Modeling Rational Agents

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Modeling Human Behavior

- What is (mathematical) modeling?
  - A description of a system using mathematical concepts and language
  - May help to
    - explain a system
    - study the effects of different components
    - make predictions about behavior

Modeling Human Behavior

- What is game theory?
  - The study of mathematical models of conflict and cooperation between <u>intelligent rational decision-makers</u>

Modeling Human Behavior

- How can we mathematically describe the conflicts between two agents?

Game Theory

- A model of a rational agent
  - Preferences
  - Utility
  - Choice
- Conflicts between agents
  - Game theoretic models

Preferences

Economic Agent

- Which characteristics are required to model an economic agent?
  - Name, age and gender, personal history, brain structure, cognitive abilities, his emotional state etc
- In most of economic theory, an economic agent is modeled ***only by his attitude*** toward the elements in some relevant set
- His attitude is expressed in the form of ***preferences***

Preferences

- Which object in the set ***X*** do you prefer?

X

Preferences

- A description of preferences should <u>fully specify</u> the attitude of the agent toward each pair of elements in ***X***

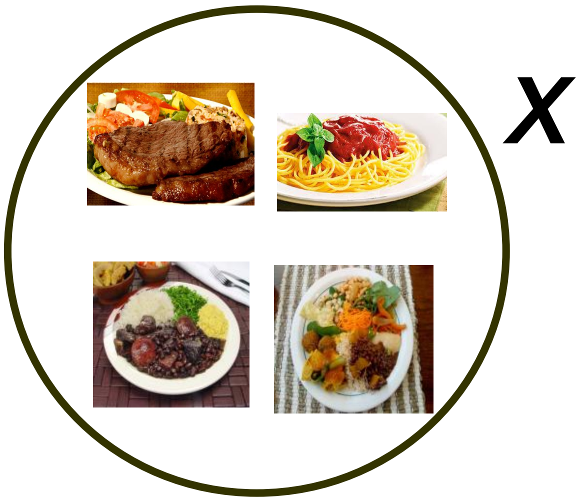
X

Questionnaire Q

Q(x,y) (for all distinct x and y in X):

How do you compare x and y? Tick one and only one of the following three options:

- ☐ I prefer x to y (this answer is denoted as $x \succ y$).
- ☐ I prefer y to x (this answer is denoted by $y \succ x$).
- ☐ I am indifferent (this answer is denoted by I).



Questionnaire Q

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How do you compare x and y? Tick one and only one of the following three options:

- ☐ I prefer x to y (this answer is denoted as $x \succ y$).
- ☐ I prefer y to x (this answer is denoted by $y \succ x$).
- ☐ I am indifferent (this answer is denoted by I).

- A "***legal***" answer to the questionnaire is a response in which exactly one of the boxes is ticked in each question

Questionnaire Q

- Exclusion of responses that demonstrate a ***lack of ability to compare***, such as:

☐ They are incomparable.

Questionnaire Q

- Exclusion of responses that demonstrate a ***dependence of other factors***, such as:

☐ It depends on what my parents think.

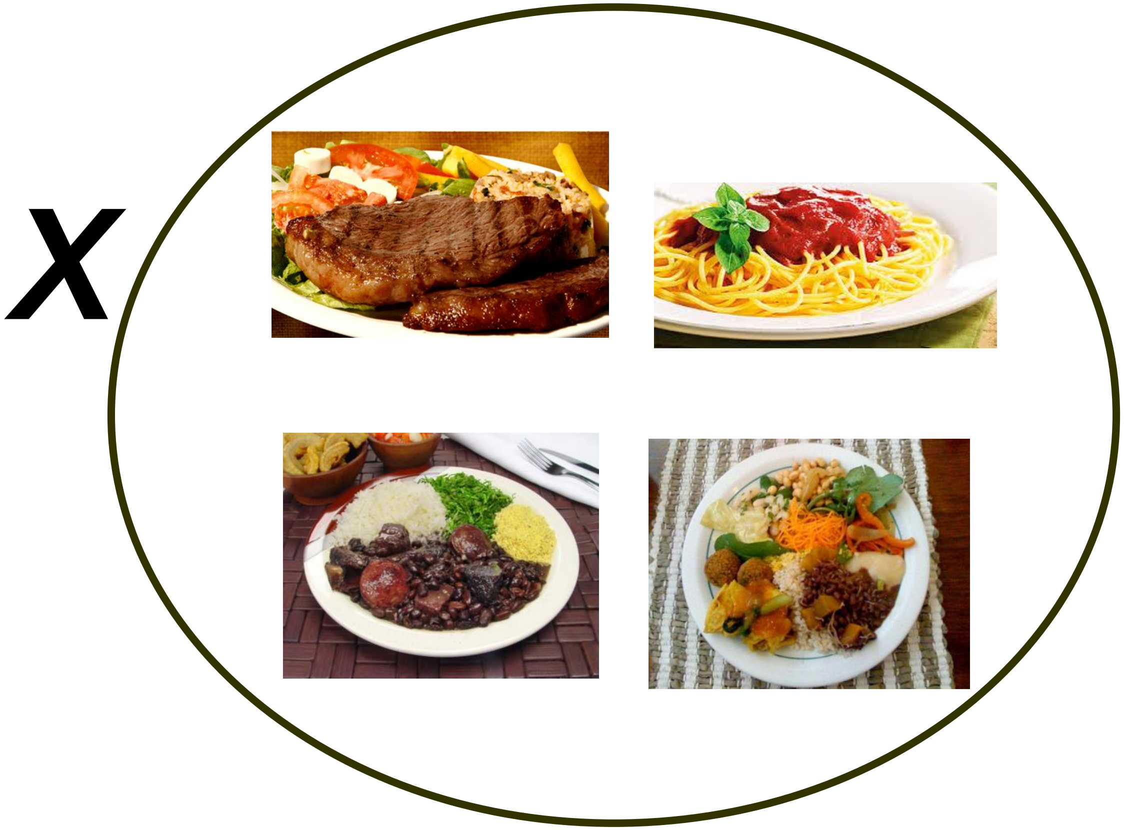
Questionnaire Q

- Exclusion of responses that demonstrate an ***intensity of preferences***, such as:

☐ I somewhat prefer x.

Questionnaire Q

- The elements in the set ***X*** are all comparable and the **intensity** of preferences are ignored



Questionnaire Q

A legal answer to the questionnaire can be formulated as a function ***f***,

which assigns to any pair ***(x, y)*** of distinct elements in ***X*** exactly one of the three "values",

- $x \succ y$ or
- $y \succ x$ or
- ***I***,

with the interpretation that ***f(x, y)*** is the answer to the question ***Q(x, y)***

Questionnaire Q

$$Q(x, y) \rightarrow f(x, y) \begin{cases} x \succ y \\ y \succ x \\ I \end{cases}$$

Preference symbol

$$y \succ x$$

Preferences

- **Definition 1**
  - Preferences on a set $\boldsymbol{X}$ are a function $\boldsymbol{f}$
  - that assigns to any pair $\boldsymbol{(x, y)}$ of distinct elements in $\boldsymbol{X}$ exactly one of the three "values"
  - $\boldsymbol{x \succ y}$, $\boldsymbol{y \succ x}$, or $\boldsymbol{I}$
  - so that for any three different elements $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ in $\boldsymbol{X}$, the following two properties hold:
    - No order effect: $\boldsymbol{f(x, y) = f(y, x)}$
    - Transitivity:
      - if $\boldsymbol{f(x, y) = x \succ y}$ and $\boldsymbol{f(y, z) = y \succ z}$, then $\boldsymbol{f(x, z) = x \succ z}$ and
      - if $\boldsymbol{f(x, y) = I}$ and $\boldsymbol{f(y, z) = I}$, then $\boldsymbol{f(x, z) = I}$

A discussion of transitivity

- How would you react if somebody told you she/he prefers $\boldsymbol{x}$ to $\boldsymbol{y}$, $\boldsymbol{y}$ to $\boldsymbol{z}$, and $\boldsymbol{z}$ to $\boldsymbol{x}$?

A discussion of transitivity

Questionnaire

Consider the travel options bellow and answer: which one do you prefer?

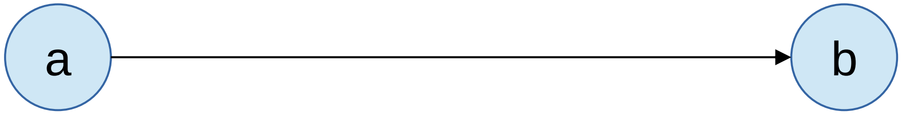
Open a text file on you computer and write an answer file in the following format. Each line contains your answer for a question. Write ***1*** if you prefer the first option, ***2*** if you prefer the second, or ***0*** if you are indifferent between the options. There is an example of an answer file at the end of this document.

1) A weekend at a 3 star hotel in New York with friends for $574 OR a weekend for $574 with friends at a 3 star hotel in Paris?

2) A weekend in New York with friends at a 3 star hotel for $574 OR a weekend at a 5 star hotel in New York for $712 with family?

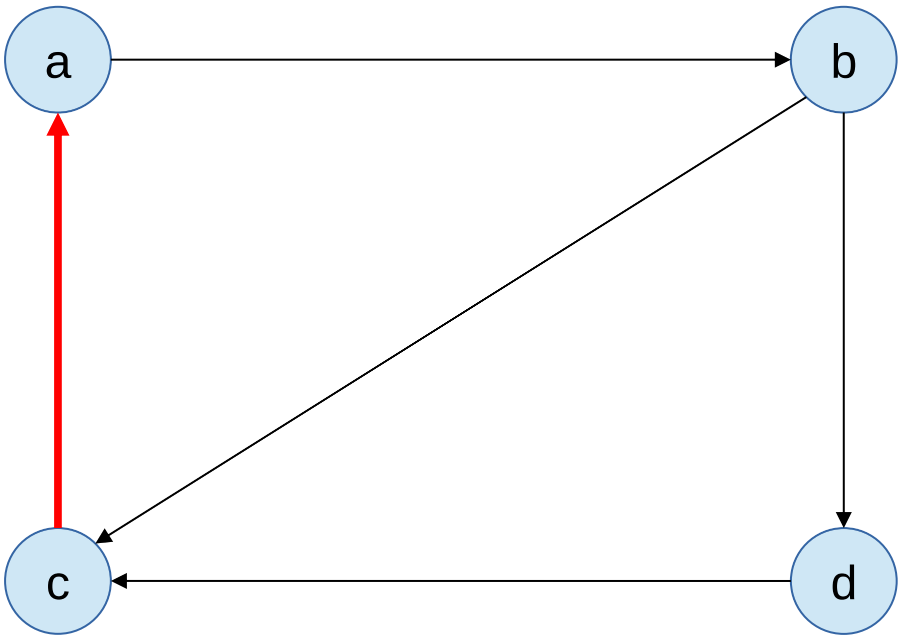
A discussion of transitivity

– There is a direct edge from ***i*** to ***j*** if $\boldsymbol{j} \succ \boldsymbol{i}$



A discussion of transitivity

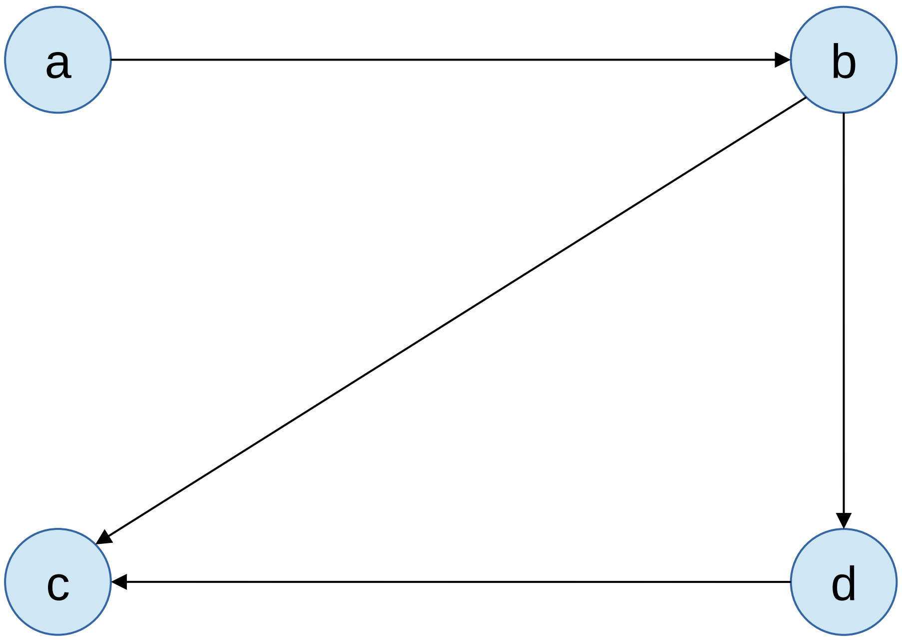
- How to check intransitivities from this questionnaire?



intransitivity detected!

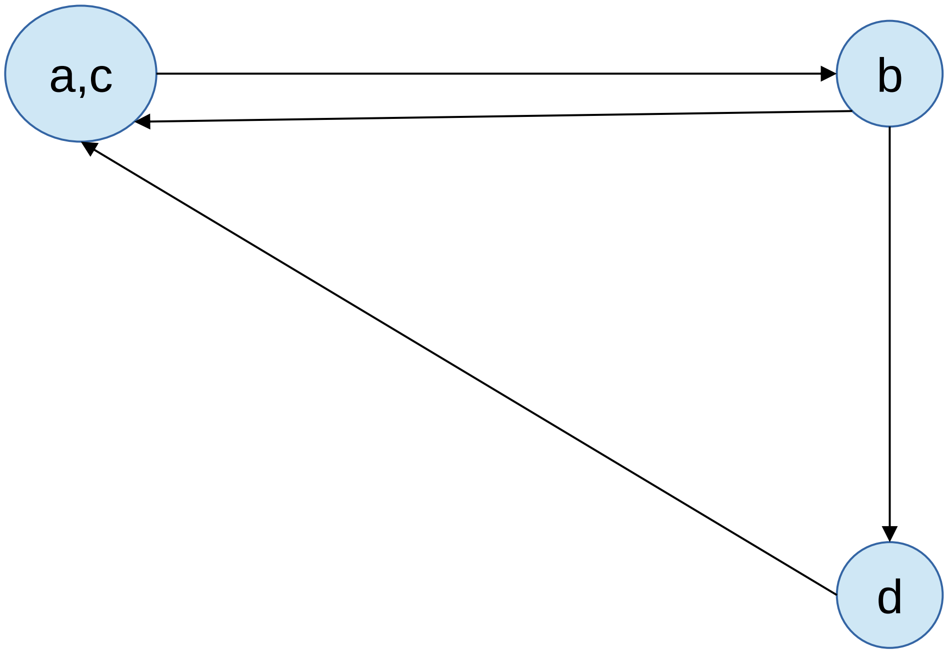
A discussion of transitivity

- And if I am indifferent between ***a*** and ***c***?



A discussion of transitivity

- And if I am indifferent between ***a*** and ***c***?



A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of **6** students who responded questionnaire **Q1** in 2025/01…
- ...3 (50%) had no intransitivities
- The median number intransitivities per student was 0.5
- The mean was 1.17

Easy questions?

6) A weekend at a 5 star hotel with romance for $842 in New York OR a weekend at a 5 star hotel for $574 in New York with romance?

30) A weekend in New York at a 3 star hotel for $842 with friends OR a weekend at a 5 star hotel for $574 in New York with friends?

31) A weekend with friends at a 5 star hotel in New York for $842 OR a weekend for $842 with friends in New York at a 3 star hotel?

37) A weekend with friends in Paris at a 5 star hotel for $574 OR a weekend at a 3 star hotel for $574 with friends in Paris?

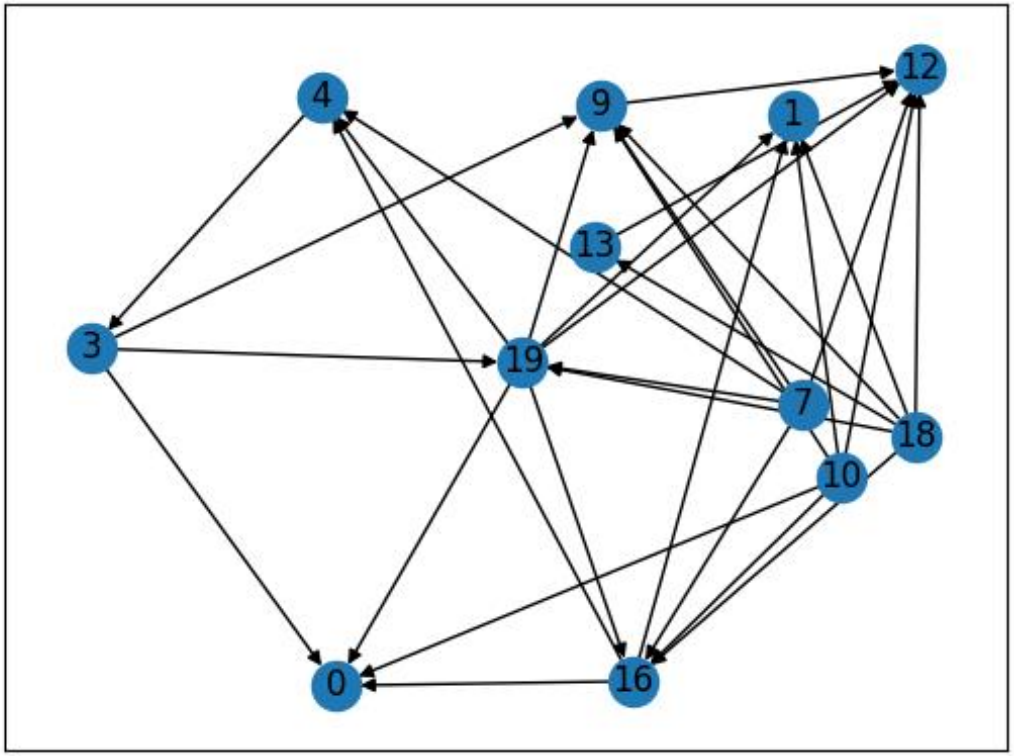
38) A weekend with friends at a 3 star hotel for $574 in Paris OR a weekend at a 3 star hotel with friends for $842 in Paris?

39) A weekend for $574 with romance at a 5 star hotel in New York OR a weekend for $842 at a 3 star hotel in New York with romance?

Easy questions?

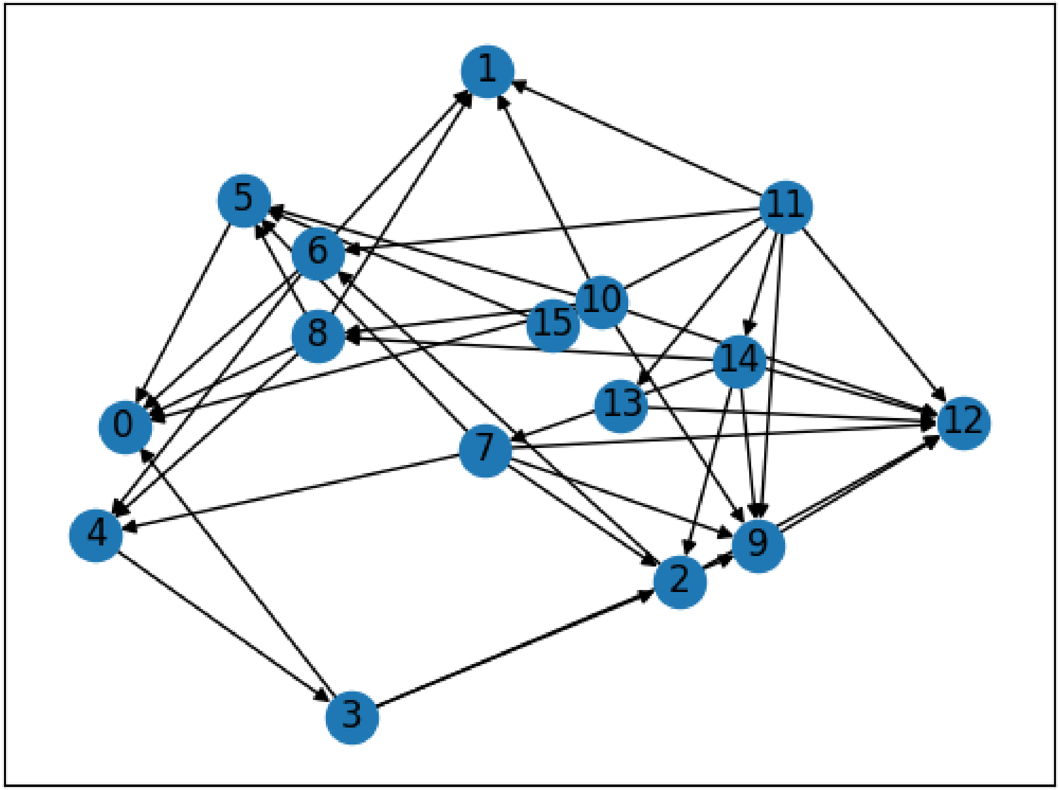
- How would you react if somebody fails to answer those easy questions?
- Out of 6 students who responded questionnaire Q1 in 2025/01…
- ...2 (~33%) gave unreasonable answers to the easy questions

Thiago Assis



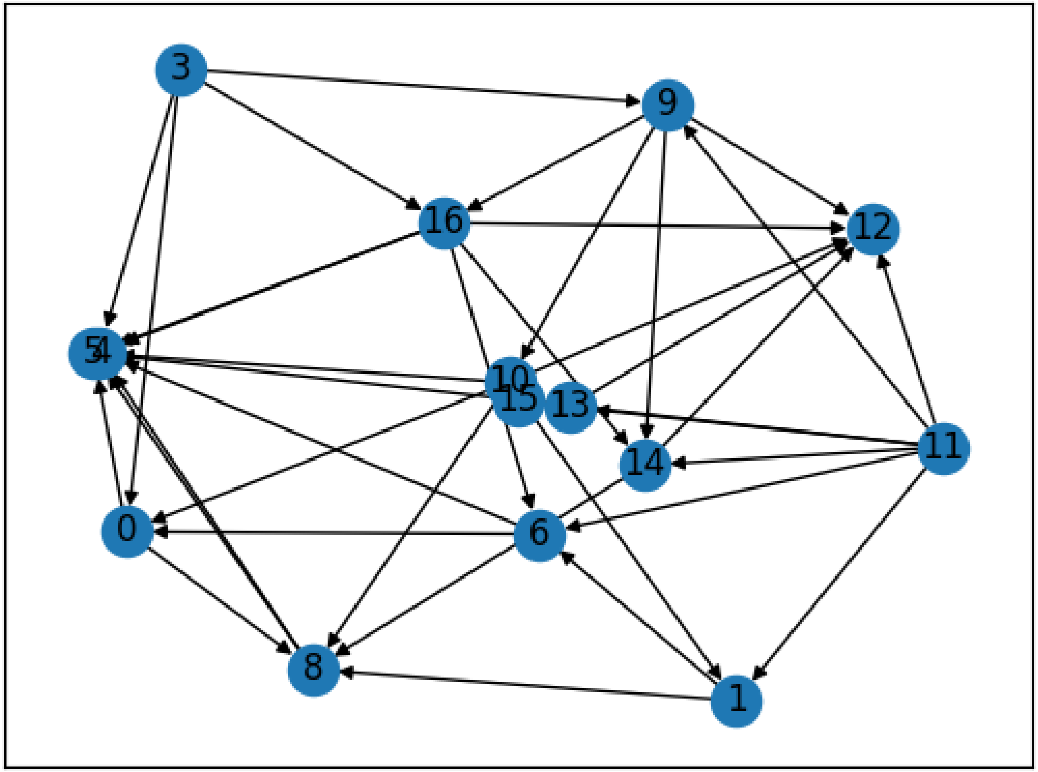
Number of nodes: 12
Number of cycles: 2
Cycles: [[16, 4, 3, 19], [3, 19, 4]]
Number of edges 31
Number of dominated choices: 2

Laila Melo



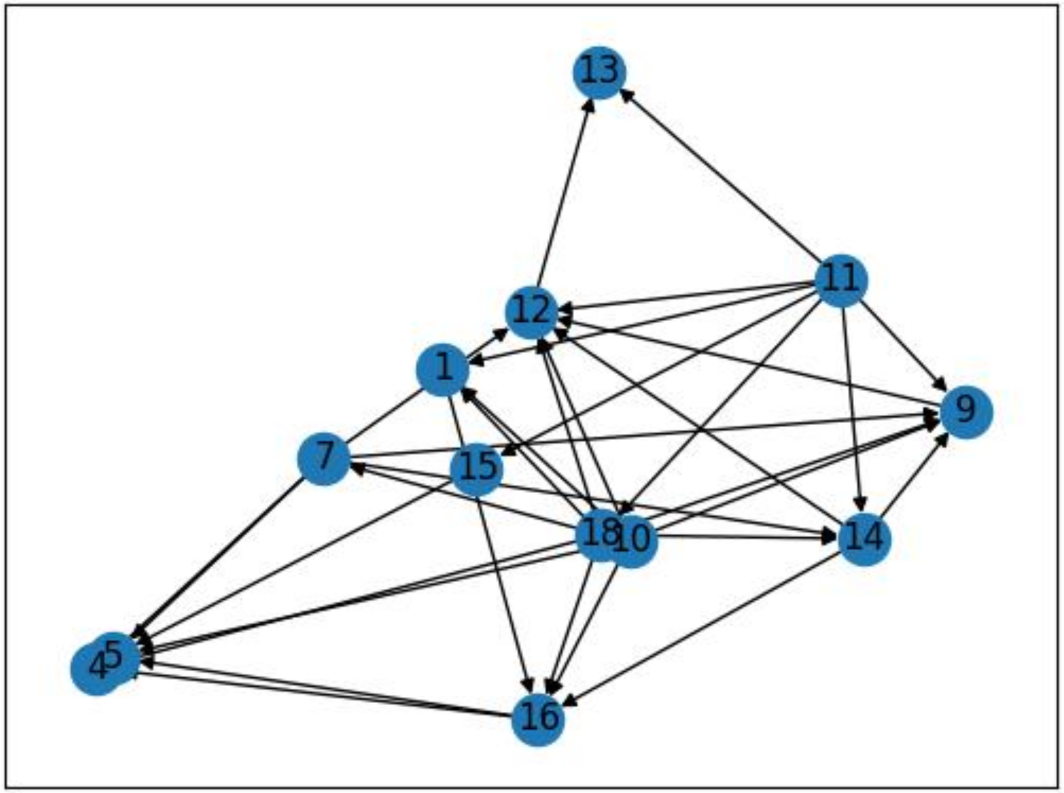
Number of nodes: 16
Number of cycles: 1
Cycles: [[2, 6, 4, 3]]
Number of edges 40
Number of dominated choices: 0

Henrique Magalhães



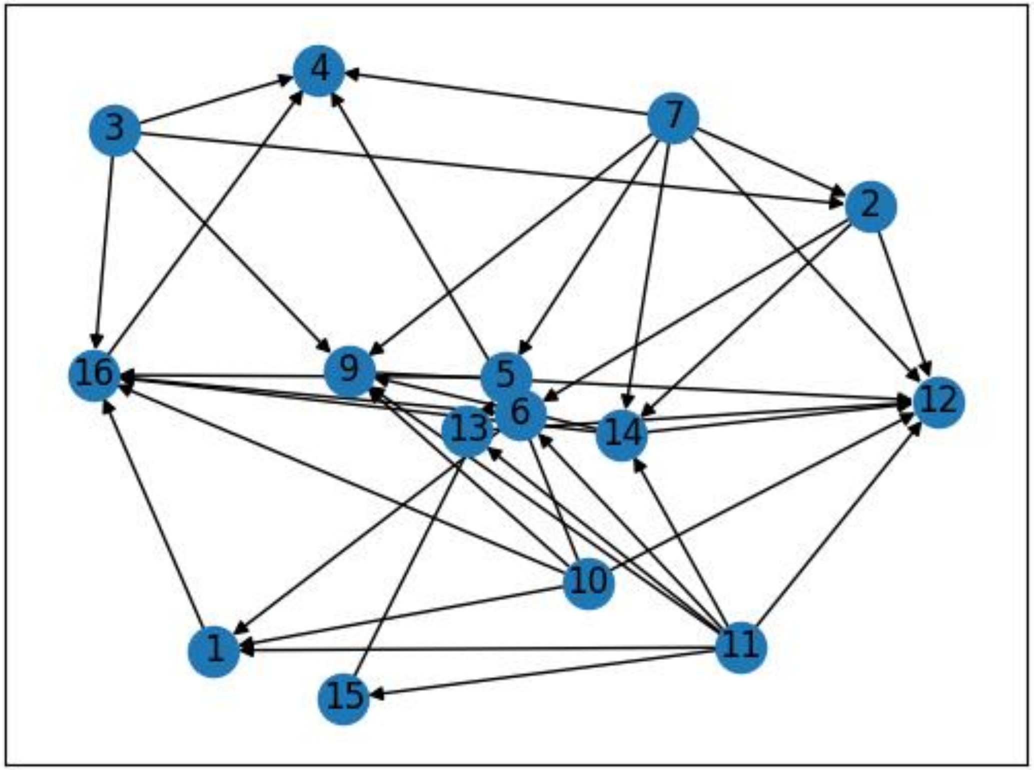
Number of nodes: 15
Number of cycles: 0
Cycles: []
Number of edges 37
Number of dominated choices: 0

Haniel Botelho



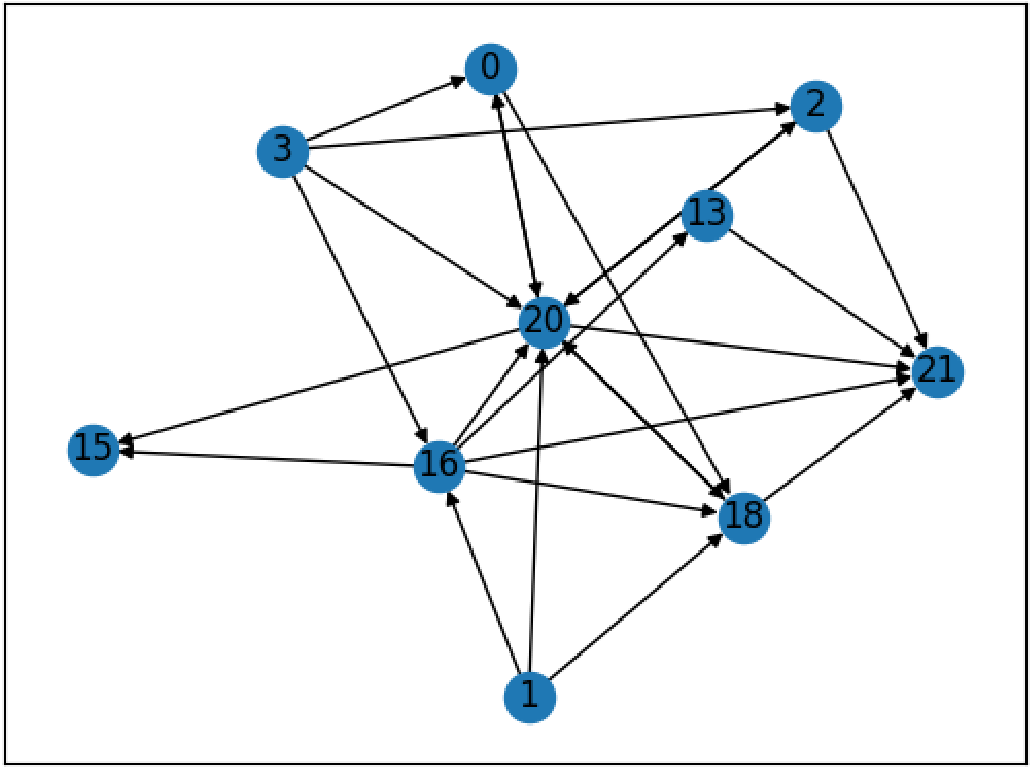
Number of nodes: 13
Number of cycles: 0
Cycles: []
Number of edges 33
Number of dominated choices: 3

Lorenzo Correa



Number of nodes: 15
Number of cycles: 0
Cycles: []
Number of edges 37
Number of dominated choices: 0

Matheus Farnese



Number of nodes: 10
Number of cycles: 4
Cycles: [[0, 20], [0, 18, 20], [2, 20], [18, 20]]
Number of edges 24
Number of dominated choices: 0

Semestres Anteriores

A discussion of transitivity

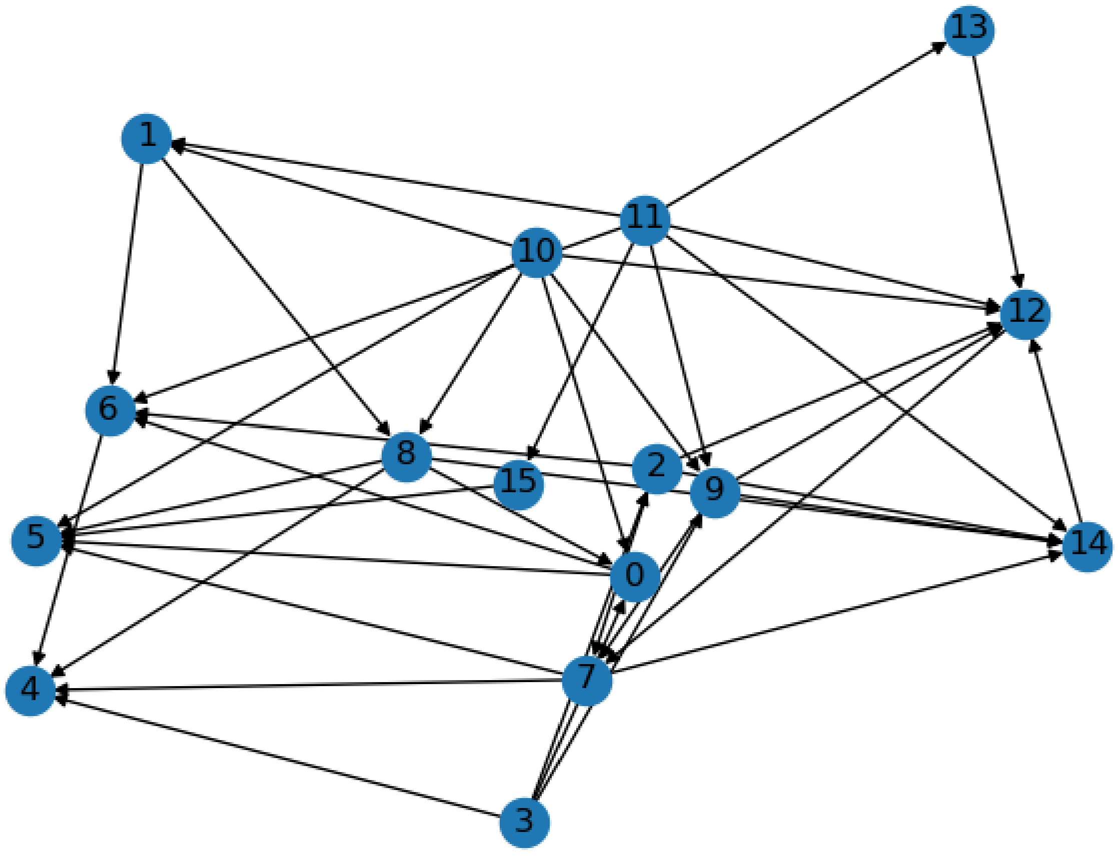
- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of **13** students who responded questionnaire **Q1** in 2024/01…
- ...6 (~46%) had no intransitivities
- The median number intransitivities per student was 1
- The mean was 5.15

Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 13 students who responded questionnaire Q1 in 2024/01…
- ...1 (~8%) gave unreasonable answers to the easy questions

A discussion of transitivity

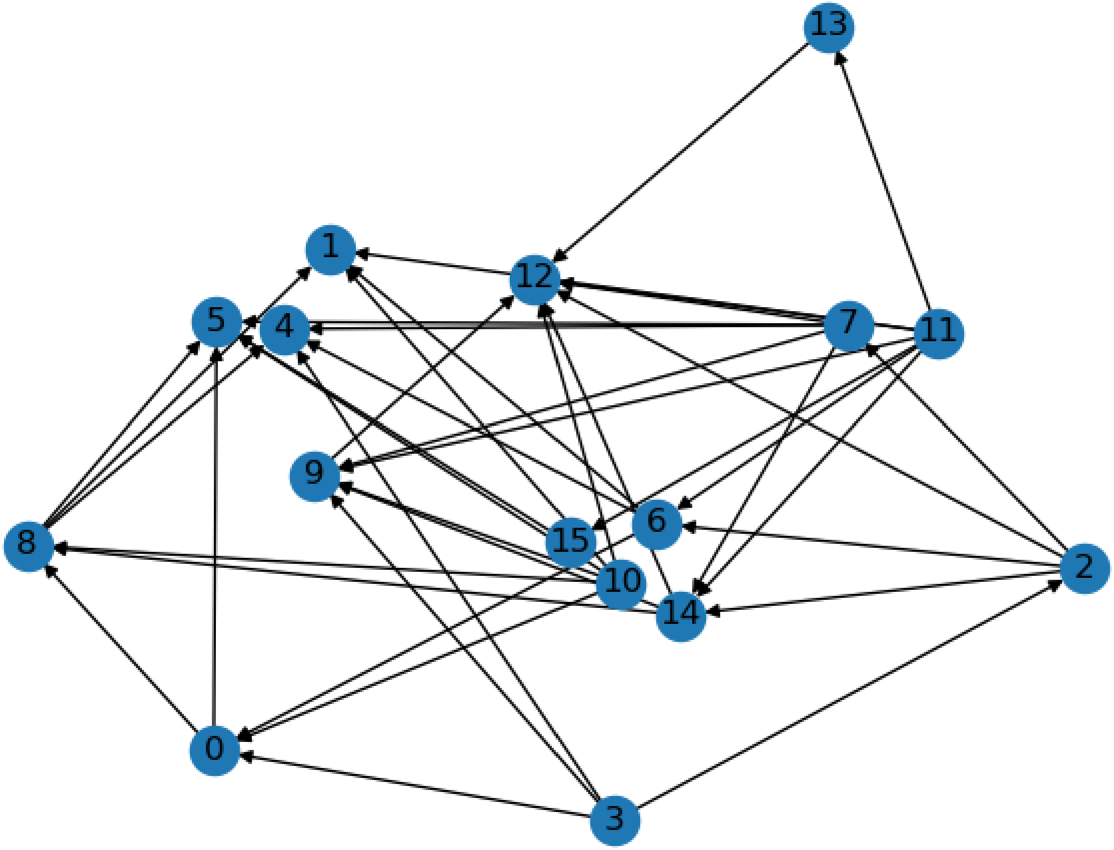
- # of intransitivities: 1 (AV)



[12, 7, 14]

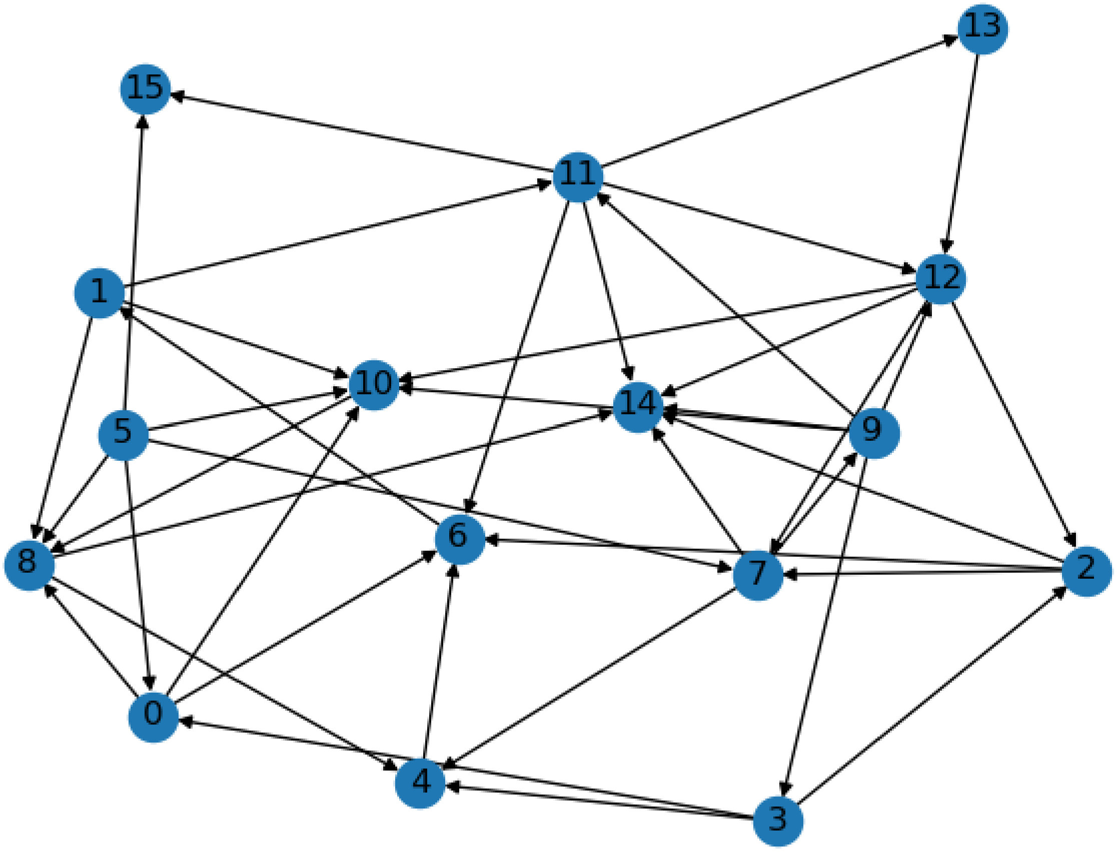
A discussion of transitivity

- # of intransitivities: 0 (AA)



A discussion of transitivity

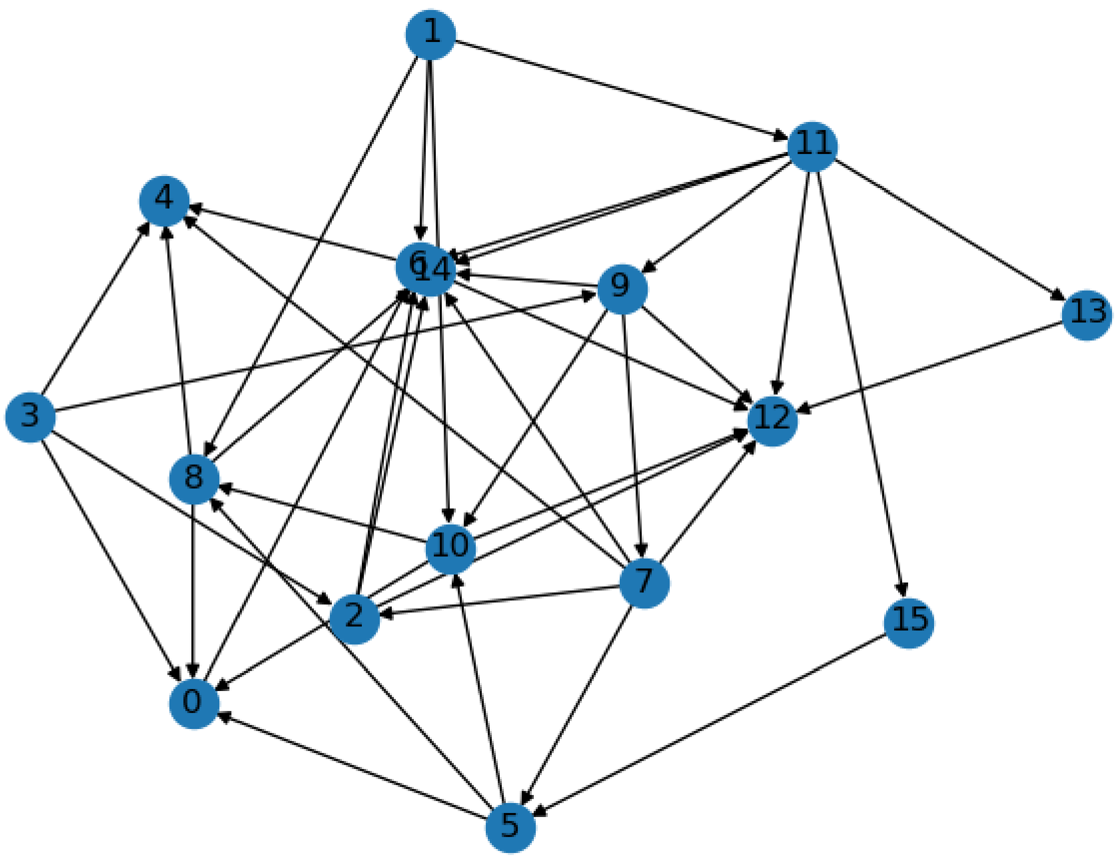
- # of intransitivities: 40 (AB)



[[0, 10, 8, 4, 6, 1, 11, 13, 12, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 12, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 12, 2, 7, 9, 3], [0, 8, 4, 6, 1, 11, 13, 12, 7, 9, 3], [0, 8, 4, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 8, 4, 6, 1, 11, 12, 7, 9, 3], [0, 8, 4, 6, 1, 11, 12, 2, 7, 9, 3], [0, 6, 1, 11, 13, 12, 7, 9, 3], [0, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 6, 1, 11, 12, 7, 9, 3], [0, 6, 1, 11, 12, 2, 7, 9, 3], [1, 11, 13, 12, 7, 4, 6], [1, 11, 13, 12, 7, 9, 10, 8, 4, 6], [1, 11, 13, 12, 7, 9, 3, 4, 6], [1, 11, 13, 12, 7, 9, 3, 2, 6], [1, 11, 13, 12, 10, 8, 4, 6], [1, 11, 13, 12, 2, 7, 4, 6], [1, 11, 13, 12, 2, 7, 9, 10, 8, 4, 6], [1, 11, 13, 12, 2, 7, 9, 3, 4, 6], [1, 11, 13, 12, 2, 6], [1, 11, 12, 7, 4, 6], [1, 11, 12, 7, 9, 10, 8, 4, 6], [1, 11, 12, 7, 9, 3, 4, 6], [1, 11, 12, 7, 9, 3, 2, 6], [1, 11, 12, 10, 8, 4, 6], [1, 11, 12, 2, 7, 4, 6], [1, 11, 12, 2, 7, 9, 10, 8, 4, 6], [1, 11, 12, 2, 7, 9, 3, 4, 6], [1, 11, 12, 2, 6], [1, 11, 6], [1, 8, 4, 6], [1, 10, 8, 4, 6], [2, 7, 9, 11, 13, 12], [2, 7, 9, 11, 12], [2, 7, 9, 3], [2, 7, 9, 12], [7, 9, 11, 13, 12], [7, 9, 11, 12], [7, 9, 12]]

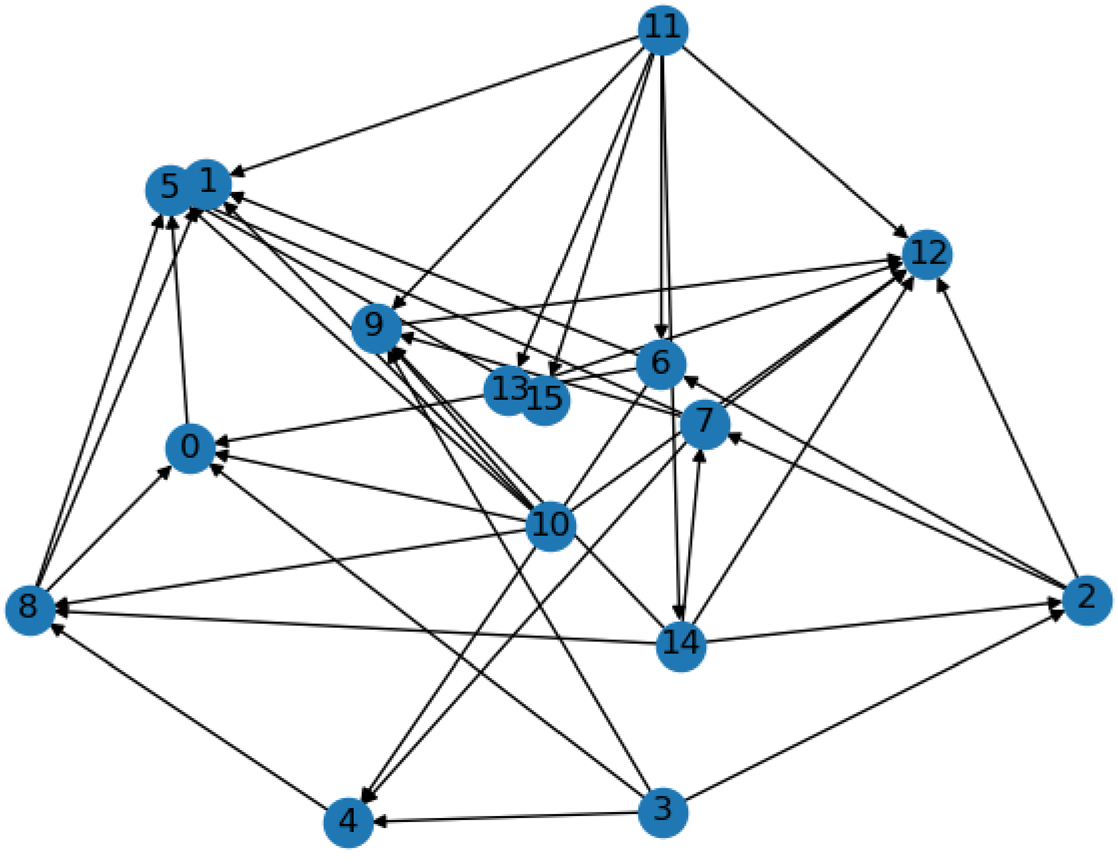
A discussion of transitivity

- # of intransitivities: 0 (AD)



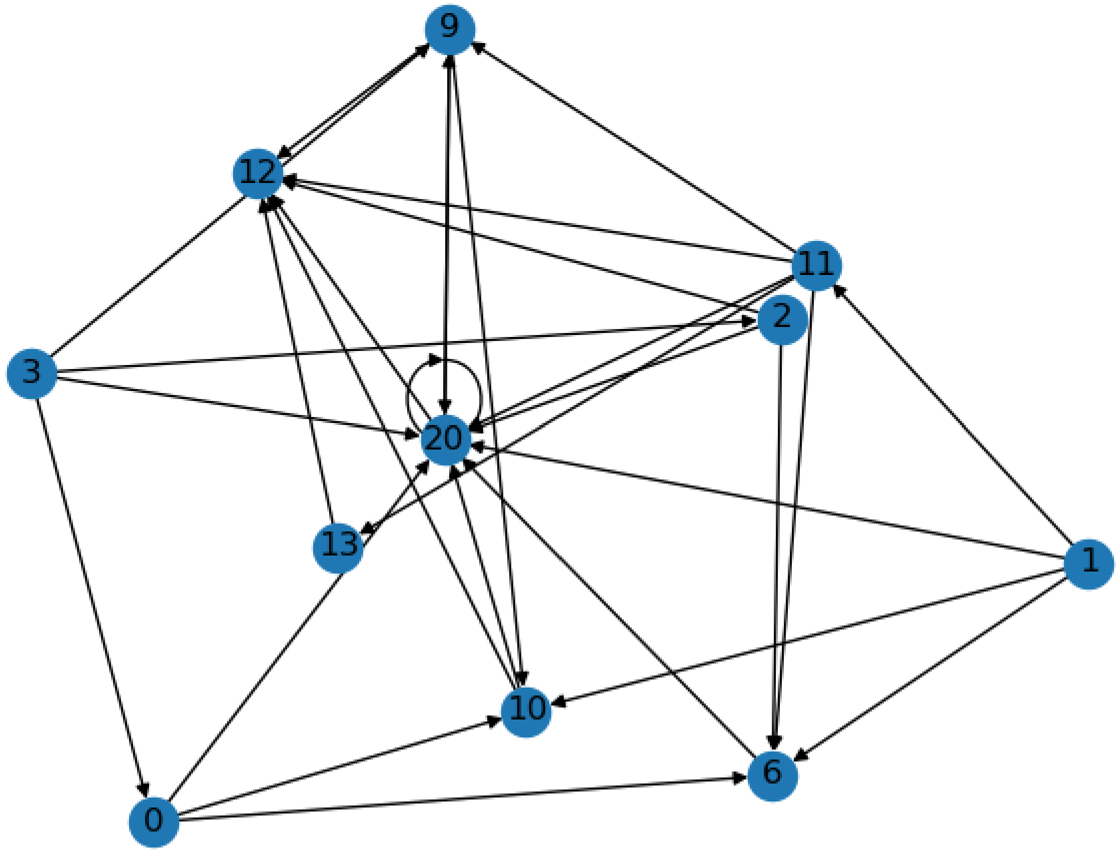
A discussion of transitivity

- # of intransitivities: 0 (GL)



A discussion of transitivity

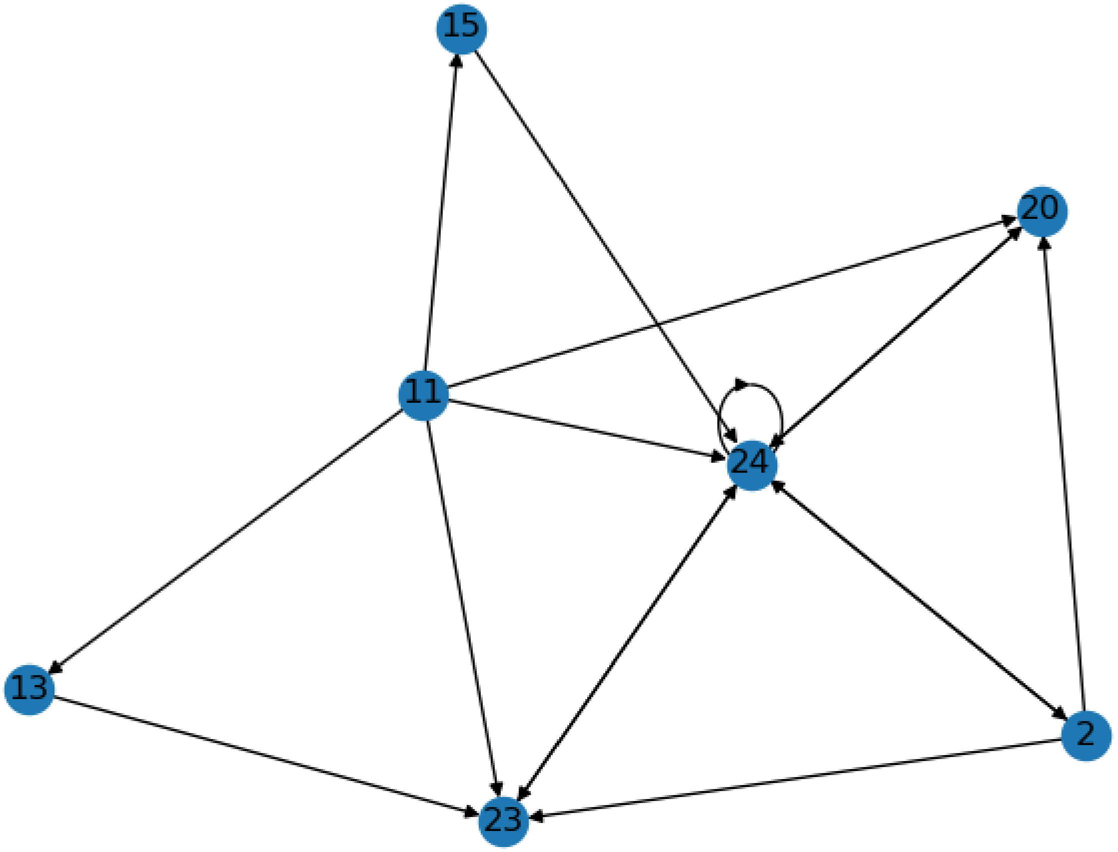
- # of intransitivities: 3 (KN)



[20]
[9, 20]
[9, 10, 20]

A discussion of transitivity

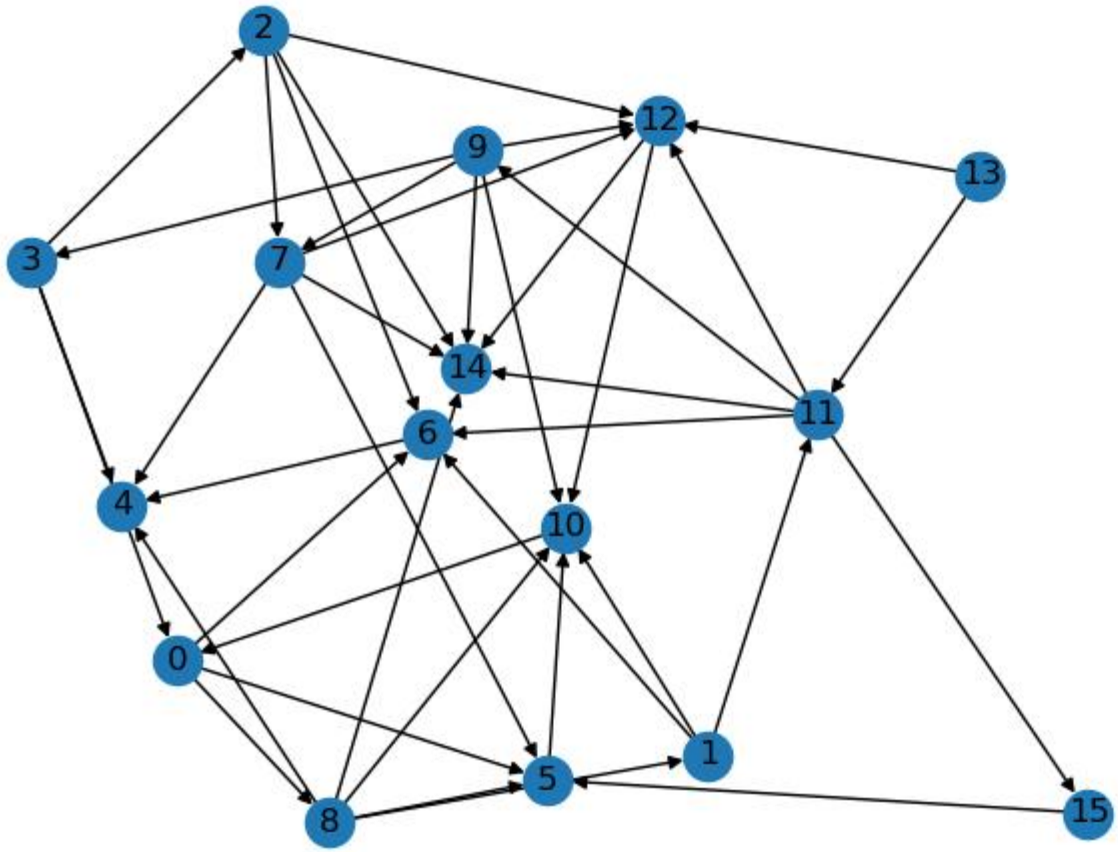
- # of intransitivities: 6 (LD)



[24], [24,23]
[24, 20], [24, 2]
[24, 2, 23],
[24, 2, 20]

A discussion of transitivity

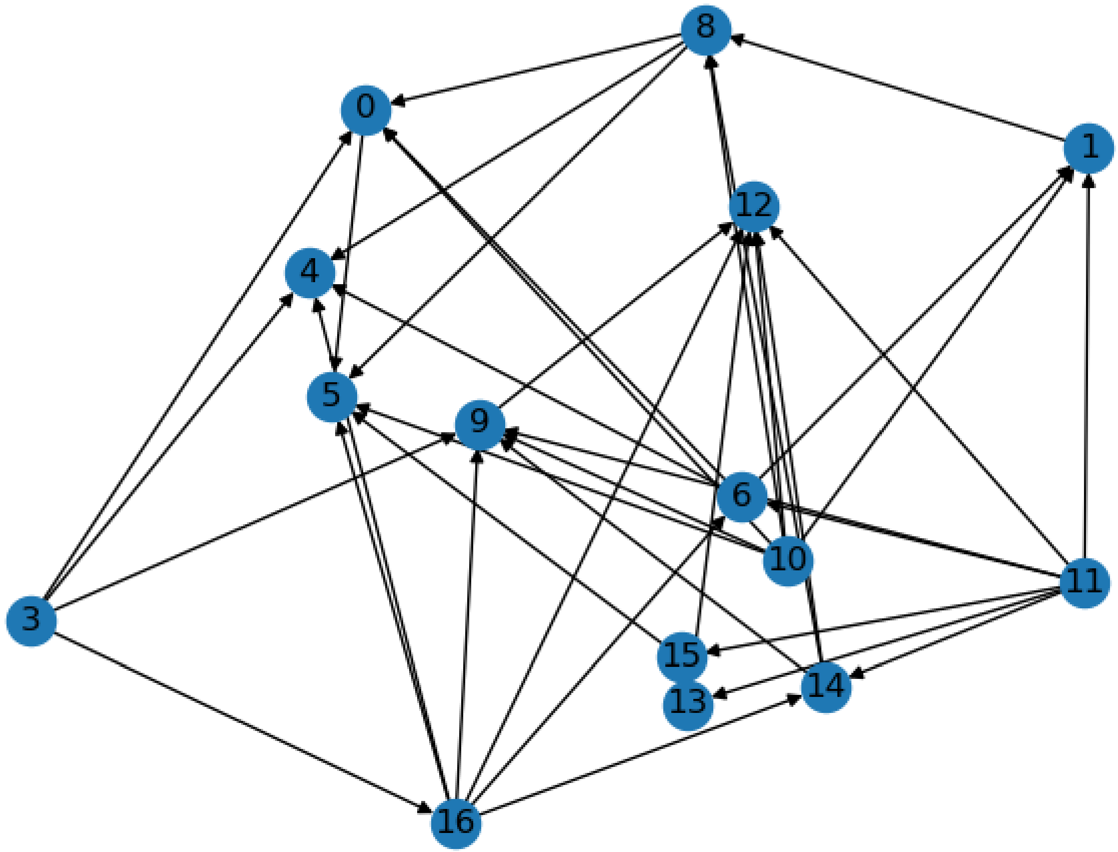
- # of intransitivities: 14 (LR)



[24], [24,23]
[24, 20], [24, 2]
[24, 2, 23],
[24, 2, 20]

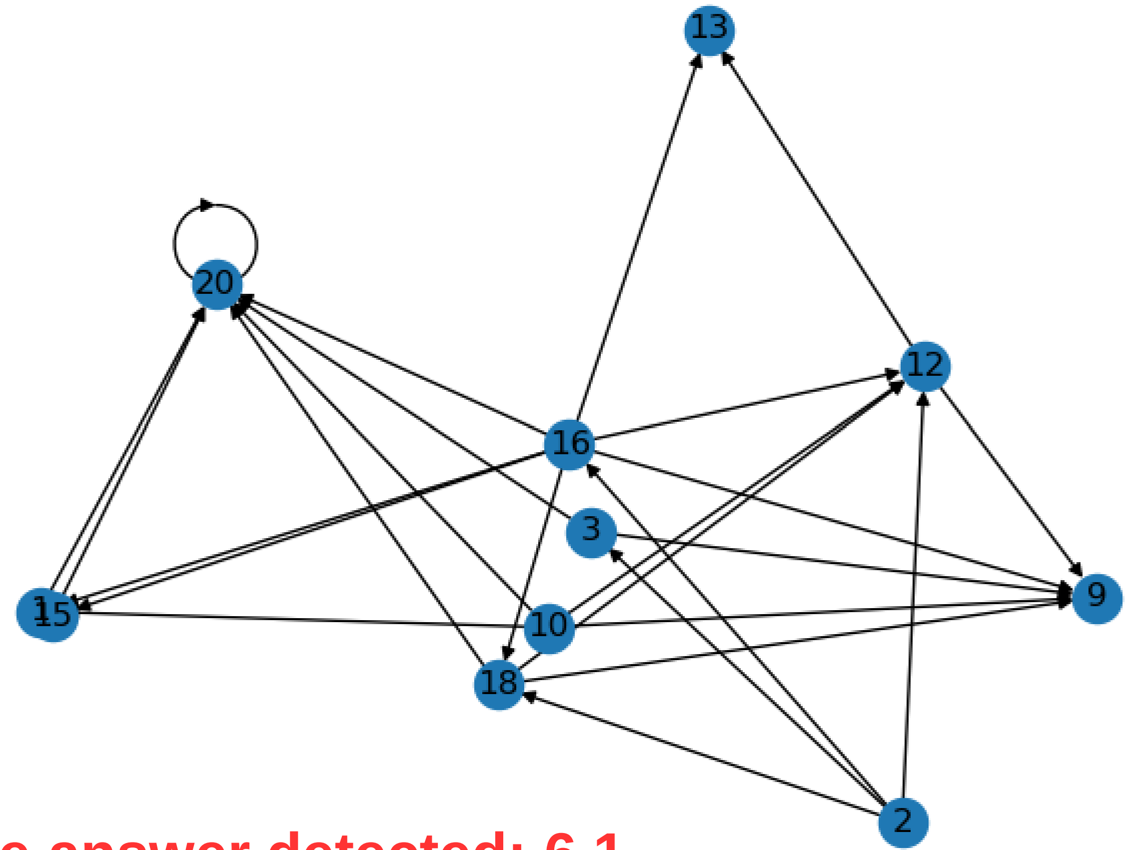
A discussion of transitivity

- # of intransitivities: 0 (LR)



A discussion of transitivity

- # of intransitivities: 1 (MM)

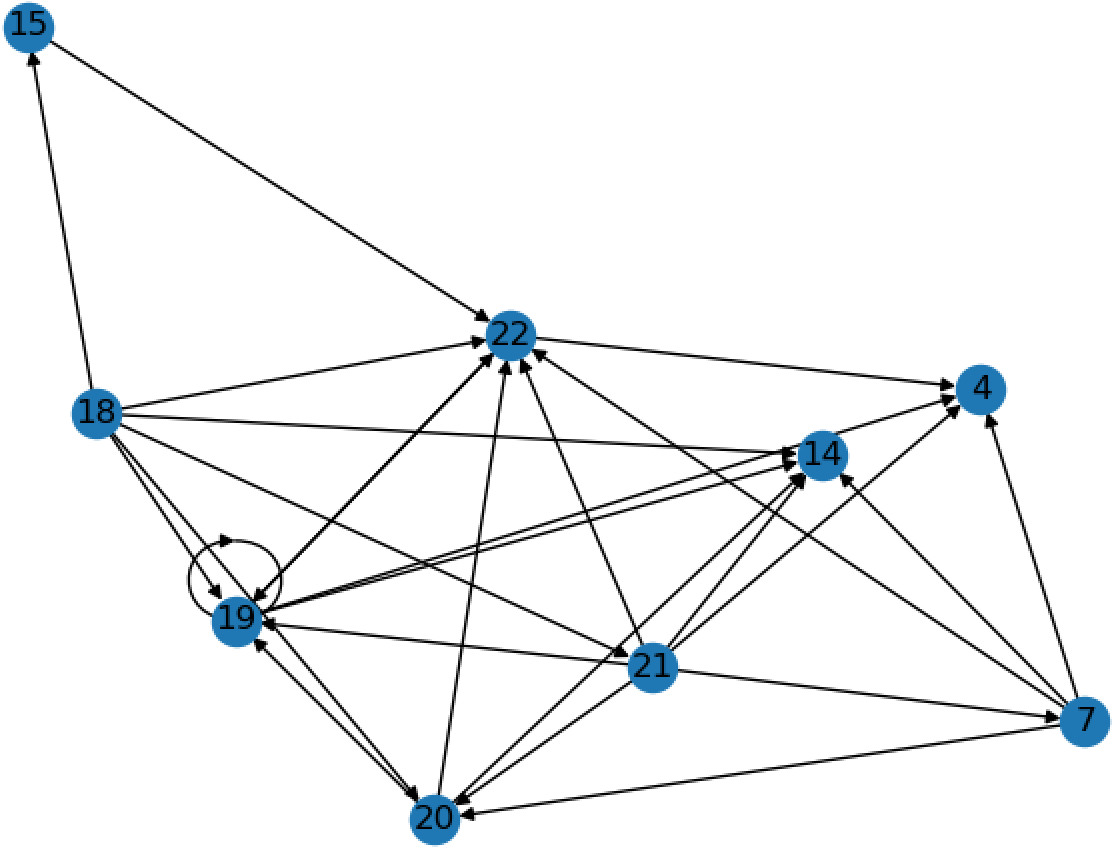


[20]

unreasonable answer detected: 6 1
unreasonable answer detected: 38 2
unreasonable answer detected: 39 2

A discussion of transitivity

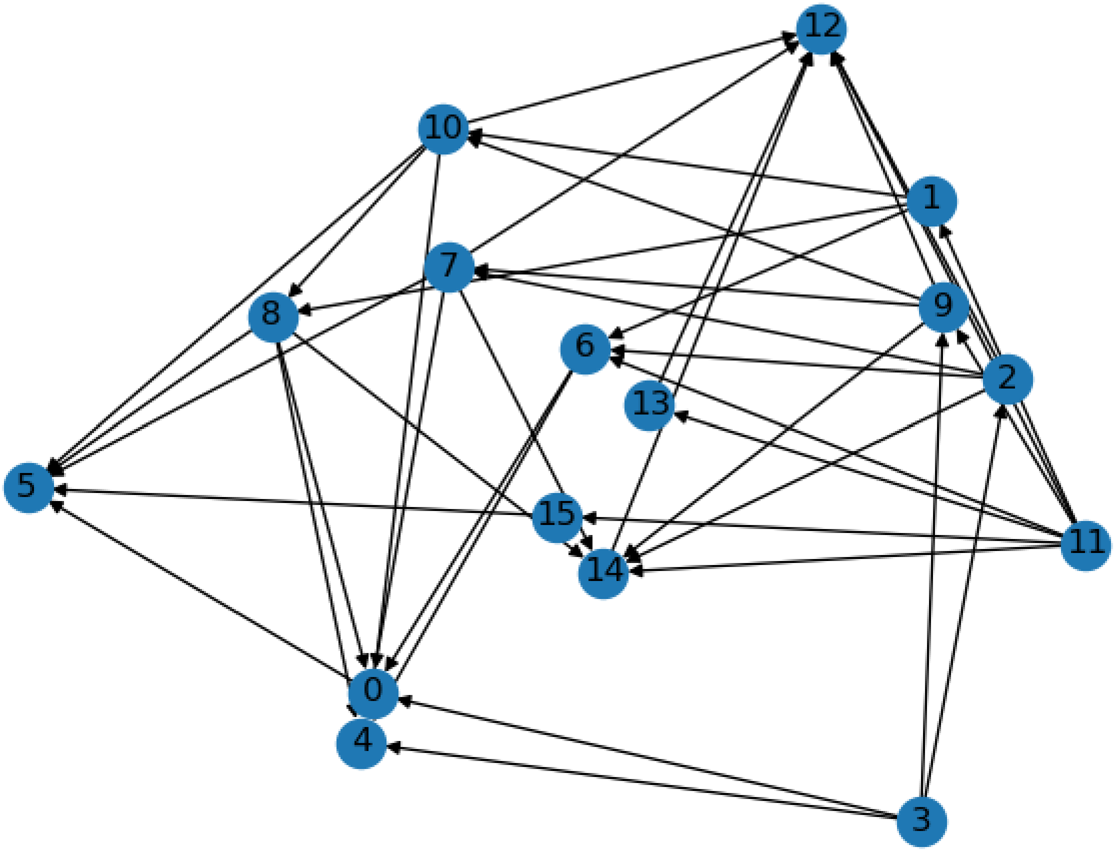
- # of intransitivities: 2 (MI)



[19]
[19, 22]

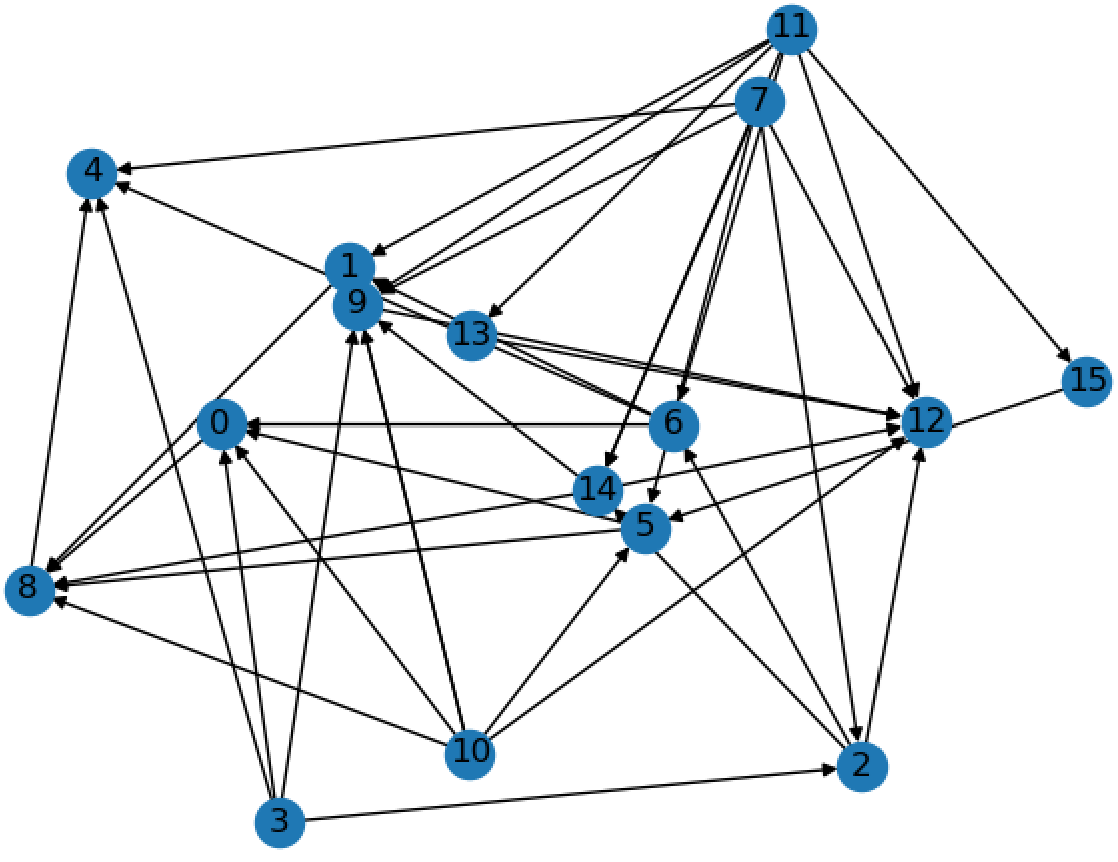
A discussion of transitivity

- # of intransitivities: 0 (PF)



A discussion of transitivity

- # of intransitivities: 0 (PS)



Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 15 students who responded questionnaire Q1 in 2022/01…
- ...3 (20%) gave unreasonable answers to the easy questions

A discussion of transitivity

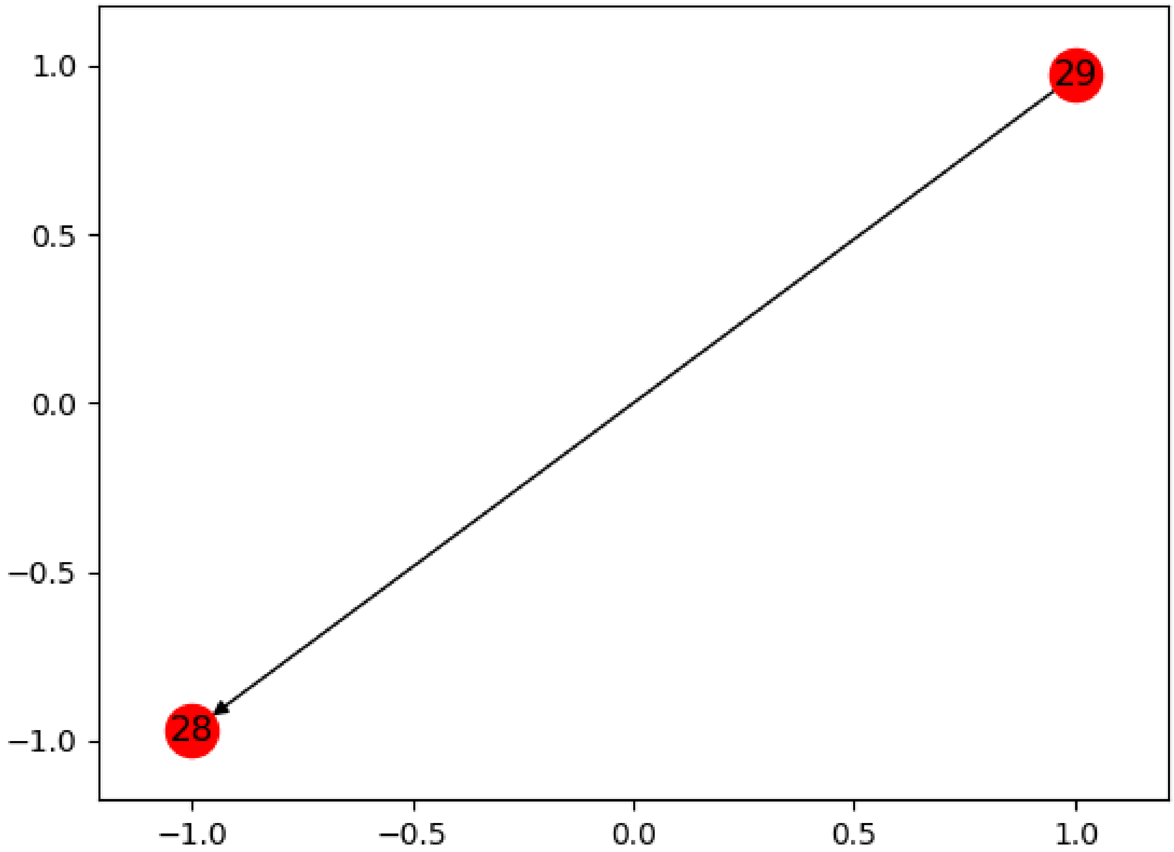
- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 63 students who responded questionnaire Q1 in 2019/02…
- ...31 (49%) had no intransitivities
- The median number intransitivities per student was 1
- The mean was 6.68 (!)

Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 63 students who responded questionnaire Q1 in 2019/02…
- ...13 (21%) gave unreasonable answers to the easy questions

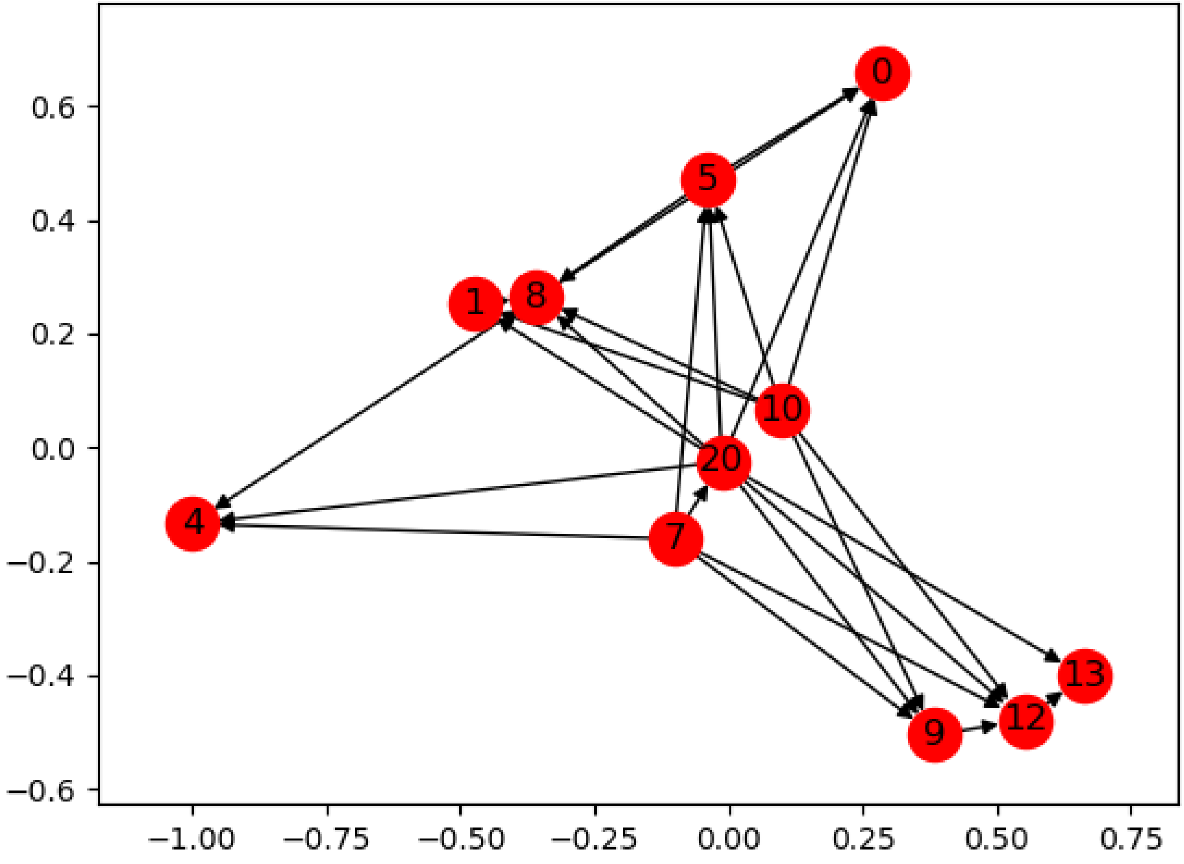
A discussion of transitivity

- # of intransitivities: 0 (I. R.)



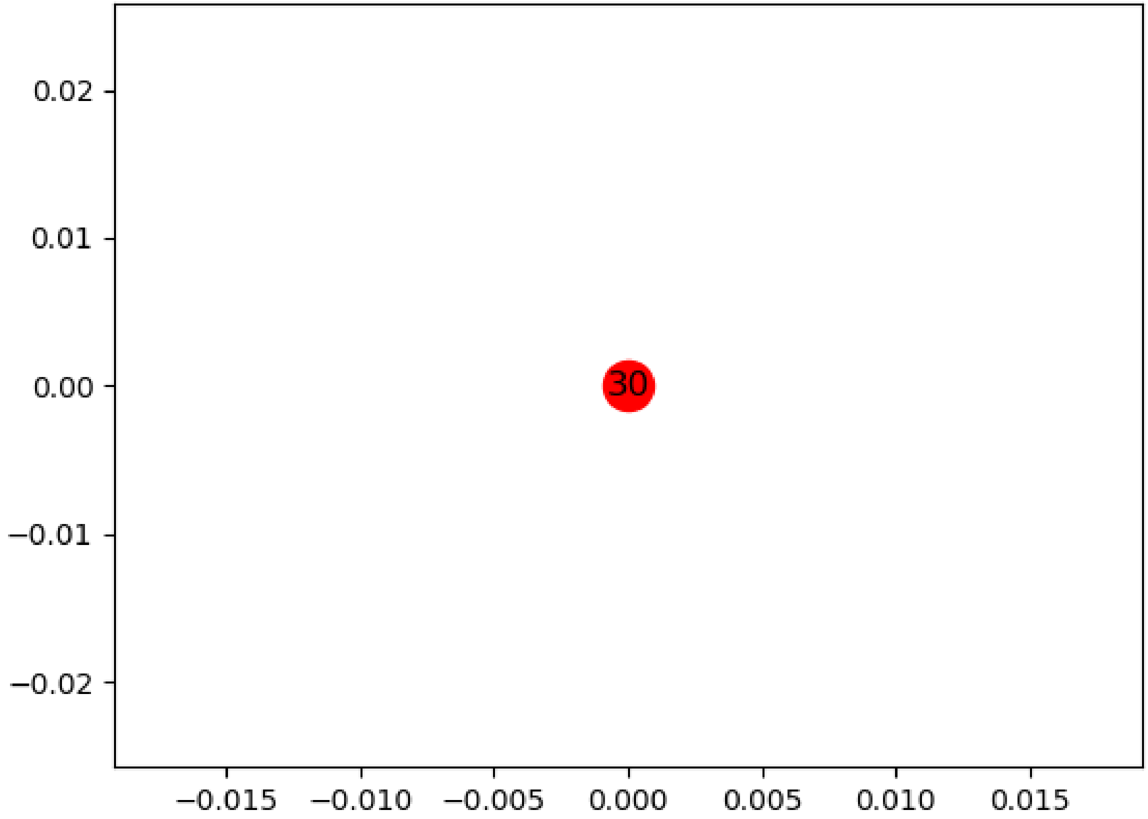
A discussion of transitivity

- # of intransitivities: 0 (R. C.)
  - 5/6 unreasonable answers



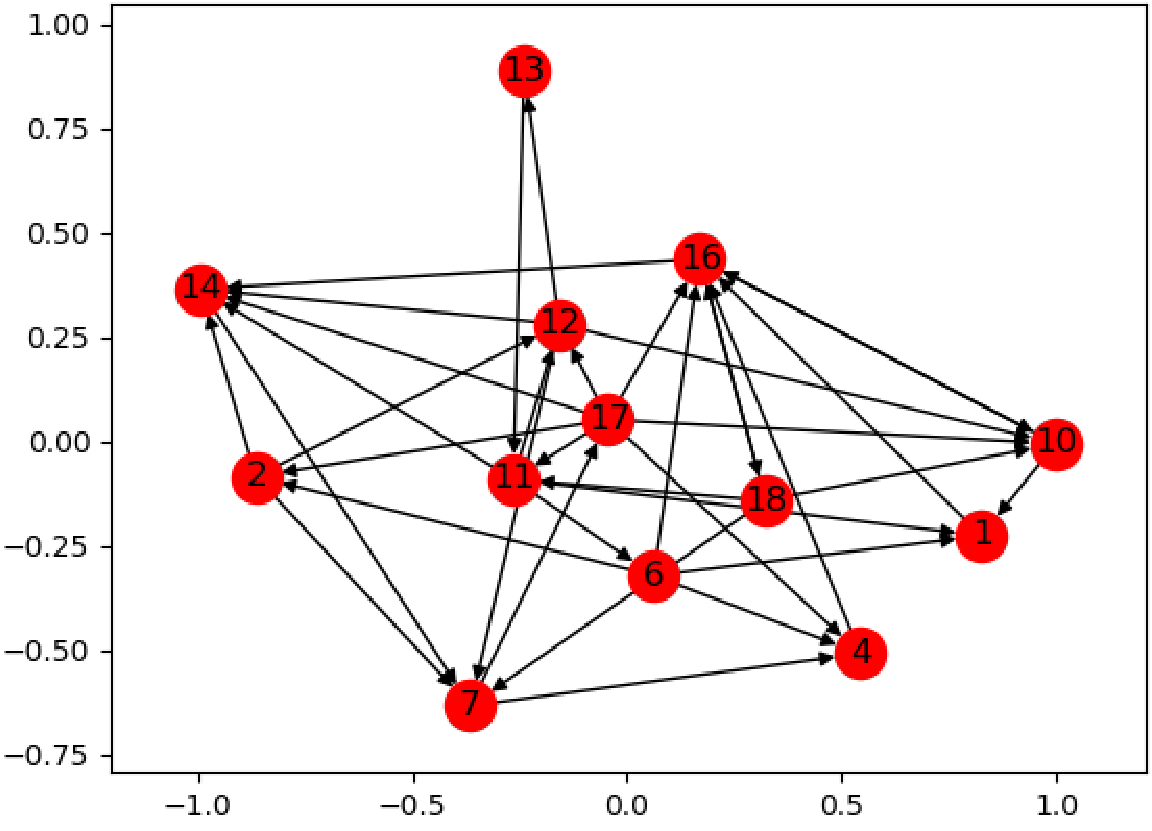
A discussion of transitivity

- # of intransitivities: 1 (T. S.)
  - 2/6 unreasonable answers



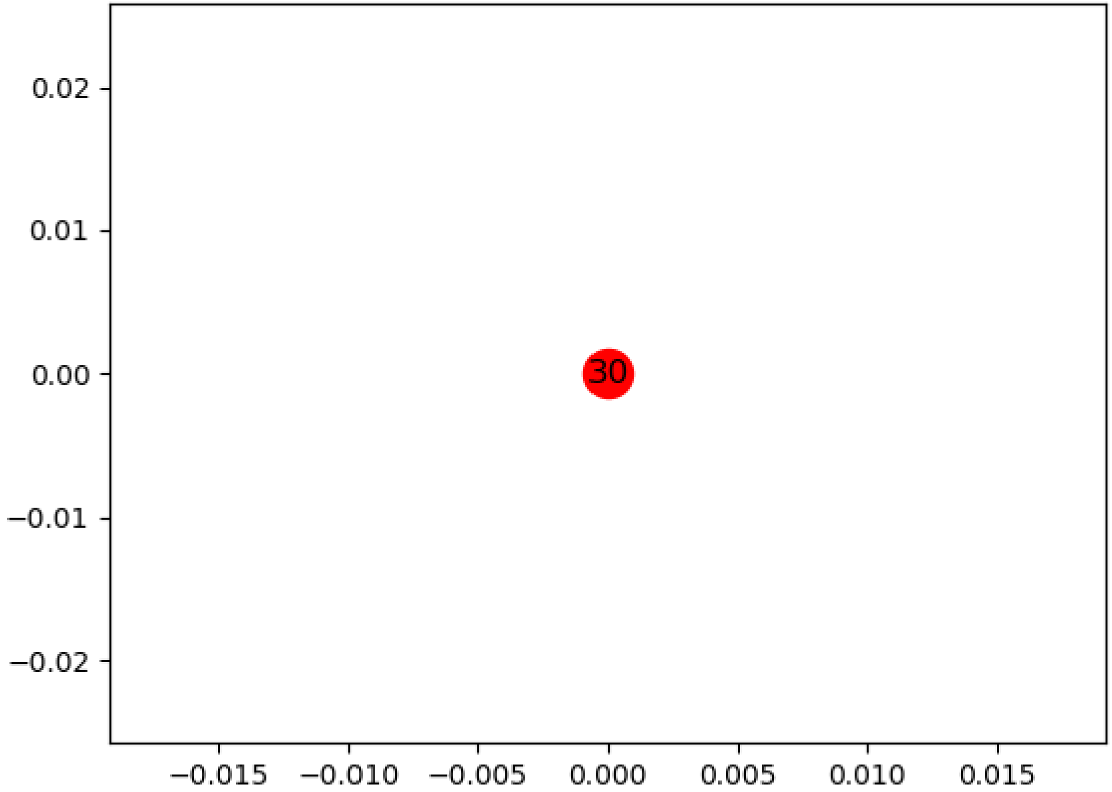
A discussion of transitivity

- # of intransitivities: 126 (A. F.)
  - 3/6 unreasonable answers



A discussion of transitivity

- # of intransitivities: 0 (G. F.)
  - 6/6 unreasonable answers



A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 38 students who responded questionnaire Q1 in 2017/02…
- ...only 4 (10.5%) had no intransitivities
- The median number intransitivities per student was 2.5
- The mean was 13.18 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 39 students who responded questionnaire Q1 in 2016/02…
- ...only 3 (7.7%) had no intransitivities
- The median number intransitivities per student was 4
- The mean was 12.1 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 27 students who responded questionnaire Q1 in 2015/02…
- ...only 4 (14.8%) had no intransitivities
- The median number intransitivities per student was 3
- The mean was 15.4 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 458 students who responded a simple preference questionnaire (details in the book),
- only 57 (12.44%) had no intransitivities in their answers,
- The median number intransitivities per student was 7

Procedures that cause violations of transitivity

- Any ideas?

Procedures that cause violations of transitivity

- Being crazy?

Procedures that cause violations of transitivity

- Being lazy?

Procedures that cause violations of transitivity

- Aggregation of considerations as a source of intransitivity
  - ***X = {a, b, c}*** and the individual has three primitive considerations in mind (eg: price, taste, quality)
  - The individual finds an alternative ***x*** better than an alternative ***y*** if a majority of considerations supports ***x***
  - If the three considerations rank the alternatives as $\boldsymbol{a \succ_1 b \succ_1 c}$, $\boldsymbol{b \succ_2 c \succ_2 a}$, and $\boldsymbol{c \succ_3 a \succ_3 b}$, then…
  - … the individual determines ***a*** to be preferred over ***b***, ***b*** over ***c***, and ***c*** over ***a***, thus violating transitivity

Procedures that cause violations of transitivity

- The use of similarities as an obstacle to transitivity
  - In some cases, an individual may express ***indifference*** in a comparison between two elements that are ***too "close"*** to be distinguishable
  - Let $\boldsymbol{X}$ be the set of real numbers
  - Consider an individual whose attitude toward the alternatives is ***"the larger the better"***, but he cannot determine whether $\boldsymbol{a}$ is greater than $\boldsymbol{b}$ unless the difference is at least $\boldsymbol{1}$
  - He will assign $\boldsymbol{f(x, y) = x \succ y}$ if $\boldsymbol{x \geq y + 1}$ and $\boldsymbol{f(x, y) = I}$ if $\boldsymbol{|x - y| < 1}$
  - Is this function always transitive?
  - This is not a preference relation because $\boldsymbol{1.5 \sim 0.8}$ and $\boldsymbol{0.8 \sim 0.3}$, but it is not true that $\boldsymbol{1.5 \sim 0.3}$

Preferences

- **Definition 1**
  - Preferences on a set $\boldsymbol{X}$ are <u>a function</u> $\boldsymbol{f}$
  - that assigns to any pair $\boldsymbol{(x, y)}$ of distinct elements in $\boldsymbol{X}$ exactly one of the three "values"
  - $\boldsymbol{x \succ y}$, $\boldsymbol{y \succ x}$, or $\boldsymbol{I}$
  - so that for any three different elements $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ in $\boldsymbol{X}$, the following two properties hold:
    - <u>No order effect</u>: $\boldsymbol{f(x, y) = f(y, x)}$
    - <u>Transitivity</u>:
      - if $\boldsymbol{f(x, y) = x \succ y}$ and $\boldsymbol{f(y, z) = y \succ z}$, then $\boldsymbol{f(x, z) = x \succ z}$ and
      - if $\boldsymbol{f(x, y) = I}$ and $\boldsymbol{f(y, z) = I}$, then $\boldsymbol{f(x, z) = I}$

A discussion of transitivity

- Is this definition weak?
- For example, if $\boldsymbol{f(x, y) = x \succ y}$ and $\boldsymbol{f(y, z) = I}$, can $\boldsymbol{f(x, z)}$ be different than $\boldsymbol{x \succ z}$?
- No! Proof in the book

Questionnaire R

$\boldsymbol{R(x,y)}$ (for all $x, y \in X$, not necessarily distinct):

Is x at least as preferred as y? Tick one and only one of the following two options:

- ☐ Yes
- ☐ No

Questionnaire R

- By a “legal” response we mean that the respondent ticks exactly one of the boxes in each question
- To qualify as preferences, a legal response must also satisfy two conditions:
  - The answer to at least one of the questions ***R(x, y)*** and ***R(y, x)*** must be ***Yes***
  - For every $x, y, z \in X$, if the answers to the questions ***R(x, y)*** and ***R(y, z)*** are ***Yes***, then so is the answer to the question ***R(x, z)***

The equivalence of the two definitions

- If I get a questionnaire Q from “Smith”, can I fill questionnaire R for “Smith”?

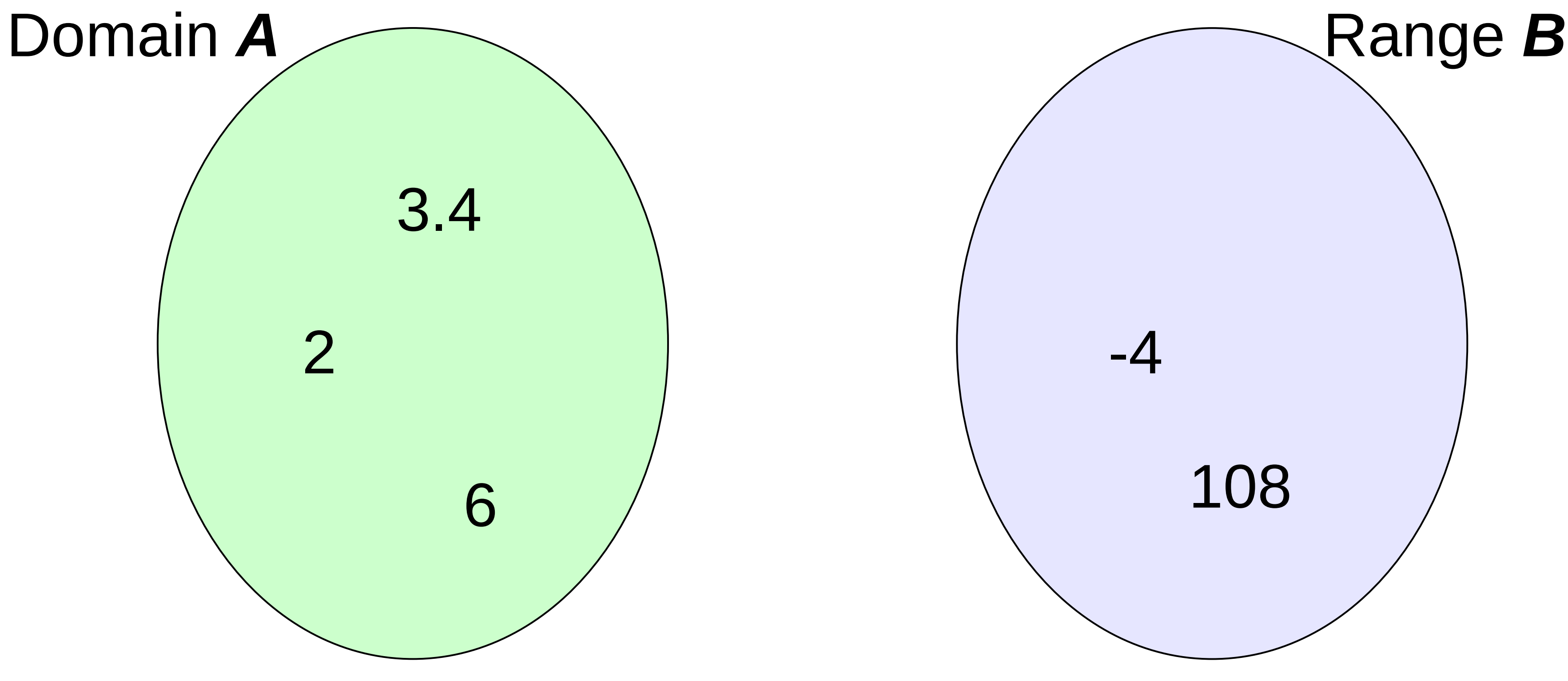
Q(x,y) (for all distinct x and y in X):
How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

R(x,y) (for all $x, y \in X$, not necessarily distinct):
Is x at least as preferred as y? Tick one and only one of the following two options:

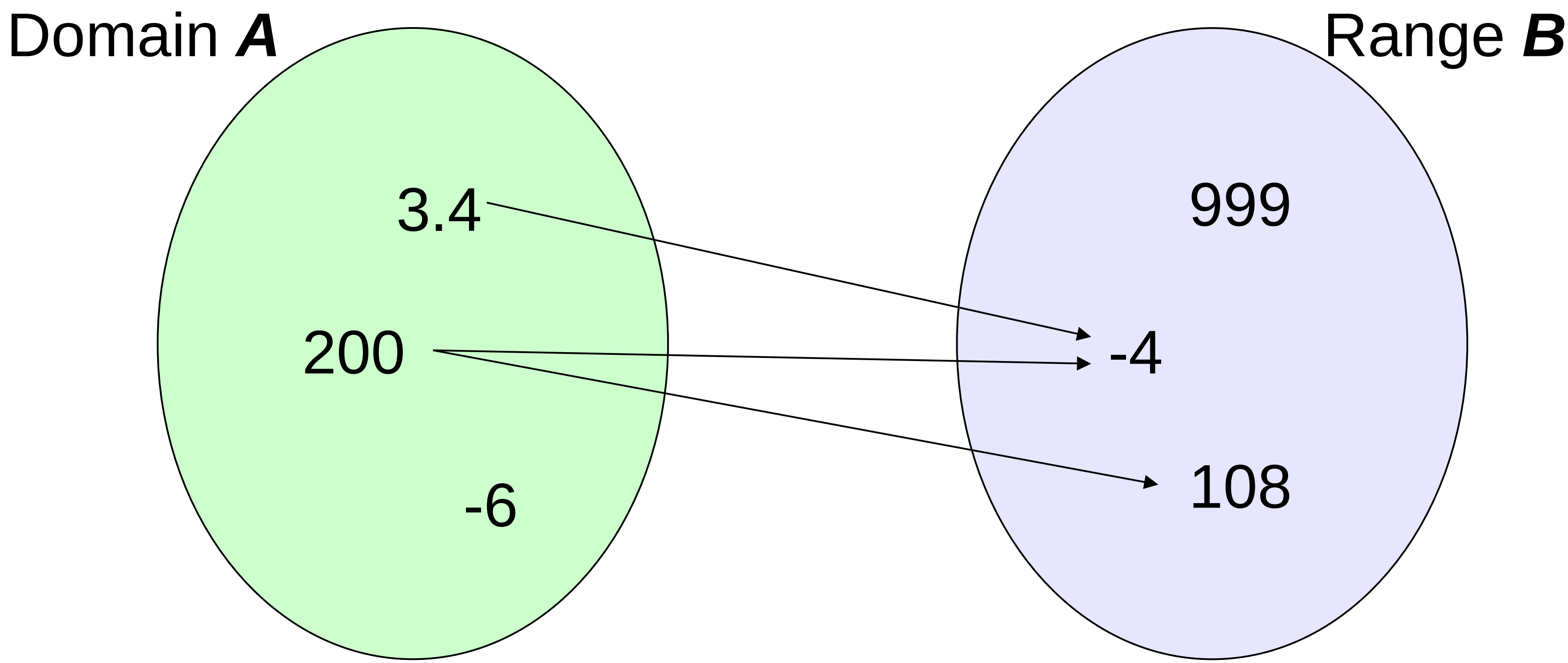
☐ Yes
☐ No

Reminder: relation



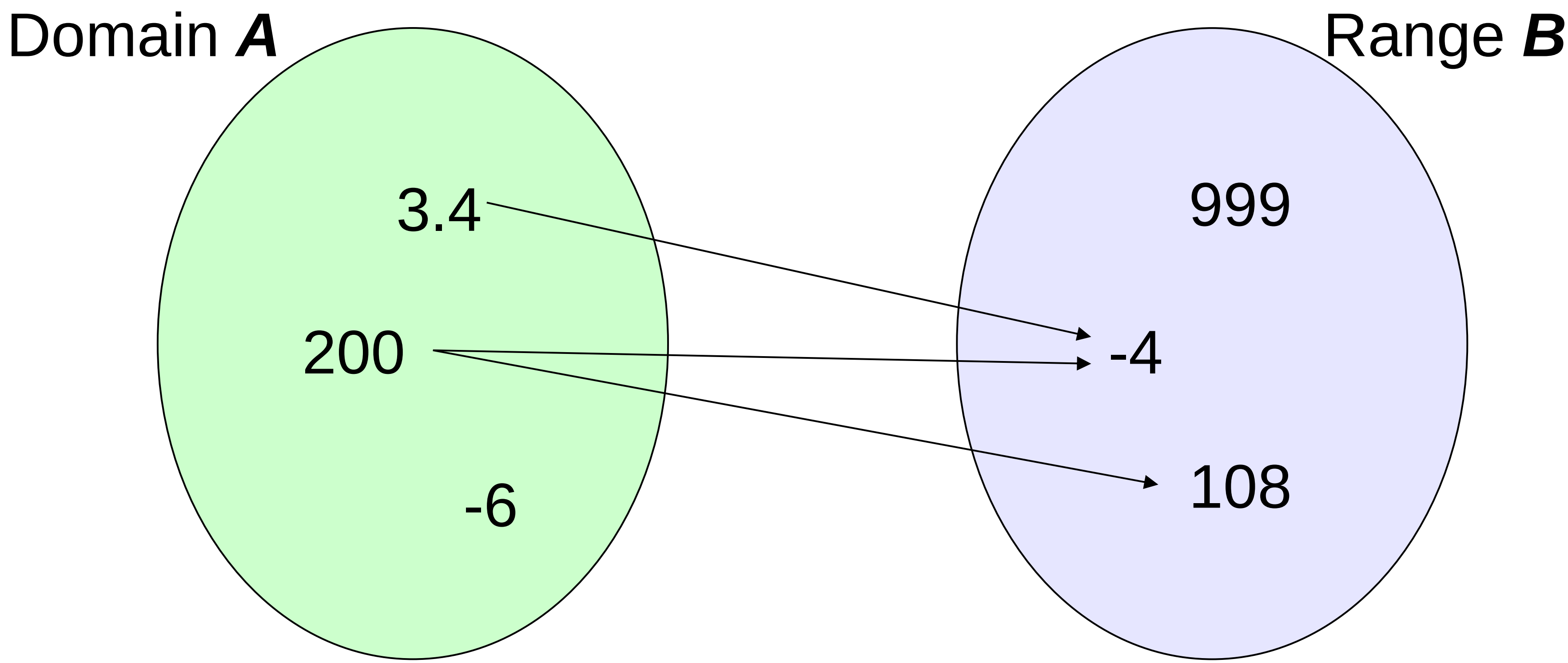
A relation *R* is a set of ordered pairs

Reminder: relation



$$R \subseteq A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Reminder: relation



R = {(3.4,-4), (200,-4), (200,108)}

Questionnaire R

- We identify a response to this questionnaire with the **binary relation** $\succsim$ on the set ***X*** defined by ***x*** $\succsim$ ***y*** if the answer to the question ***R(x, y)*** is ***Yes***
- Ex: If ***x*** is at least as preferred as ***y***, then ***x*** $\succsim$ ***y***

Reminder

- An n-ary relation on $\boldsymbol{X}$ is a subset of $\boldsymbol{X^n}$
- Examples:
  - "Being a parent of" is a binary relation on the set of human beings
  - "being a hat" is an unary relation on the set of objects
  - "$\boldsymbol{x + y = z}$" is a 3-ary relation on the set of numbers
  - " $\boldsymbol{x}$ is better than $\boldsymbol{y}$ more than $\boldsymbol{x'}$ is better than $\boldsymbol{y'}$ " is 4-ary relation on a set of alternatives

Reminder

- An n-ary relation on ***X*** can be thought of as a response to a questionnaire regarding all n-tuples of elements of ***X*** where each question can get only a ***Yes*** answer
- Ex: is $a_1 \succsim a_2 \succsim a_3 \succsim \ldots \succsim a_n$? (Yes)

Preferences

- Definition 2
  - Preferences on a set $\boldsymbol{X}$ is a binary relation $\succsim$ on $\boldsymbol{X}$ satisfying:
    - ***Completeness***: For any $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X}$, $\boldsymbol{x} \succsim \boldsymbol{y}$, or $\boldsymbol{y} \succsim \boldsymbol{x}$
    - ***Transitivity***: For any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}$, if $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$, then $\boldsymbol{x} \succsim \boldsymbol{z}$

The equivalence of the two definitions

- If I get a questionnaire Q from "Smith", can I fill questionnaire R for "Smith"?

Q(x,y) (for all distinct x and y in X):
How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

R(x,y) (for all $x, y \in X$, not necessarily distinct):
Is x at least as preferred as y? Tick one and only one of the following two options:

☐ Yes
☐ No

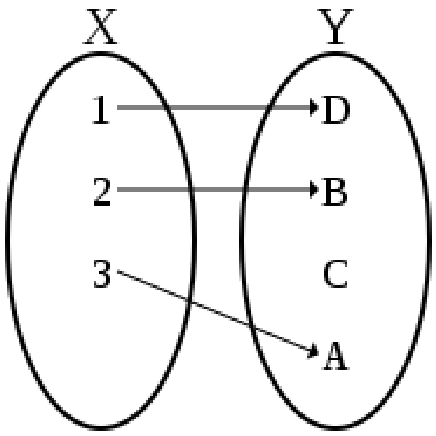
Reminder

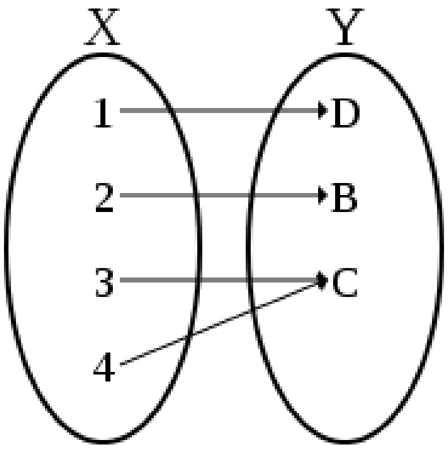
- The function $f : X \to Y$ is a ***one-to-one*** function (or injection) if $f(x) = f(y)$ implies that $x = y$
  - Ex: Brazilians → CPF number

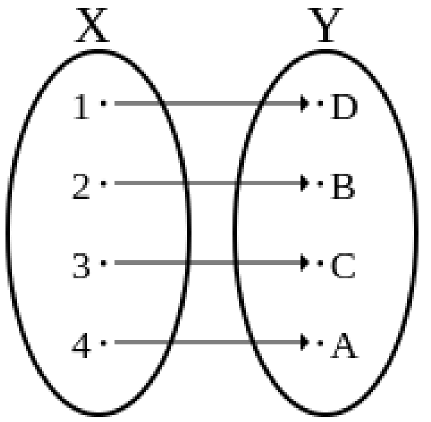
- The function $f : X \to Y$ is an ***onto*** function (or surjection) if for every $y \in Y$ there is an $x \in X$ such that $f(x) = y$
  - Ex: people → country of birth

- The function $f : X \to Y$ is a ***one-to-one and onto*** function (or bijection, or one-to-one correspondence) if for every $y \in Y$ there is a unique $x \in X$ such that $f(x) = y$
  - Ex: Brazilians→ Passport

Reminder







The equivalence of the two definitions

- If I get a questionnaire Q from “Smith”, can I fill questionnaire R for “Smith”?
- We need to construct a ***<u>one-to-one and onto</u>*** function answers to Q and answers to R, such that the correspondence preserves the meaning of the responses to the two questionnaires

The equivalence of the two definitions

A response to: $Q(x,y)$ and $Q(y,x)$	A response to: $R(x,y)$ and $R(y,x)$	
$x \succ y$	Yes	No
I	Yes	Yes
$y \succ x$	No	Yes

Summary

- Preferences on $\boldsymbol{X}$ are a binary relation $\succsim$ on a set $\boldsymbol{X}$ satisfying completeness and transitivity
- Notate $\boldsymbol{x} \succ \boldsymbol{y}$ when both $\boldsymbol{x} \succsim \boldsymbol{y}$ and <u>not</u> $\boldsymbol{y} \succsim \boldsymbol{x}$, and $\boldsymbol{x} \sim \boldsymbol{y}$ when $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{x}$

Summary

- Now, with one single relation ($\succsim$), we can describe the full preference relation towards the items in ***X***
  - With questionnaire ***Q***, we needed two relations: $\succ$ and ***I***

Fun problem

- Let's listen to the Shepard tone
  - https://www.youtube.com/watch?v=BzNzgsAE4F0
- Can you think of any economic analogies?

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800
L3	$800	$900	$1000	$500	$600	$700

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800
L3	$800	$900	$1000	$500	$600	$700
L4	$700	$800	$900	$1000	$500	$600
L5	$600	$700	$800	$900	$1000	$500
L6	$500	$600	$700	$800	$900	$1000