Choice

Choice functions

- Here, we start modeling "agent behavior"

Choice functions

- Where should I do my PhD in computer science?

Choice functions

- Consider a grand set ***X*** of possible alternatives
- We view a ***choice problem*** as a nonempty subset of ***X***, and we refer to a choice from ***A*** $\subseteq$ ***X*** as specifying one of ***A***'s members

Choice functions

- A rational agent considers only the set of alternatives available to him
  - if the alternatives appear in a list, he ignores:
    - the **order** in which they are presented
    - the **number of times** an alternative appears in the list
    - an alternative with a **default status**

Choice functions

X = the set of all CS graduate programs in the world

Choice functions

X = the set of all CS graduate programs in the world

Choice functions

A = the set containing the items I can choose

Choice functions

A choice function ***C*** assigns to each set ***A*** a unique element of ***A*** with the interpretation that ***C(A)*** is the chosen element from the set ***A***

Inria
INVENTEURS DU MONDE NUMÉRIQUE

Stony Brook University

LELAND STANFORD JUNIOR UNIVERSITY
DIE LUFT DER FREIHEIT WEHT
1891

UFMG

KAIST

Carnegie Mellon University

Choice functions

C chooses the most difficult program to be admitted

Choice functions

C chooses the most difficult program to be admitted

C(A)

Choice functions

C chooses the program that is the closest to home

C(A)

Choice functions

C chooses the program that is located in the most fun city to live

C(A)

Choice functions

- Where did you go for lunch today?
- Is this your favorite restaurant in the world?
- How can you model your choice function?

Choice functions

- In some contexts, not all choice problems are relevant
- Therefore we allow that the agent's behavior be defined only on a set ***D*** of <u>subsets</u> of ***X***
- We will refer to a pair ***(X,D)*** as a <u>***context***</u>
  - <u>Where should we go for lunch?</u>
    - ***D*** can be thought of all the subsets of restaurants that are within walking distance from a given coordinate

Choice functions

- Agent's behavior as response to a <u>questionnaire</u> that contains questions of the following type, one for each $\boldsymbol{A \in D}$:
  - ***Q(A)***: Assume you must choose from a set of alternatives ***A***. Which alternative do you choose?
- A permissible response to this questionnaire requires that the agent select a <u>unique</u> element in ***A*** for every question ***Q(A)***
- We implicitly assume that the agent <u>cannot give any other answer</u> such as "I choose either ***a*** or ***b***"; "the probability of my choosing $\boldsymbol{a \in A}$ is ***p(a)***"; or "I don't know"

Rational choice functions

- Choice is an outcome of "rational deliberation"
  - The decision maker has in mind a preference relation $\succsim$ on the set ***X***
  - Given any choice problem ***A*** in ***D***, he chooses an element in ***A*** that is $\succsim$***-optimal***

Rational choice functions

- The induced choice function $\boldsymbol{C}_{\succsim}$ assigns to every nonempty set $\boldsymbol{A} \in \boldsymbol{D}$ the ***$\succsim$-best*** element of $\boldsymbol{A}$
- Note that the preference relation is fixed, that is, it is <u>independent of the choice set</u> being considered

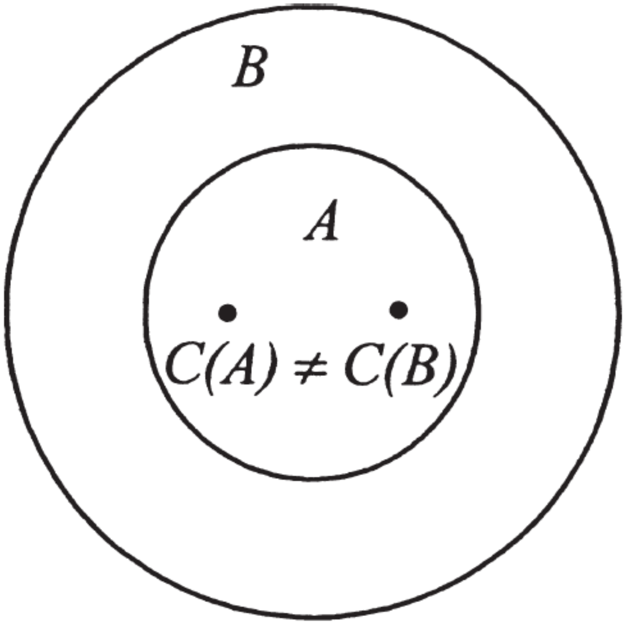
Rationalizing

- The decision maker's behavior can be described as if he is <u>maximizing</u> some preference relation
- A choice function $\boldsymbol{C}$ can be rationalized if there is a preference relation $\succsim$ on $\boldsymbol{X}$ so that $\boldsymbol{C = C_{\succsim}}$
  - $\boldsymbol{C(A) = C_{\succsim}(A)}$ for any $\boldsymbol{A}$ in the domain of $\boldsymbol{C}$
  - Always choose the $\succsim$***-best*** element in $\boldsymbol{A}$

Rationalizing

- **Condition *α***:
  - We say that ***C*** satisfies condition ***α*** if for any two problems $\boldsymbol{A,B \in D}$, if $\boldsymbol{A \subset B}$ and $\boldsymbol{C(B) \in A}$, then…
  - $\boldsymbol{C(A) = C(B)}$

Violates condition α!



Rationalizing

- Example of a choice procedure that does not satisfy condition ***α***?
- The <u>second-best procedure</u>
  - the decision maker has in mind an ordering $\succsim$ of ***X*** and for any given choice problem set ***A*** chooses the ***$\succsim$-maximal*** from the non-optimal alternatives
- If ***A*** contains all the elements in ***B*** besides the ***$\succsim$-best***, then $\boldsymbol{C(B) \in A \subset B}$ but $\boldsymbol{C(A) \neq C(B)}$

Rationalizing

- ***Condition α*** is a sufficient condition for a choice function to be formulated as if the decision maker is maximizing some preference relation

Rationalizing

- **Proposition:**
  - Assume that $\boldsymbol{C}$ is a choice function with a domain containing at least all subsets of $\boldsymbol{X}$ of size $\boldsymbol{2}$ or $\boldsymbol{3}$. If $\boldsymbol{C}$ satisfies ***condition α***, then there is a preference $\succsim$ on $\boldsymbol{X}$ so that $\boldsymbol{C = C_{\succsim}}$

Rationalizing

- **Proposition:**
  - Assume that C is a choice function with a domain containing at least all subsets of X of size 2 or 3. If C satisfies ***condition α***, then there is a preference $\succsim$ on X so that $C = C_{\succsim}$
- ***Proof:***
  - Define $\succsim$ by $x \succsim y$ if $x = C(\{x, y\})$
  - Is the relation $\succsim$ a preference relation?
    - **Completeness:** $C(\{x, y\})$ is always well defined
    - **Transitivity:** If $x \succsim y$ and $y \succsim z$, $C(\{x, y\}) = x$ and $C(\{y, z\}) = y$. If $C(\{x, z\}) \neq x$, $C(\{x, z\}) = z$. By **condition α** and $C(\{x, z\}) = z$, $C(\{x, y, z\}) \neq x$. By **condition α** and $C(\{x, y\}) = x$, $C(\{x, y, z\}) \neq y$, and by **condition α** and $C(\{y, z\}) = y$, $C(\{x, y, z\}) \neq z$. A contradiction to $C(\{x, y, z\}) \in \{x, y, z\}$
  - Is $C(B) = C_{\succsim}(B)$?
    - Assume that $C(B) = x$ and $C_{\succsim}(B) \neq x$, ie. there is $y \in B$ so that $y \succ x$
    - By definition of $\succsim$, this means $C(\{x, y\}) = y$, contradicting **condition α**

Rationalizing

- **Proposition:**
  - Let ***C*** be a choice function with a domain ***D*** satisfying that if $\boldsymbol{A,B \in D}$, then $\boldsymbol{A \cup B \in D}$
  - If ***C*** satisfies **condition α**, then there is a preference relation $\succsim$ on ***X*** such that $\boldsymbol{C = C_{\succsim}}$
- **Proof:**
  - in the book

Rationalizing

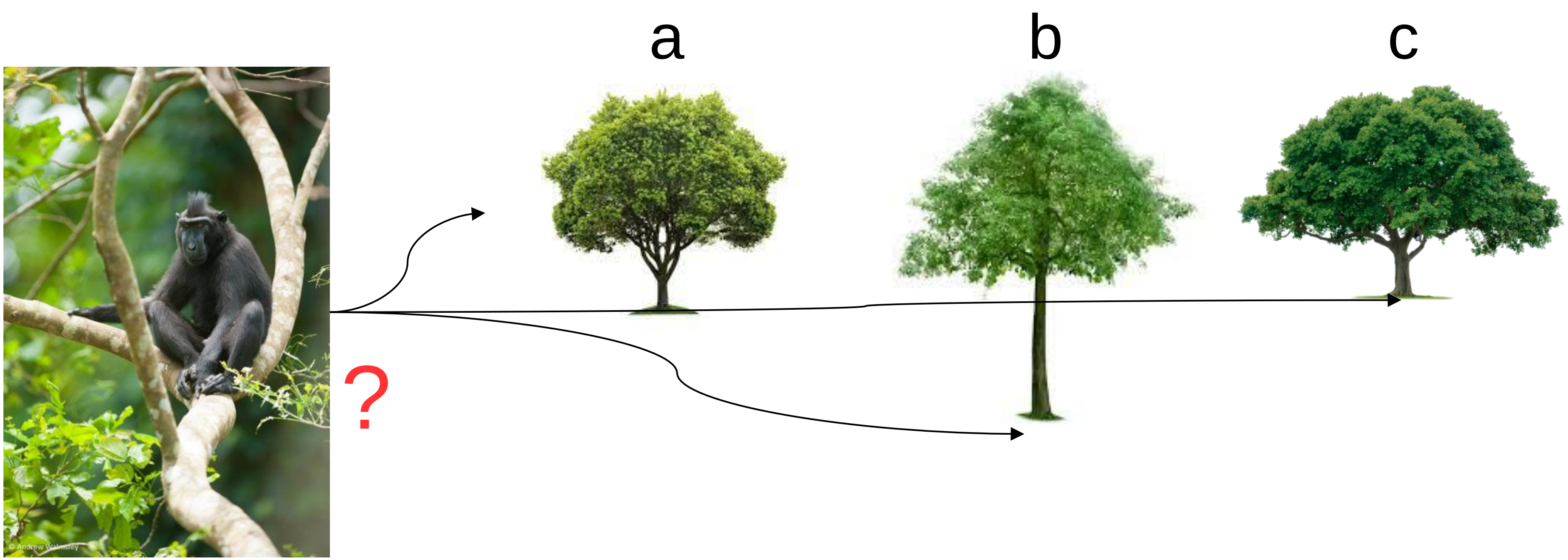
- ***Condition α*** is a sufficient condition for a choice function to be formulated as if the decision maker is maximizing some preference relation
  - The agent will always select the ***≿-best*** element available

Dutch Book Arguments

- **Claim:**
  - An economic agent who behaves according to a choice function that is **not** induced from maximization of a preference relation will not survive
- People should be rational in this sense and, if they are not, they should convert to reasoning of this type

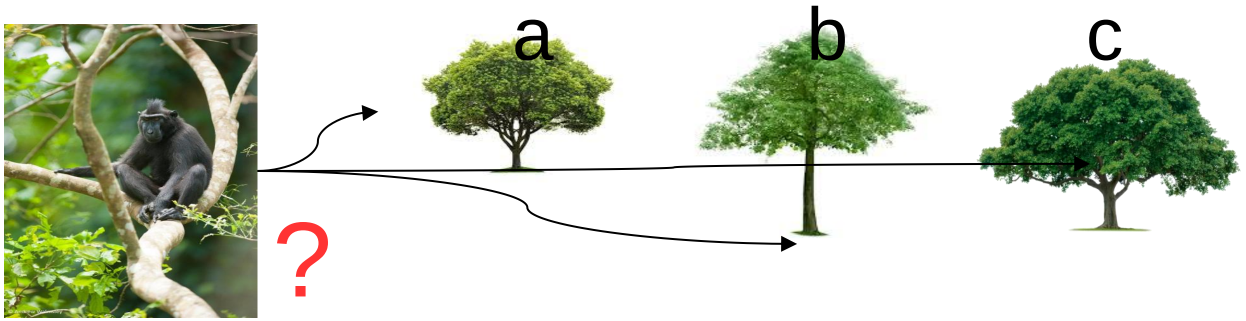
Example

- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***C({a, b}) = b, C({b, c}) = c, C({a, c}) = a***



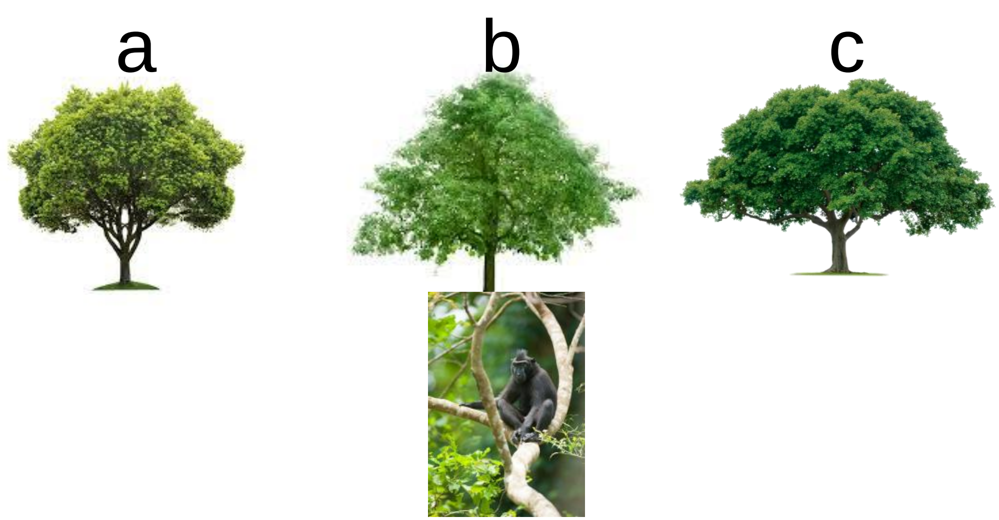
Example

- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***C({a, b}) = b, C({b, c}) = c, C({a, c}) = a***
  - This choice function can be derived from a preference relation?



Example

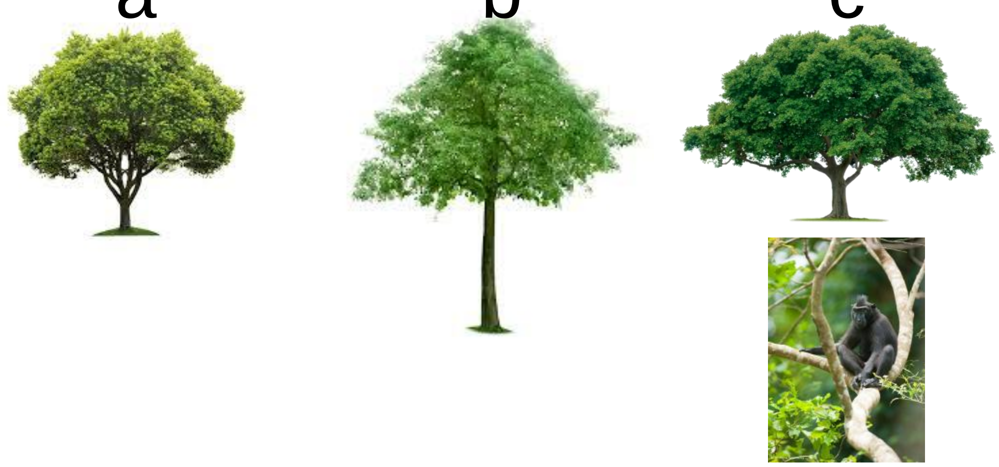
- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***$C(\{a, b\}) = b$, $C(\{b, c\}) = c$, $C(\{a, c\}) = a$***
  - ***$C(\{a,b\}) = b$***



Example

- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***C({a, b}) = b, C({b, c}) = c, C({a, c}) = a***
  - ***C({b,c}) = c***

a b c



Example

- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***C({a, b}) = b, C({b, c}) = c, C({a, c}) = a***
  - ***C({a,c}) = a***

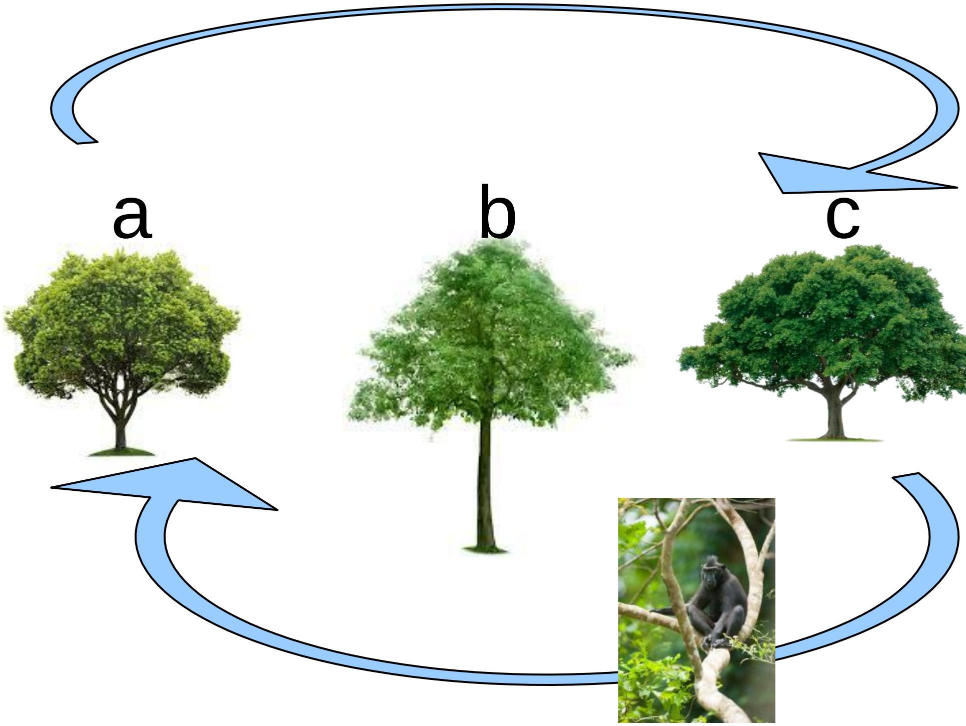
a

b

c

Example

- A monkey should pick a tree out of three to sleep in
  - Assume that the monkey can assess only two alternatives at a time and that his choice function is ***C({a, b}) = b, C({b, c}) = c, C({a, c}) = a***
  - Never sleeps! :(



Dutch book argument

- What if a person ***X*** behaves like this monkey in a market towards three alternatives ***A***, ***B*** and ***C***?
  - "Money pump argument": an agent can offer ***B*** to ***X*** for ***A+ε***, later ***C*** for ***B+ε***, then ***A*** for ***C+ε***, until ***X*** has no money

Alert!

- Be careful with the alternatives' description!

What is an alternative

- Suppose the following
  - Smith chooses ***chicken*** from the menu:
    - ***steak tartare*, *chicken***
  - Smith chooses ***steak tartare*** from the menu:
    - ***steak tartare*, *chicken*, *frog legs***
- Is Smith rational?

What is an alternative

- Now suppose the following
  - Smith chooses ***chicken*** from the menu:
    - ***steak tartare in a restaurant where frog legs are not served*, *chicken***
  - Smith chooses ***steak tartare in a restaurant where frog legs are served*** from the menu:
    - ***steak tartare in a restaurant where frog legs are served*, *chicken*, *frog legs***

- Is Smith rational?

Choice Functions as Internal Equilibria

- What is your favourite movie?

Choice Functions as Internal Equilibria

- If the decision maker follows the rational man procedure using a preference relation with ***indifferences***, the previously defined induced choice function ***$C_{\succsim}(A)$*** might be undefined
  - For some choice problems there would be more than one optimal element

Choice Functions as Internal Equilibria

- A choice correspondence $\boldsymbol{C}$ is required to assign to every nonempty $\boldsymbol{A} \in \boldsymbol{D}$ a nonempty subset of $\boldsymbol{A}$, that is, $\boldsymbol{\varnothing \neq C(A) \subseteq A}$
- $\boldsymbol{C(A)}$ is the set of all elements in $\boldsymbol{A}$ that are satisfactory
  - If the decision maker is about to make a decision and choose $\boldsymbol{a \in C(A)}$, he has <u>no desire</u> to move away from it

Choice Functions as Internal Equilibria

- In other words, the ***induced choice correspondence*** reflects an "internal equilibrium":
  - If the decision maker facing ***A*** considers an alternative outside ***C(A)***, he will continue searching for another alternative
  - If he happens to consider an alternative inside ***C(A)***, he will take it

Choice Functions as Internal Equilibria

- Given a preference relation $\succsim$ we define the ***<u>induced choice correspondence</u>*** (assuming it is never empty) as
  - $C_{\succsim}(A) = \{x \in A \mid x \succsim y \text{ for all } y \in A\}$

- When $x, y \in A$ and $x \in C(A)$, we say that x is <u>revealed</u> to be **at least as good** as y
- If, in addition, $y \notin C(A)$, we say that x is <u>revealed</u> to be **strictly better** than y

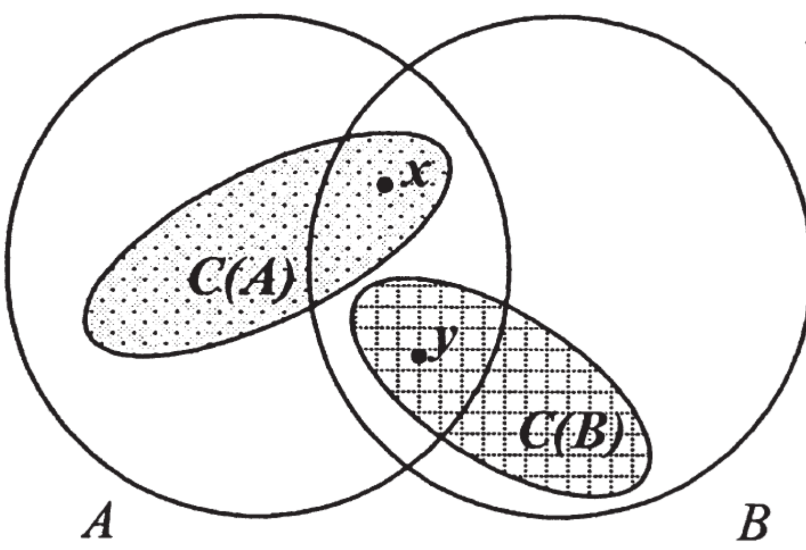
Choice Functions as Internal Equilibria

- **The Weak Axiom of Revealed Preference (WA):**
  - We say that ***C*** satisfies ***WA*** if whenever $x, y \in A \cap B$, $x \in C(A)$, and $y \in C(B)$, it is also true that ***x***

$x \in C(A)$

$y \in C(B)$

$x \notin C(B)$



Violates WA!

Choice Functions as Internal Equilibria

- The Weak Axiom trivially implies two properties:
  - **Condition α**: If $\boldsymbol{a \in A \subset B}$ and $\boldsymbol{a \in C(B)}$, then

    $\boldsymbol{a \in C(A)}$

  - **Condition β**: If $\boldsymbol{a, b \in A \subset B, a \in C(A)}$, and $\boldsymbol{b \in C(B)}$, then

    $\boldsymbol{a \in C(B)}$ and $\boldsymbol{b \in C(A)}$

- Notice that if ***C(A)*** contains all elements that are <u>maximal</u> according to some preference relation, then ***C*** satisfies **WA**

The Satisfying Procedure

- How do you proceed when you have to buy a laptop?

The Satisfying Procedure

- You need to buy a laptop which has
  - at least 256 Gb of SSD,
  - at least 8 Gb of RAM,
  - dedicated graphics processor,
  - and weights at most 2.0 Kg

- How do you proceed in this case?

The Satisfying Procedure

- Consider the following "decision scheme"
  - Let $\boldsymbol{v : X \to \mathbb{R}}$ be a valuation of the elements in ***X***
  - Let $\boldsymbol{v* \in \mathbb{R}}$ be a threshold of satisfaction
  - Let ***O*** be an ordering of the alternatives in ***X***
    - e.g. price
  - Given a set ***A***, the decision maker arranges the elements of this set in a list ***L(A,O)*** according to the ordering ***O***
  - He then chooses the first element in ***L(A,O)*** that has a ***v-value*** at least as large as ***v∗***
  - If there is no such element in ***A***, the decision maker chooses the last element in ***L(A,O)***

The Satisfying Procedure

- Reminder:
  - **Condition α:** we say that ***C*** satisfies ***condition α*** if for any two problems ***A,B*** $\in$ ***D***, if ***A*** $\subset$ ***B*** and ***C(B)*** $\in$ ***A***, then ***C(A) = C(B)***
- Does the choice function induced by this procedure satisfy condition α?
- YES!

The Satisfying Procedure

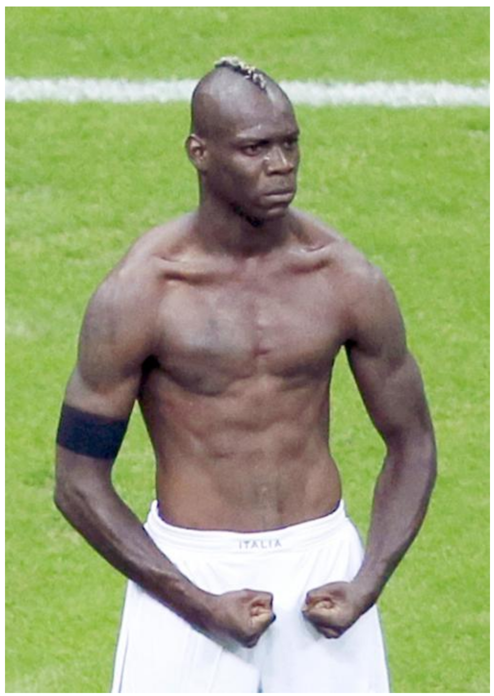
- Assume that ***a*** is chosen from ***B*** and is also a member of $\boldsymbol{A \subset B}$
- The list ***L(A,O)*** is obtained from ***L(B,O)*** by eliminating all elements in $\boldsymbol{B - A}$
- If $\boldsymbol{v(a) \geq v*}$, then ***a*** is the first satisfactory element in ***L(B,O)*** and is also the first satisfactory element in ***L(A,O)***
- Thus, ***a*** is chosen from ***A***
- If all elements in ***B*** are unsatisfactory, then ***a*** must be the last element in ***L(B,O)***
- Since ***A*** is a subset of ***B***, all elements in ***A*** are unsatisfactory and ***a*** is the last element in ***L(A,O)***
- Thus, ***a*** is chosen from ***A***

The Satisfying Procedure

- Summary
  - The satisfying procedure, proposed by Herbert Simon, though it is stated in a way that <u>seems unrelated to the maximization of a preference relation or utility function</u>, can be described as if the decision maker maximizes a preference relation

Choice Functions as Internal Equilibria

- Who would you pick to be your teammate?



Choice Functions as Internal Equilibria

- ***C(A)*** may be chosen under any of many possible particular circumstances not included in the description of the set ***A***
- Formally, let ***(A, f)*** be an ***extended choice set*** where ***f*** is the frame that accompanies the set ***A***
  - e.g.: the default alternative or the order of the alternatives
- Let ***c(A, f)*** be the choice of the decision maker from the choice set ***A*** given the frame ***f***

Choice Functions as Internal Equilibria

- The ***<u>(extended) choice function</u>*** ***c*** induces a choice correspondence by

  $$\boldsymbol{C(A) = \{x|x = c(A, f) \text{ for some } f\}}$$

Psychological Motives Not Included within the Framework

- Framing
- Simplifying the choice problem and the use of similarities
- Reason-based choice
- Mental accounting

Framing

- An outbreak of disease is expected to cause 600 deaths in the United States
- Two mutually exclusive programs are expected to yield the following results:
  - a. Exactly 400 people will die
  - b. With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- Which program would you choose?

Framing

- In essence, ***framing theory*** suggests that how something is presented to the audience (called “the frame”) influences the choices people make about how to process that information

Framing

- An outbreak of disease is expected to cause 600 deaths in the United States
- Two mutually exclusive programs are expected to yield the following results:
  - A. Exactly 200 people will be saved
  - B. With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- Which program would you choose?

Framing

- ***Results in our class***
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 20% chose ***a (1/5)***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 86% chose ***A (6/7)***

Framing

- ***Results in 2024/01***
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 43% chose ***a (3/7)***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 83% chose ***A (5/6)***

Framing

- ***Results in 2022/01***
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 50% chose ***a (4/8)***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 75% chose ***A (6/8)***

Framing

- ***Results in 2019/02***
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 30% chose ***a (11/37)***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 48% chose ***A (14/29)***

Framing

- ***Results in 2017/02***
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 19% chose ***a (5/26)***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 59% chose ***A (10/17)***

Framing

- ***Results in 2016/02***
    - ***a.*** 400 people will die
    - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 22% chose ***a***
    - ***A.*** 200 people will be saved
    - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 56% chose ***A***

Framing

- **Results in 2015-2**
  - ***a.*** 400 people will die
  - ***b.*** With probability 1/3, 0 people will die, and with probability 2/3, 600 people will die
- 27% chose ***a***
  - ***A.*** 200 people will be saved
  - ***B.*** With probability 1/3, all 600 will be saved, and with probability 2/3, none will be saved
- 57% chose ***A***

Framing

- Experiment conducted by Tversky and Kahneman (1986)
- Whereas only 22% of the first group chose ***a***, 72% of the second group chose ***A***
- These are "problematic" results since by any reasonable criterion ***a*** and ***A*** are identical alternatives, as are ***b*** and ***B***
- Thus, the choice from ***{a, b}*** should be consistent with the choice from ***{b, B}***

Framing

- Overall, the results expose the sensitivity of choice to the ***framing*** of the alternatives
- What is more basic to rational decision making than taking the same choice when only the manner in which the problems are stated is different?

Simplifying the Choice Problem and the Use of Similarities

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15
	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

Simplifying the Choice Problem and the Use of Similarities

- Which roulette do you prefer, ***a*** or ***b***?

(c)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize \$	0	45	30	-15	-15

(d)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize \$	0	45	45	-10	-15

Results for our class

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 0% of the students chose ***a (0/3)***

Results for our class

- Which roulette do you prefer, ***c*** or ***d***?

(c)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	30	−15	−15

(d)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	45	−10	−15

- 0% of the students chose ***c (0/9)***

Results for 2024/01

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 29% of the students chose ***a (2/7)***

Results for 2024/01

- Which roulette do you prefer, ***c*** or ***d***?

	Color	White	Red	Green	Blue	Yellow
(c)	Chance %	90	6	1	1	2
	Prize $	0	45	30	-15	-15

	Color	White	Red	Green	Blue	Yellow
(d)	Chance %	90	6	1	1	2
	Prize $	0	45	45	-10	-15

- 0% of the students chose ***c (0/6)***

Results for 2022/01

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15
	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 44% of the students chose ***a (4/9)***

Results for 2022/01

- Which roulette do you prefer, ***c*** or ***d***?

	Color	White	Red	Green	Blue	Yellow
(c)	Chance %	90	6	1	1	2
	Prize \$	0	45	30	-15	-15

	Color	White	Red	Green	Blue	Yellow
(d)	Chance %	90	6	1	1	2
	Prize \$	0	45	45	-10	-15

- 0% of the students chose ***c (0/9)***

Results in 2019/02

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15
	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 42% of the students chose ***a (13/31)***

Results in 2019/02

- Which roulette do you prefer, ***c*** or ***d***?

	Color	White	Red	Green	Blue	Yellow
(c)	Chance %	90	6	1	1	2
	Prize $	0	45	30	-15	-15

	Color	White	Red	Green	Blue	Yellow
(d)	Chance %	90	6	1	1	2
	Prize $	0	45	45	-10	-15

- 9% of the students chose ***c (3/33)***

Results in 2017/2

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 30% of the students chose ***a (7/23)***

Results in 2017/2

- Which roulette do you prefer, ***c*** or ***d***?

(c)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize $	0	45	30	-15	-15

(d)	Color	White	Red	Green	Blue	Yellow
	Chance %	90	6	1	1	2
	Prize $	0	45	45	-10	-15

- 0% of the students chose ***c (0/20)***

Results in 2016/2

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 55% of the students chose ***a***

Results in 2016/2

- Which roulette do you prefer, ***c*** or ***d***?

(c)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	30	−15	−15

(d)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	45	−10	−15

- 0% of the students chose ***c***

Results for 2015-2

- Which roulette do you prefer, ***a*** or ***b***?

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15
	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

- 40% of the students chose ***a***

Results for 2015-2

- Which roulette do you prefer, ***c*** or ***d***?

(c)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	30	−15	−15

(d)

Color	White	Red	Green	Blue	Yellow
Chance %	90	6	1	1	2
Prize $	0	45	45	−10	−15

- 5% of the students chose ***c***

Simplifying the Choice Problem and the Use of Similarities

- Which roulette do you prefer, ***a*** or ***b***?
- Also in Tversky and Kahneman (1986)
  - 58% of the subjects in the first group chose a
  - Nobody in the second group chose c
- Students in A. Rubinstein class
  - two problems were presented, one after the other, to about 1,350 students
  - 52% chose a
  - 7% chose c

Simplifying the Choice Problem and the Use of Similarities

- In a complicated choice problem, we often transfer the complicated problem into a simpler one by “canceling” similar elements
- Although ***d*** clearly dominates ***c***, the comparison between ***a*** and ***b*** is not as easy

Simplifying the Choice Problem and the Use of Similarities

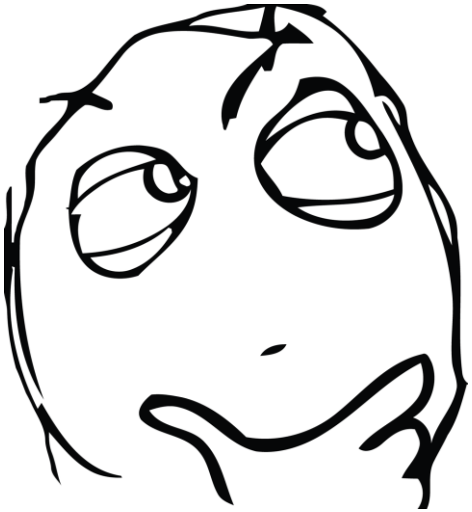
- Many subjects "cancel" the probabilities of White, Yellow, and Red and are left with comparing the prizes of Green, a process that leads them to choose a

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize $	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize $	0	45	-10	-15

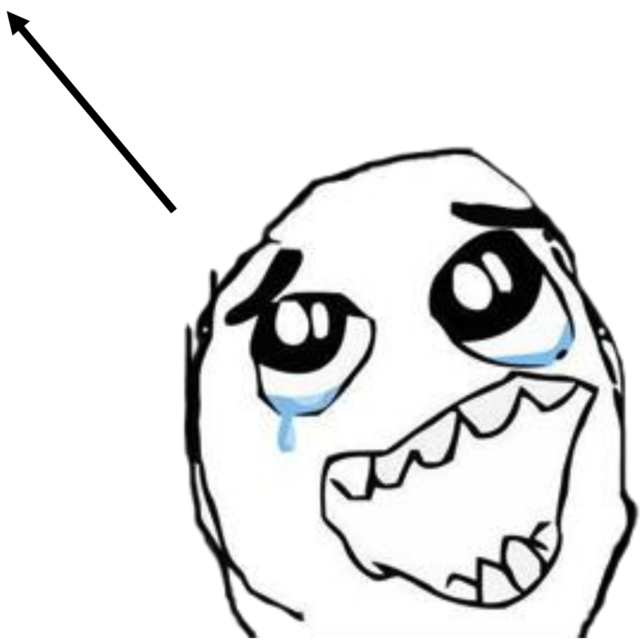
Reason-Based Choice

- Derp must choose between two universities



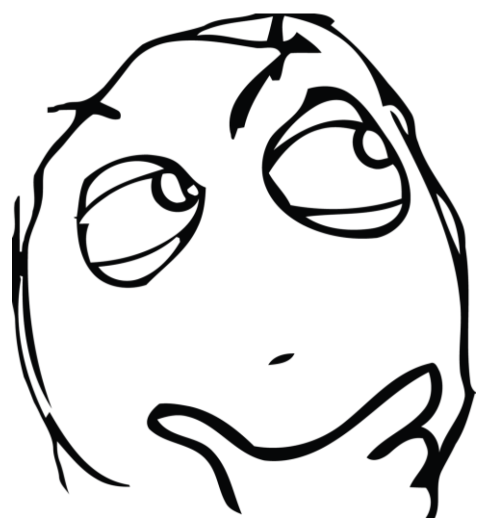
Reason-Based Choice

- C({Stanford, UFMG}) = Stanford



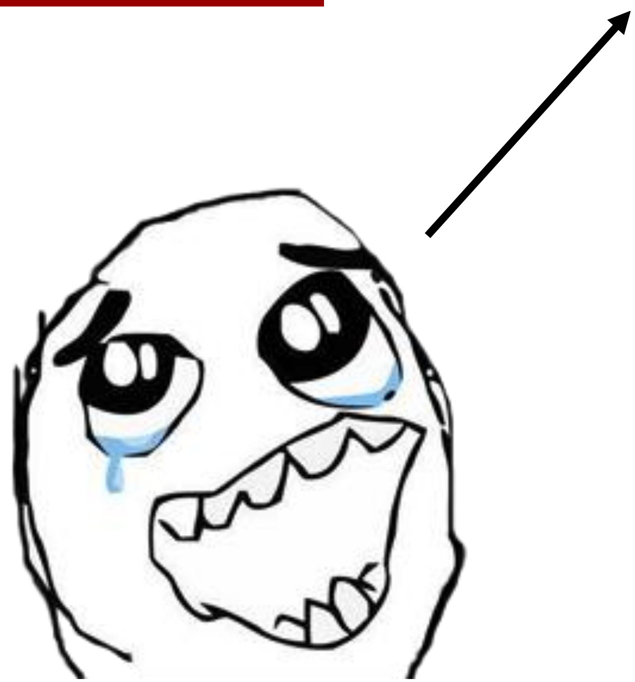
Reason-Based Choice

- Derp must choose between **three** universities



Reason-Based Choice

- C({Stanford, CMU, UFMG}) = UFMG



Reason-Based Choice

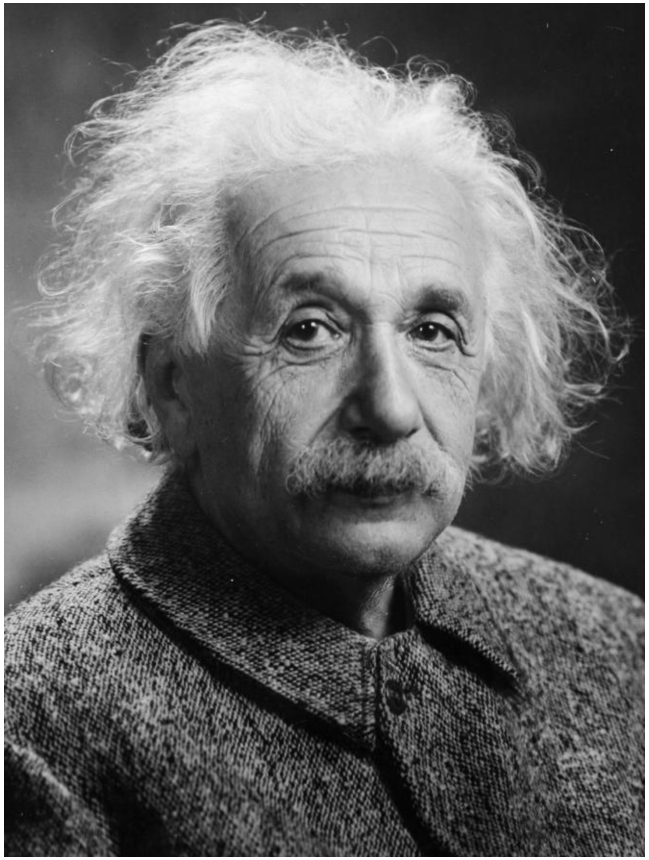
- Does his choice satisfy ***condition α***?
  - No!
- Is the reason related to a careless specification of the alternatives (as in the restaurant's menu example discussed previously)?
  - No!

Reason-Based Choice

- What happened, then?
- His explanation is that he prefers an American university so long as he does not have to choose between American schools – a choice he deems harder
- His reasoning involves an attempt to avoid the difficulty of making a decision

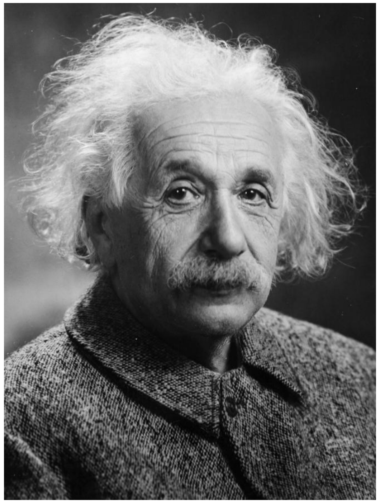
Reason-Based Choice

- As another example, imagine a handsome guy called Albert, who is looking for a date to take to a party



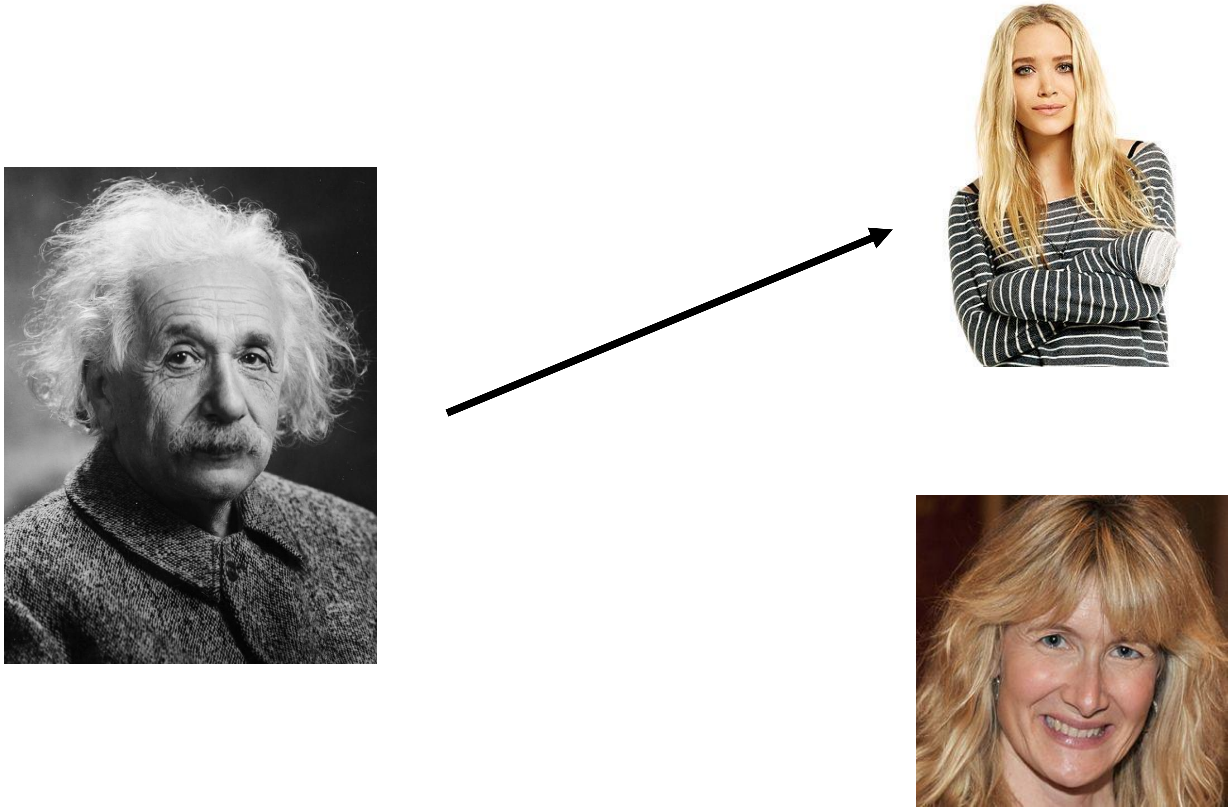
Reason-Based Choice

- Albert knows two girls that are crazy about him, both of whom would love to go to the party
- The two girls are called Mary and Laura



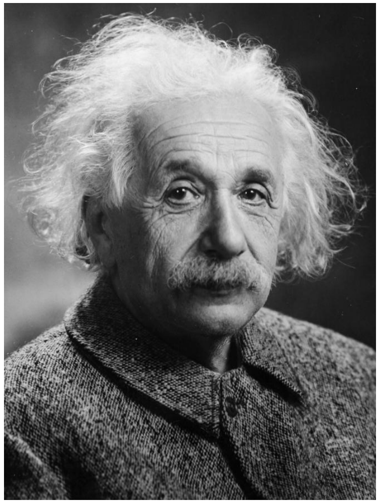
Reason-Based Choice

- Of the two, Albert prefers Mary



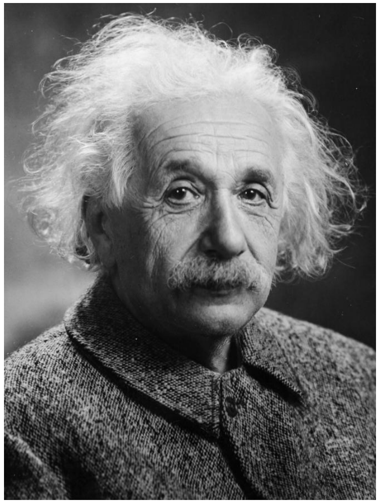
Reason-Based Choice

- Now imagine that Mary has a sister, and this sister is also crazy about Albert



Reason-Based Choice

- Which girl should Albert choose?
  - Is the reason related to a careless specification of the alternatives?




Reason-Based Choice

- Another example follows Huber, Payne, and Puto (1982)
  - Let ***a* = (*a1*, *a2*)** be "a holiday package of ***a1*** days in Paris and ***a2*** days in London"
  - Choose one of the four vectors
    - ***a*** = (7, 4), ***b*** = (4, 7), ***c*** = (6, 3), and ***d*** = (3, 6)
  - All subjects in the experiment agreed that a day in Paris and a day in London are desirable goods

Reason-Based Choice

- Choose one of the four vectors
  - ***a*** = (7, 4), ***b*** = (4, 7), ***c*** = (6, 3), and ***d*** = (3, 6)
- Some had to choose between ***a***, ***b***, and ***c***
- Others had to choose between ***a***, ***b***, and ***d***

Results for our class

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***c*** = (6, 3)

33% (2/6) chose **(7,4)**
67% (4/6) chose **(4,7)**
0% chose **(6,3)**

Results for our class

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***d*** = (3, 6)

20% (2/10) chose **(7,4)**
80% (8/10) chose **(4,7)**
0% chose **(3,6)**

Results for 2024/01

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***c*** = (6, 3)

43% chose **(7,4)**
57% chose **(4,7)**
0% chose **(6,3)**

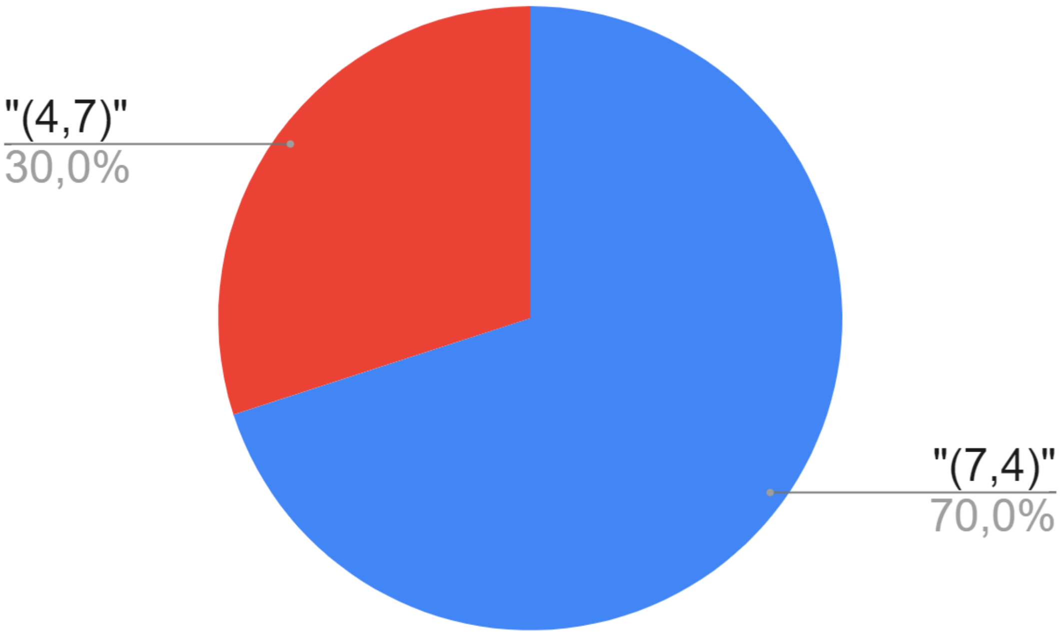
Results for 2024/01

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***d*** = (3, 6)

50% chose **(7,4)**
50% chose **(4,7)**
0% chose **(3,6)**

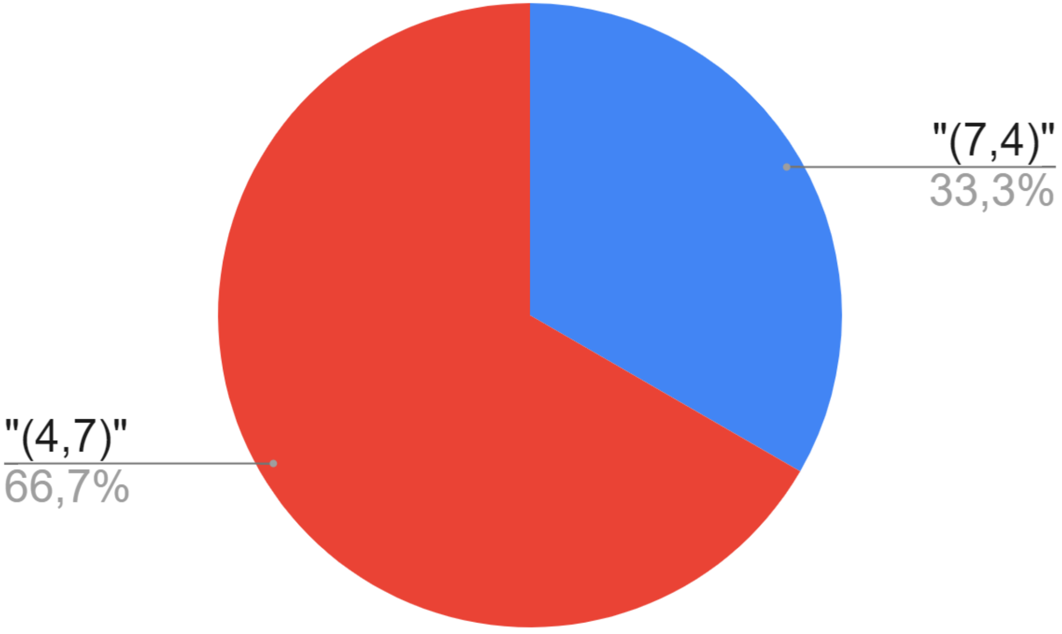
Results for 2022/01

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***c*** = (6, 3)



Results for 2022/01

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***d*** = (3, 6)

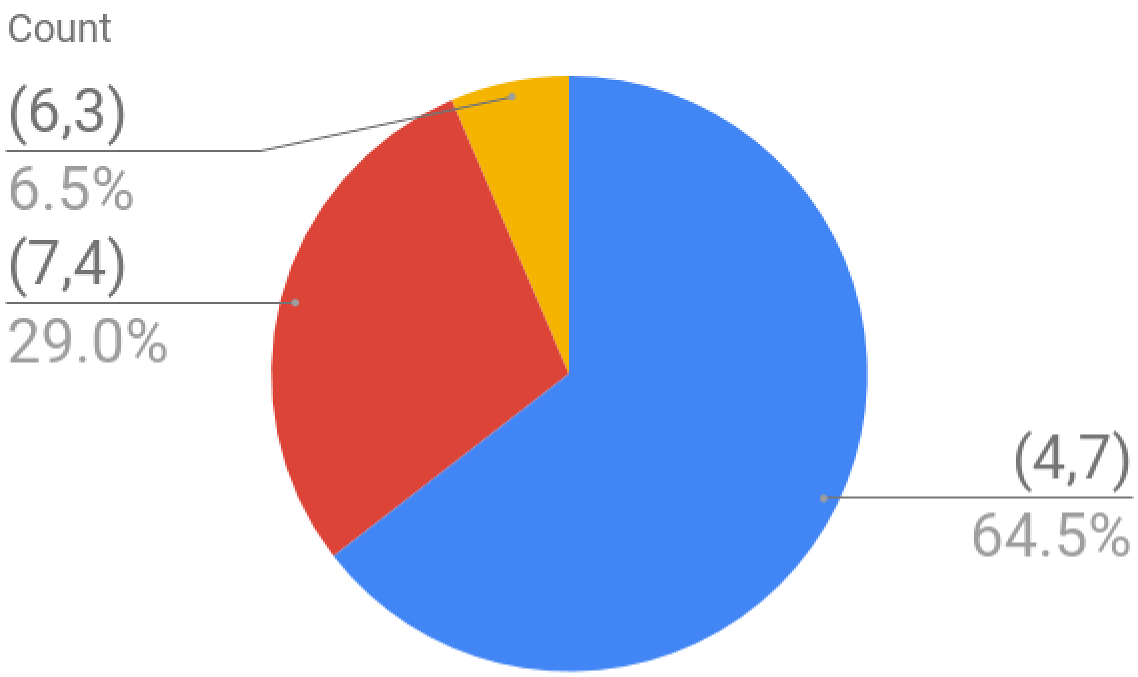


Reason-Based Choice

- Choose one of the four vectors
  - ***a*** = (7, 4), ***b*** = (4, 7), ***c*** = (6, 3), and ***d*** = (3, 6)
- Some had to choose between ***a***, ***b***, and ***c***
- Others had to choose between ***a***, ***b***, and ***d***
- **Conclusion:**
  - The subjects exhibited a clear tendency toward choosing ***a*** out of the set ***{a, b, c}*** and choosing ***b*** out of the set ***{a, b, d}***

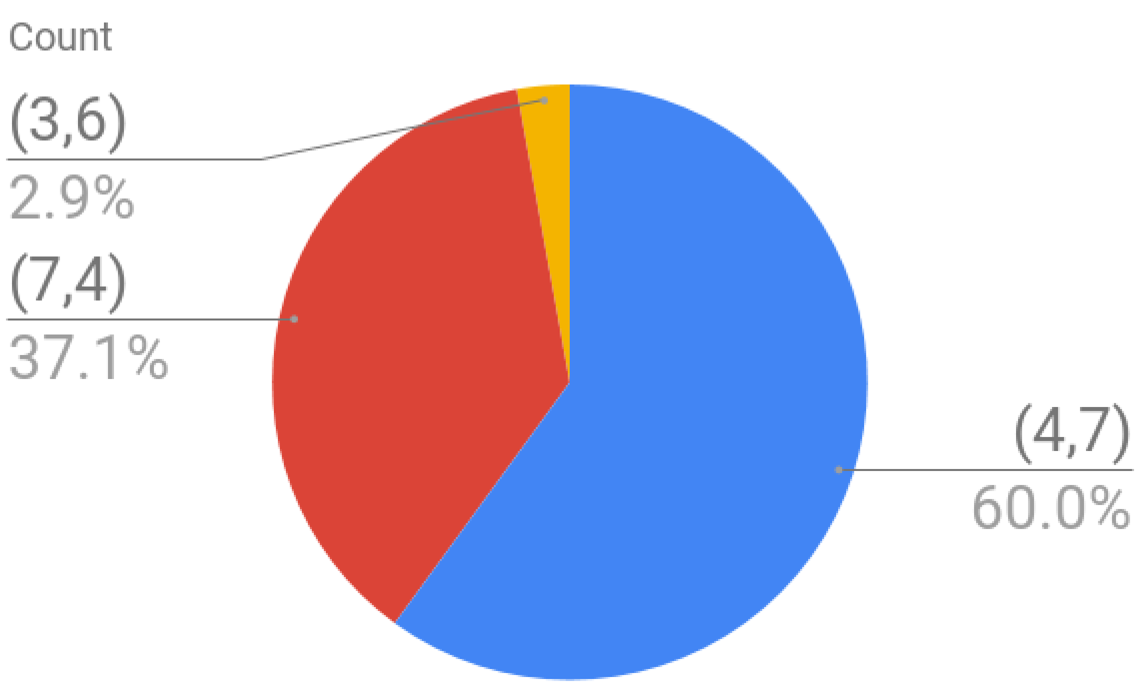
Results in 2019/02

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***c*** = (6, 3)



Results in 2019/02

- Choose one of the three vectors
  - ***a*** = (7, 4), ***b*** = (4, 7) and ***d*** = (3, 6)




FAIL

Reason-Based Choice

- Decision makers look for reasons to prefer one alternative over the other
- Typically, making decisions by using “external reasons” will not cause violations of rationality
- However, applying “internal reasons” such as “I prefer the alternative ***a*** over the alternative ***b*** since ***a*** clearly dominates the other alternative ***c*** while ***b*** does not” might cause conflicts with ***<u>condition α</u>***.

Reason-Based Choice

- “This paper considers the role of reasons and arguments in the making of decisions. It is proposed that, when faced with the need to choose, decision makers often seek and construct reasons in order to resolve the conflict and justify their choice, to themselves and to others.”

  Shafir, Eldar, Itamar Simonson, and Amos Tversky. "Reason-based choice." Cognition 49.1-2 (1993): 11-36.

Mental Accounting

- From Kahneman and Tversky (1984)
  - Group 1
    - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?
  - Group 2
    - You have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?

Mental Accounting

- Group 1
  - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?
- Group 2
  - You have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?

- Should you give different answers to these questions?

Mental Accounting

- Should you give different answers to these questions?
  - If you care only about seeing the play and your wealth, then no
  - Nonetheless, only 46% said they would buy another ticket after they had lost the first one, whereas 88% said they would buy a ticket after losing the banknote
  - Many of those who decided not to buy another ticket after losing the first one attributed a price of $20 to the ticket rather than $10

Results for our class

- Group 1
  - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?

75% (6/8) chose **yes**
25% (2/8) chose **no**

Results for our class

- Group 2
  - You have decided to see a play where the admission is \$10 per ticket. As you arrive at the theater, you discover that you have lost a \$10 bill. Would you still pay \$10 for a ticket for the play?

87,50% (7/8) chose **yes**
12,50% (1/8) chose **no**

Results for 2024/01

- Group 1
  - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?

100% chose **yes**

0% chose **no**

Results for 2024/01

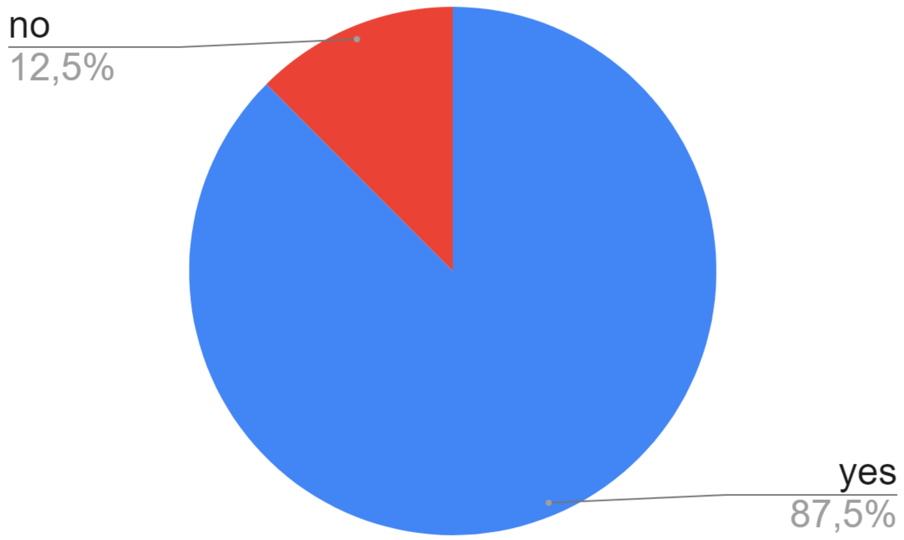
- Group 2
  - You have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?

100% chose **yes**

0% chose **no**

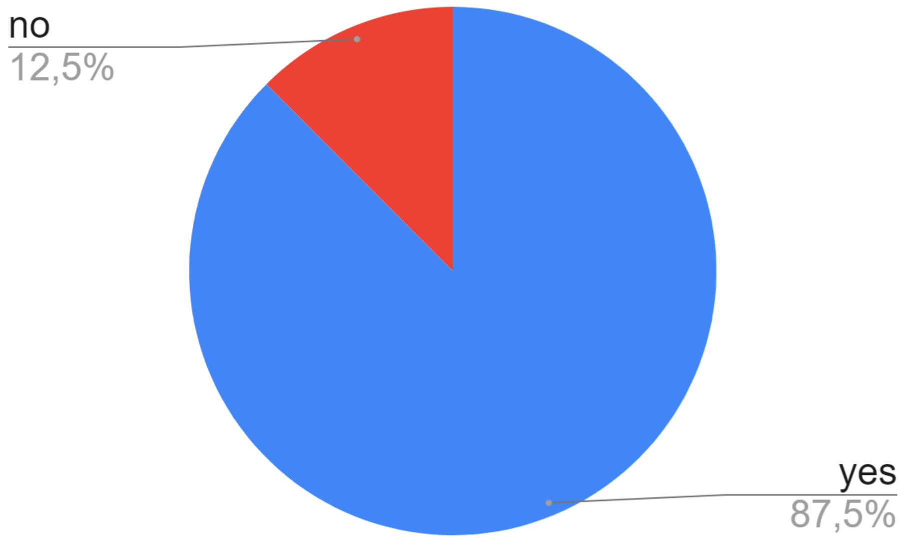
Results for 2022/01

- Group 1
  - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?



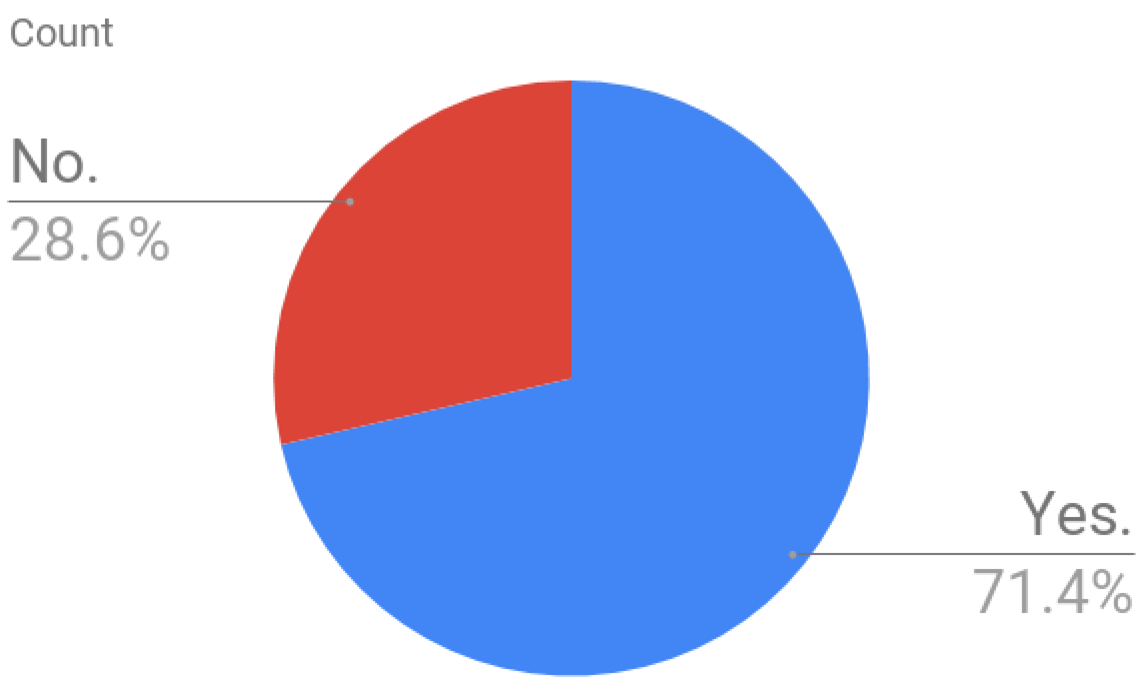
Results for 2022/01

- Group 2
  - You have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?



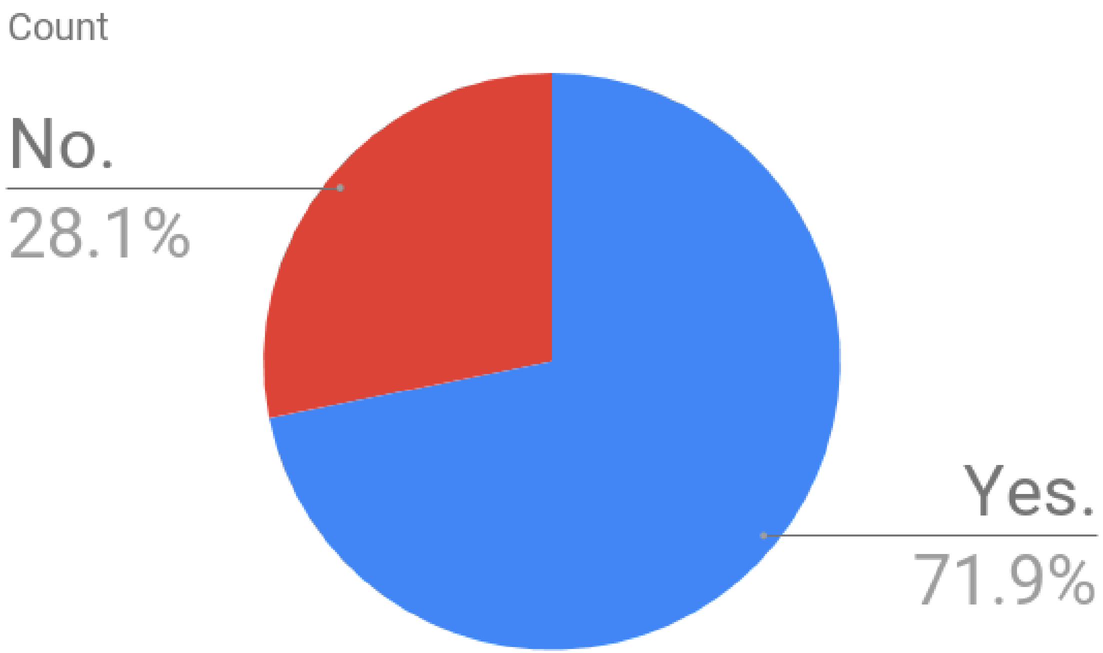
Results in 2019/02

- Group 1
  - You have decided to see a play and paid the admission price of $10 per ticket. As you approach the theater, you discover that you have lost the ticket. The seat was not marked and the ticket cannot be recovered. Would you pay $10 for another ticket?



Results in 2019/02

- Group 2
  - You have decided to see a play where the admission is $10 per ticket. As you arrive at the theater, you discover that you have lost a $10 bill. Would you still pay $10 for a ticket for the play?




FAIL

Mental Accounting

- This example demonstrates that decision makers may conduct “mental calculations” that are inconsistent with rationality

Modeling Choice Procedures

- There is a large body of evidence showing that decision makers systematically use procedures of choice that violate the classical assumptions and that the rational man paradigm is lacking
- More and more economic models in which economic agents are assumed to follow alternative procedures of choice

Modeling Choice Procedures

- Now we will include not only the set of alternatives but <u>additional information</u> as well
- This additional information is considered to be <u>irrelevant</u> to the interests of the decision maker but may nevertheless <u>affect</u> his choice

Modeling Choice Procedures

- Another is a case in which the additional information consists of a <u>default</u> option

Modeling Choice Procedures

- The statement ***c(A, a) = b*** means that when facing the choice problem ***A*** with a default alternative ***a*** the decision maker chooses the alternative ***b***
- Experimental evidence and introspection tell us that a <u>default option</u> is often viewed positively by decision makers
- This phenomenon is known as the ***<u>status quo bias</u>***

Modeling Choice Procedures

- Any examples in the real life?
  - reelection?

Modeling Choice Procedures

- Let ***X*** be a finite set of alternatives
- Define an ***extended choice function*** to be a function that assigns a unique element in ***A*** to every pair ***(A, a)*** where $\boldsymbol{A \subseteq X}$ and $\boldsymbol{a \in A}$
  - ***a*** can be any element of ***A***
- Let's see some examples...

Modeling Choice Procedures

- The decision maker has in mind a vector of orderings ($\succ_i$), which are interpreted to be criteria, and an additional ordering $\succ$ interpreted to be the real preference relation of the decision maker
- The alternative ***C(A, a)*** is the ***$\succ$-best*** element in the set of alternatives which are as good as ***a*** by all criteria
  - $C(A, a) = \{x \mid x \succ_i a \text{ for all } i\}$
- Example
  - replacing the smartphone

Modeling Choice Procedures

- Let d be a distance function on X
- The decision maker has in mind a preference relation
- The element $C(A, a)$ is the best alternative that is not too far from a
  - $C(A, a) = \{x \mid d(x, a) \leq d*\}$ for some $d*$
- Example
  - Bummer! Our favorite pizza place is closed! Where should we go now?

Modeling Choice Procedures

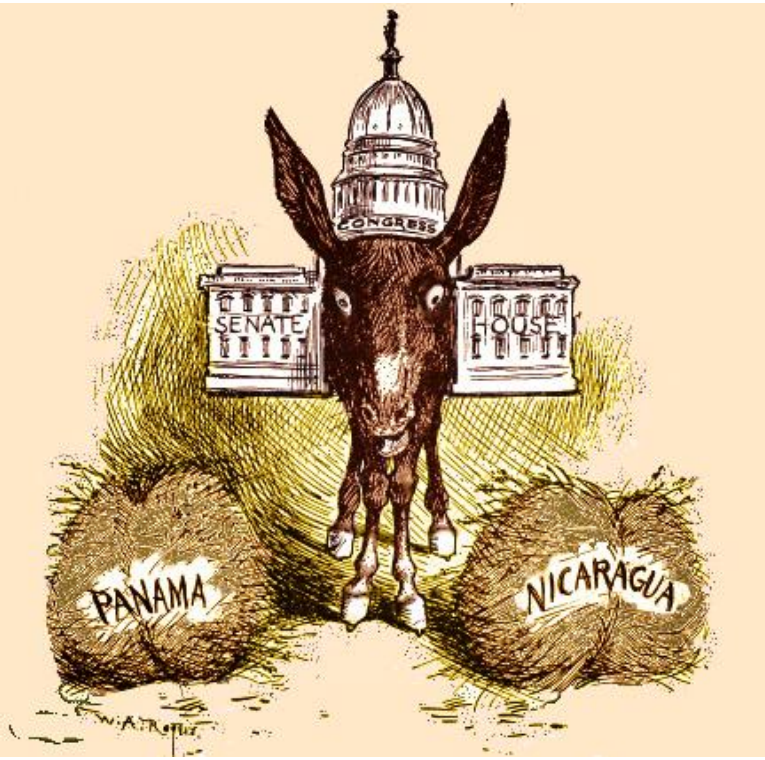
- The decision maker has two criteria in mind
- If there is a ***unique*** alternative which is "Pareto optimal" and "Pareto dominates" the default, it is chosen
- If not, then the decision maker stays with the default option
- He cannot make up his mind facing a dilemma

Modeling Choice Procedures

- Buridan's donkey

Modeling Choice Procedures

- Political cartoon c. 1900, showing the United States Congress as Buridan's ass (the two hay piles version), hesitating between a Panama route or a Nicaragua route for an Atlantic-Pacific canal



Modeling Choice Procedures

- A ***default bias***
  - The decision maker is characterized by a utility function ***u*** and a “bias function” $\boldsymbol{\beta}$, which assigns a non-negative number to each alternative
  - The function ***u*** is interpreted as representing the “true” preferences. The number $\boldsymbol{\beta(x)}$ is interpreted as the bonus attached to ***x*** when it is a default alternative

Modeling Choice Procedures

- A ***default bias***
  - Given a extended choice problem ***(A, a)***, the procedure denoted by ***DBP***$_{u,\beta}$ selects:

$$DBP_{u,\beta}(A,a) = \begin{cases} x \in A - \{a\} & if \quad u(x) > u(a) + \beta(a) \ and \ u(x) > u(y) \\ & \qquad for \ any \ y \in A - \{a, x\} \\ a & if \quad u(a) + \beta(a) > u(x), \forall x \in A - \{a\} \end{cases}$$

Modeling Choice Procedures

- A ***default bias***
  - How we characterize the set of extended choice functions that can be described as $\boldsymbol{DBP_{u,\beta}}$ for some $\boldsymbol{u}$ and $\boldsymbol{\beta}$?
- We will adopt two assumptions:
  - The Weak Axiom (WA)
  - Default Tendency (DT)

Modeling Choice Procedures

- A ***default bias***
  - The Weak Axiom (WA)
    - We say that an extended choice function $\boldsymbol{c}$ satisfies the Weak Axiom if there are no sets $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{A} \cap \boldsymbol{B}$, $\boldsymbol{a} \neq \boldsymbol{b}$ and $\boldsymbol{x}, \boldsymbol{y} \notin \{\boldsymbol{a}, \boldsymbol{b}\}$ ($\boldsymbol{x}$ and $\boldsymbol{y}$ are not necessarily distinct) such that:
    - 1. $\boldsymbol{c(A, a) = a}$ and $\boldsymbol{c(B, a) = b}$ or
      - If $\boldsymbol{a}$ is revealed to be better than $\boldsymbol{b}$ in a choice problem where $\boldsymbol{a}$ is the default, then there cannot be any choice problem in which $\boldsymbol{b}$ is revealed to be better than $\boldsymbol{a}$ when $\boldsymbol{a}$ is the default

Modeling Choice Procedures

- A ***default bias***
  - The Weak Axiom (WA)
    - We say that an extended choice function c satisfies the Weak Axiom if there are no sets $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{a}$, $\boldsymbol{b} \in \boldsymbol{A} \cap \boldsymbol{B}$, $\boldsymbol{a} \neq \boldsymbol{b}$ and $\boldsymbol{x}, \boldsymbol{y} \notin \{\boldsymbol{a}, \boldsymbol{b}\}$ (x and y are not necessarily distinct) such that:
    - 1. $\boldsymbol{c(A, a) = a}$ and $\boldsymbol{c(B, a) = b}$ or
    - 2. $\boldsymbol{c(A, x) = a}$ and $\boldsymbol{c(B, y) = b}$
      - If ***a*** is revealed to be better than ***b*** in a choice problem where neither ***a*** nor ***b*** is a default, there cannot be any choice problem in which ***b*** is revealed to be better than ***a*** when again neither ***a*** nor ***b*** is the default

Modeling Choice Procedures

- A ***default bias***
  - The Weak Axiom (WA)
    - WA implies that for every ***a*** there is a preference relation $\succ_a$ such that ***c(A, a)*** is the $\succ_a$***-maximal*** element in ***A***

Modeling Choice Procedures

- A ***default bias***
  - Default Tendency (DT)
    - If $c(A, x) = a$, then $c(A, a) = a$
    - If the decision maker chooses ***a*** from a set ***A*** when $x \neq a$ is the default, he does not change his mind if ***x*** is replaced by ***a*** as the default alternative

Modeling Choice Procedures

- A ***default bias***
  - Proposition:
    - An extended choice function ***c*** satisfies ***WA*** and ***DT*** if and only if it is a default-bias procedure ($\boldsymbol{DBP_{u,\beta}}$)
    - Proof in the book