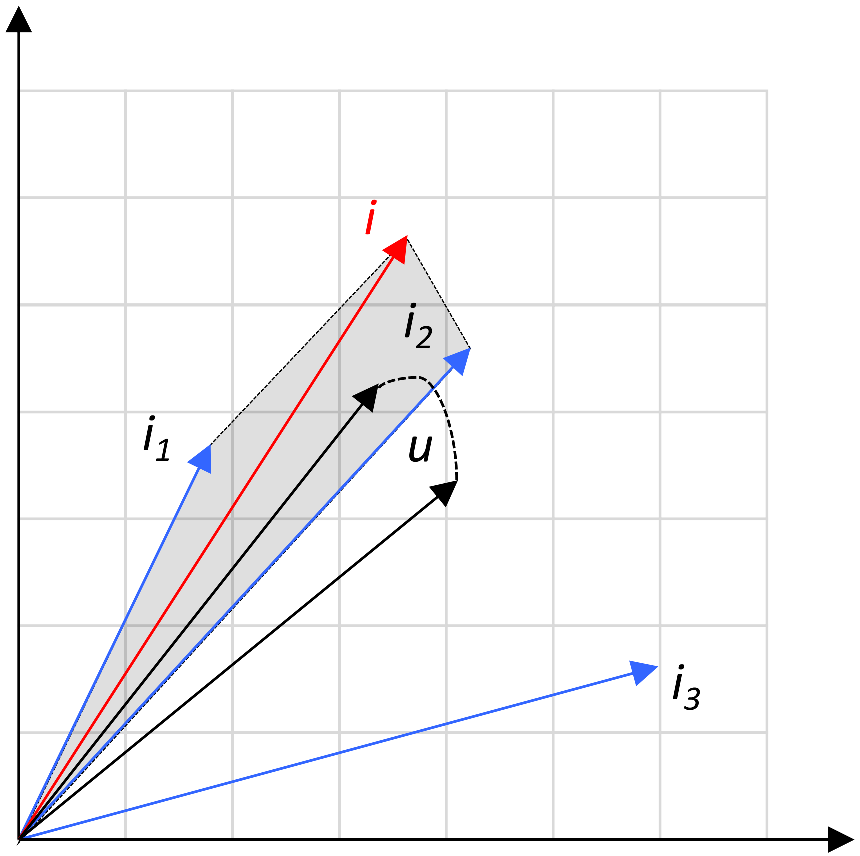
User has rated

- i_1: ★★★
- i_2: ★★★★★
- i_3: ★★★

User prototype

$$\vec{u} = 3\vec{i}_1 + 5\vec{i}_2 + 3\vec{i}_3$$

Representing the user



User has rated

- i_1: ★★★
- i_2: ★★★★★
- i_3: ★★★

User prototype

$$\vec{u} = 3\vec{i}_1 + 5\vec{i}_2 + 3\vec{i}_3$$

k-NN recommendation

Each item is a vector $\vec{\imath}$

- One component for each term

Each user is a vector $\vec{u}$

- $\vec{u}_i = \frac{1}{|N_{ui}|} \sum_{j \in N_{ui}} r_{uj}\, \vec{\jmath}$
  - N_{ui}: neighbors of i rated by u
  - r_{uj}: rating of user u to item j

Prediction score

- $\mathrm{sim}(\vec{u}, \vec{\imath}) = \cos(\vec{u}_i, \vec{\imath})$

Summary

Rocchio is a ***nearest centroid*** recommender

- Items are matched against the user centroid
- Different items will use the same centroid

k-NN is a ***nearest neighbor*** recommender

- Neighbors are chosen on-demand for each item
- Different items will have different neighbors

References

Recommender Systems: An Introduction (Sec. 3.3)

Recommender Systems Handbook (Sec. 3.2)

Recommender Systems: The Textbook (Sec. 4.4)