QUESTÃO 1

Give an example of preferences over a countable set in which the preferences cannot be represented by a utility function that returns only integers as values.

Let $X = \mathbb{N}$, which is countable. Define preferences to be such that

$$1 \succ 3 \succ 5 \succ \ldots \succ 2 \succ 4 \succ \ldots.$$

By contradiction, assume that there exists a utility function $u : X \to \mathbb{Z}$ that represents $\succsim$. Then $u(1) = N$ and $u(2) = n$ for some $n, N \in \mathbb{Z}$. But there are an infinite number of odd numbers, implying that u maps to an infinite number of integers between n and N, a contradiction.

QUESTÃO 3

a. Is the statement "if both U and V represent $\succsim$, then there is a strictly monotonic function $f : \Re \to \Re$ such that $V(x) = f(U(x))$" correct?

False: Let $X = \Re$ and preferences be represented by the utility functions

$$V(x) = x \quad \text{and} \quad U(x) = \begin{cases} x & \text{if } x \leq 0 \\ x+1 & \text{if } x > 0. \end{cases}$$

The only increasing function $f : \Re \to \Re$ that satisfies $V(x) = f(U(x))$ is

$$f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 0 & \text{if } 0 < x \leq 1 \\ x-1 & \text{if } x > 1 \end{cases}$$

which is not *strictly* increasing.

QUESTÃO 4

b. Can a continuous preference relation be represented by a discontinuous utility function?

True: The preferences ($x \succsim y$ if $x \geq y$) is represented by U in (a) are continuous, though U is discontinuous.

QUESTÃO 5

c. Show that in the case of $X = \Re$, the preference relation that is represented by the discontinuous utility function $u(x) = [x]$ (the largest integer n such that $x \geq n$) is not a continuous relation.

$1 \succ 1/2$, but $1 - \epsilon \sim 1/2$ for $\epsilon > 0$ small, violating C1.

QUESTÃO 6

The following are descriptions of decision making procedures. Discuss whether the procedures can be described in the framework of the choice model discussed in this lecture and whether they are compatible with the "rational man" paradigm.

a. The DM chooses an alternative in order to maximize another person's suffering.

Assuming that the relation "the other person suffers more from x than he does from y" is complete and transitive, the DM is maximizing a well-defined preference relation.

b. The DM asks his two children to rank the alternatives and then chooses the alternative that is the best "on average".

The question is, of course, what does the expression "on average" mean. If the DM ranks all alternatives in X and uses the ranking to attach the number to each alternative a in any set A (independently of A), then the DM's behavior is consistent with rationality. But if the score of an alternative is recalculated for every choice set then his behavior may be inconsistent with the rational man paradigm. For example, assume that one child ranks the alternatives a, d, e, b, c and the other as b, c, a, d, e. Then, the element a is chosen from the set $\{a, b, c, d, e\}$ while b is chosen from $\{a, b, c\}$.

c. The DM has an ideal point in mind and chooses the alternative that is closest to it.

Let x be the ideal point and $d(a, b)$ the distance function between $a, b \in X$. The behavior is rationalized by the preferences represented by $u(a) = -d(a, x)$.

d. The DM looks for the alternative that appears most often in the choice set.

A choice function C is not well-defined. The DM's behavior is different when faced with the group of elements (a, a, b) than when faced with the group (a, b, b), even though in both cases he chooses from the set $\{a, b\}$.

e. The DM has an ordering in mind and always chooses the median element.

C violates condition α. Assume that the order of the grand set $X = \{a, b, c, d, e\}$ is alphabetical. Then, $C(\{a, b, c, d, e\}) = \{c\}$ but $C(\{a, b, c\}) = \{b\}$.

QUESTÃO 7

Consider the following choice procedure: A decision maker has a strict ordering $\succsim$ over the set X and assigns to each $x \in X$ a natural number $class(x)$ to be interpreted as the "class" of x. Given a choice problem A, he chooses the best element in A from those belonging to the most common class in A (i.e., the class that appears in A most often). If there is more than one most common class, he picks the best element from the members of A that belong to a most common class with the highest class number.

a. Is this procedure consistent with the "rational man" paradigm?

No. Let $a \succ b \succ c \succ d \succ e$, $class(a) = class(b) = class(c) = 1$ and $class(d) = class(e) = 2$. $C(\{a,b,c,d,e\}) = a$ but $C(\{a,d,e\}) = d$, thus violating α.

b. Define the relation $\mathbf{xPy}$ if $\mathbf{x}$ is chosen from $\{\mathbf{x},\mathbf{y}\}$. Show that the relation P is a strict ordering (complete, asymmetric and transitive).

By definition, P is complete and asymmetric. We will see that it is also transitive. That is, if xPy and yPz, then xPz.

If xPy and yPz, then $[class(x) > class(y)$ or $class(x) = class(y)$ and $x \succsim y]$, and $[class(y) > class(z)$ or $class(y) = class(z)$ and $y \succsim z]$. If either $class(x) > class(y)$ or $class(y) > class(z)$, then $class(x) > class(z)$ and $C(\{x,z\}) = \{x\}$. Otherwise, $class(x) = class(z)$ and $x \succsim z$ and thus $x \in C(\{x,z\})$.

Alternatively: Note that P is identical to the lexicographic preferences with first priority given to class and second priority to the relation $\succsim$.

QUESTÃO 11

Adam lives in the Garden of Eden and eats only apples. Time in the garden is discrete ($t = 1,2,\ldots$) and apples are eaten only in discrete units. Adam possesses preferences over the set of streams of apple consumption. Assume that:

a) Adam likes to eat up to 2 apples a day and cannot bear to eat 3 apples a day.

b) Adam is impatient. He would be delighted to increase his consumption on day t from 0 to 1 or from 1 to 2 apples at the expense of an apple he is promised a day later.

c) In any day in which he does not have an apple, he prefers to get one apple immediately in exchange for two apples tomorrow.

d) Adam expects to live for 120 years.

Show that if (poor) Adam is offered a stream of 2 apples starting in day 4 for the rest of his expected life, he would be willing to exchange that offer for one apple right away.

The following is a sequence of streams, in an increasing ordering:

(0,0,0,2,2,......,2)

(0,0,1,0,2,......,2). and continuing in this way until:

(0,0,1,1,1,......1,0)

(0,0,2,0,2,0...,2,0,0)

(0,1,0,1,0,....1,0,1,0,0,0) and "folding from the end":

(0,1,0,1,0,.1,0..2,0,0,0,0,0)

(0,1,0,1,0,.1,1,0,0,0,0,0,0)...until we reach:

(0,2,0,...0)

(1,0,....)