Risk Aversion

Risk Aversion

- Who is this guy?

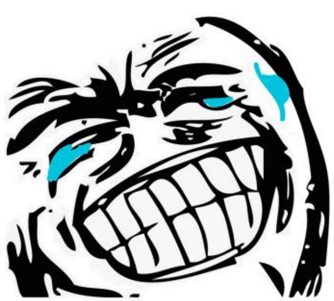
Risk Aversion

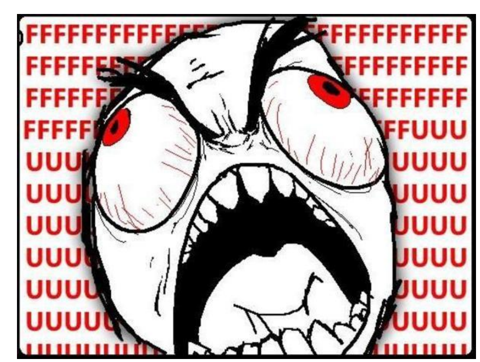
- Lets watch a portion of a video…
  - 2:45 (specially)

Risk Aversion

- Imagine a situation where River Plate plays against Boca Juniors and falls to 2^{nd} division iff it loses

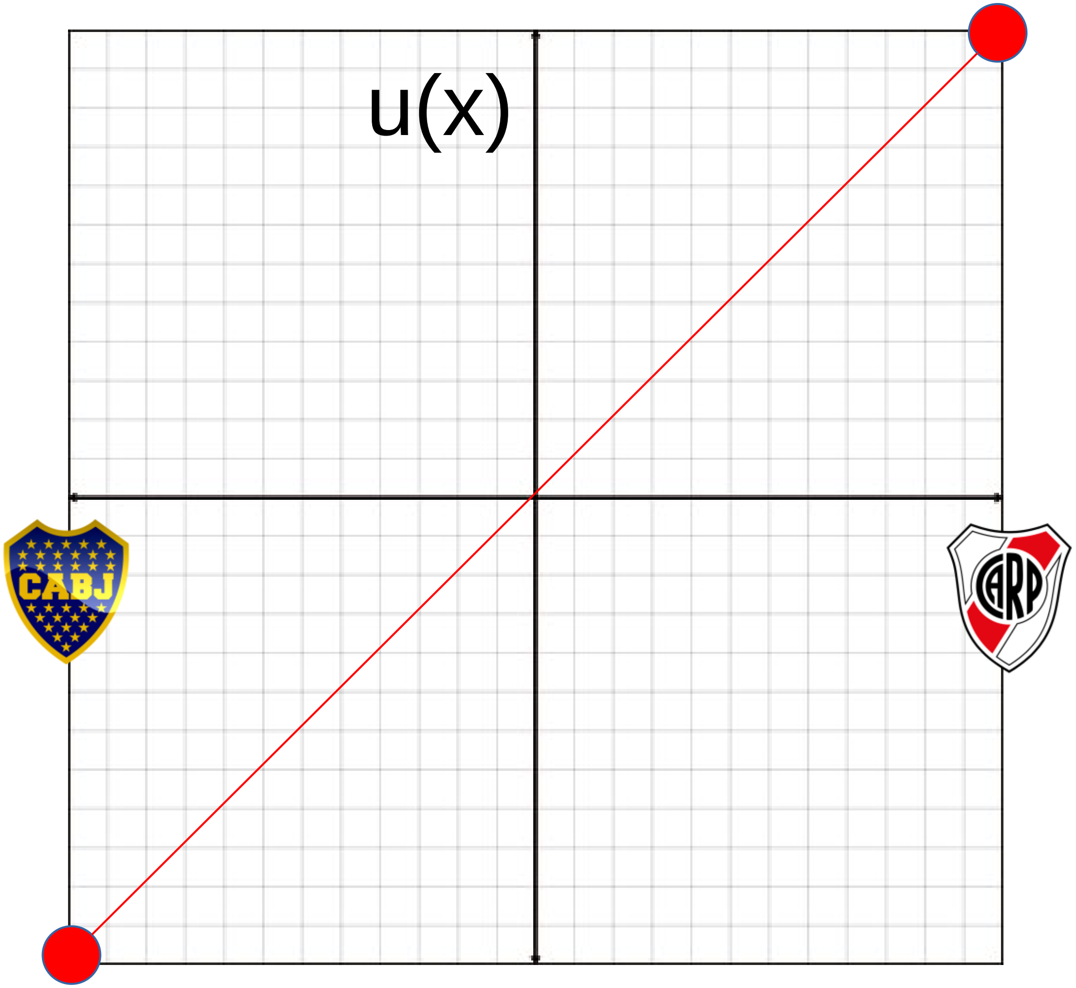
vs.

u(W,D) = 

u(L) = 

Risk Aversion

- Utilities for this lottery



Risk Aversion

- Imagine a situation where River Plate plays against Boca Juniors and falls to 2nd division iff it loses

vs.

$u(W,D) = \$105$ $u(L) = -\$105$

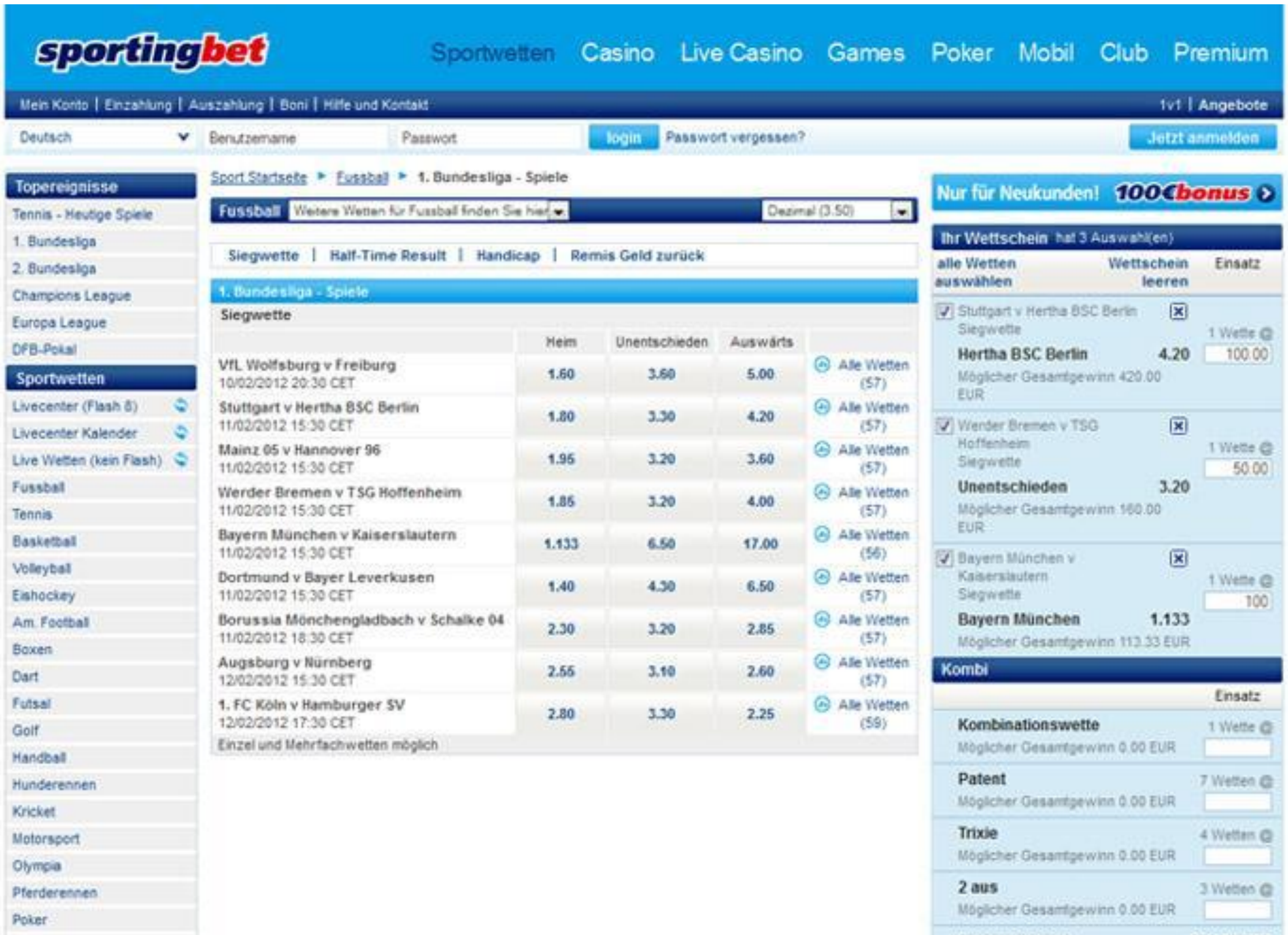
Risk Aversion

- What should I do if I am extremely risk averse?

vs.

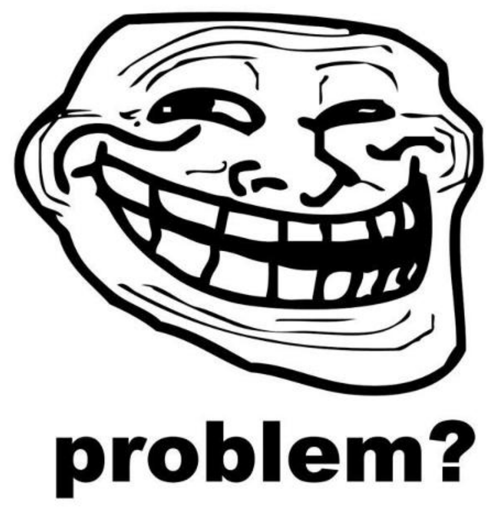
$u(W,D) = \$105$ $u(L) = -\$105$

Risk Aversion



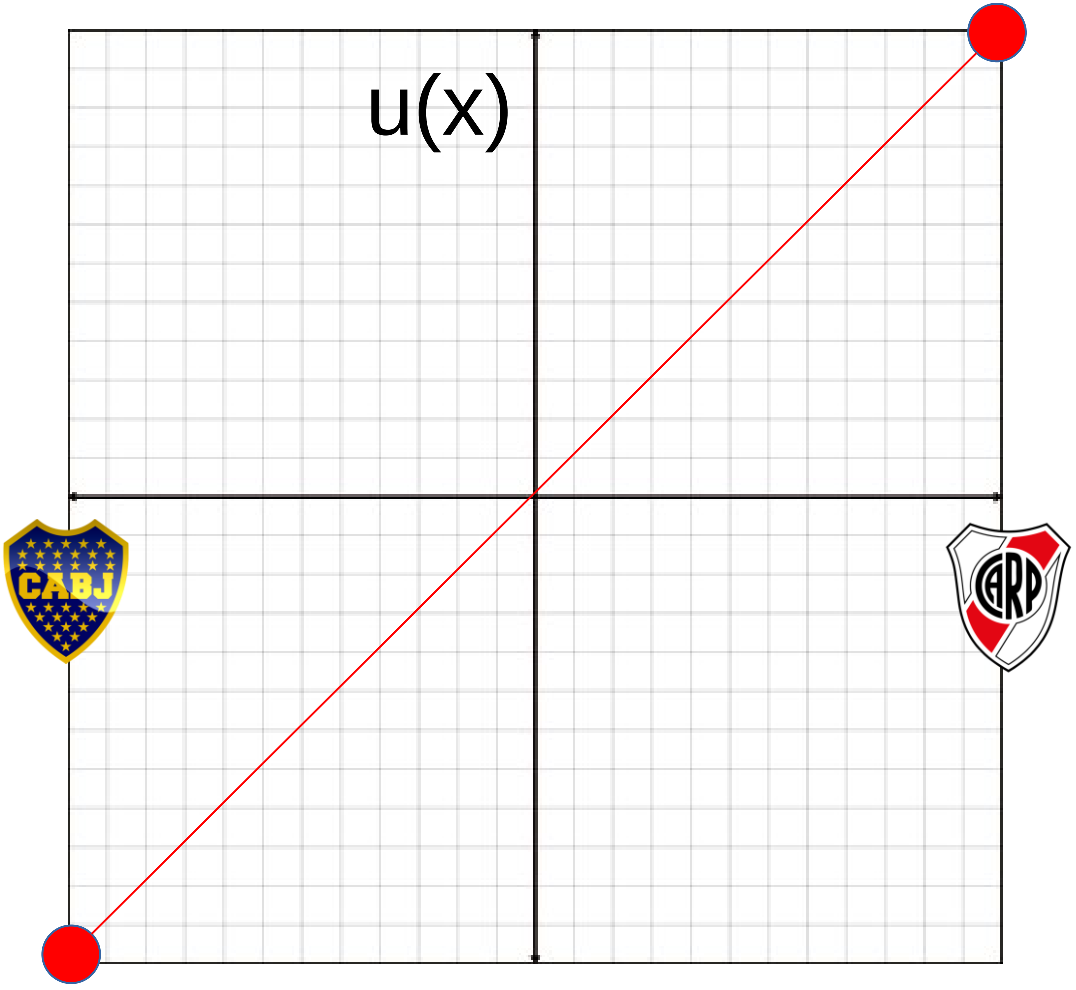
Risk Aversion

- Suppose a site is paying $\boldsymbol{r = 3}$ dollars for each dollar I put on Boca Juniors
- Then...
  - let $\boldsymbol{x}$ be the amount of dollars I put on Boca Juniors
  - then, if $\boldsymbol{u(W,D) - x = u(L) + (r-1)x}$, I have nothing to worry about!
  - $105 - x = -105 + 3x - x$
  - $3x = 210$
  - $x = \$70$



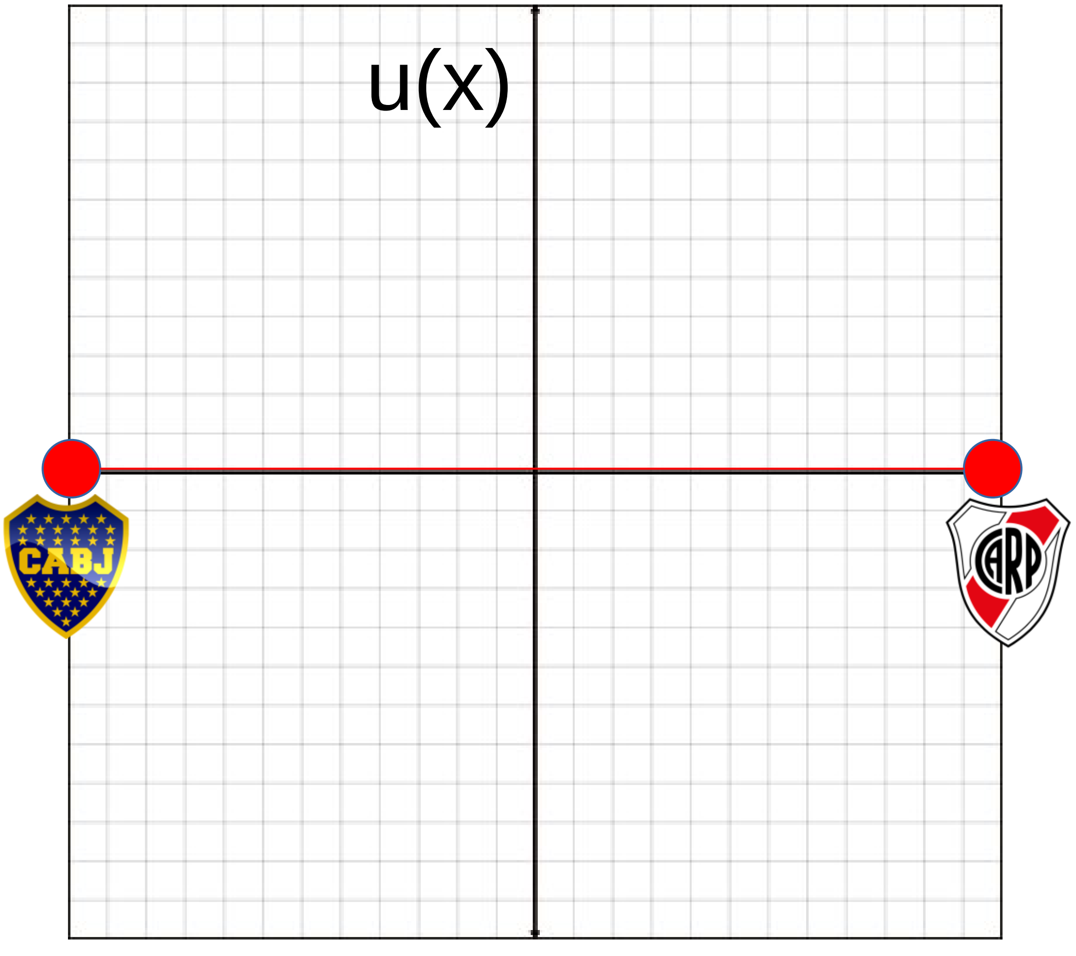
Risk Aversion

- Utilities for this lottery



Risk Aversion

- Utilities for this lottery



Risk Aversion

- … as if I am paying $70 for not playing this lottery

Lotteries with monetary prize

a

Smart TV Gamer LED 3D 40" Samsung UN40J6400 - Full HD 4 HDMI 3 USB 2 Óculos
de R$ 2.549,00
por R$ 1.999,00
em até 10x de R$ 199,90 sem juros
ou **R$ 1.799,10 à vista**
disponível sob consulta
☐ Ver com outros produtos

b

Smart TV Gamer LED 4k Ultra HD 75" Samsung - UN75JU6500 4 HDMI 3 USB Wi-Fi
R$ 15.999,00
em até 12x de R$ 1.333,25 sem juros
ou **R$ 14.399,10 à vista**
☐ Ver com outros produtos

c

Smart TV OLED Curva 3D 55" LG 55EA9850 Full HD - Conversor Integrado 4 HDMI 3 USB Wi-Fi 4 Óculos
de R$ 11.990,00
por R$ 7.999,00
em até 12x de R$ 666,58 sem juros
ou **R$ 7.199,10 à vista**
☐ Ver com outros produtos

d

Smart TV Gamer LED 75" Samsung UN75J6300A - Full HD Conversor Integrado 4 HDMI 3 USB Wi-Fi
R$ 11.499,00
em até 12x de R$ 958,25 sem juros
ou **R$ 10.349,10 à vista**
☐ Ver com outros produtos

u(c) = $7.999,00

Lotteries with monetary prize

a

b

u(a) = $5,00

u(b) = $1.299,00

Lotteries with monetary prize

- Preference relations $\succsim$ over the space of lotteries for which there is a continuous function ***u***, such that $\succsim$ is represented by

$$Eu(p) = \sum_{z \in Z} p(z)u(z)$$

- The function ***Eu*** assigns to the lottery ***p*** the expectation of the random variable that receives the value ***u(x)*** with a probability ***p(x)***

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	$30	$50	$70
p	50%	0%	50%
q	0%	100%	0%

Eu(p) = $50,00

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	$0	$50	$100
p	50%	0%	50%
q	0%	100%	0%

Eu(p) = $50,00

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	$0	$30	$100
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	$0	$30M	$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Let's play a game...

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	$0	$30M	$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Is there an ***x*** value that would make you choose ***p***?

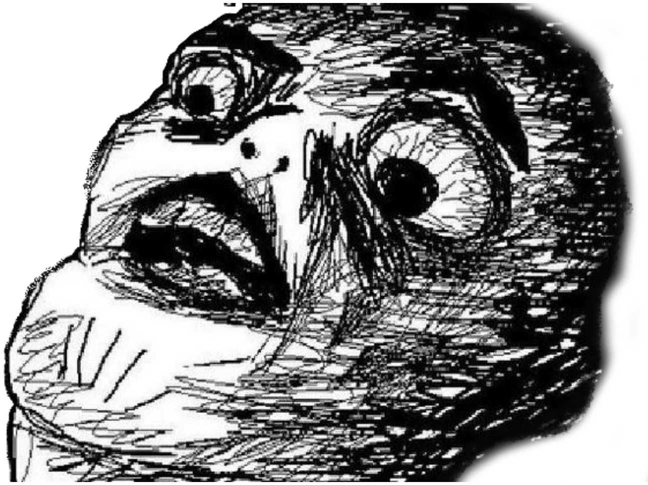
lottery	**$0**	**$30M**	**$[x]M**
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- If yes, let's imagine the following situation…
  - Choose x_1 that will make you choose ***p***

lottery	$0	$30M	$[$x_1$]M
p	50%	0%	50%
q	0%	100%	0%

  - Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation…
  - Choose x_1 that will make you choose p

lottery	\$0	\$30M	\$[x_1]M
p	50%	0%	50%
q	0%	100%	0%

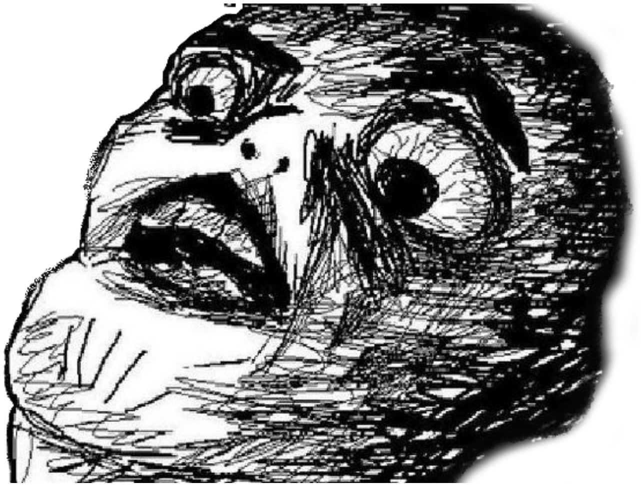
  - YOU WON!

Lotteries with monetary prize

- If yes, let's imagine the following situation…
  - Choose x_2 that will make you choose ***p***

lottery	**$0**	**$[$x_1$]M**	**$[$x_2$]M**
p	50%	0%	50%
q	0%	100%	0%

  - Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation…
  - Choose now a $\boldsymbol{x_2}$ that will make you choose ***p***

lottery	**$0**	**$[$x_1$]M**	**$[$x_2$]M**
p	50%	0%	50%
q	0%	100%	0%

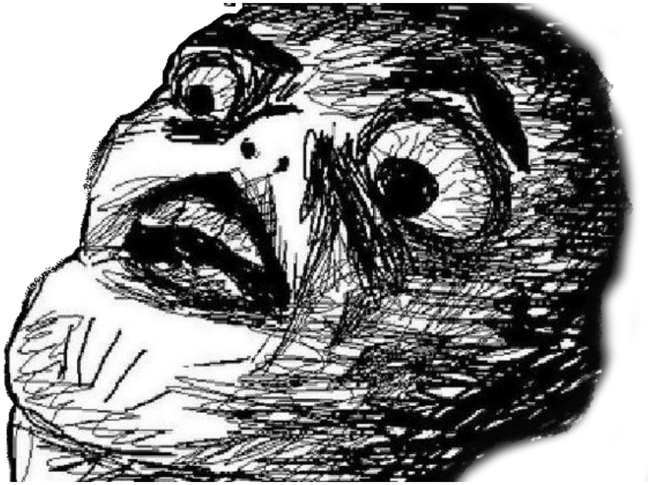
  - YOU WON!

Lotteries with monetary prize

- If yes, let's imagine the following situation…
  - Choose $\boldsymbol{x_3}$ that will make you choose ***p***

lottery	**$0**	**$[$x_2$]M**	**$[$x_3$]M**
p	50%	0%	50%
q	0%	100%	0%

  - Now flip a coin



Lotteries with monetary prize

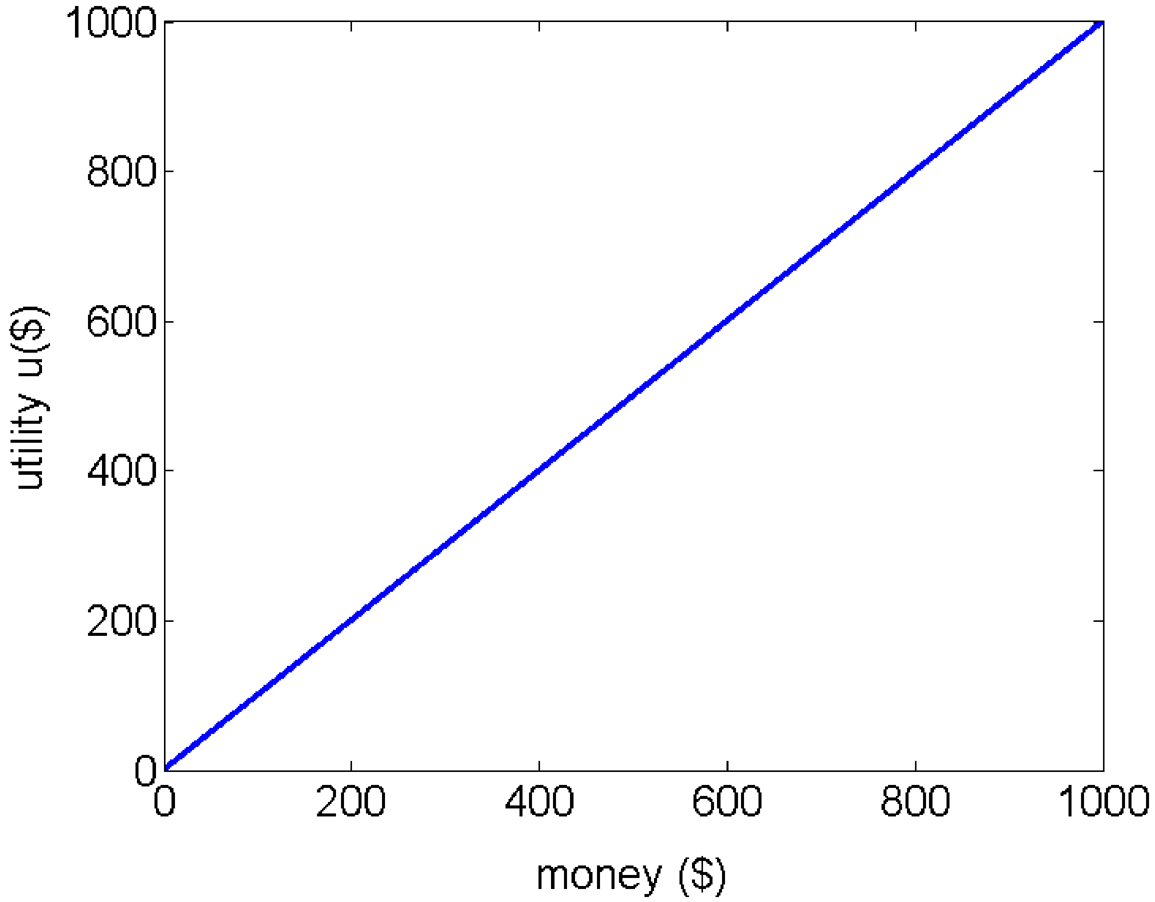
- If utility is <u>not bounded</u>, this game goes on forever
- More important:
  - What is the probability of our decision maker being left with no money?
  - 1!
- St. Petersburg paradox

Lotteries with monetary prize

- St. Petersburg paradox, proposed by Nicolaus Bernoulli more than 300 years ago

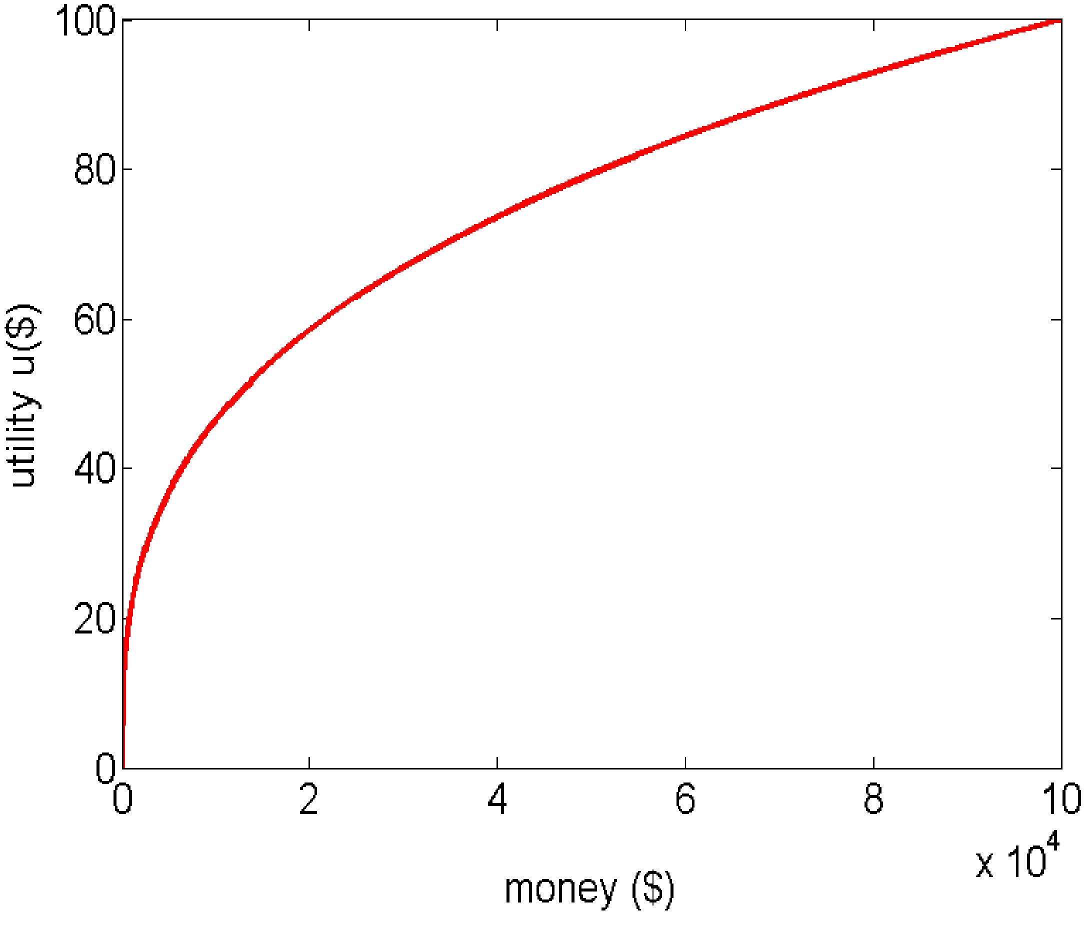
Lotteries with monetary prize

- Money vs. Utility over money



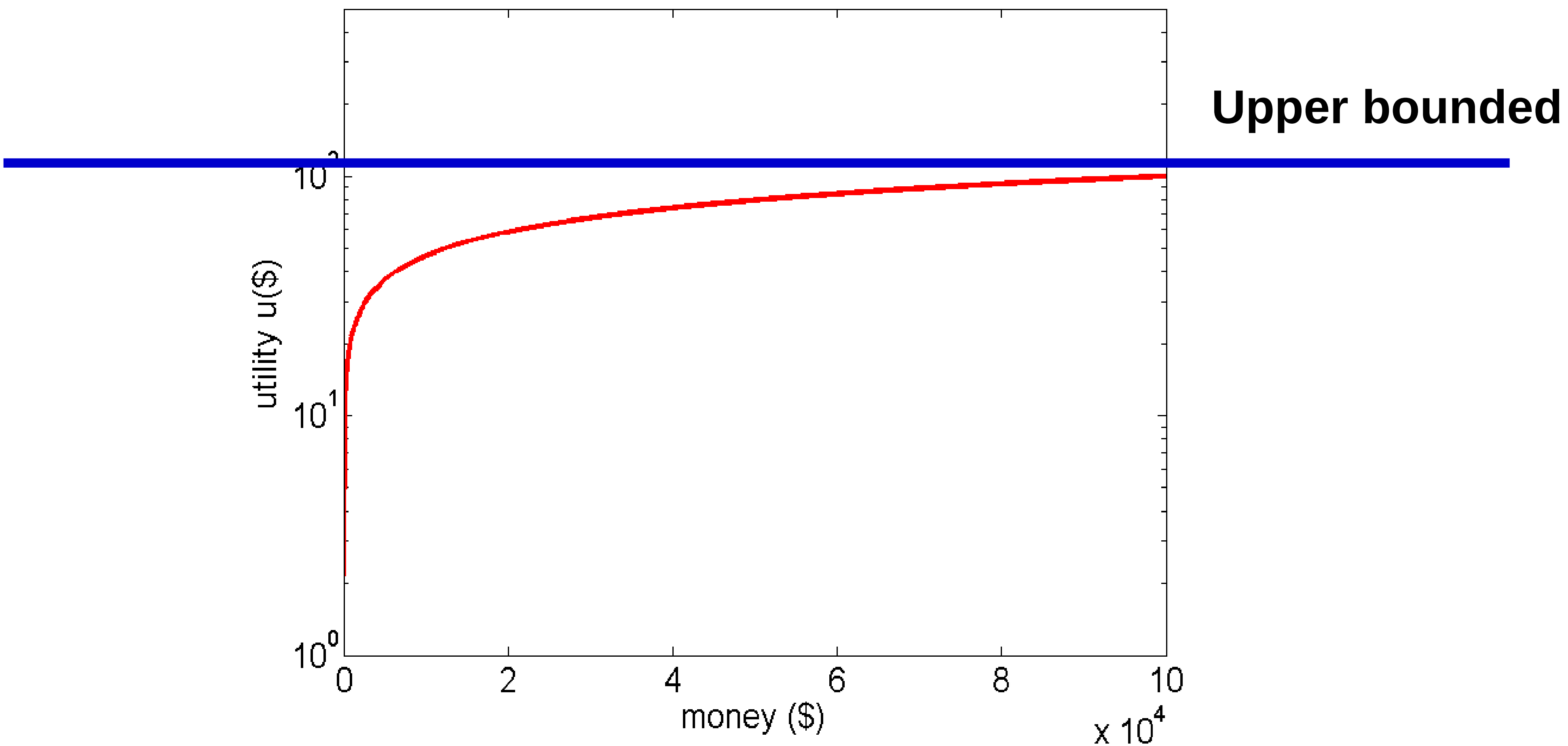
Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Daniel Bernoulli (cousin of Nicolaus) explains :
  - “The value of an item must not be based upon its price, but rather on the utility it yields.
  - The price of the item is dependent only on the thing itself and is equal for everyone.
  - The utility, however, is dependent on the particular circumstances of the person making the estimate.”

Lotteries with monetary prize

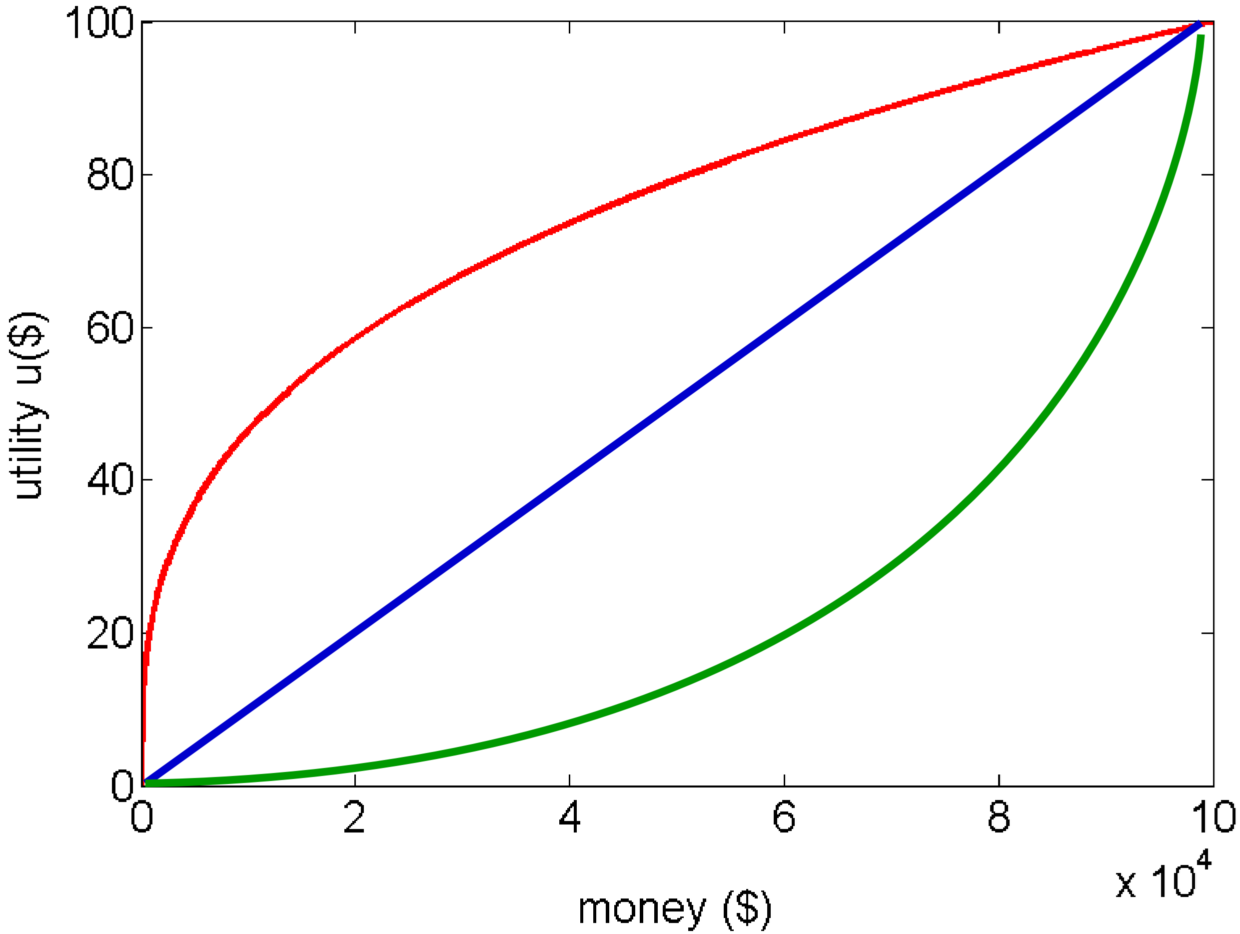
- Two insights about the St. Petersburg paradox
  - The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others
  - The utility from gaining an additional dollar would decrease with wealth
    - Utility increases as wealth increases and at a declining rate

Lotteries with monetary prize

- The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others

Risk Aversion

- How to measure?



Risk Aversion

- How to measure?
  - Every prize should be converted to money
  - Define a utility function ***u($)*** over money
  - For a lottery ***p*** over prizes, calculate:
    - ***E(p)***: the expected amount of money of lottery ***p***
    - ***Eu(p):*** the expected utility of lottery ***p***
    - ***u(E(p)):*** the utility of the expected amount of money of ***p***
    - ***CE(p):*** the <u>certainty equivalent</u>
      - the amount of money I am willing to pay to play lottery ***p***

Risk Aversion

- How much would you pay to play lottery ***p***?
  - Let ***CE(p)***, the certainty equivalent, be the amount of money I am willing to pay to play lottery ***p***
- How much ***A*** and ***B*** value to you?
  - ***u(A)*** and ***u(B)*** (or ***u($20)*** and ***u($80)***)
  - **Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u($20) + 0.75 u($80)**
  - **E(p) = p(A)A + p(B)B = 0.25 $20 + 0.75 $80 = $65**
- If I am risk neutral
  - ***u(E(p)) = Eu(p)*** and ***E(p) = CE(p) = $65***

lottery	A($20)	B($80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery ***p***?
  - Let ***CE(p)***, the certainty equivalent, be the amount of money I am willing to pay to play lottery ***p***
- How much ***A*** and ***B*** value to you?
  - ***u(A)*** and ***u(B)*** (or ***u($20)*** and ***u($80)***)
  - **Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u($20) + 0.75 u($80)**
  - **E(p) = p(A)A + p(B)B = 0.25 $20 + 0.75 $80 = $65**
- If I am risk averse
  - ***u(E(p)) > Eu(p)*** and ***E(p) > CE(p) < $65***

lottery	A($20)	B($80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery ***p***?
  - Let ***CE(p)***, the certainty equivalent, be the amount of money I am willing to pay to play lottery ***p***
- How much ***A*** and ***B*** value to you?
  - ***u(A)*** and ***u(B)*** (or ***u($20)*** and ***u($80)***)
  - **Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u($20) + 0.75 u($80)**
  - **E(p) = p(A)A + p(B)B = 0.25 $20 + 0.75 $80 = $65**
- If I am risk seeking
  - ***u(E(p)) < Eu(p)*** and ***E(p) < CE(p) > $65***

lottery	A($20)	B($80)
p	25%	75%

Risk Aversion

- Claim:
  - Let $\succsim$ be a preference on ***L(Z)*** represented by the vNM utility function ***u,***

    the preference relation $\succsim$ is risk averse iff ***u*** is

    strictly concave
- Proof
  - In the book

Reminder: strictly concave functions

$f((1\text{-}\alpha)x + \alpha y) > (1\text{-}\alpha)f(x) + \alpha f(y)$

consider a lottery with two prizes ***x*** and ***y***

and $p(y) = \alpha$ and $p(x) = 1\text{-}\alpha$

$E(p) = (1\text{-}\alpha)x + \alpha y$

if the function ***f*** is strictly concave

$f(E(p)) > (1\text{-}\alpha)f(x) + \alpha f(y)$

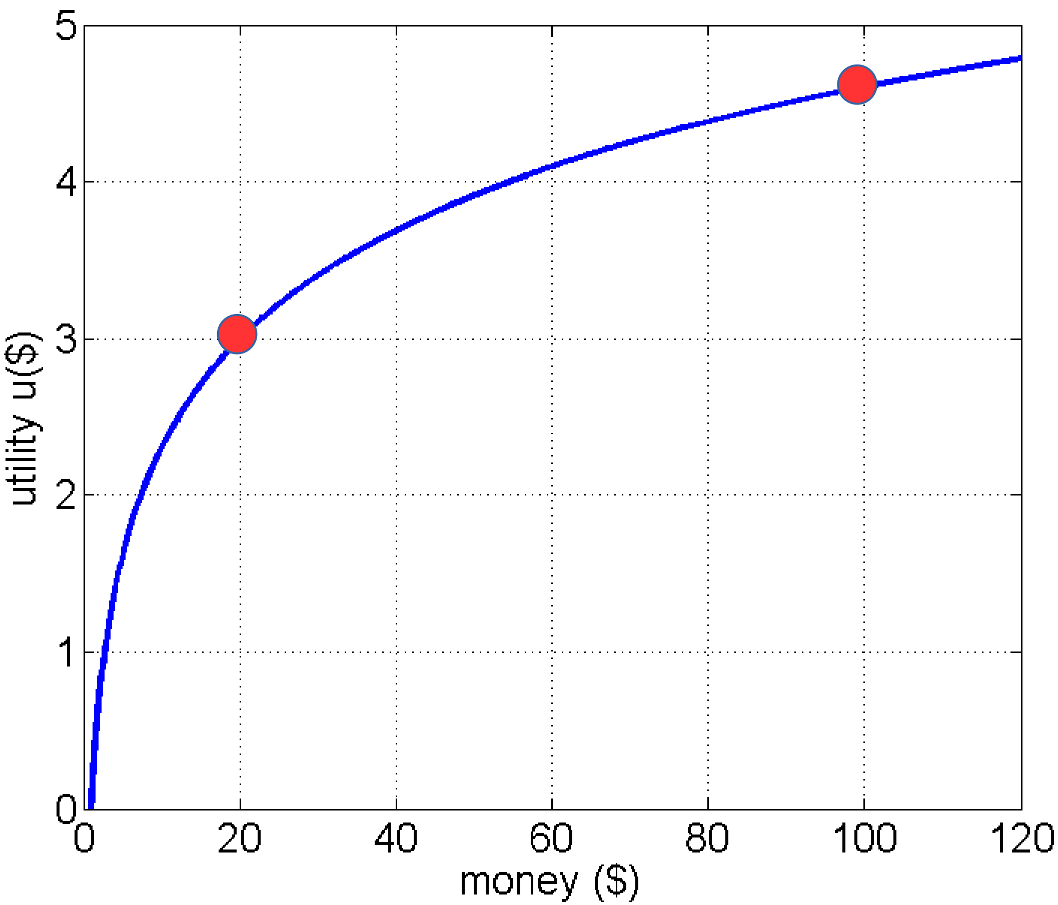
if ***f*** is the utility function

$u(E(p)) > (1\text{-}\alpha)u(x) + \alpha u(y)$

$u(E(p)) > Eu(p)$ (definition of risk aversion)

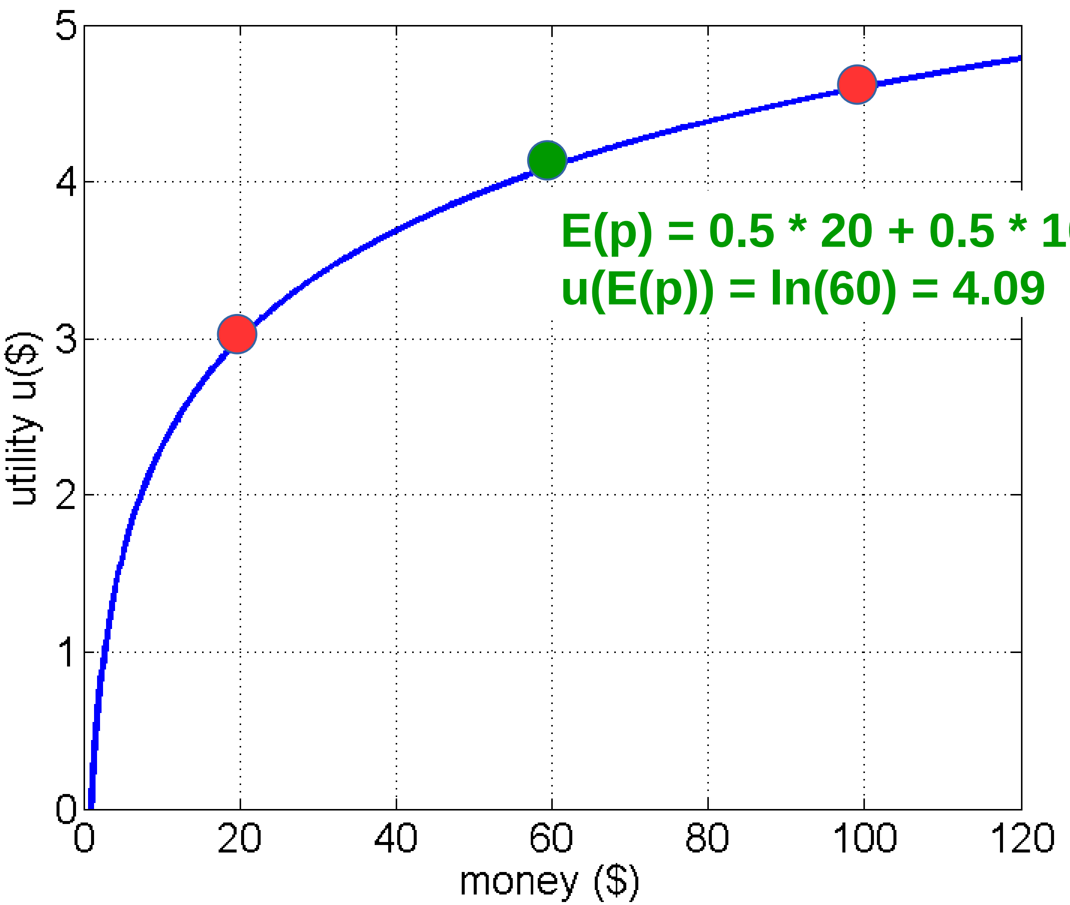
Risk Aversion

- ***u(x) = ln(x)***
- ***p = (0.5) $20 ⊕ (0.5) $100***



Risk Aversion

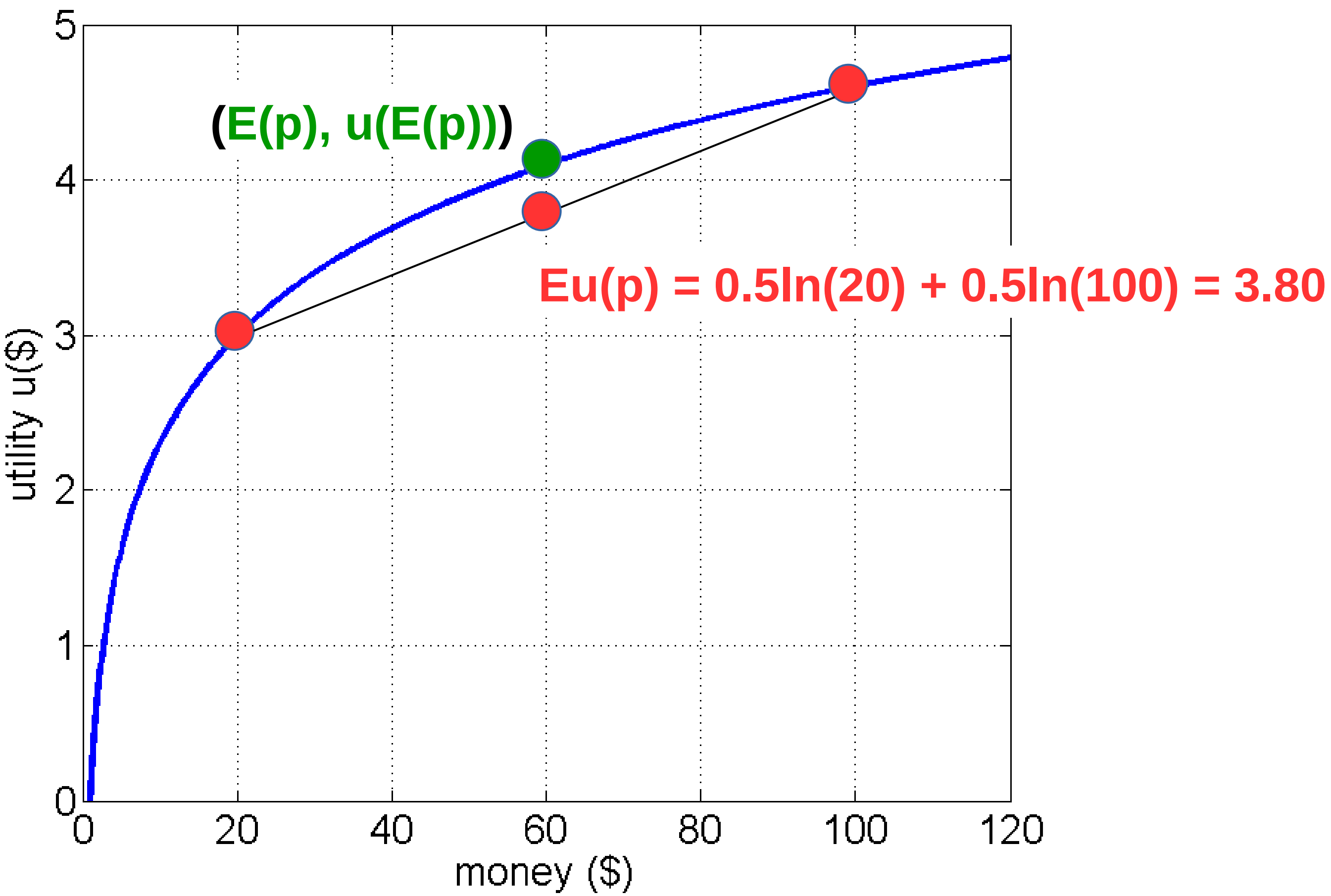
- ***u(x) = ln(x)***
- ***p = (0.5) $20 ⊕ (0.5) $100***



E(p) = 0.5 * 20 + 0.5 * 100 = 60
u(E(p)) = ln(60) = 4.09

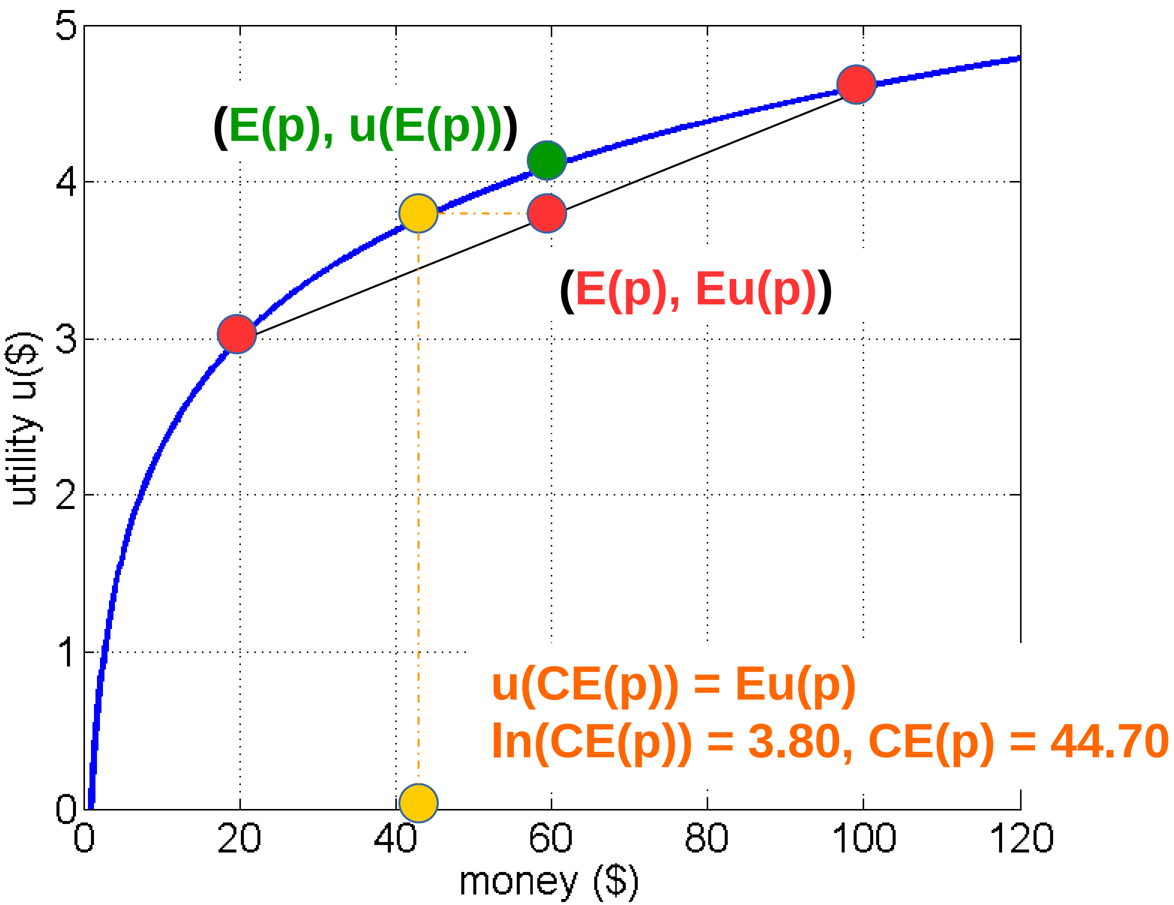
Risk Aversion

- ***u(x) = ln(x)***
- ***p = (0.5) $20 ⊕ (0.5) $100***



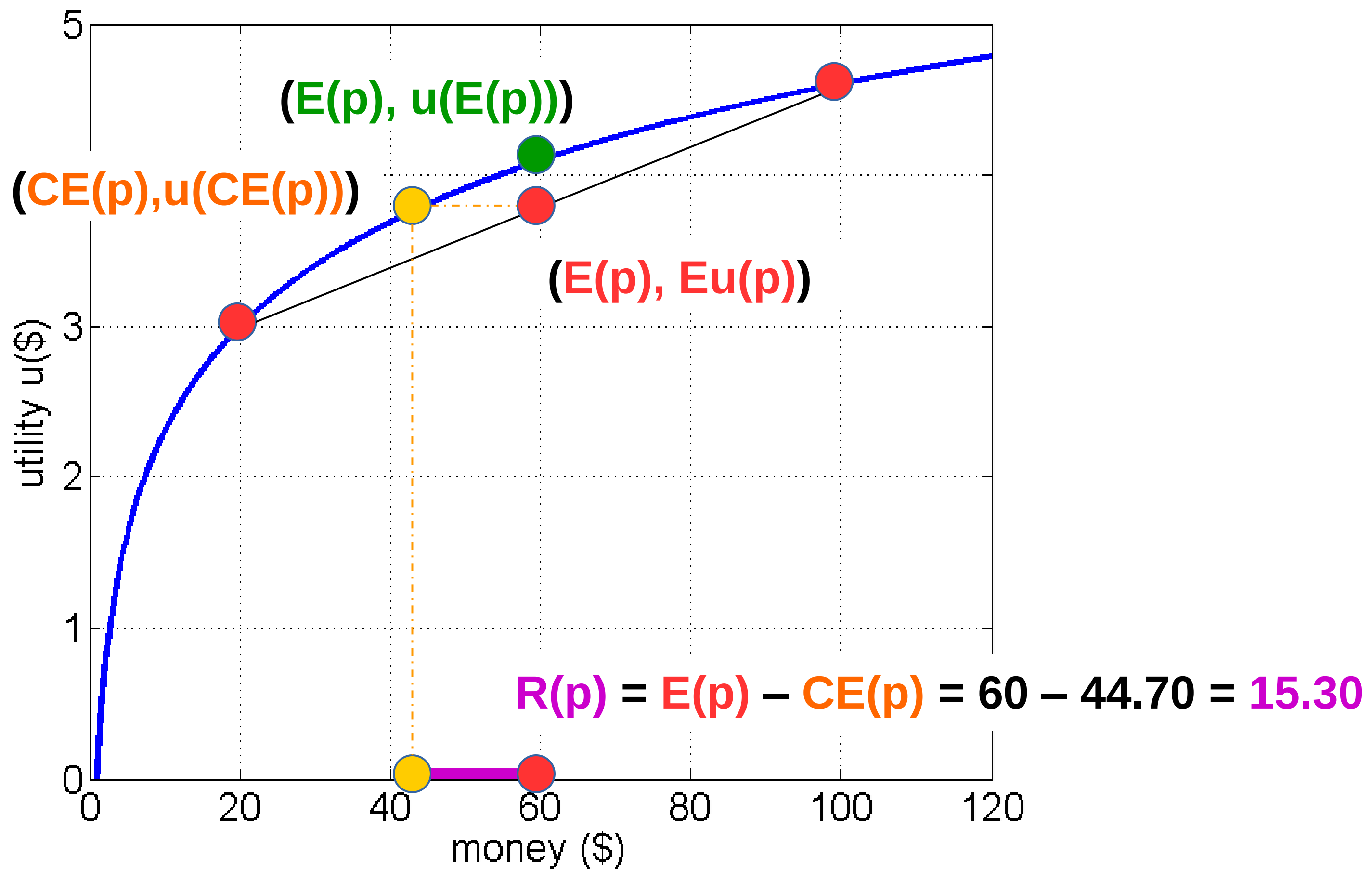
Risk Aversion

- ***u(x) = ln(x)***
- ***p = (0.5) $20 ⊕ (0.5) $100***



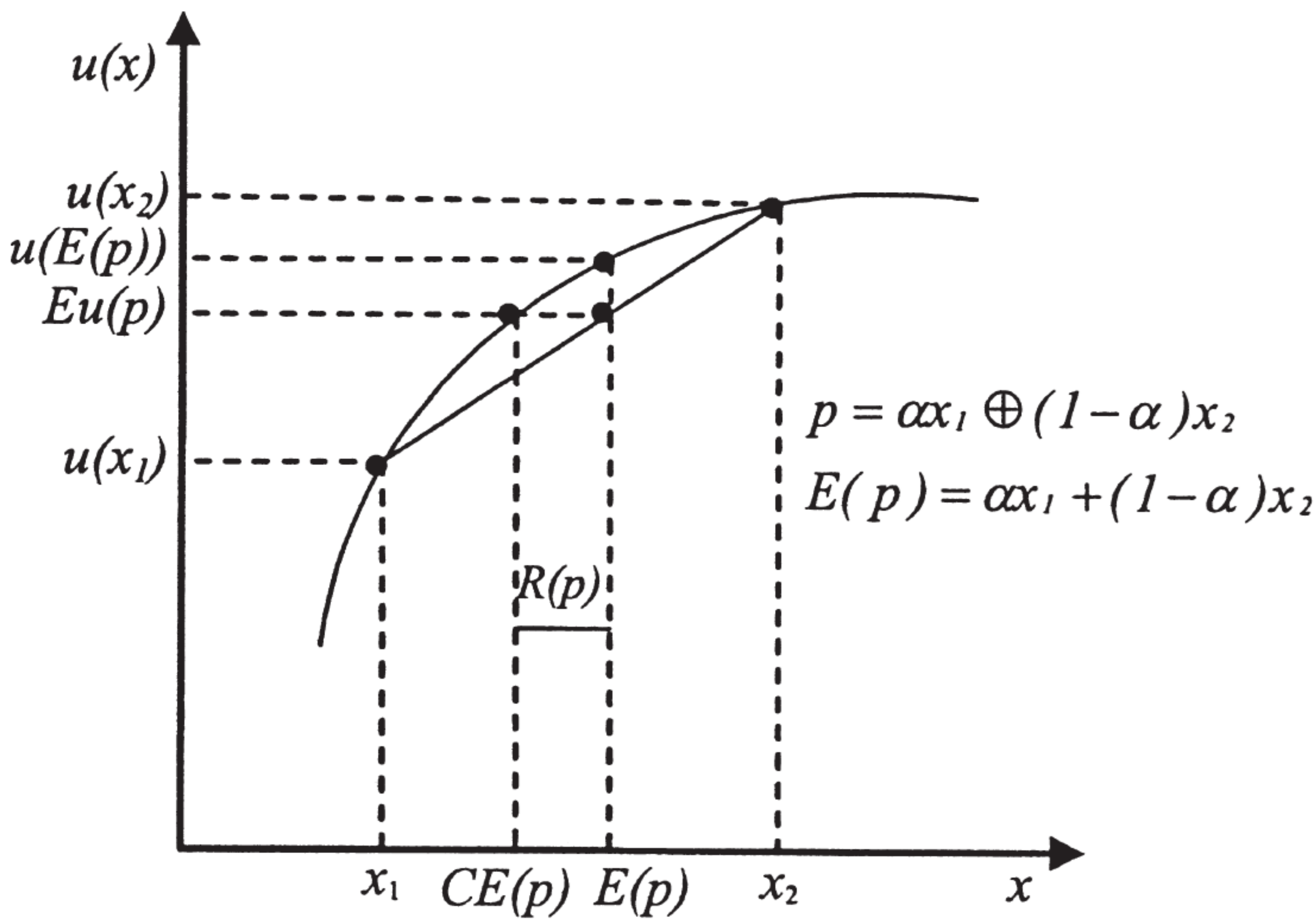
Risk Aversion

- ***u(x) = ln(x)***
- ***p = (0.5) $20 ⊕ (0.5) $100***



Risk Aversion

- Risk premium ***R(p)***
  - ***R(p) = E(p) - CE(p)***

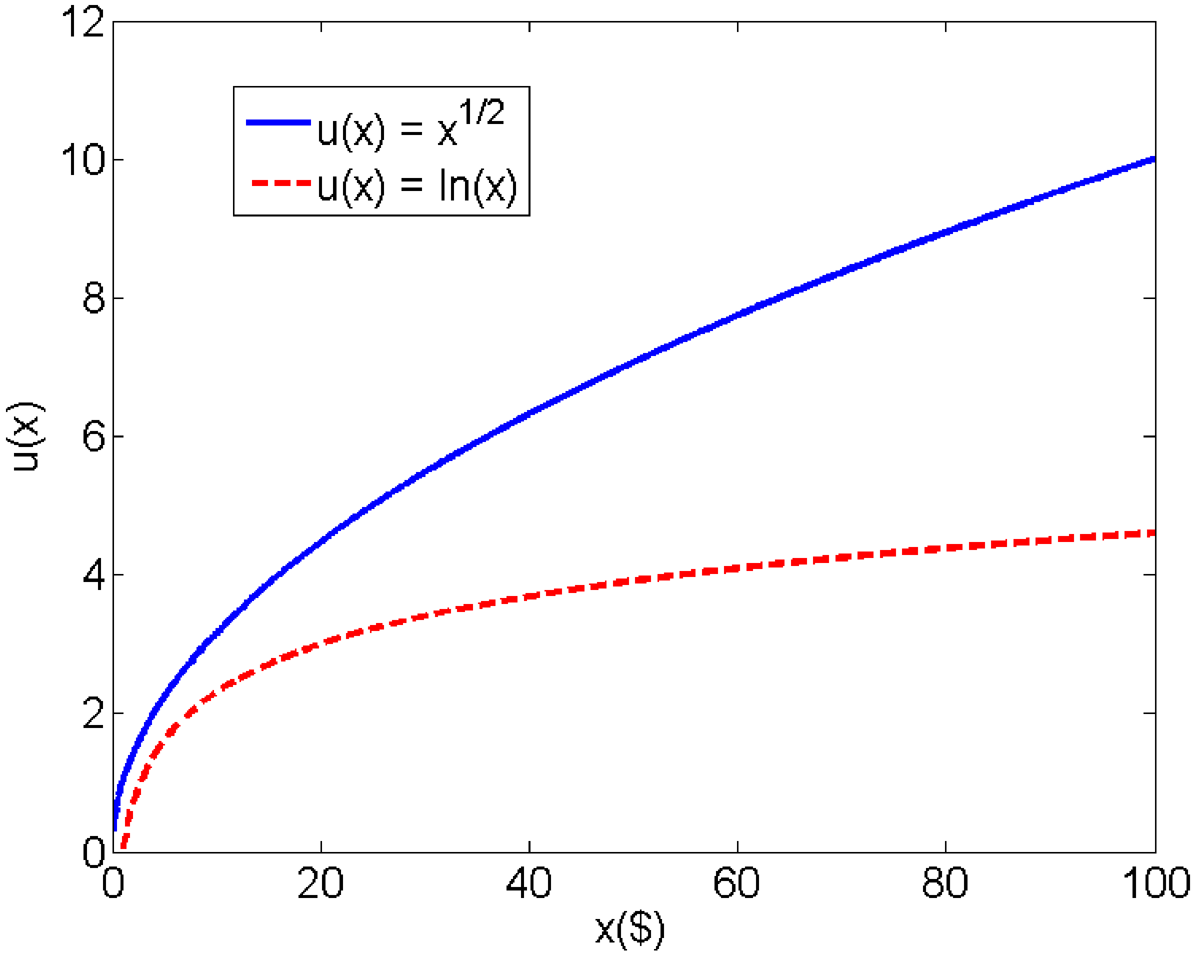


Risk Aversion

- Claim:
  - The preference relation $\boldsymbol{\succsim_1}$ is more risk averse than $\boldsymbol{\succsim_2}$ if $\boldsymbol{CE_1(p) \leq CE_2(p)}$ for all $\boldsymbol{p}$

Risk Aversion

- Which individual is more risk averse?

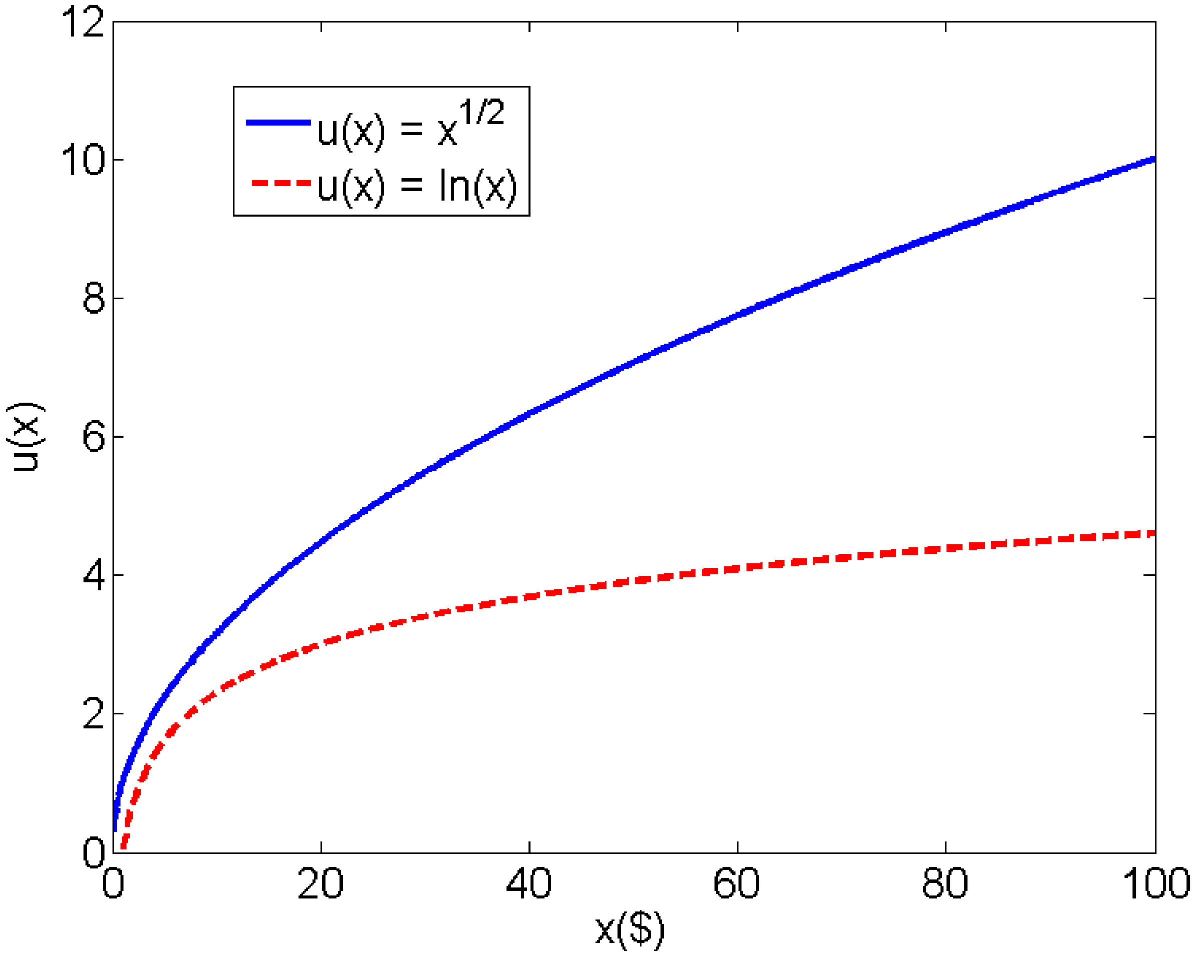


Risk Aversion

- Another definition of the relation "more risk averse" exists when vNM utility functions are twice differentiable:
  - Let u_1 and u_2 be twice differentiable vNM utility functions representing $\succsim_1$ and $\succsim_2$, respectively
  - The preference relation $\succsim_1$ is more risk averse than $\succsim_2$ if $r_1(x) \geq r_2(x)$ for all x, where
  - $r_i(x) = -u''_i(x)/u'_i(x)$
- The number $r(x)$ is called the **coefficient of absolute risk aversion** of u at x
- A higher coefficient of absolute risk aversion means a more risk-averse decision maker

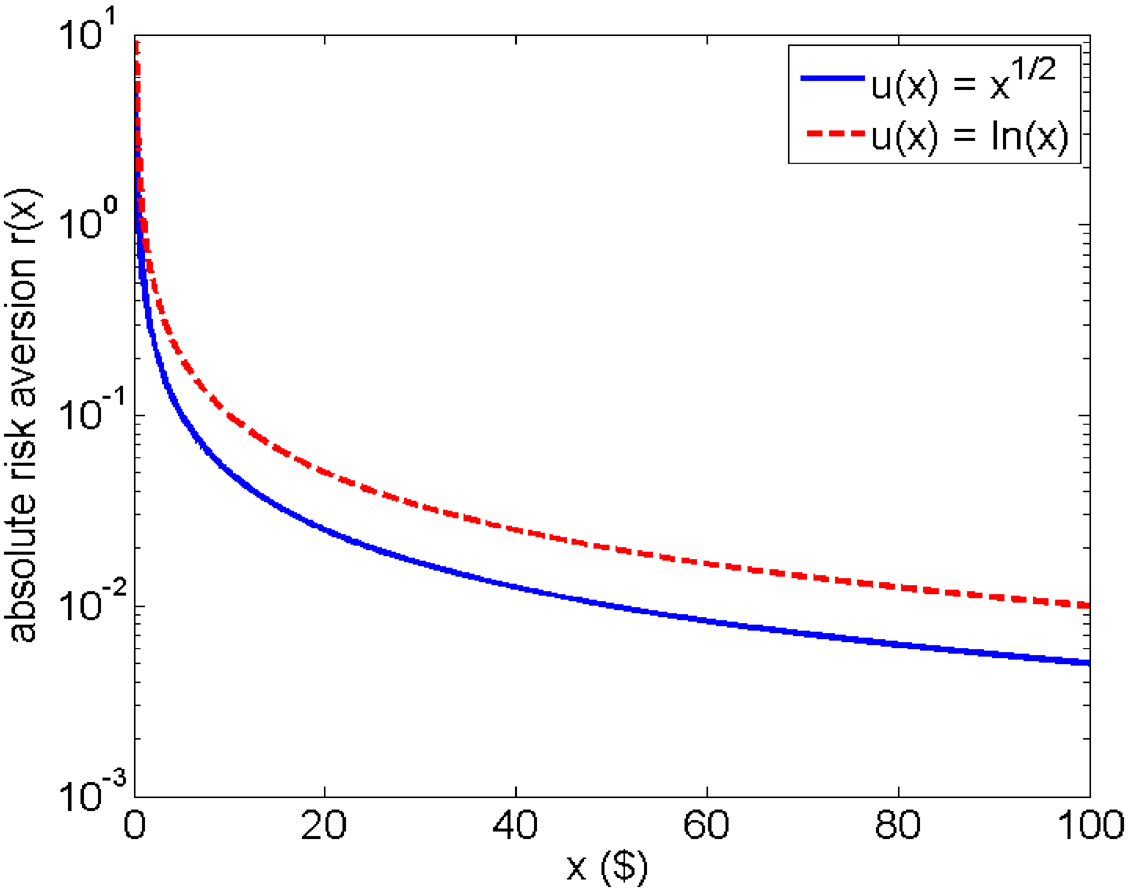
Risk Aversion

- Which individual is more risk averse?



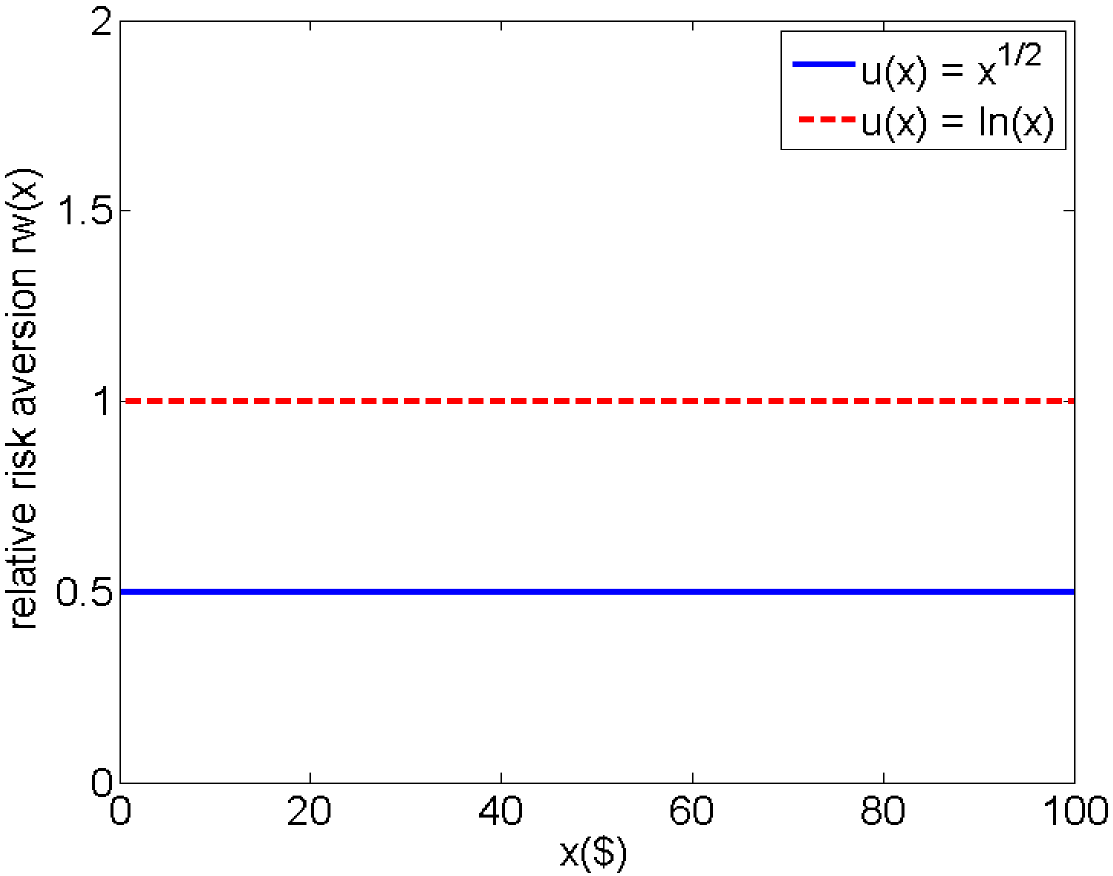
Risk Aversion

- Absolute risk aversion coefficient ***r***
  - ***r(x) = -u''(x) / u'(x)***



Risk Aversion

- Relative risk aversion coefficient ***rw***
  - ***rw(x) = x r(x)***

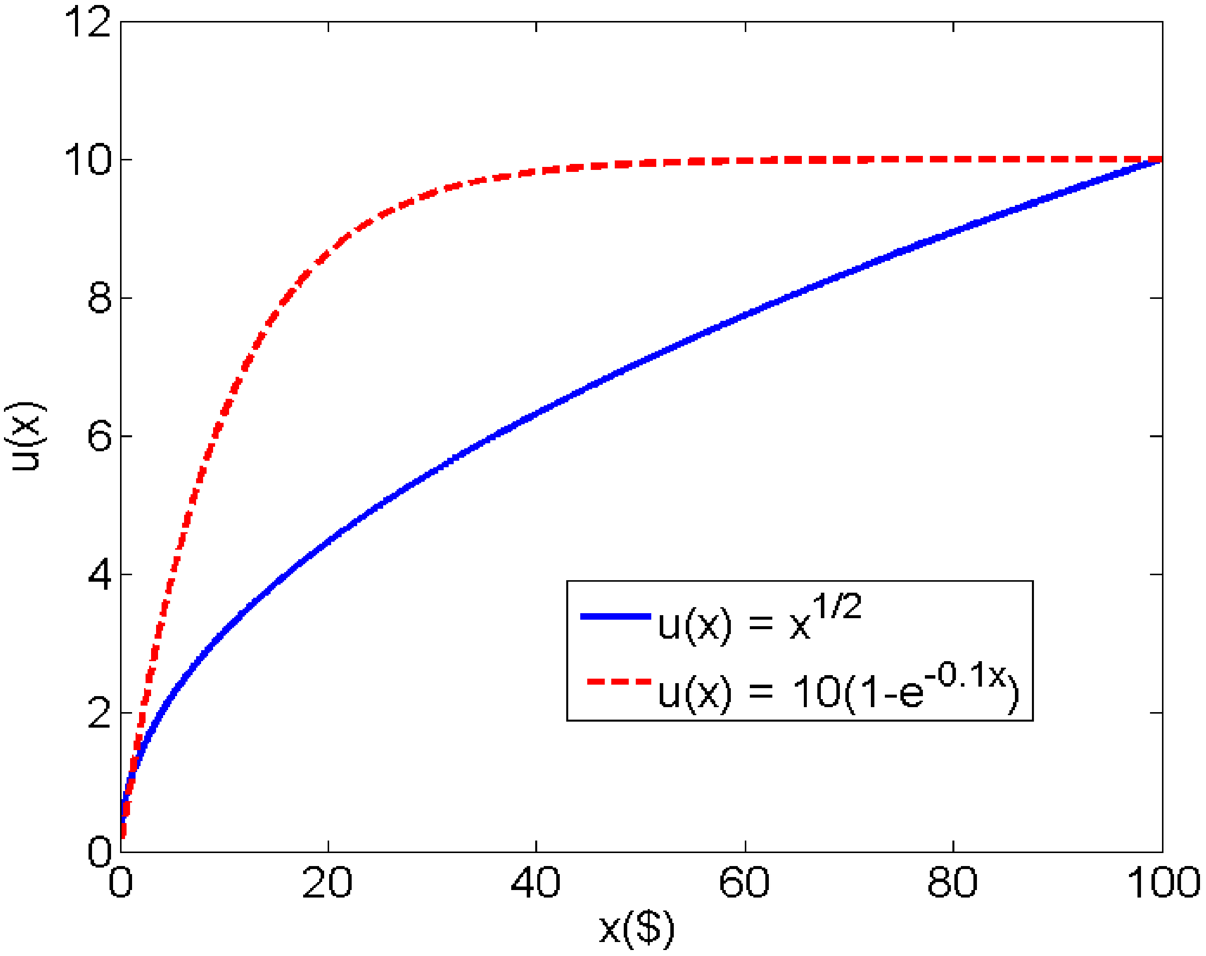


Risk Aversion

- Absolute risk aversion coefficient ***r***
  - If ***r(x)*** decreases with ***x***, then individuals will invest larger money amounts in risky assets as they get wealthier
- Relative risk aversion coefficient ***rw***
  - If ***rw(x)*** is constant with ***x***, individuals will invest the same percentage of their wealth in risky assets as they get wealthier

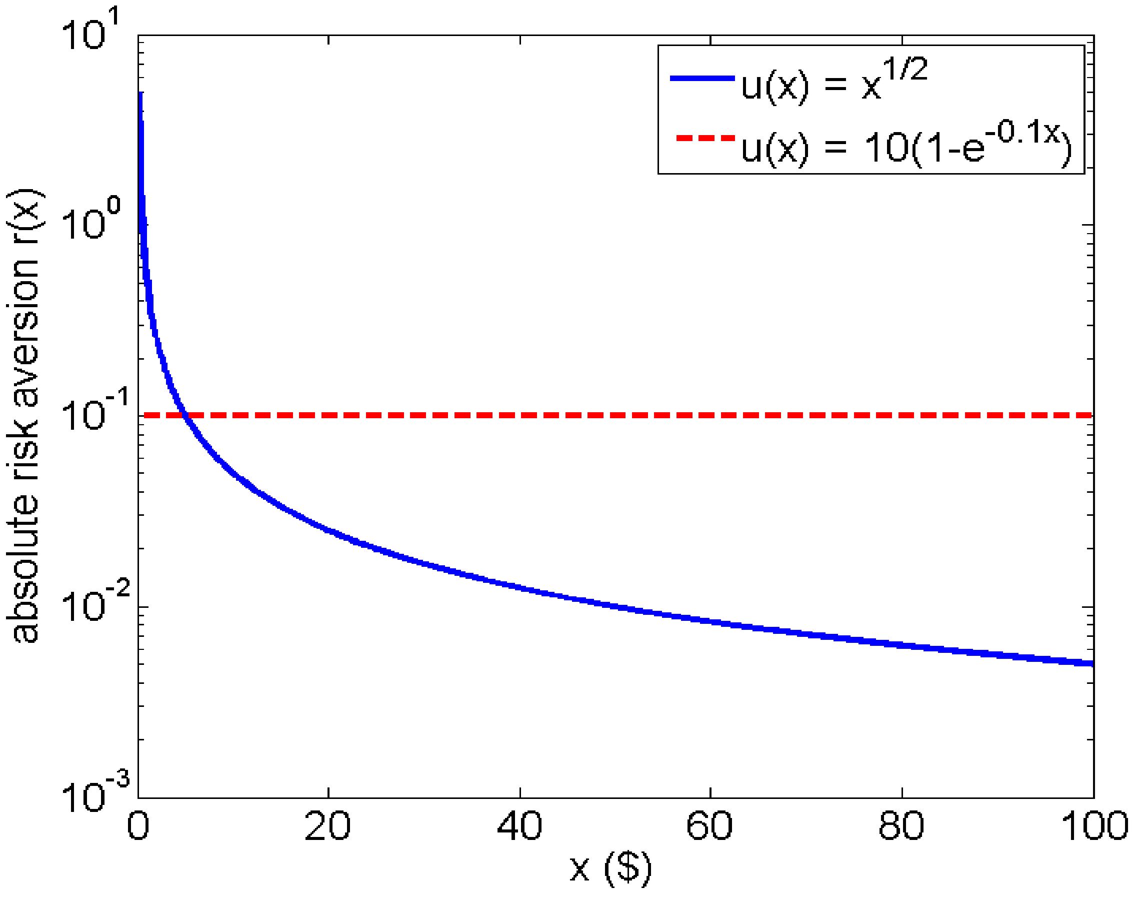
Risk Aversion

- Which individual is more risk averse?



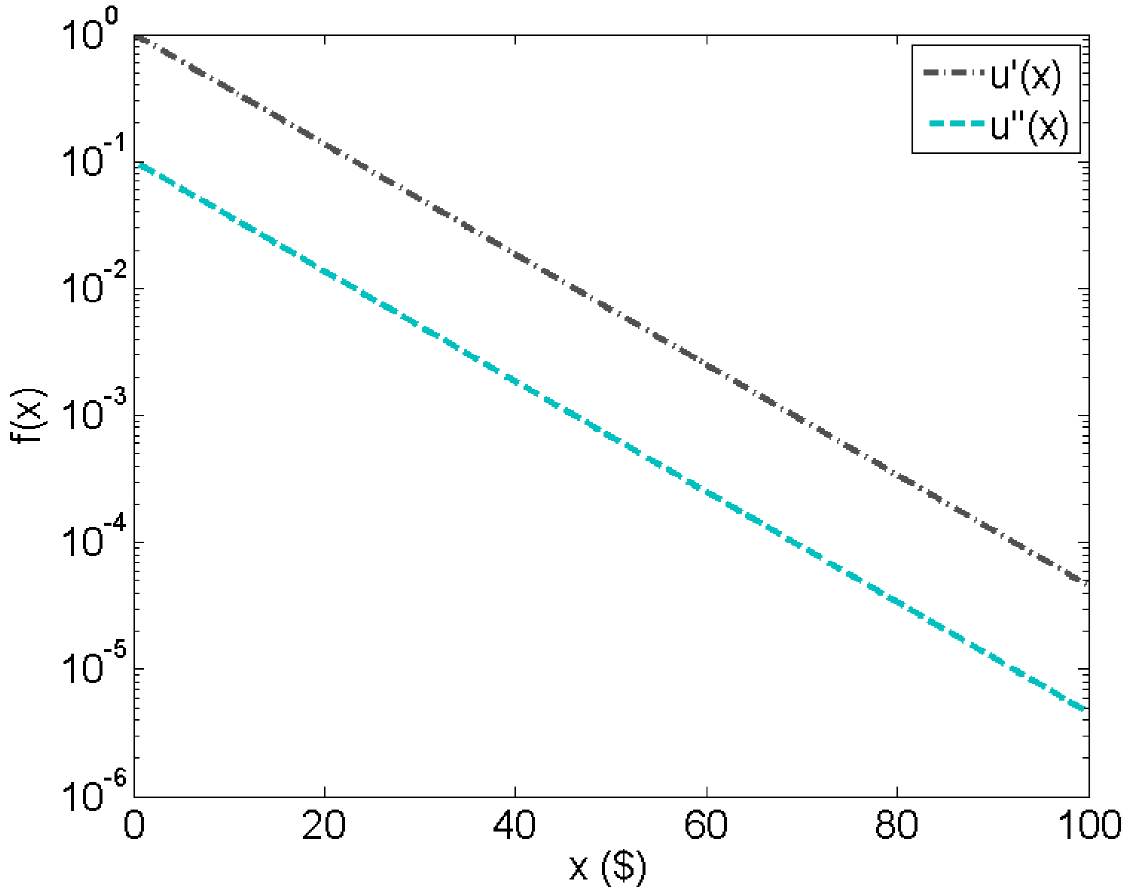
Risk Aversion

- Absolute risk aversion coefficient ***r***
  - ***r(x) = -u''(x) / u'(x)***



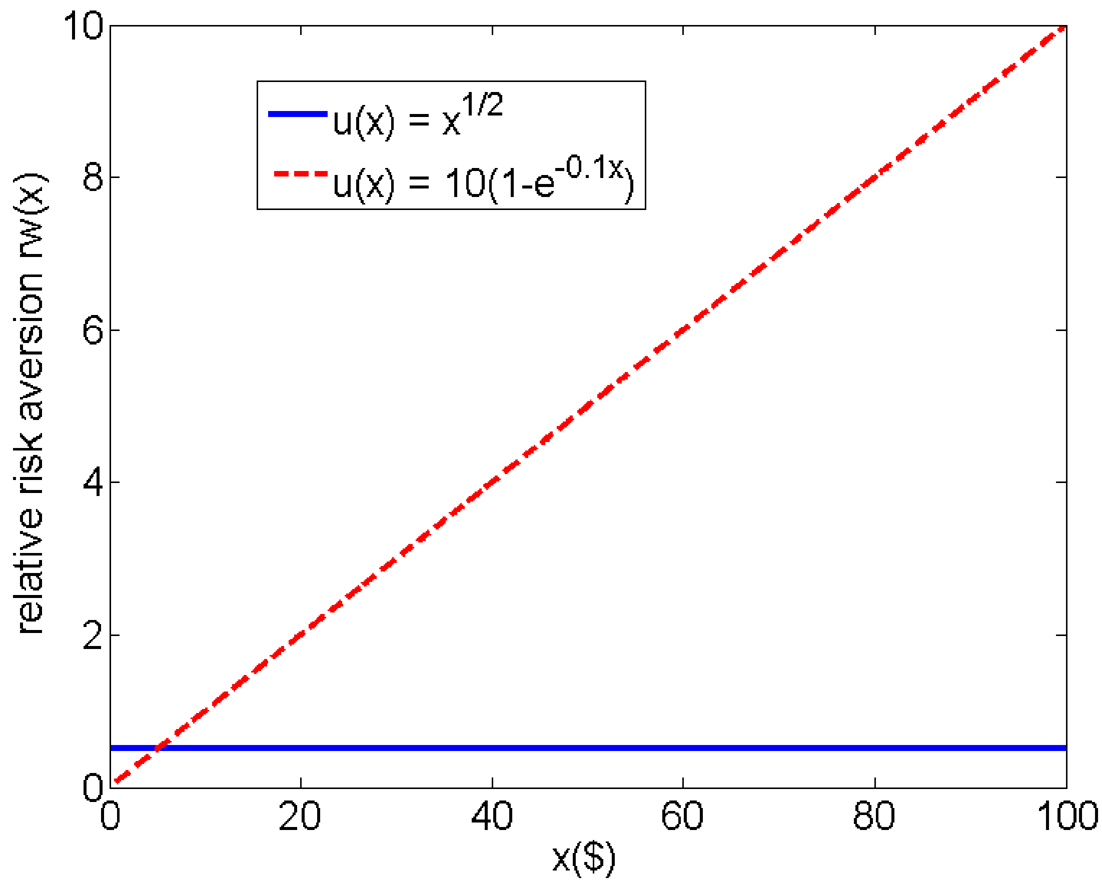
Risk Aversion

- Absolute risk aversion coefficient ***r***
  - $u(x) = 10(1-e^{-0.1x})$



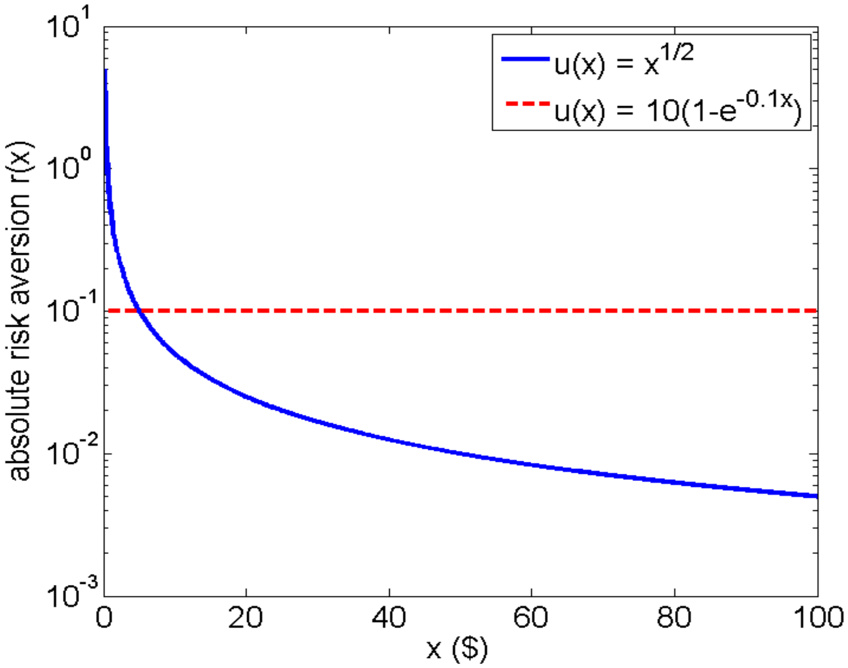
Risk Aversion

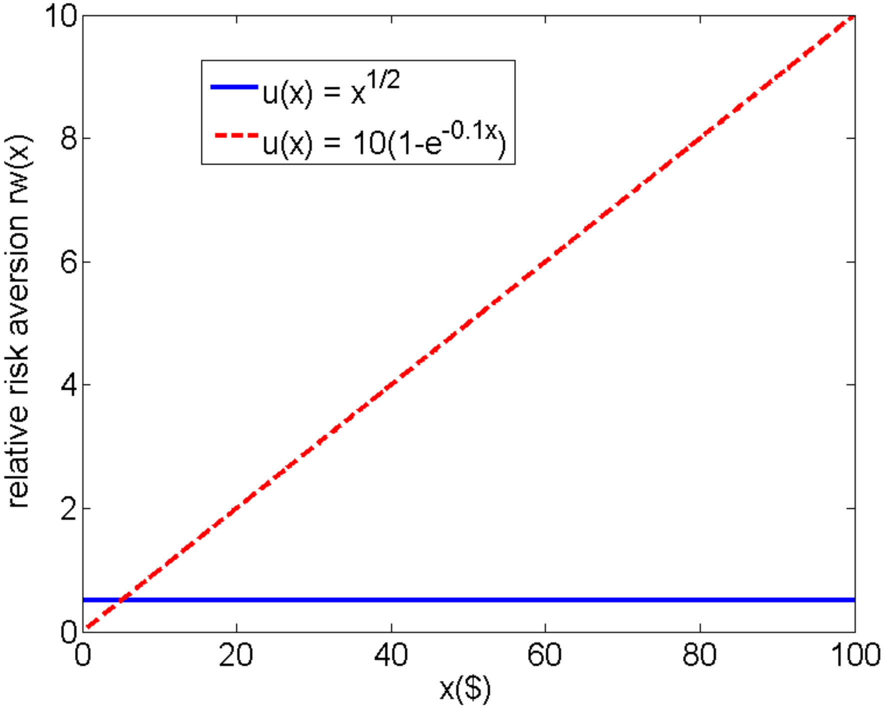
- Relative risk aversion coefficient ***rw***
  - ***rw(x) = x r(x)***



Risk Aversion

- What happens with an agent represented by the red curve?





Risk Aversion

- Absolute risk aversion coefficient ***r***
  - With constant ***r(x)***, the amount of wealth that we expose to risk remains constant as wealth increases
    - Invariance to wealth
- Relative risk aversion coefficient ***rw***
  - If ***rw(x)*** is increasing with ***x***, individuals will invest less percentage of their wealth in risky assets as they get wealthier

Invariance to Wealth

- Claim:
  - Assume that ***u*** is a vNM utility function representing preferences $\succsim$, which are monotonic and exhibit invariance to wealth
  - Then ***u*** must be exponential or linear
- Proof in the book

First-Order Stochastic Domination

- Is there other ways to compare lotteries besides using the expected utility ***Eu***?
- Now we see when a lottery ***p*** first-order stochastically dominates a lottery ***q***
  - Or $\boldsymbol{pD_1\, q}$

First-Order Stochastic Domination

- Which lottery do you prefer?

	\$0	**\$20**	**\$50**	**\$100**	**\$200**
p	0.1	0.1	0.2	0.3	0.3
q	0.15	0.05	0.25	0.35	0.20

First-Order Stochastic Domination

- Which lottery do you prefer?
  - ***$G(p,x) = \Sigma_{z \geq x}\, p(z)$***

	\$0	**\$20**	**\$50**	**\$100**	**\$200**
G(p,x)	**1** (0.1)	**0.9** (0.1)	**0.8** (0.2)	**0.6** (0.3)	**0.3** (0.3)
G(q,x)	**1** (0.15)	**0.85** (0.05)	**0.8** (0.25)	**0.55** (0.35)	**0.2** (0.20)

First-Order Stochastic Domination

- Which lottery do you prefer?
  - $F(p,x) = \Sigma_{z \leq x}\, p(z)$

	\$0	**\$20**	**\$50**	**\$100**	**\$200**
F(p,x)	**0.1** (0.1)	**0.2** (0.1)	**0.4** (0.2)	**0.7** (0.3)	**1** (0.3)
F(q,x)	**0.15** (0.15)	**0.2** (0.05)	**0.45** (0.25)	**0.80** (0.35)	**1** (0.20)

First-Order Stochastic Domination

- Claim:
  - pD_1q iff for all x, $G(p, x) \geq G(q, x)$ or
  - pD_1q iff for all x, $F(p, x) \leq F(q, x)$