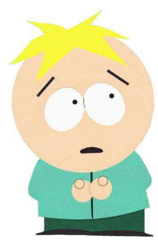
Class #7 – Part 1

Introduction no Noncooperative Game Theory: Games in Normal Form

Motivation

- Systems that include multiple autonomous entities with either diverging information or diverging interests, or both

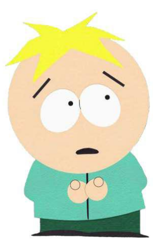


Games

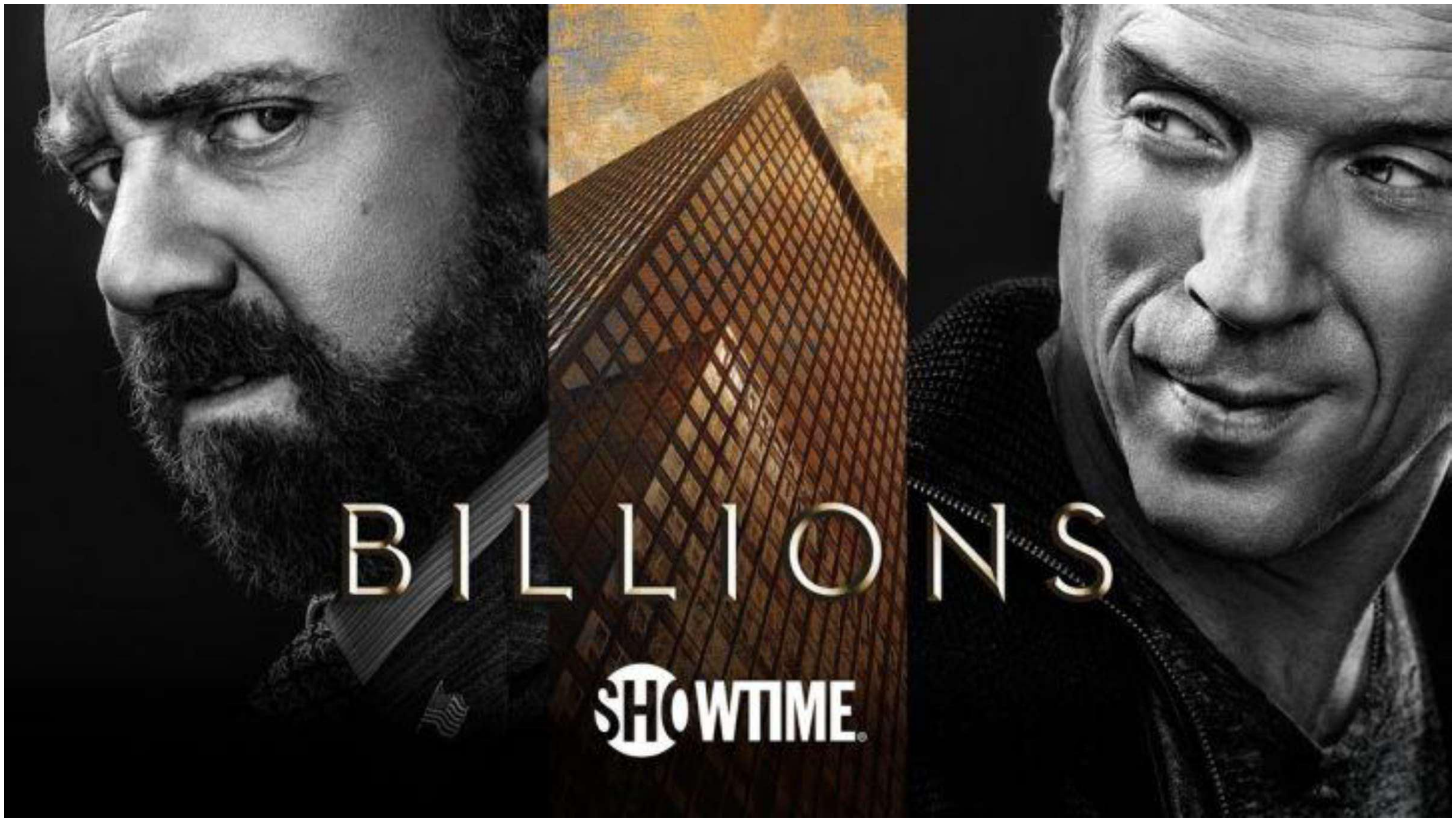
- Which cell phone Cartman should pick?

Games

- And now?



Billions



[The Problem With Game Theory – The Philosophy of Billions - YouTube]
[Decisions Matter: Game Theory in Billions – Part I – Fan Fun with Damian Lewis]

Motivation

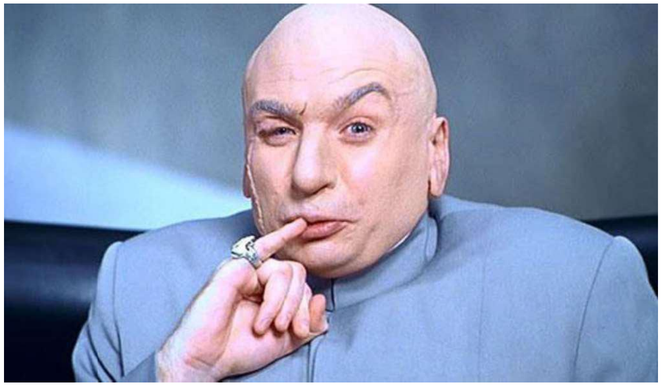
- Indeed, the **Internet** can be viewed as the ultimate platform for interaction among self-interested, distributed computational entities
  - Trading agents
  - “Interface agents” that facilitate the interaction between the user and various computational resources
  - Game-playing agents that assist (or replace) human players in a multiplayer game
  - Autonomous robots

Game Theory

- Outcome of a person's decision depends not just on her preferences, but also on the **choices made by others**
- Main question:
  - Which behaviors **tend to sustain** themselves when carried out in a larger population?

Self-Interested Agents

- What does it mean?
  - They want to cause harm to each other?
    - Not necessarily!
  - They care only about themselves?
    - Not necessarily!



Self-Interested Agents

- What does it mean?
  - Each agent has his own description of which states of the world he likes
    - which can include good things happening to other agents
  - and that he acts in an attempt to bring about these states of the world

Self-Interested Agents

- A utility function is a mapping from states of the world to real numbers
  - measures of an agent's level of happiness in the given states
- If uncertain about which state of the world he faces
  - expected value of his utility function with respect to the appropriate probability distribution over states

Self-Interested Agents

- The states can be thought of as the prizes in the context of lotteries

Example: friends and enemies

- Alice has three options: club (***c***), movie (***m***), watching a video at home (***h***)
- On her own, her utility for these three outcomes is ***100*** for ***c***, ***50*** for ***m*** and ***50*** for ***h***
- Alice also cares about Bob (<u>who she hates</u>) and Carol (<u>who she likes</u>)
- Bob is at the club ***60%*** of the time, and at the movies otherwise
- Carol is at the movies ***75%*** of the time, and at the club otherwise
- If Alice runs into Bob at the movies, she suffers disutility of ***40***; if she sees him at the club she suffers disutility of ***90***
- If Alice sees Carol, she enjoys whatever activity she's doing ***1.5*** times as much as she would have enjoyed it otherwise

Example: friends and enemies

- What should Alice do? Reminder: u(***home***) = 50

		60%	40%
		$B = c$	$B = m$
25%	$C = c$	15	150
75%	$C = m$	10	100

$A = c$

	60%	40%
	$B = c$	$B = m$
$C = c$	50	10
$C = m$	75	15

$A = m$

Alice chooses ***club***

Alice chooses ***movie***

Example: friends and enemies

- What should Alice do? Reminder: u(***home***) = 50

	60% $B = c$	40% $B = m$			60% $B = c$	40% $B = m$
$C = c$	15	150	25%	$C = c$	50	10
$C = m$	10	100	75%	$C = m$	75	15
	$A = c$				$A = m$	

Alice chooses ***club:***

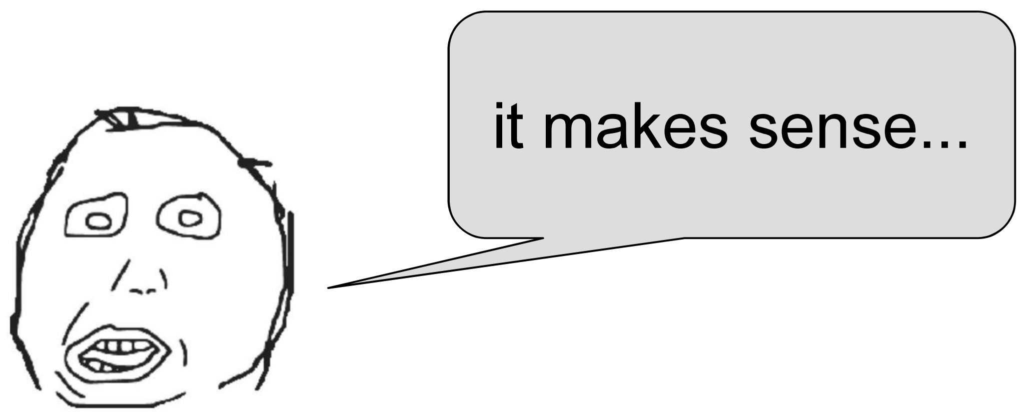
Eu(c) = 0.25(0.6 x 15 + 0.4 x 150) + 0.75(0.6 x 10 + 0.4 x 100) = 51.75

Alice chooses ***movies***

Eu(m) = 0.25(0.6 x 50 + 0.4 x 10) + 0.75(0.6 x 75 + 0.4 x 15) = 46.75

Example: friends and enemies

- What should Alice do? Reminder: u(***home***) = 50
  - Alice prefers to go to the ***club*** (though Bob is often there and Carol rarely is), and prefers staying ***home*** to going to the ***movies*** (though Bob is usually not at the ***movies*** and Carol almost always is)



Alice chooses ***club:***

Eu(c) = 0.25(0.6 x 15 + 0.4 x 150) + 0.75(0.6 x 10 + 0.4 x 100) = 51.75

Alice chooses ***movies***

Eu(m) = 0.25(0.6 x 50 + 0.4 x 10) + 0.75(0.6 x 75 + 0.4 x 15) = 46.75

What is a game?
Example 1

- Problem
  - You have an exam and a presentation tomorrow
  - You have time to prepare yourself for just one
  - The exam is individual and the presentation is together with a colleague
  - Which one should you pick?

What is a game?
Example 1

- Possible outcomes
  - Exam
    - If you study: 92
    - If you don't study: 80
  - Presentation
    - If both work: 100
    - If only one works: 92
    - If no one works: 84

  - The same outcomes are valid for your colleague

What is a game? Example 1

- Possible outcomes (summary)
  - If you study and your colleague works on the presentation
    - (92 + 92) / 2 = **92**
  - If you both study
    - (92 + 84) / 2 = **88**
  - If you work on the presentation and your colleague studies
    - (80 + 92) / 2 = **86**
  - If you both work on the presentation
    - (80 + 100) / 2 = **90**

What is a game?

- A set of **players**
  - you and your partner
- A set of possible **strategies** for each player
  - to prepare for the presentation, or to study for the exam
- A set of **payoffs** for each player and for each joint choice of strategies (the more, the better)
  - the average grade

Games in Normal Form

- A (finite, n-person) normal-form game is a tuple $\boldsymbol{(N,A,u)}$, where:
  - $\boldsymbol{N}$ is a finite set of $\boldsymbol{n}$ players, indexed by $\boldsymbol{i}$
  - $\boldsymbol{A = A_1 \times \cdots \times A_n}$, where $\boldsymbol{A_i}$ is a finite set of actions available to player $\boldsymbol{i}$
    - Each vector $\boldsymbol{a = (a_1, \ldots, a_n) \in A}$ is called an action profile
  - $\boldsymbol{u = (u_1, \ldots, u_n)}$ where $\boldsymbol{u_i : A \rightarrow \mathbb{R}}$ is a real-valued utility (or payoff) function for player $\boldsymbol{i}$
    - $\boldsymbol{u : A^n \rightarrow \mathbb{R}^n}$

What is a game?
Example 1

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

What is a game?

SÍLVIO SANTOS-PROGRAMA SETE E MEIO-DUAS EDIÇÕES JUNTOS -2002-MP4 - YouTube

What is a game?
Example 1

Payoff Matrix

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

What is a game?
Example 1

“Player 2” or “Column player”

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

“Player 1” or “Row player”

What is a game?
Example 1

Strategies A_2 of Player 2

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

Strategies A_1 of Player 1

What is a game?
Example 1

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

Payoffs of Player 1

What is a game?
Example 1

Payoffs of Player 2

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

What is a game?
Example 1

u(Exam, Presentation):
The outcome of the game if P***1*** chooses '***Exam***' and P**2** chooses '***Presentation***'

P**2** gets 86

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

P***1*** gets 92

Reasoning about Behavior in a Game

- Everything that a player cares about is summarized in the player's payoffs
  - If a player is altruistic, the payoffs should reflect it