Modeling Rational Agents


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Modeling Human Behavior

- What is (mathematical) modeling?
  - A description of a system using mathematical concepts and language
  - May help to
    - explain a system
    - study the effects of different components
    - make predictions about behavior

Modeling Human Behavior

- What is game theory?
  - The study of mathematical models of conflict and cooperation between intelligent rational decision-makers

Modeling Human Behavior

- How can we mathematically describe the conflicts between two agents?

Game Theory

- A model of a rational agent
  - Preferences
  - Utility
  - Choice
- Conflicts between agents
  - Game theoretic models

Preferences

Economic Agent

- Which characteristics are required to model an economic agent?
  - Name, age and gender, personal history, brain structure, cognitive abilities, his emotional state etc
- In most of economic theory, an economic agent is modeled ***only by his attitude*** toward the elements in some relevant set
- His attitude is expressed in the form of ***preferences***

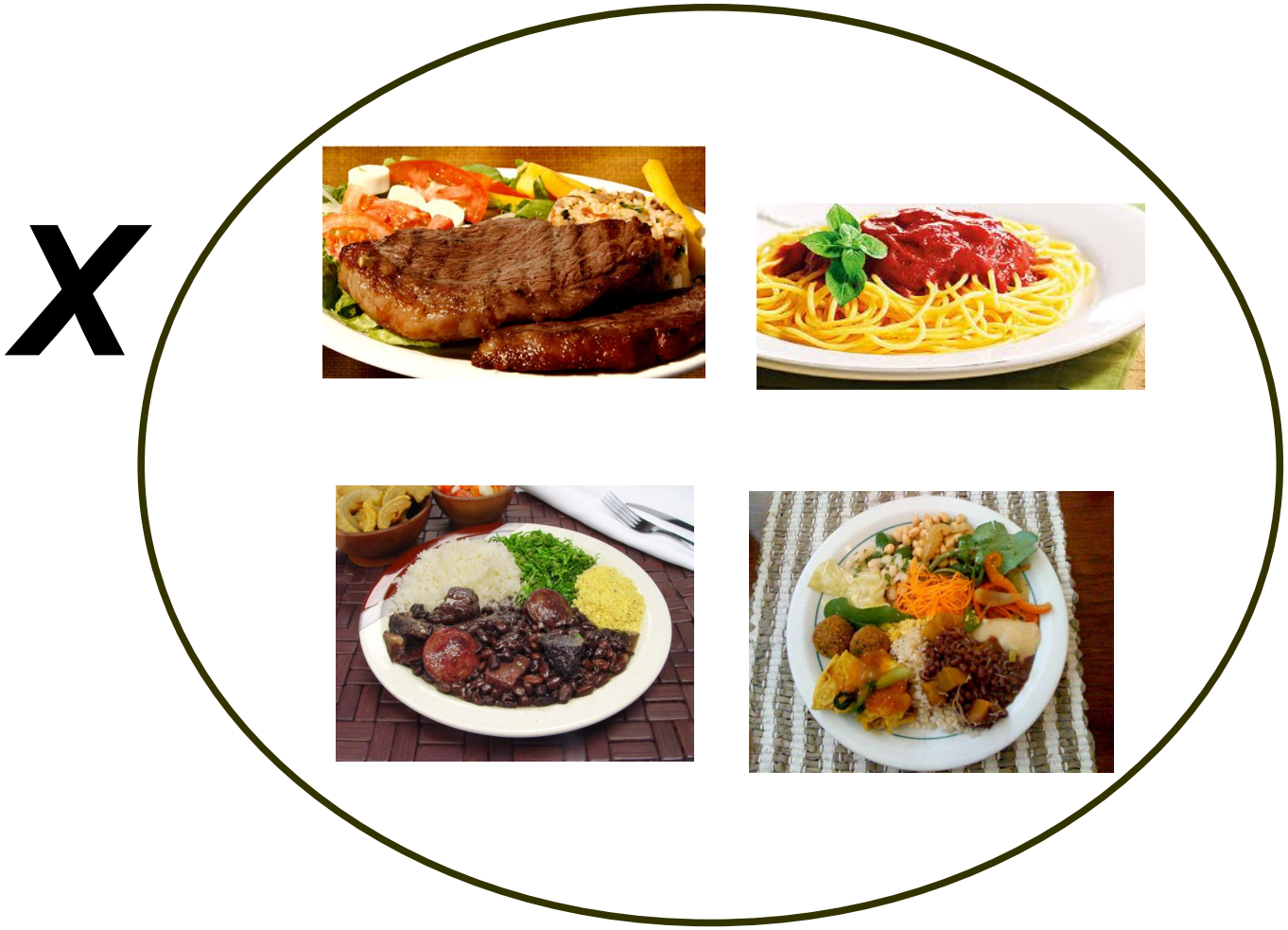
Preferences

- Which object in the set ***X*** do you prefer?

X

Preferences

- A description of preferences should fully specify the attitude of the agent toward each pair of elements in ***X***

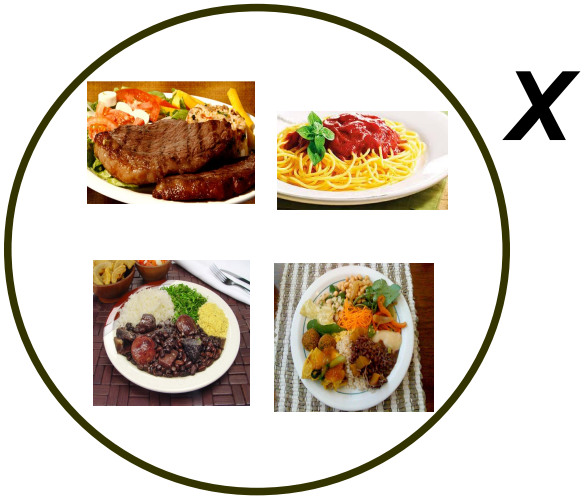


Questionnaire Q

Q(x,y) (for all distinct x and y in X):

How do you compare x and y? Tick one and only one of the following three options:

- ☐ I prefer x to y (this answer is denoted as $x \succ y$).
- ☐ I prefer y to x (this answer is denoted by $y \succ x$).
- ☐ I am indifferent (this answer is denoted by I).



Questionnaire Q

Q(x,y) (for all distinct x and y in X):

How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

- A "***legal***" answer to the questionnaire is a response in which exactly one of the boxes is ticked in each question

Questionnaire Q

- Exclusion of responses that demonstrate a ***lack of ability to compare***, such as:

☐ They are incomparable.

Questionnaire Q

- Exclusion of responses that demonstrate a ***dependence of other factors***, such as:

☐ It depends on what my parents think.

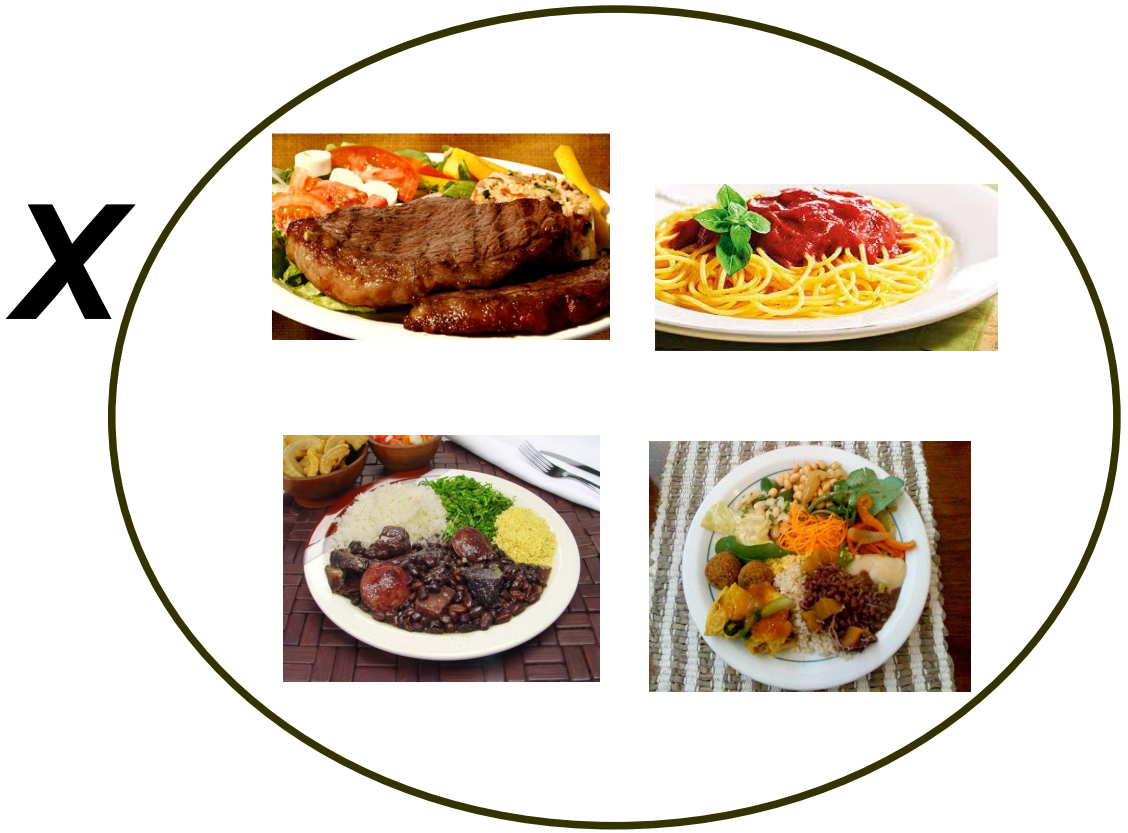
Questionnaire Q

- Exclusion of responses that demonstrate an ***intensity of preferences***, such as:

☐ I somewhat prefer x.

Questionnaire Q

- The elements in the set ***X*** are all comparable and the **intensity** of preferences are ignored



Questionnaire Q

A legal answer to the questionnaire can be formulated as a function ***f***,

which assigns to any pair ***(x, y)*** of distinct elements in ***X*** exactly one of the three “values”,

- $x > y$ or
- $y > x$ or
- ***I***,

with the interpretation that ***f(x, y)*** is the answer to the question ***Q(x, y)***

Questionnaire Q

$$Q(x, y) \rightarrow f(x, y) \begin{cases} x \succ y \\ y \succ x \\ I \end{cases}$$

Preference symbol

$$y \succ x$$

Preferences

- **Definition 1**
  - Preferences on a set ***X*** are a function ***f***
  - that assigns to any pair ***(x, y)*** of distinct elements in ***X*** exactly one of the three "values"
  - ***x*** $\succ$ ***y***, ***y*** $\succ$ ***x***, or ***I***
  - so that for any three different elements ***x***, ***y***, and ***z*** in ***X***, the following two properties hold:
    - No order effect: ***f(x, y) = f(y, x)***
    - Transitivity:
      - if ***f(x, y) = x*** $\succ$ ***y*** and ***f(y, z) = y*** $\succ$ ***z***, then ***f(x, z) = x*** $\succ$ ***z*** and
      - if ***f(x, y) = I*** and ***f(y, z) = I***, then ***f(x, z) = I***

A discussion of transitivity

- How would you react if somebody told you she/he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?

A discussion of transitivity

Questionnaire

Consider the travel options bellow and answer: which one do you prefer?

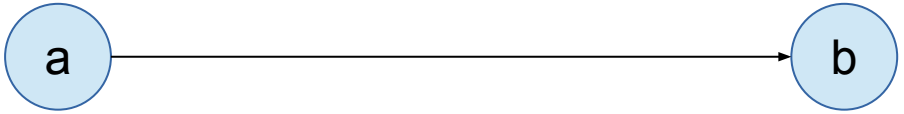
Open a text file on you computer and write an answer file in the following format. Each line contains your answer for a question. Write ***1*** if you prefer the first option, ***2*** if you prefer the second, or ***0*** if you are indifferent between the options. There is an example of an answer file at the end of this document.

1) A weekend at a 3 star hotel in New York with friends for $574 OR a weekend for $574 with friends at a 3 star hotel in Paris?

2) A weekend in New York with friends at a 3 star hotel for $574 OR a weekend at a 5 star hotel in New York for $712 with family?

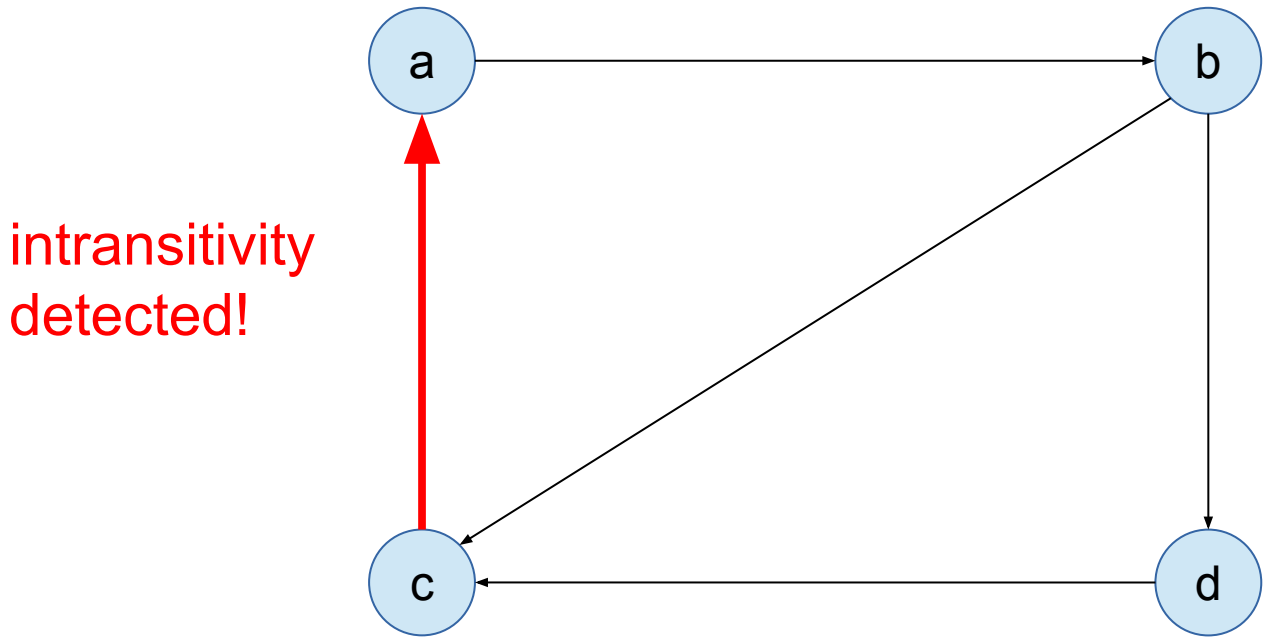
A discussion of transitivity

- There is a direct edge from ***i*** to ***j*** if $j \succ i$



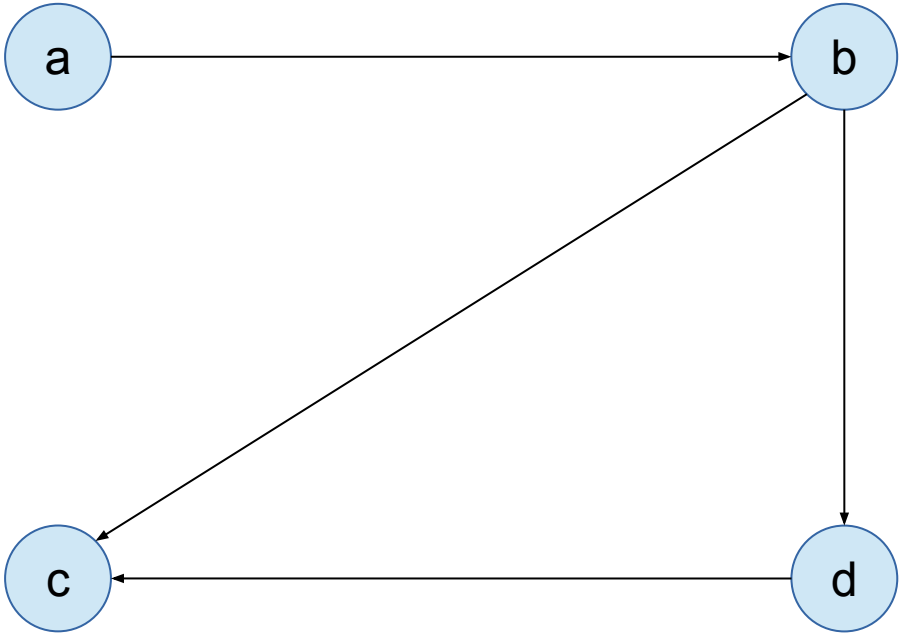
A discussion of transitivity

- How to check intransitivities from this questionnaire?



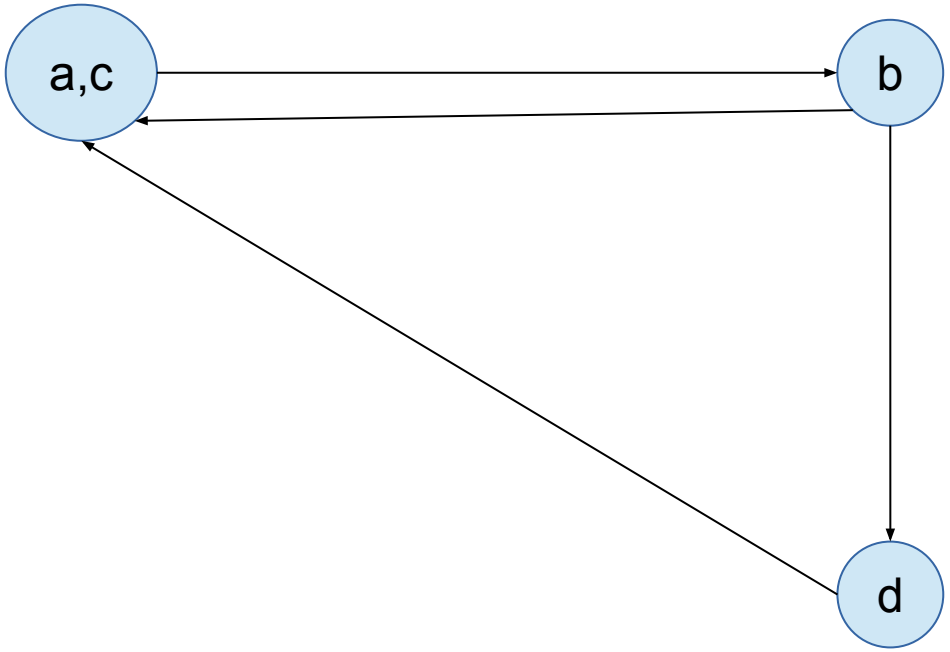
A discussion of transitivity

- And if I am indifferent between ***a*** and ***c***?



A discussion of transitivity

- And if I am indifferent between ***a*** and ***c***?



A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of **6** students who responded questionnaire **Q1** in 2025/01…
- ...3 (50%) had no intransitivities
- The median number intransitivities per student was 0.5
- The mean was 1.17

Easy questions?

6) A weekend at a 5 star hotel with romance for $842 in New York OR a weekend at a 5 star hotel for $574 in New York with romance?

30) A weekend in New York at a 3 star hotel for $842 with friends OR a weekend at a 5 star hotel for $574 in New York with friends?

31) A weekend with friends at a 5 star hotel in New York for $842 OR a weekend for $842 with friends in New York at a 3 star hotel?

37) A weekend with friends in Paris at a 5 star hotel for $574 OR a weekend at a 3 star hotel for $574 with friends in Paris?

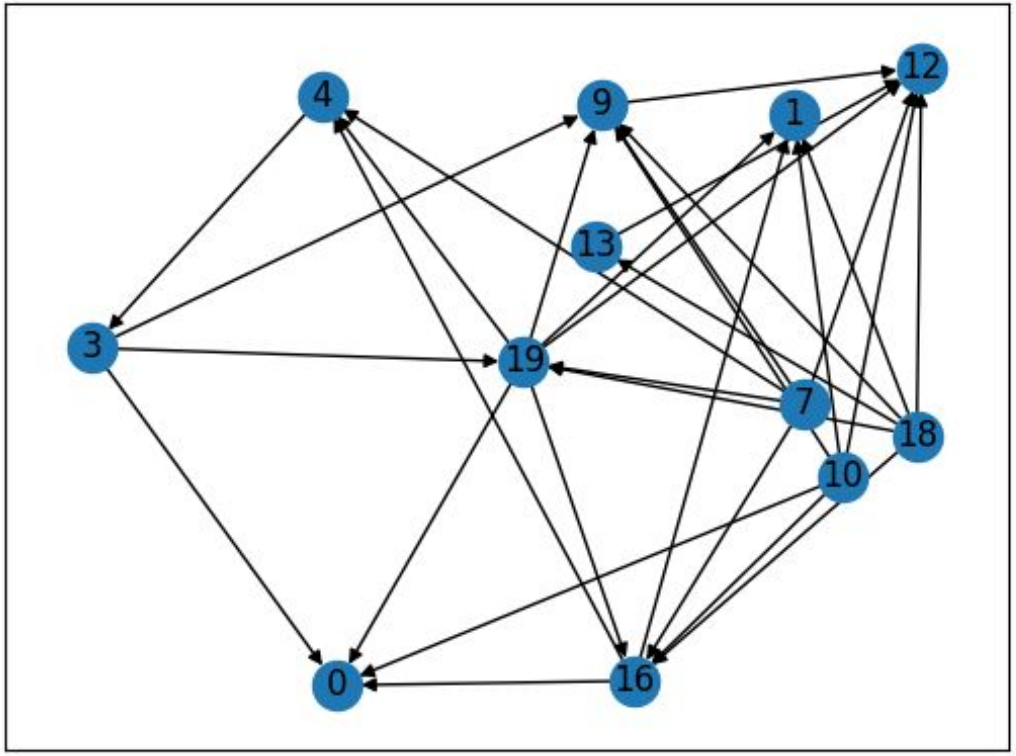
38) A weekend with friends at a 3 star hotel for $574 in Paris OR a weekend at a 3 star hotel with friends for $842 in Paris?

39) A weekend for $574 with romance at a 5 star hotel in New York OR a weekend for $842 at a 3 star hotel in New York with romance?

Easy questions?

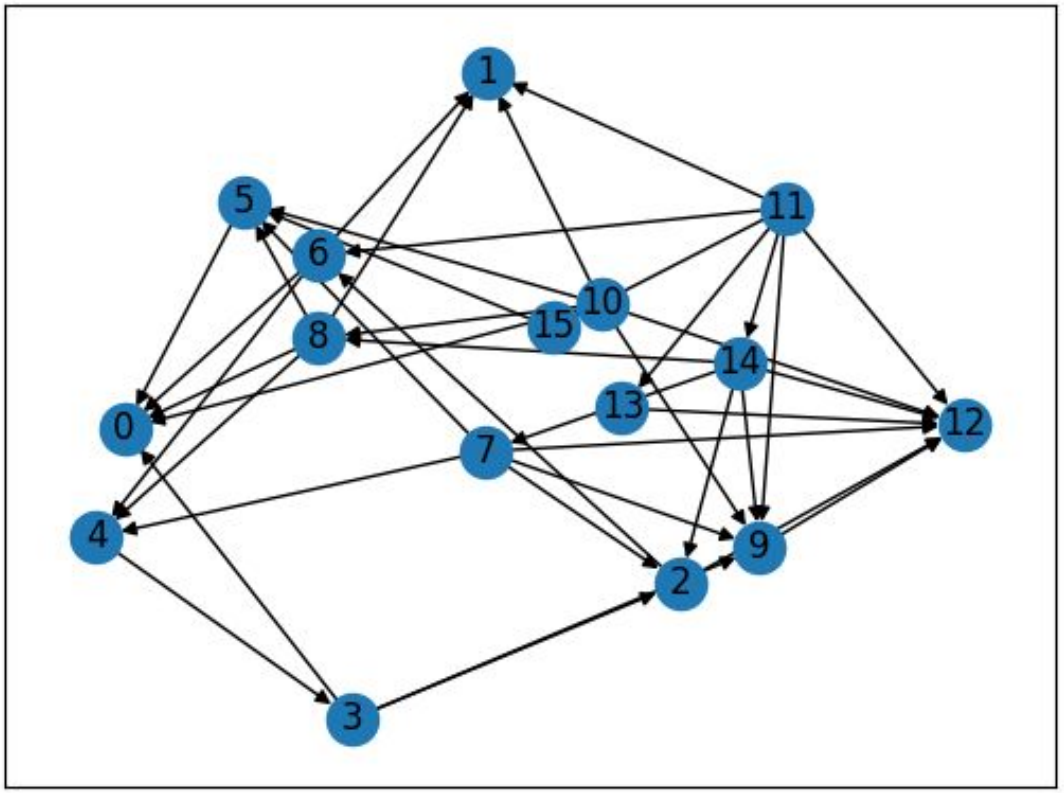
- How would you react if somebody fails to answer those easy questions?
- Out of 6 students who responded questionnaire Q1 in 2025/01…
- ...2 (~33%) gave unreasonable answers to the easy questions

Thiago Assis



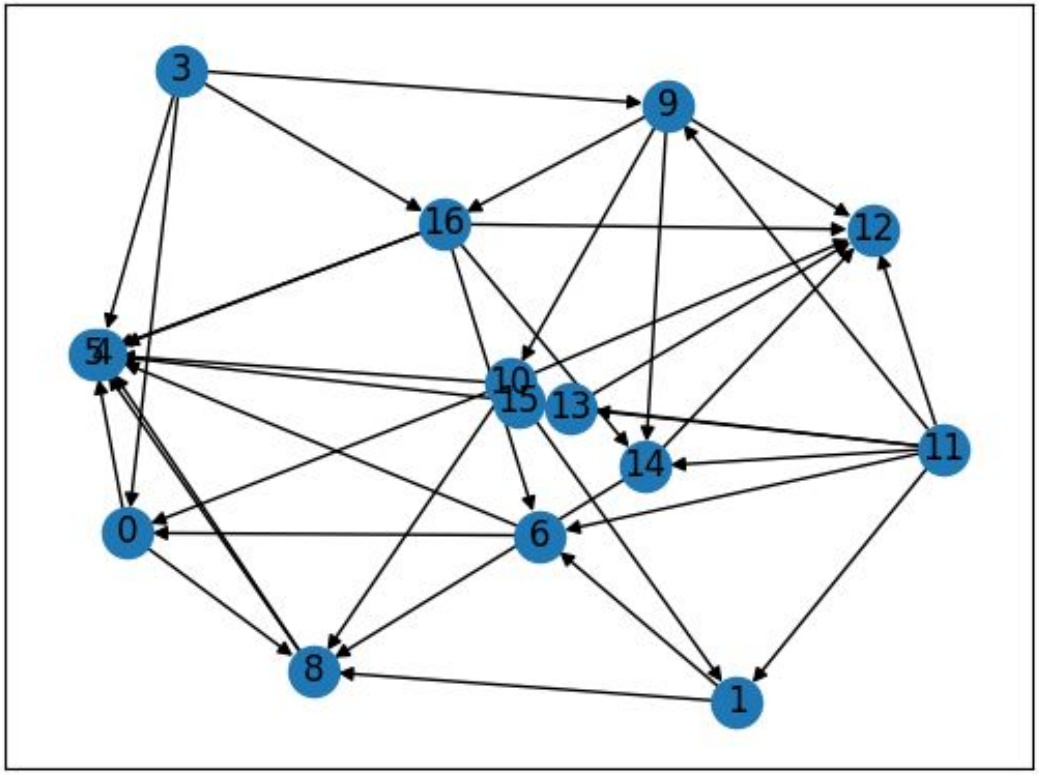
Number of nodes: 12
Number of cycles: 2
Cycles: [[16, 4, 3, 19], [3, 19, 4]]
Number of edges 31
Number of dominated choices: 2

Laila Melo



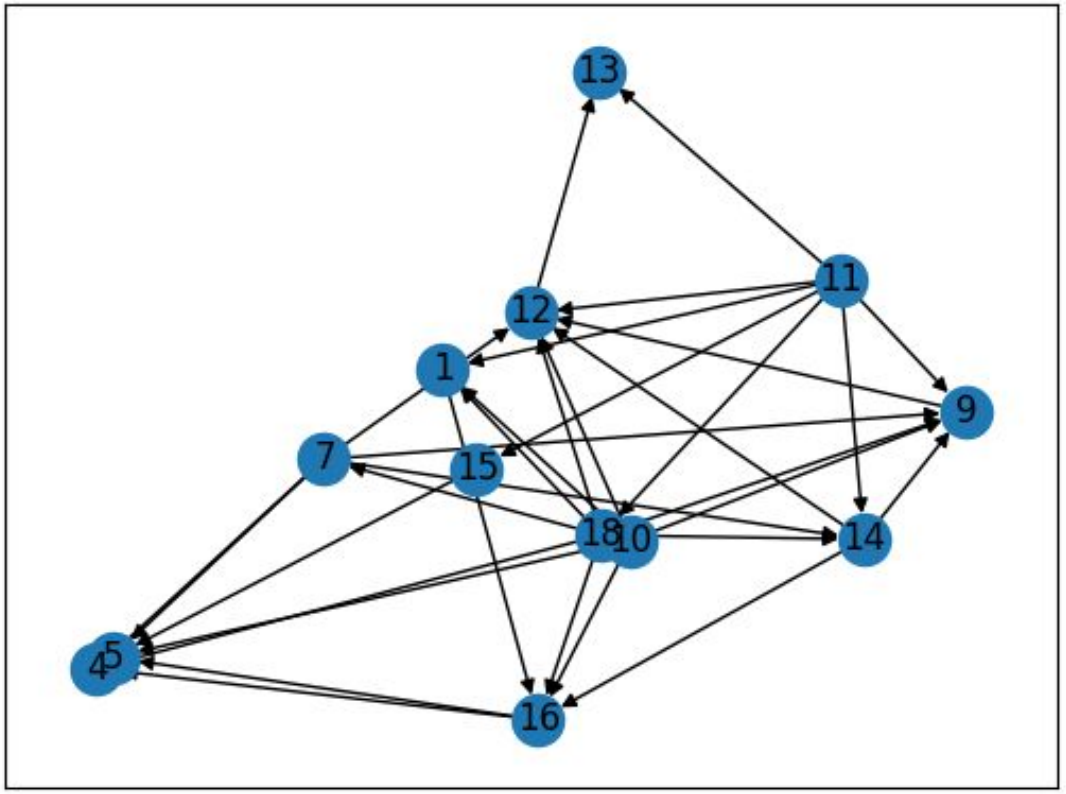
Number of nodes: 16
Number of cycles: 1
Cycles: [[2, 6, 4, 3]]
Number of edges 40
Number of dominated choices: 0

Henrique Magalhães



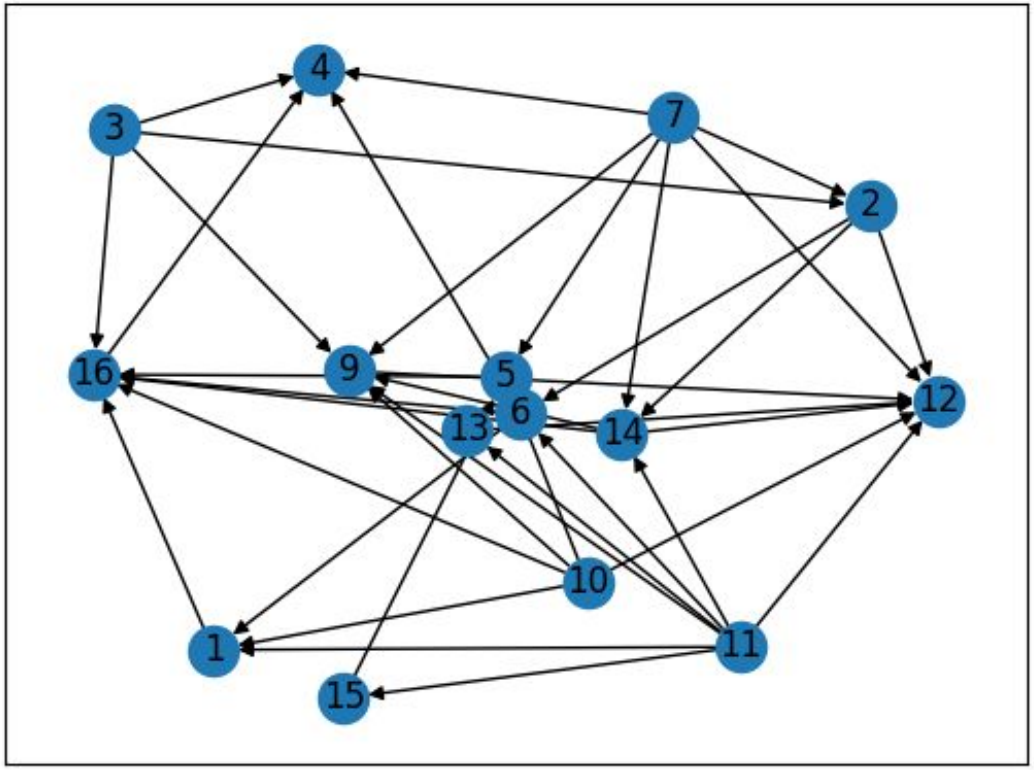
Number of nodes: 15
Number of cycles: 0
Cycles: []
Number of edges 37
Number of dominated choices: 0

Haniel Botelho



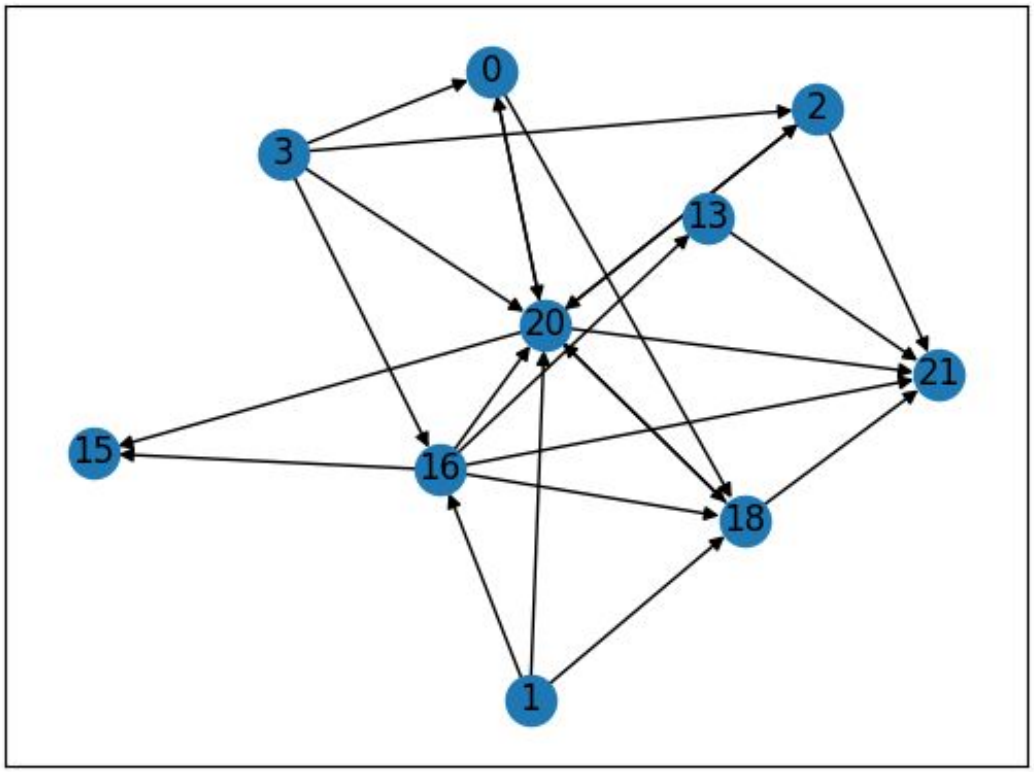
Number of nodes: 13
Number of cycles: 0
Cycles: []
Number of edges 33
Number of dominated choices: 3

Lorenzo Correa



Number of nodes: 15
Number of cycles: 0
Cycles: []
Number of edges 37
Number of dominated choices: 0

Matheus Farnese



Number of nodes: 10
Number of cycles: 4
Cycles: [[0, 20], [0, 18, 20], [2, 20], [18, 20]]
Number of edges 24
Number of dominated choices: 0

Semestres Anteriores

A discussion of transitivity

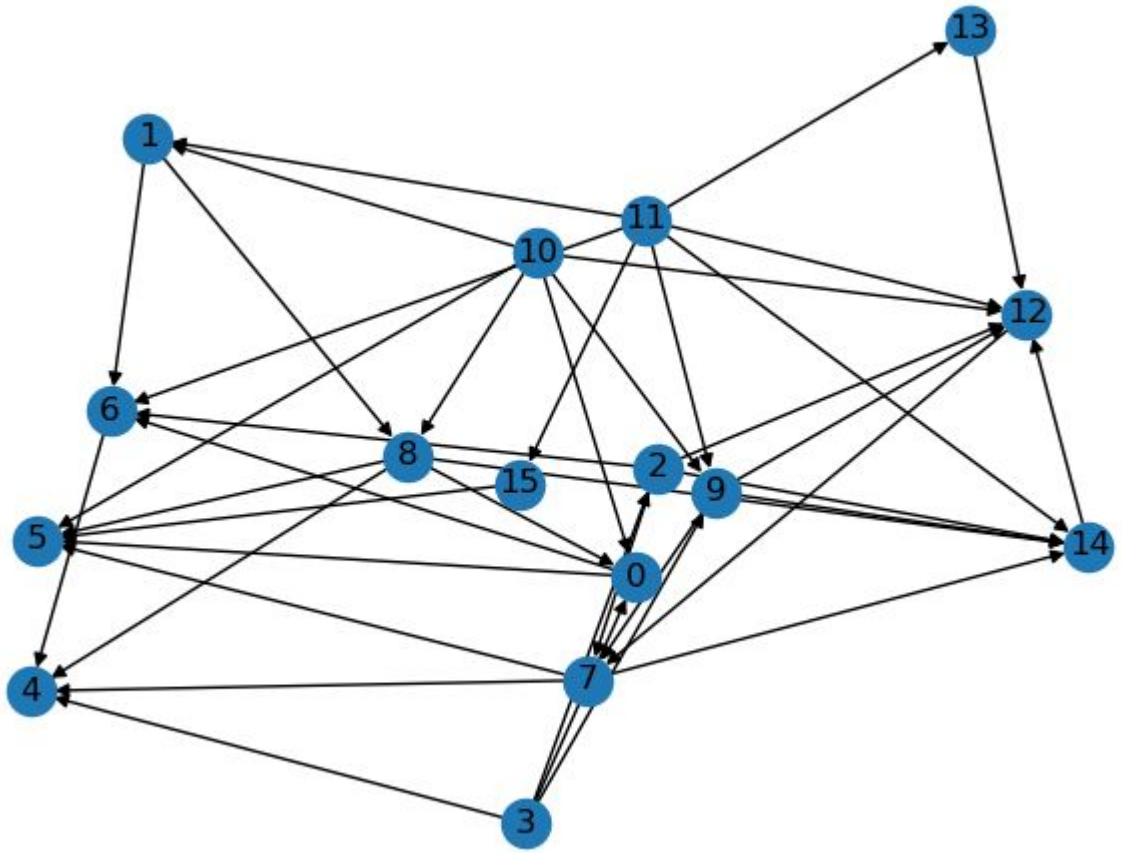
- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of **13** students who responded questionnaire **Q1** in 2024/01…
- ...6 (~46%) had no intransitivities
- The median number intransitivities per student was 1
- The mean was 5.15

Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 13 students who responded questionnaire Q1 in 2024/01…
- ...1 (~8%) gave unreasonable answers to the easy questions

A discussion of transitivity

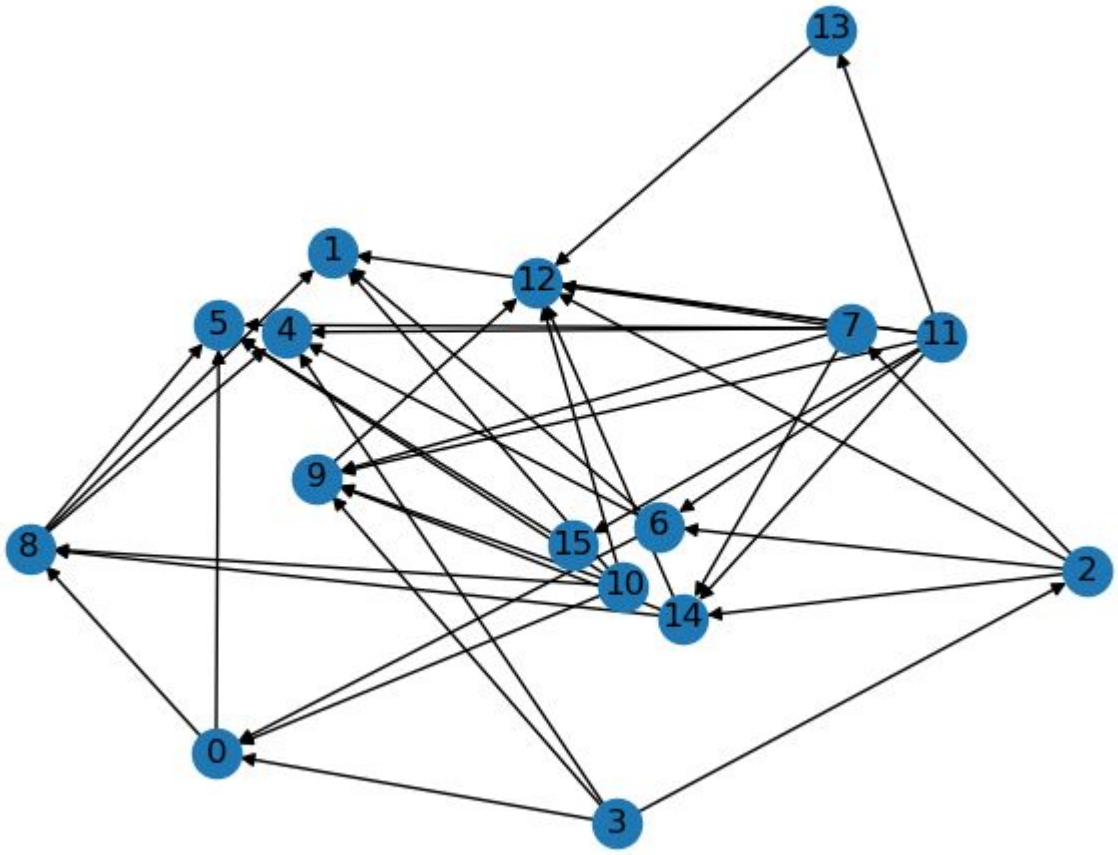
- # of intransitivities: 1 (AV)



[12, 7, 14]

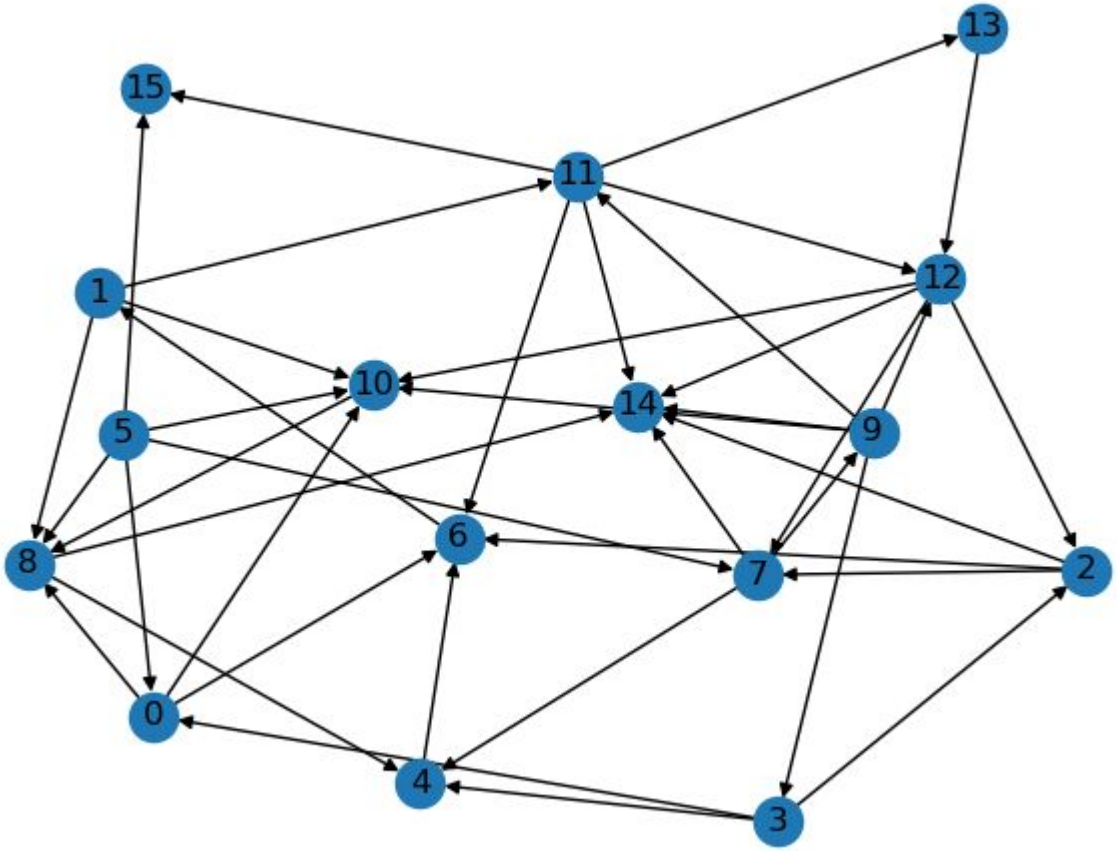
A discussion of transitivity

- # of intransitivities: 0 (AA)



A discussion of transitivity

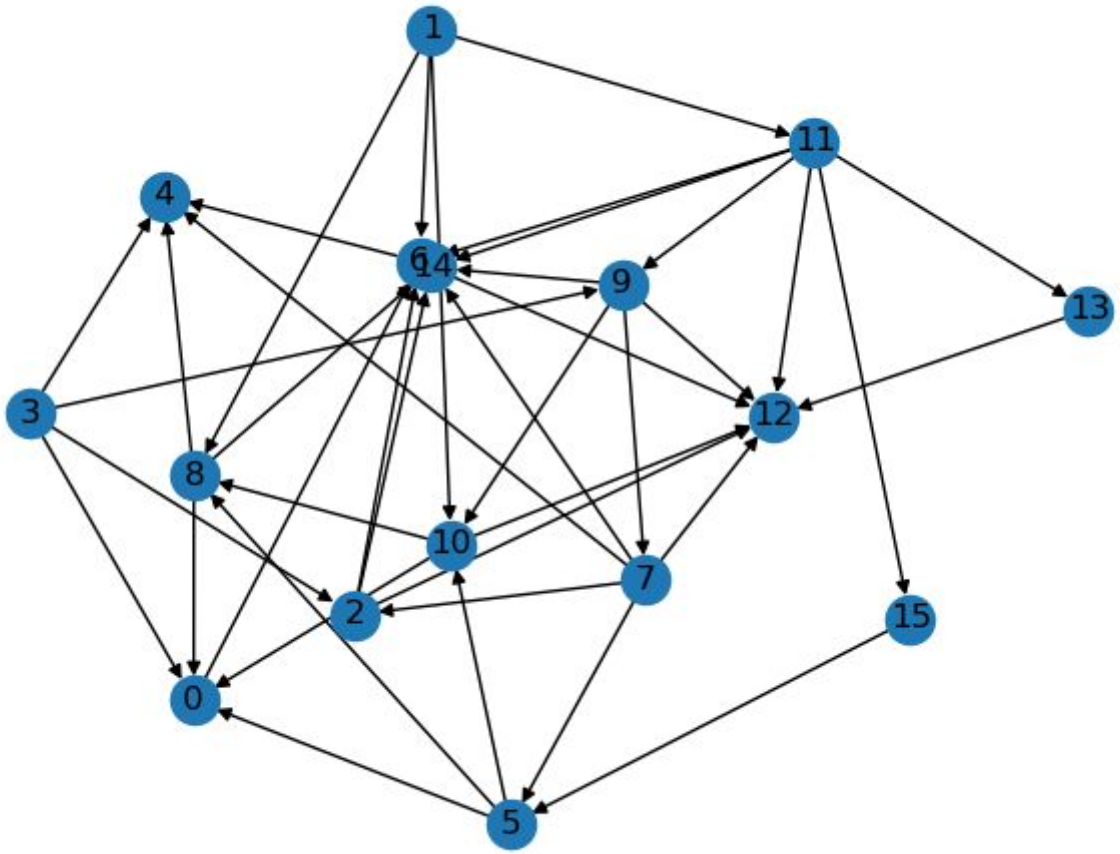
- # of intransitivities: 40 (AB)



[[0, 10, 8, 4, 6, 1, 11, 13, 12, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 12, 7, 9, 3], [0, 10, 8, 4, 6, 1, 11, 12, 2, 7, 9, 3], [0, 8, 4, 6, 1, 11, 13, 12, 7, 9, 3], [0, 8, 4, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 8, 4, 6, 1, 11, 12, 7, 9, 3], [0, 8, 4, 6, 1, 11, 12, 2, 7, 9, 3], [0, 6, 1, 11, 13, 12, 7, 9, 3], [0, 6, 1, 11, 13, 12, 2, 7, 9, 3], [0, 6, 1, 11, 12, 7, 9, 3], [0, 6, 1, 11, 12, 2, 7, 9, 3], [1, 11, 13, 12, 7, 4, 6], [1, 11, 13, 12, 7, 9, 10, 8, 4, 6], [1, 11, 13, 12, 7, 9, 3, 4, 6], [1, 11, 13, 12, 7, 9, 3, 2, 6], [1, 11, 13, 12, 10, 8, 4, 6], [1, 11, 13, 12, 2, 7, 4, 6], [1, 11, 13, 12, 2, 7, 9, 10, 8, 4, 6], [1, 11, 13, 12, 2, 7, 9, 3, 4, 6], [1, 11, 13, 12, 2, 6], [1, 11, 12, 7, 4, 6], [1, 11, 12, 7, 9, 10, 8, 4, 6], [1, 11, 12, 7, 9, 3, 4, 6], [1, 11, 12, 7, 9, 3, 2, 6], [1, 11, 12, 10, 8, 4, 6], [1, 11, 12, 2, 7, 4, 6], [1, 11, 12, 2, 7, 9, 10, 8, 4, 6], [1, 11, 12, 2, 7, 9, 3, 4, 6], [1, 11, 12, 2, 6], [1, 11, 6], [1, 8, 4, 6], [1, 10, 8, 4, 6], [2, 7, 9, 11, 13, 12], [2, 7, 9, 11, 12], [2, 7, 9, 3], [2, 7, 9, 12], [7, 9, 11, 13, 12], [7, 9, 11, 12], [7, 9, 12]]

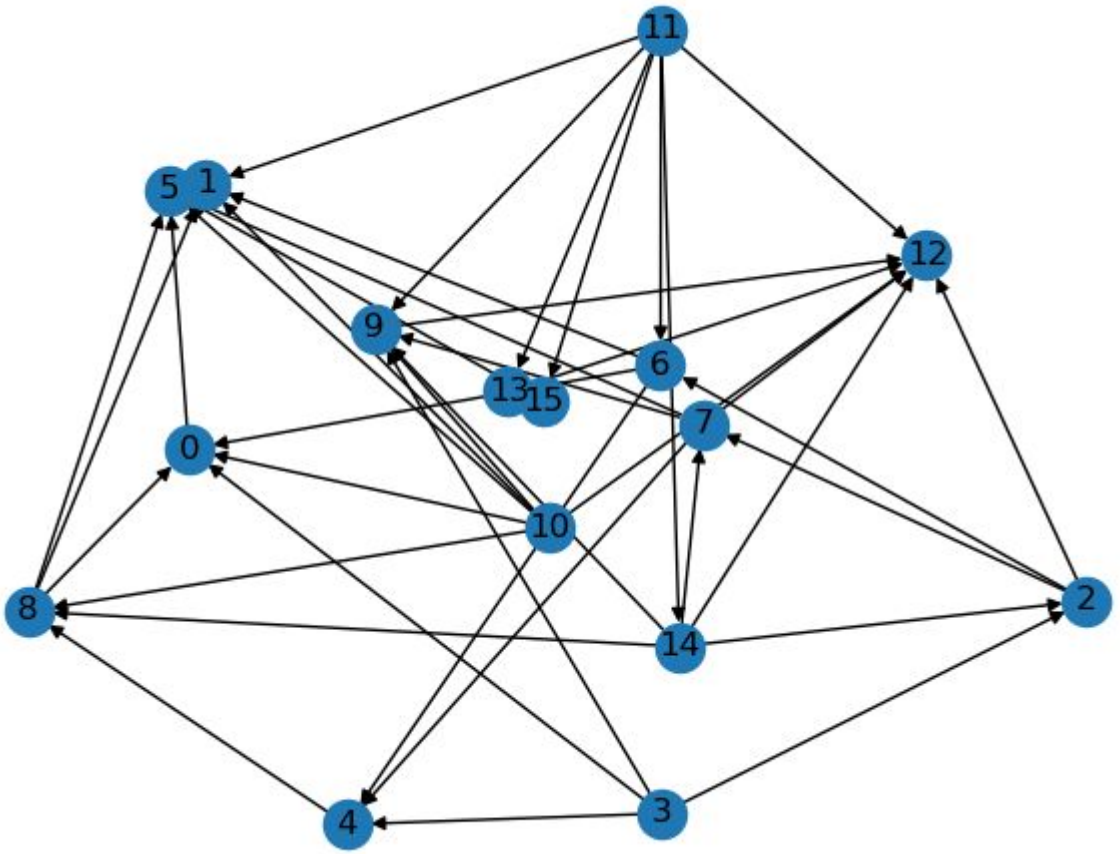
A discussion of transitivity

- # of intransitivities: 0 (AD)



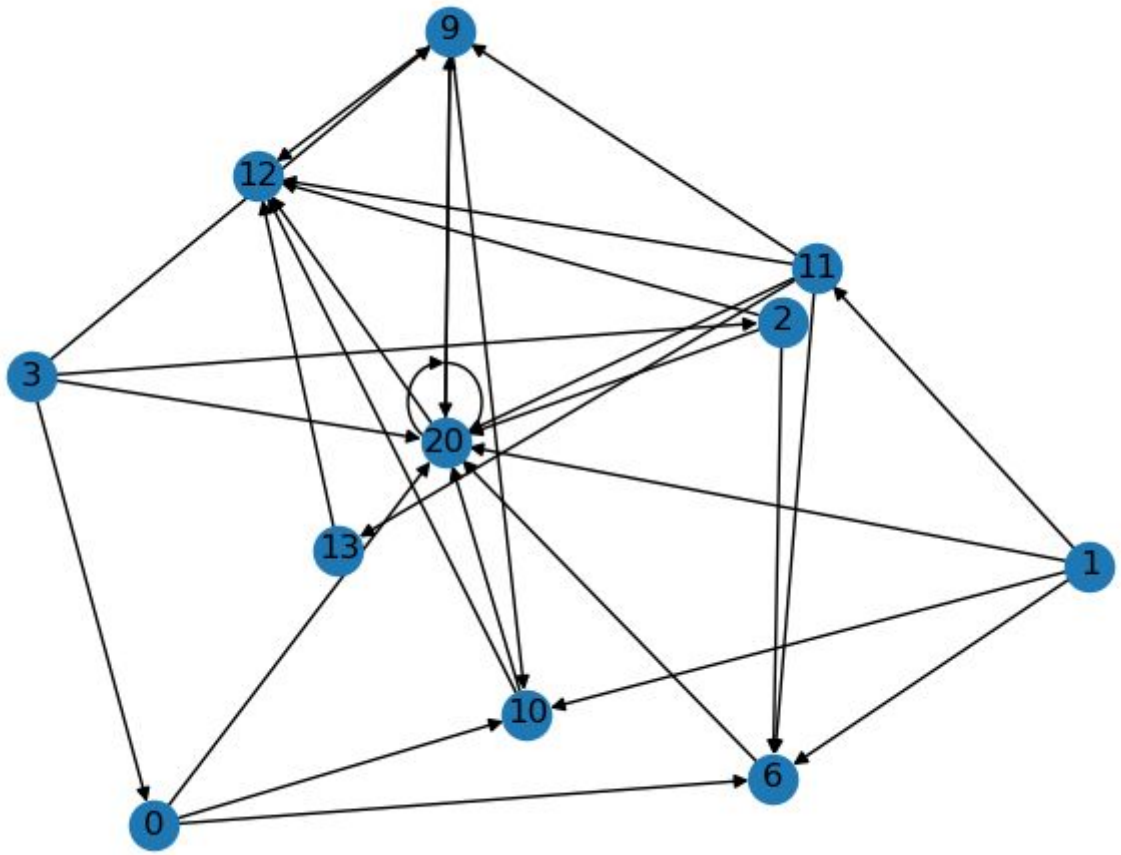
A discussion of transitivity

- # of intransitivities: 0 (GL)



A discussion of transitivity

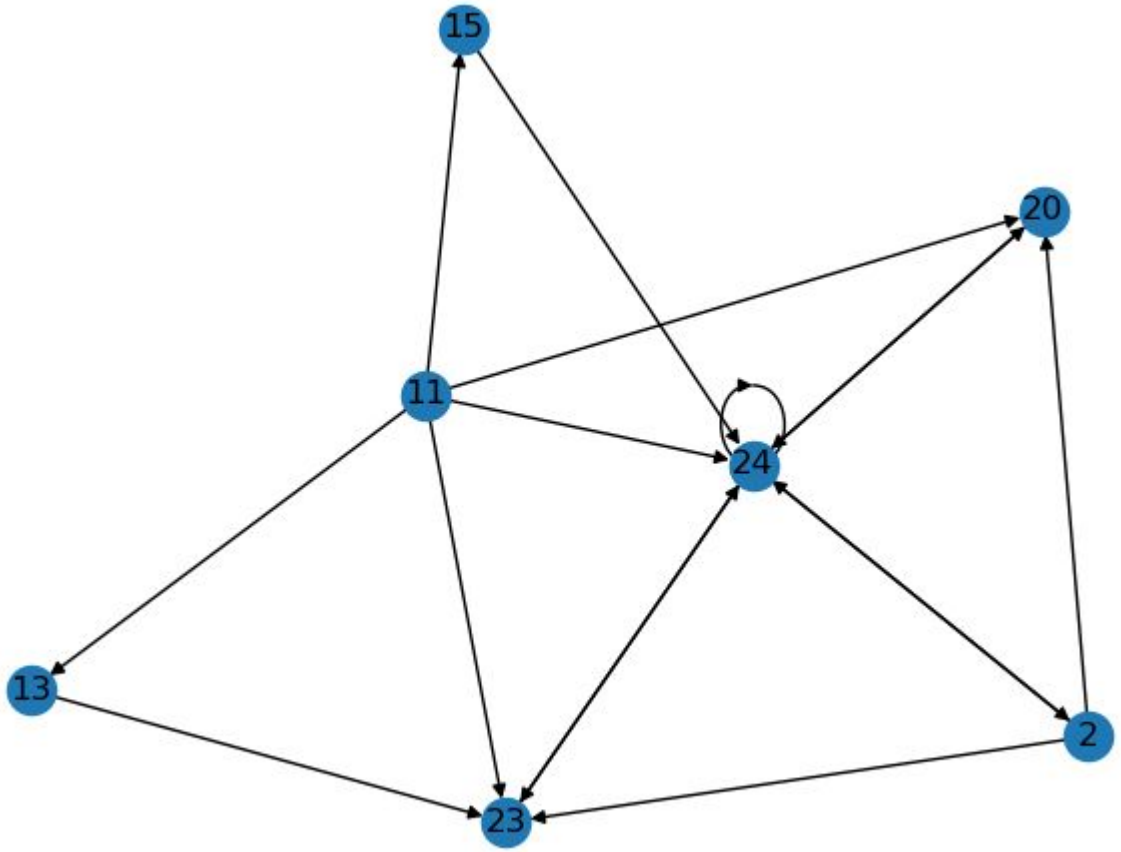
- # of intransitivities: 3 (KN)



[20]
[9, 20]
[9, 10, 20]

A discussion of transitivity

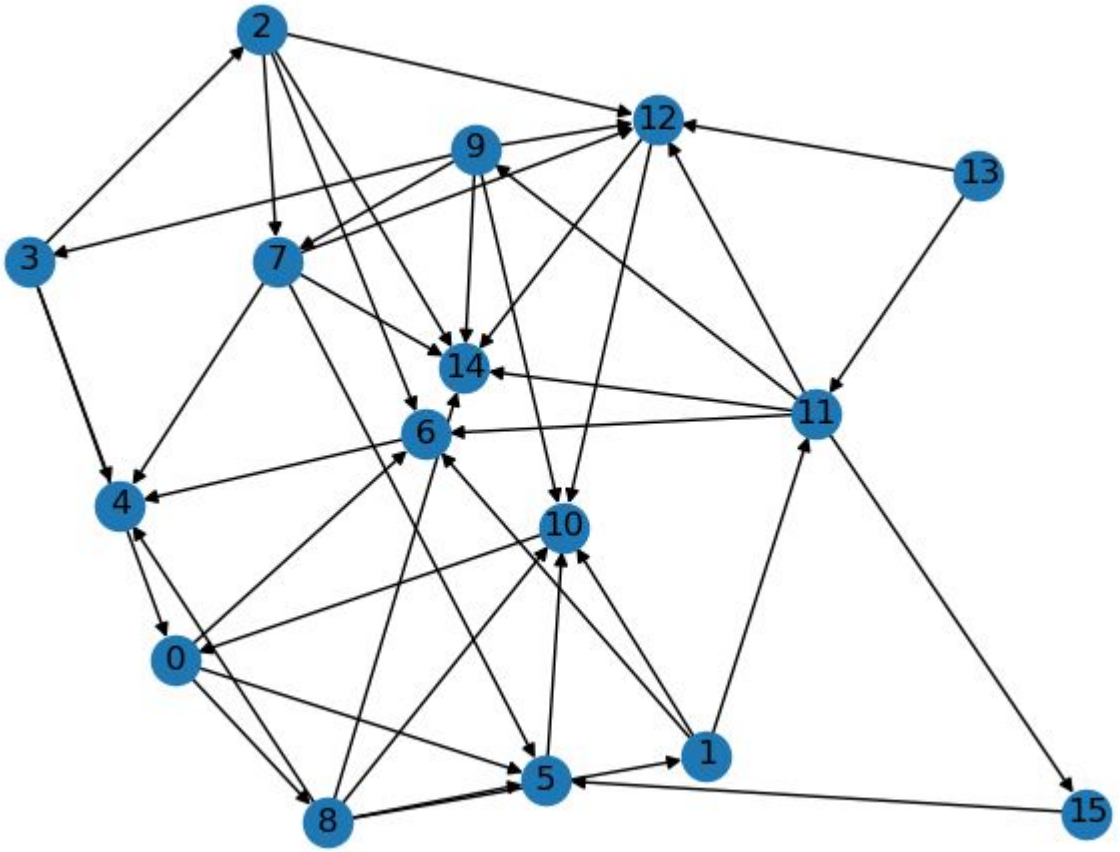
- # of intransitivities: 6 (LD)



[24], [24,23]
[24, 20], [24, 2]
[24, 2, 23],
[24, 2, 20]

A discussion of transitivity

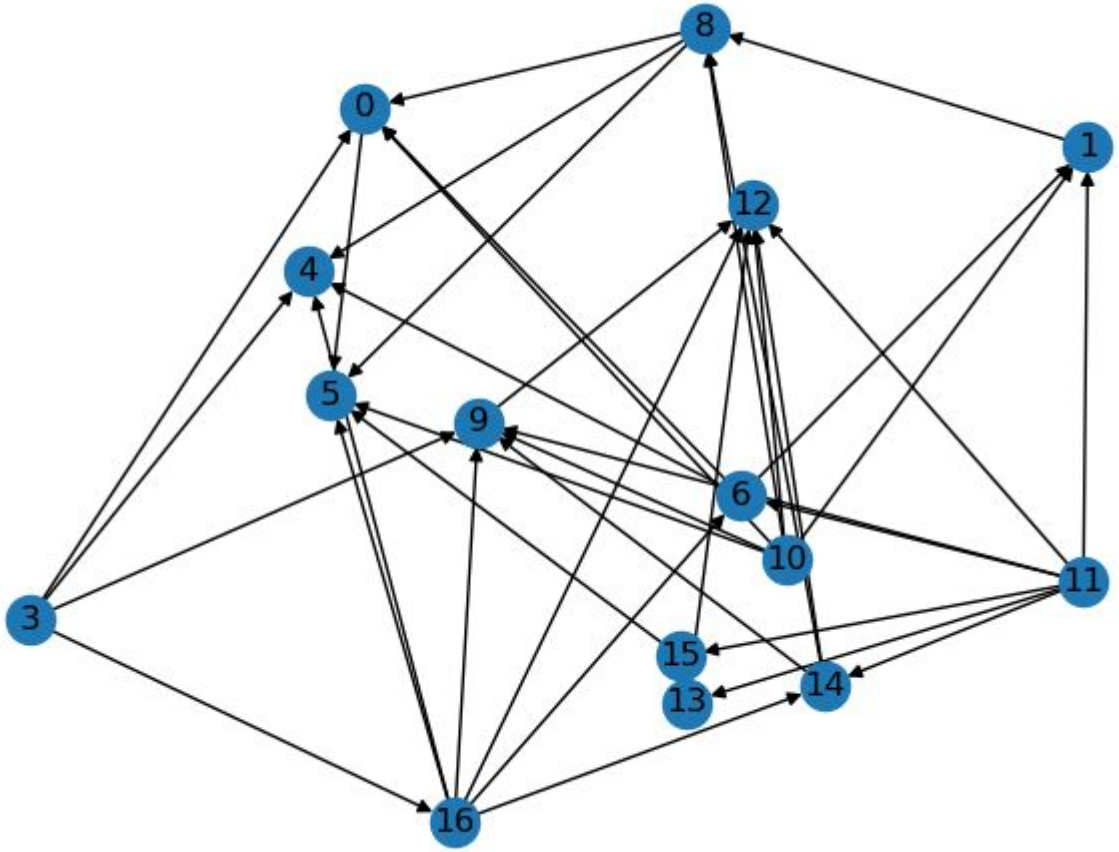
- # of intransitivities: 14 (LR)



[24], [24,23]
[24, 20], [24, 2]
[24, 2, 23],
[24, 2, 20]

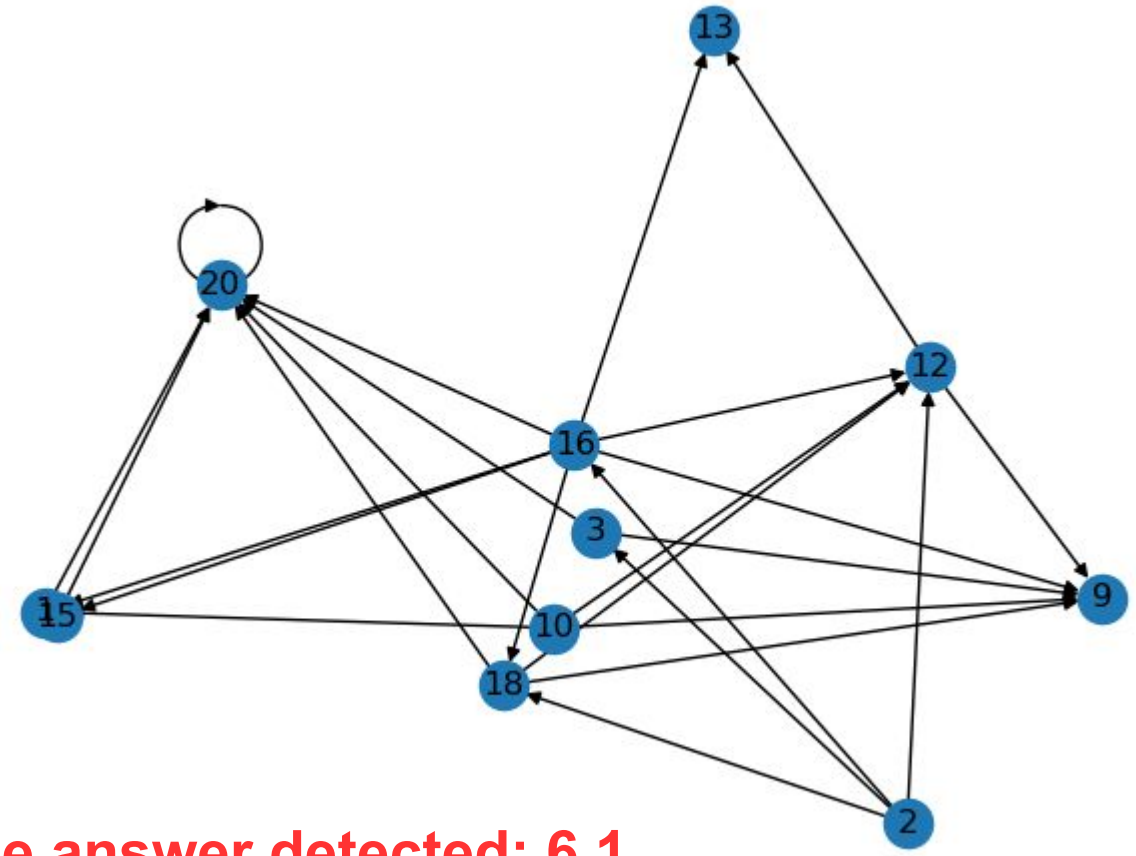
A discussion of transitivity

- # of intransitivities: 0 (LR)



A discussion of transitivity

- # of intransitivities: 1 (MM)

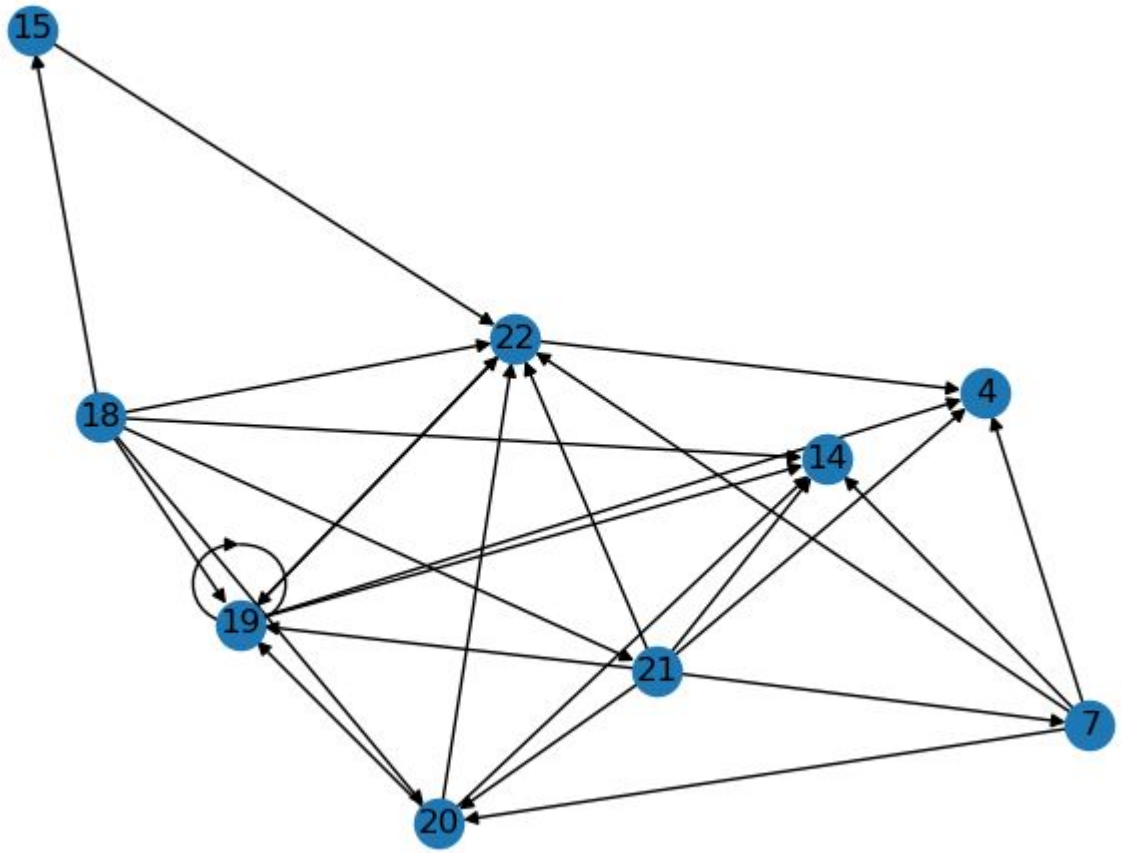


[20]

unreasonable answer detected: 6 1
unreasonable answer detected: 38 2
unreasonable answer detected: 39 2

A discussion of transitivity

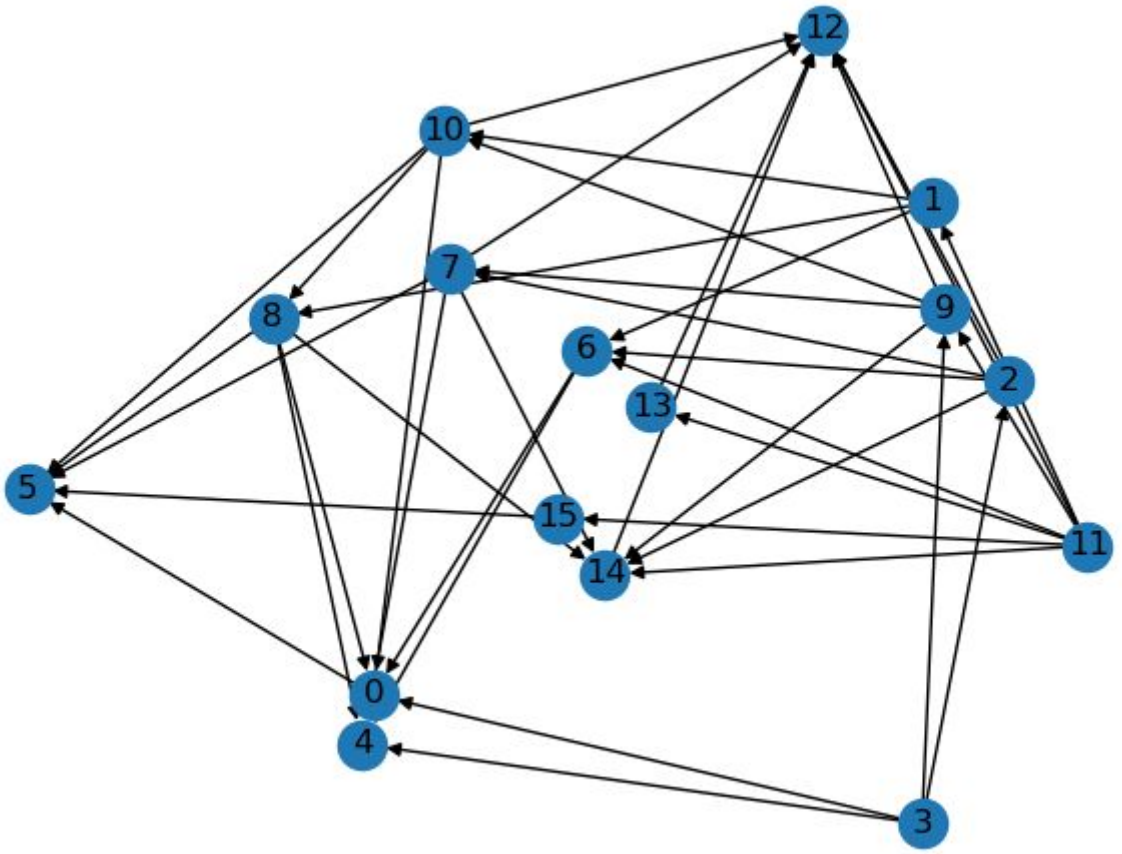
- # of intransitivities: 2 (MI)



[19]
[19, 22]

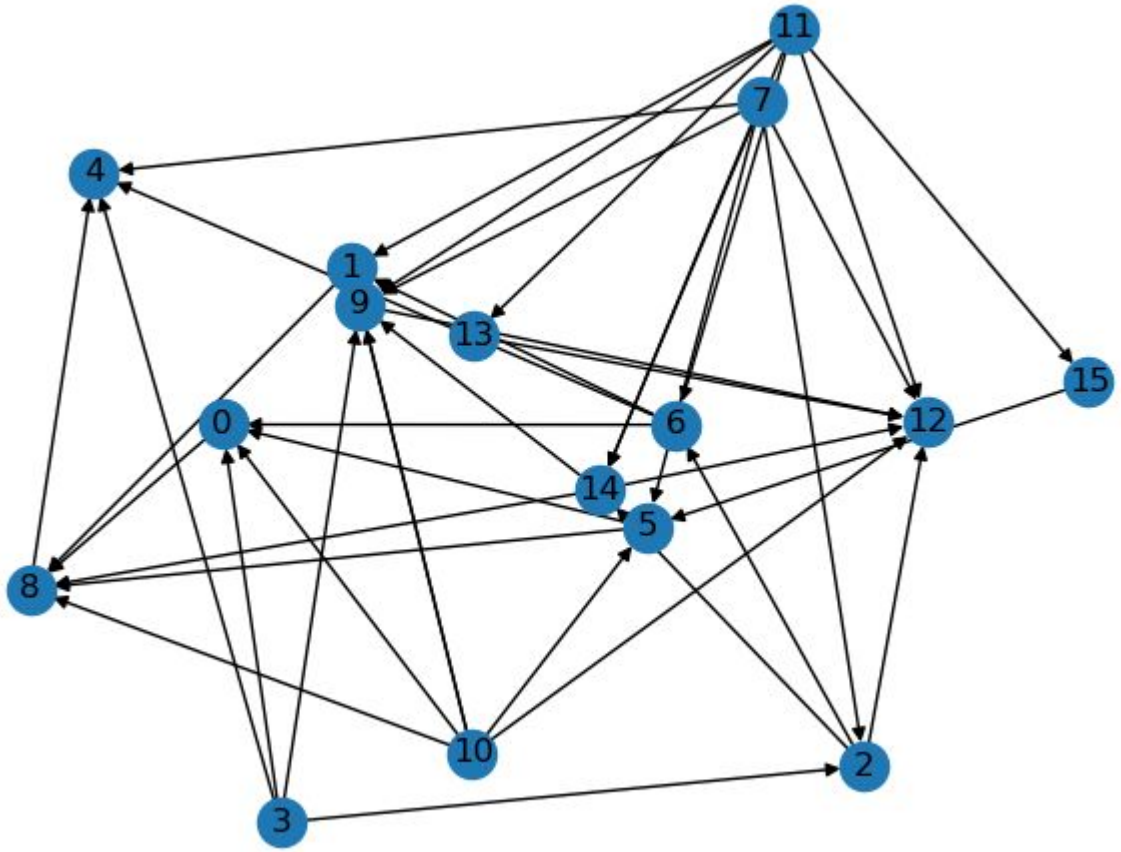
A discussion of transitivity

- # of intransitivities: 0 (PF)



A discussion of transitivity

- # of intransitivities: 0 (PS)

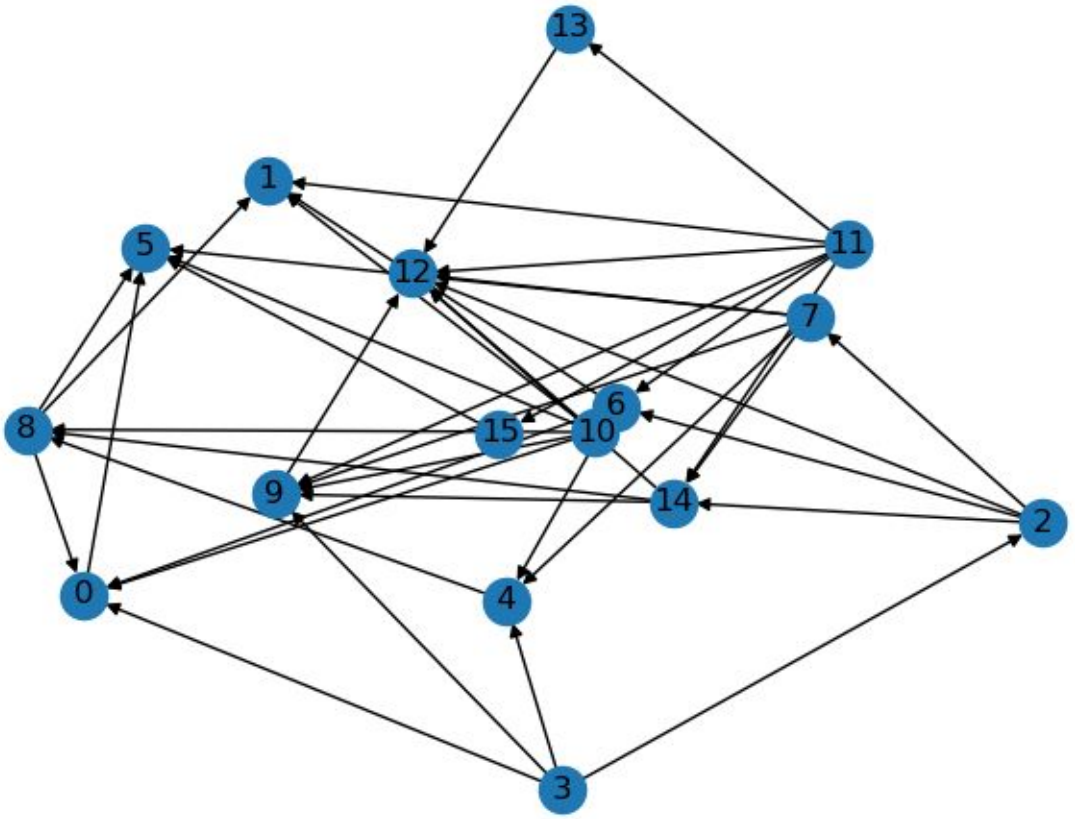


Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 15 students who responded questionnaire Q1 in 2022/01…
- ...3 (20%) gave unreasonable answers to the easy questions

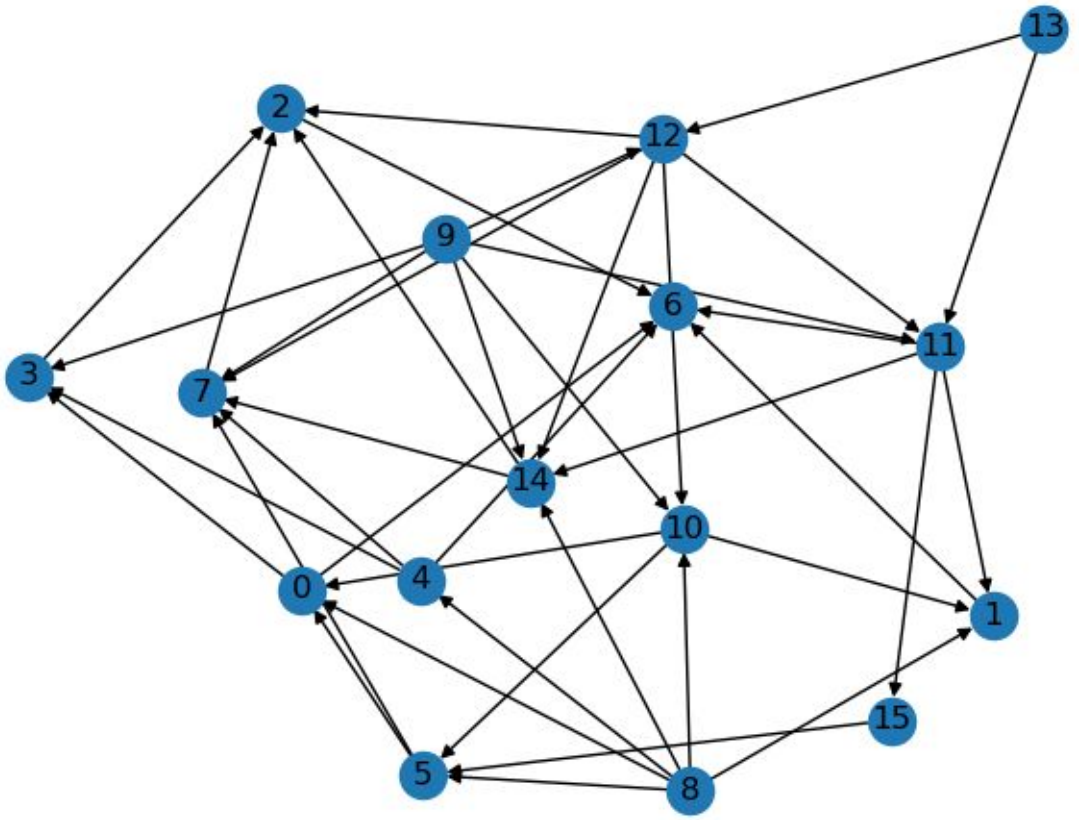
A discussion of transitivity

- # of intransitivities: 0 (AR)



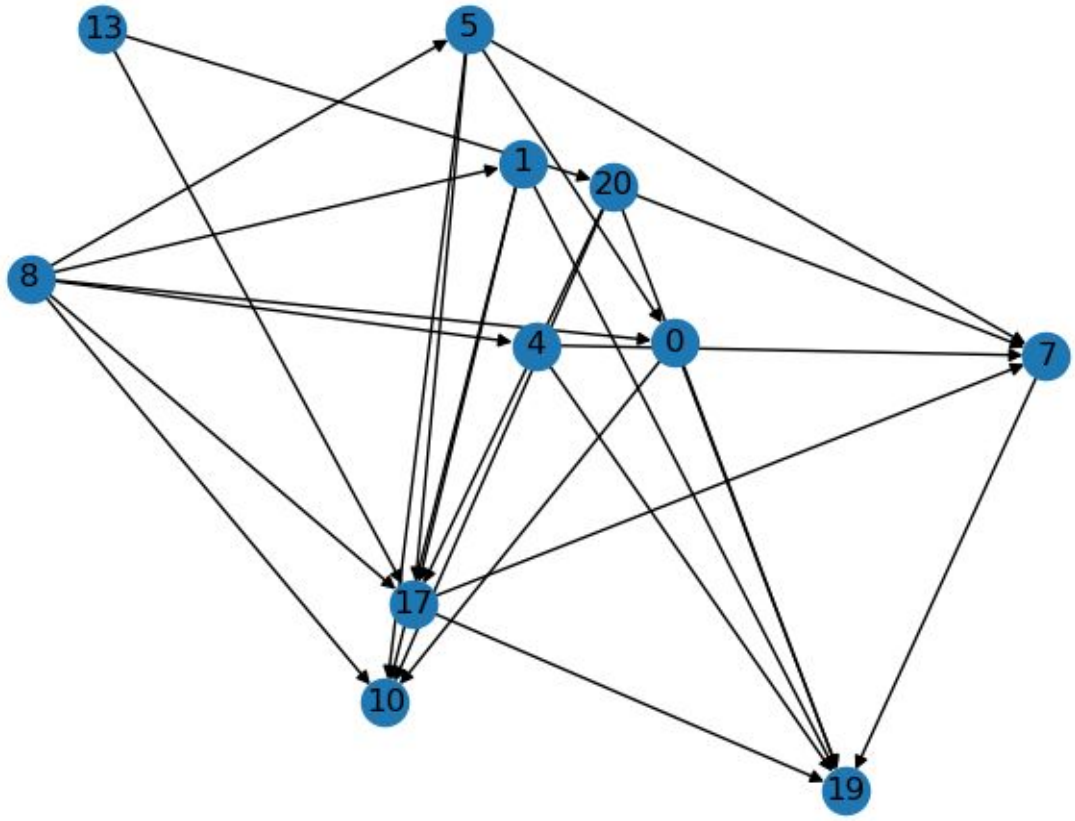
A discussion of transitivity

- # of intransitivities: 0 (AP)



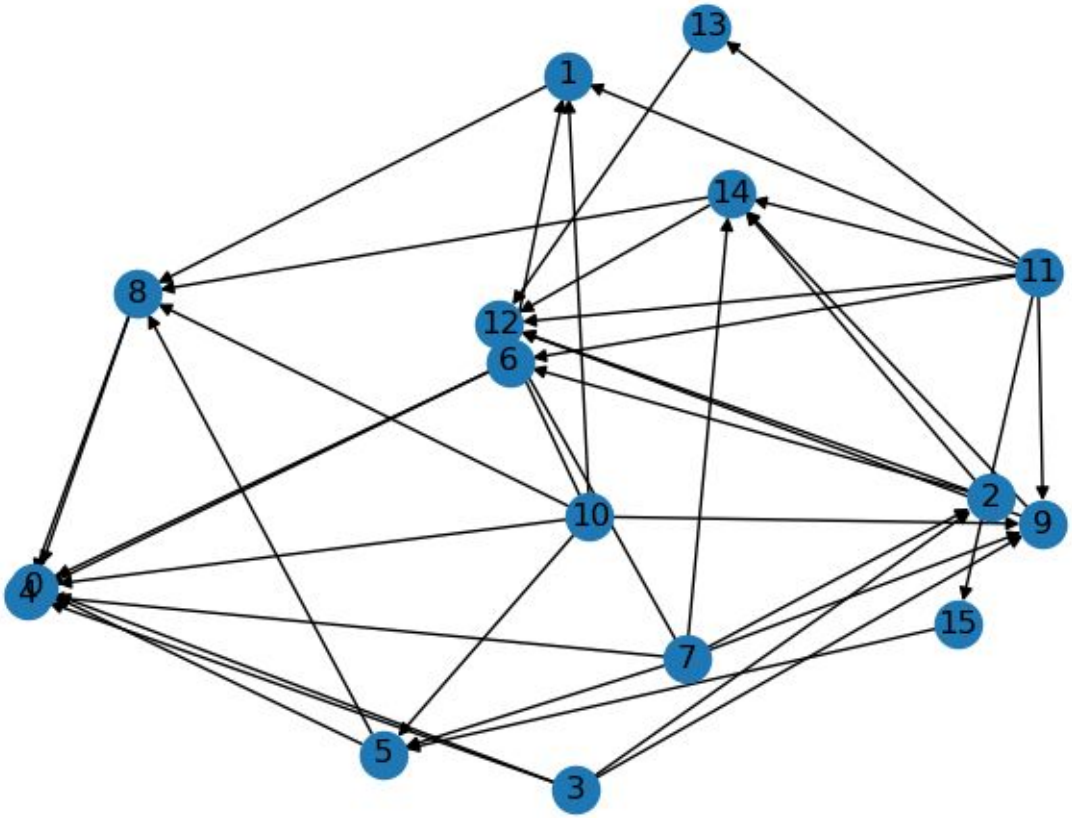
A discussion of transitivity

- # of intransitivities: 0 (BM)
  - 5/6 *unreasonable* answers



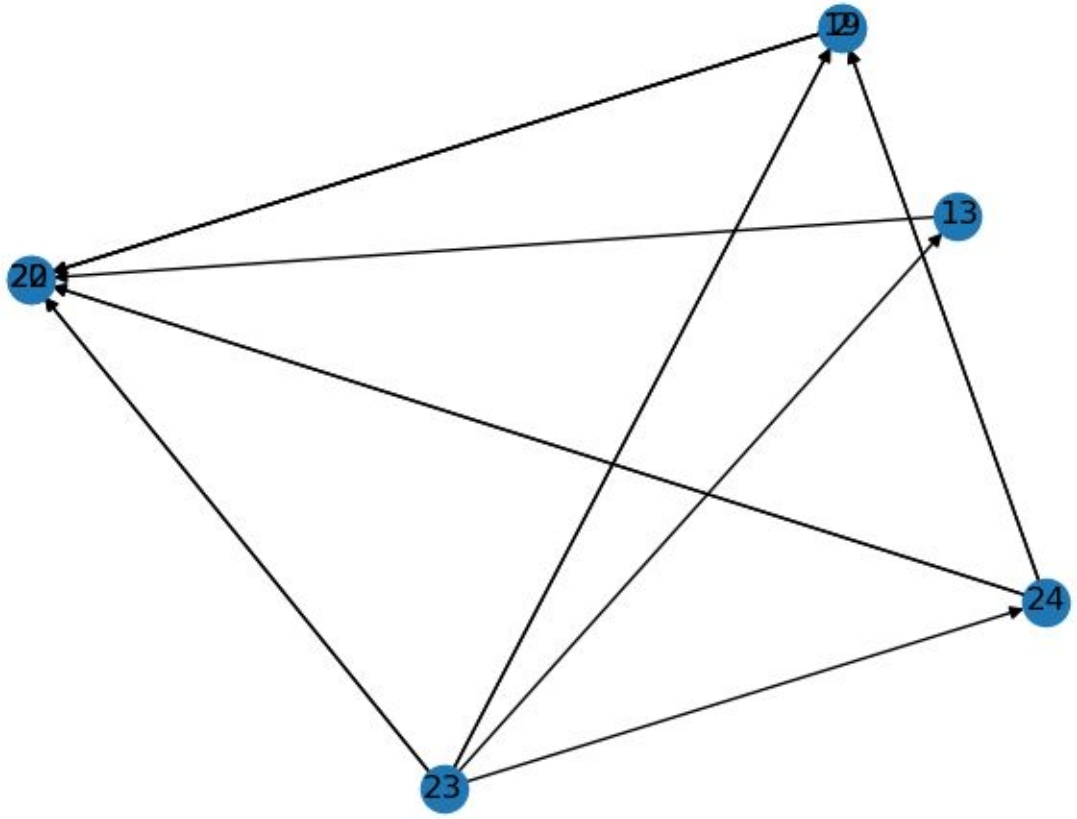
A discussion of transitivity

- # of intransitivities: 0 (EM)



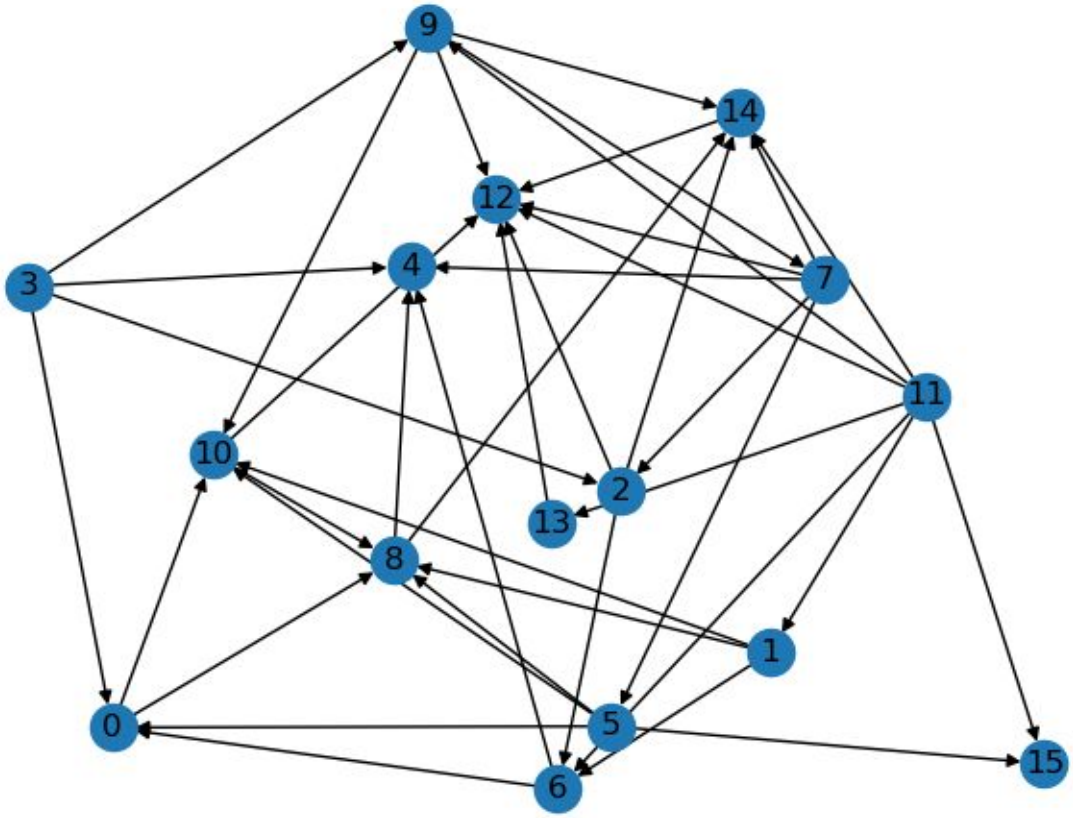
A discussion of transitivity

- # of intransitivities: 0 (EV)



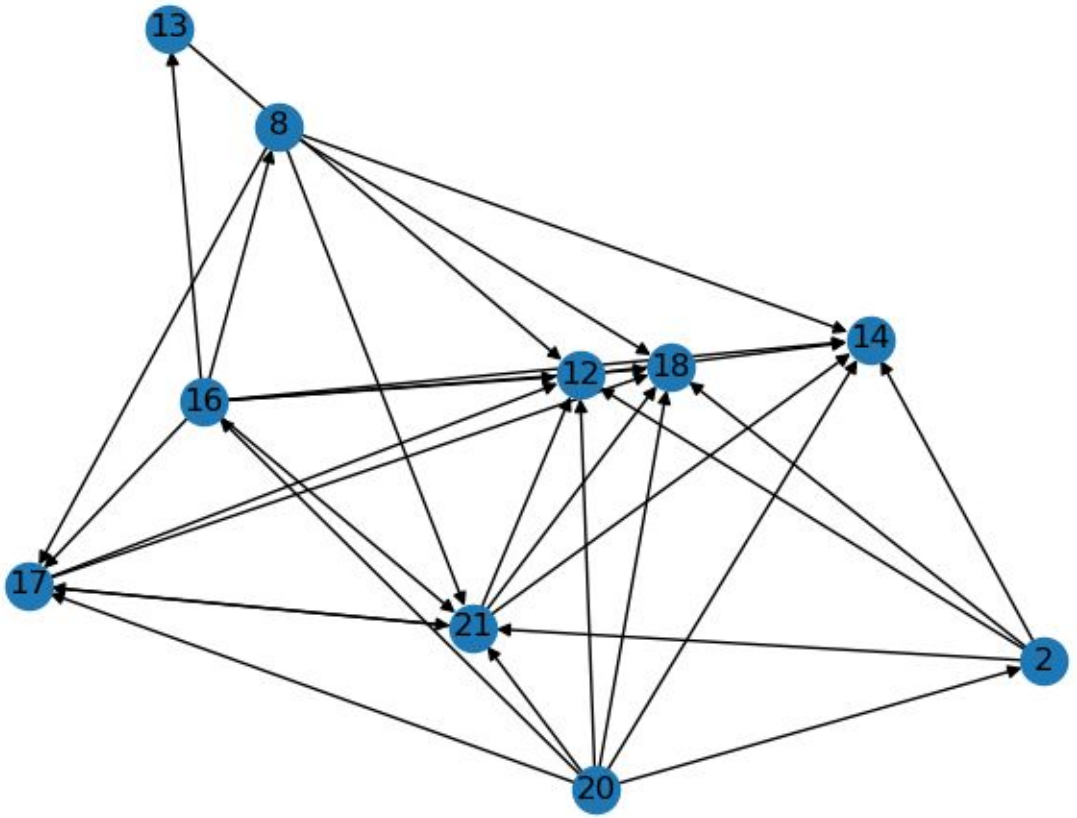
A discussion of transitivity

- # of intransitivities: 0 (GU)



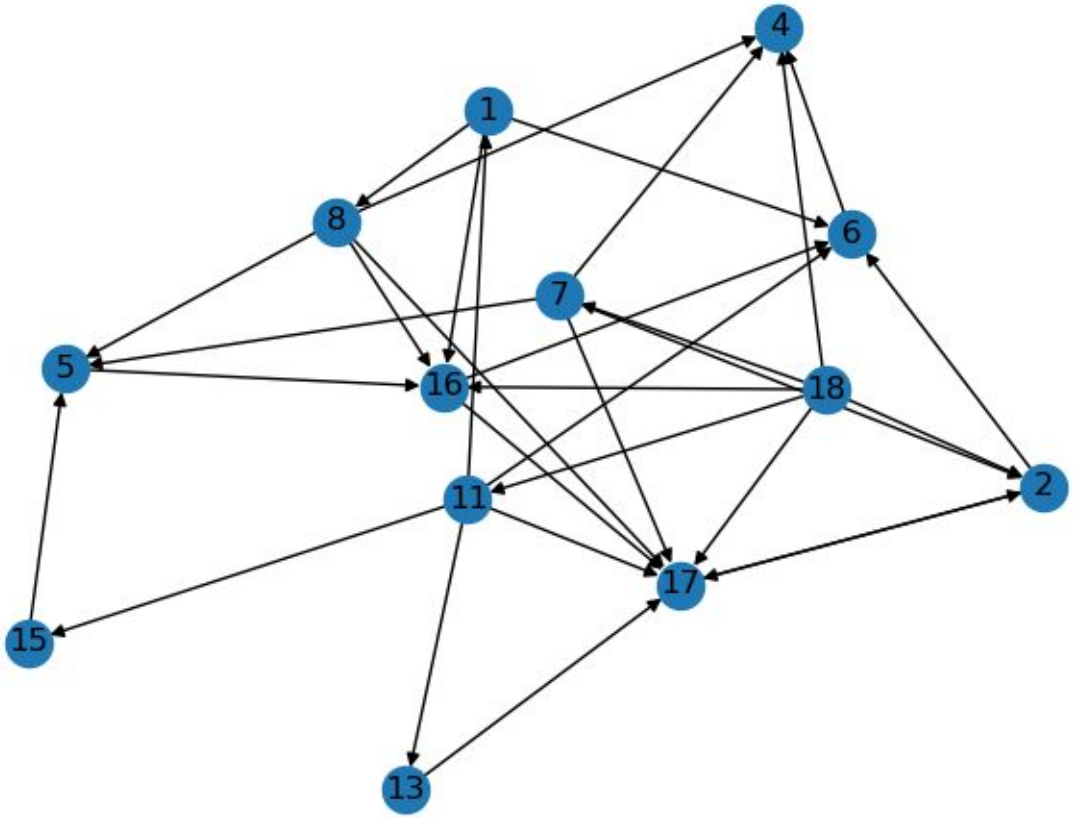
A discussion of transitivity

- # of intransitivities: 1 (GD)



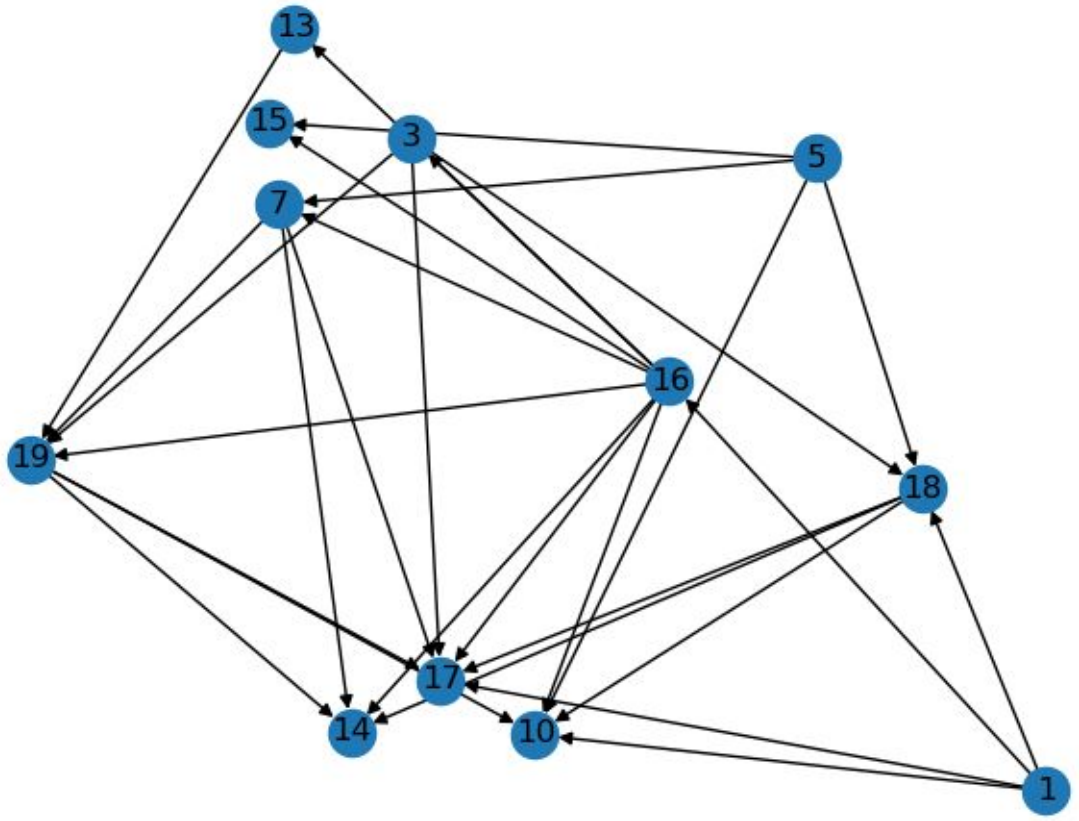
A discussion of transitivity

- # of intransitivities: 1 (GG)



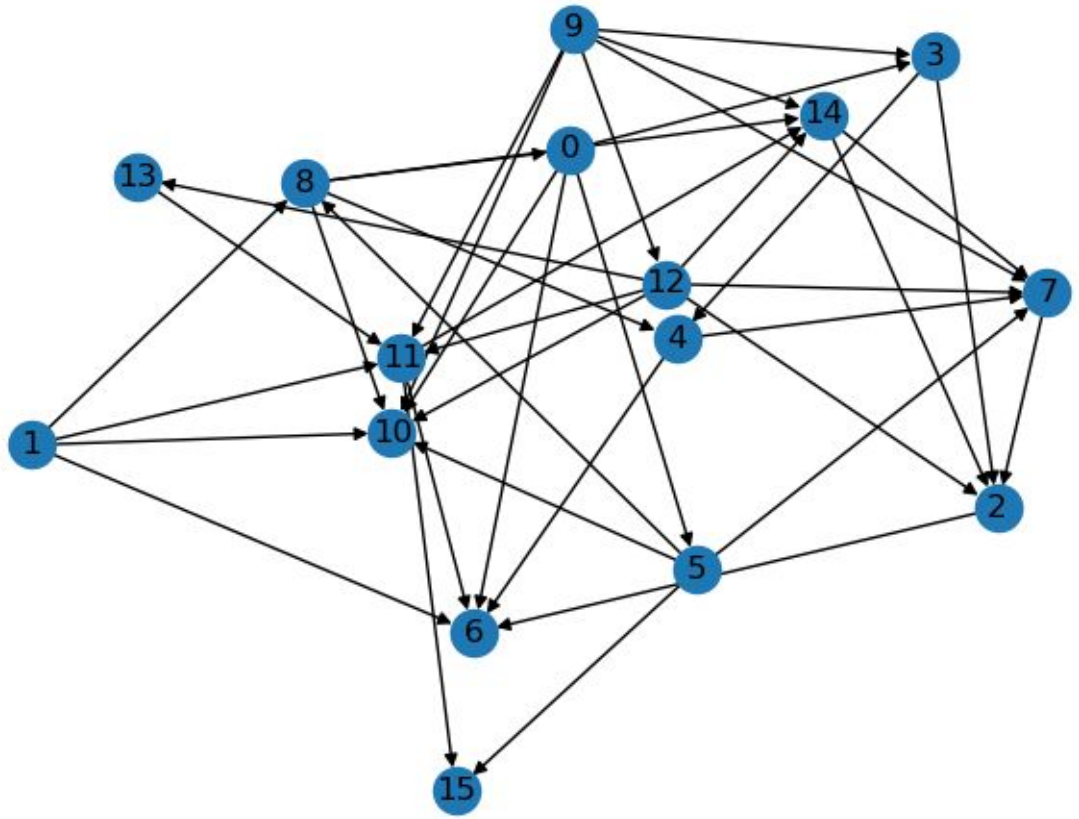
A discussion of transitivity

- # of intransitivities: 0 (LG)



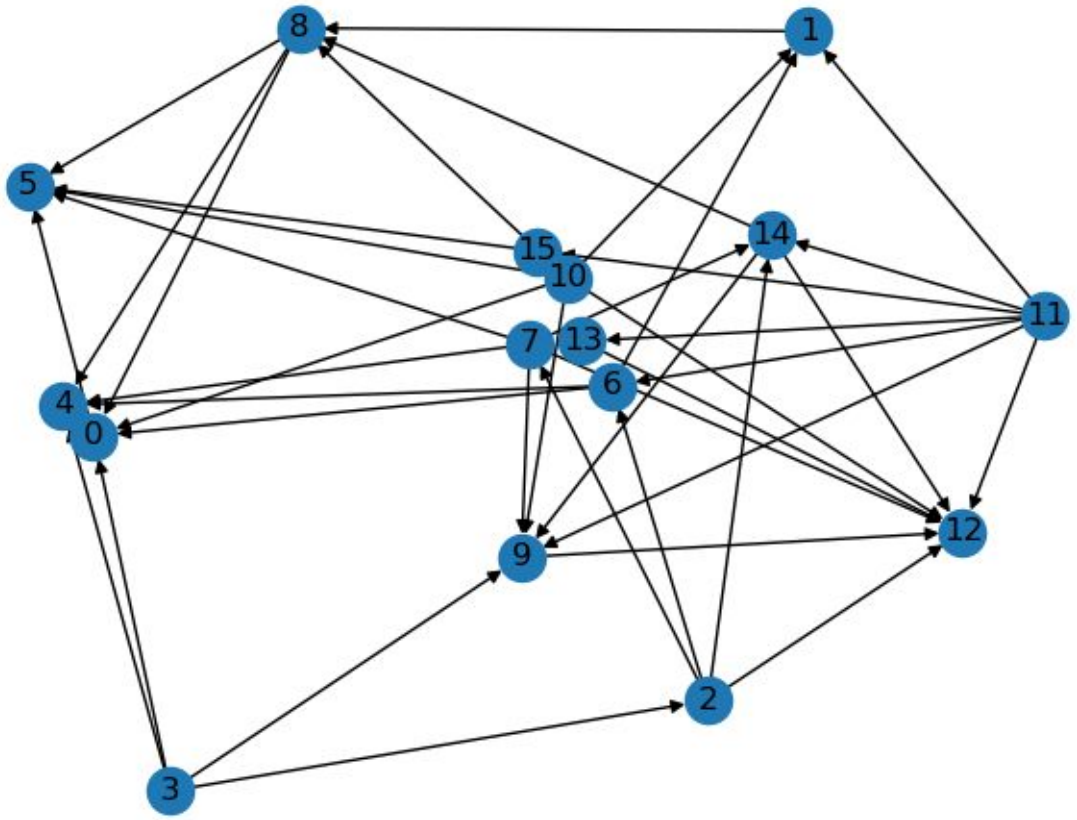
A discussion of transitivity

- # of intransitivities: 0 (LA)
  - 1/1 unreasonable answers



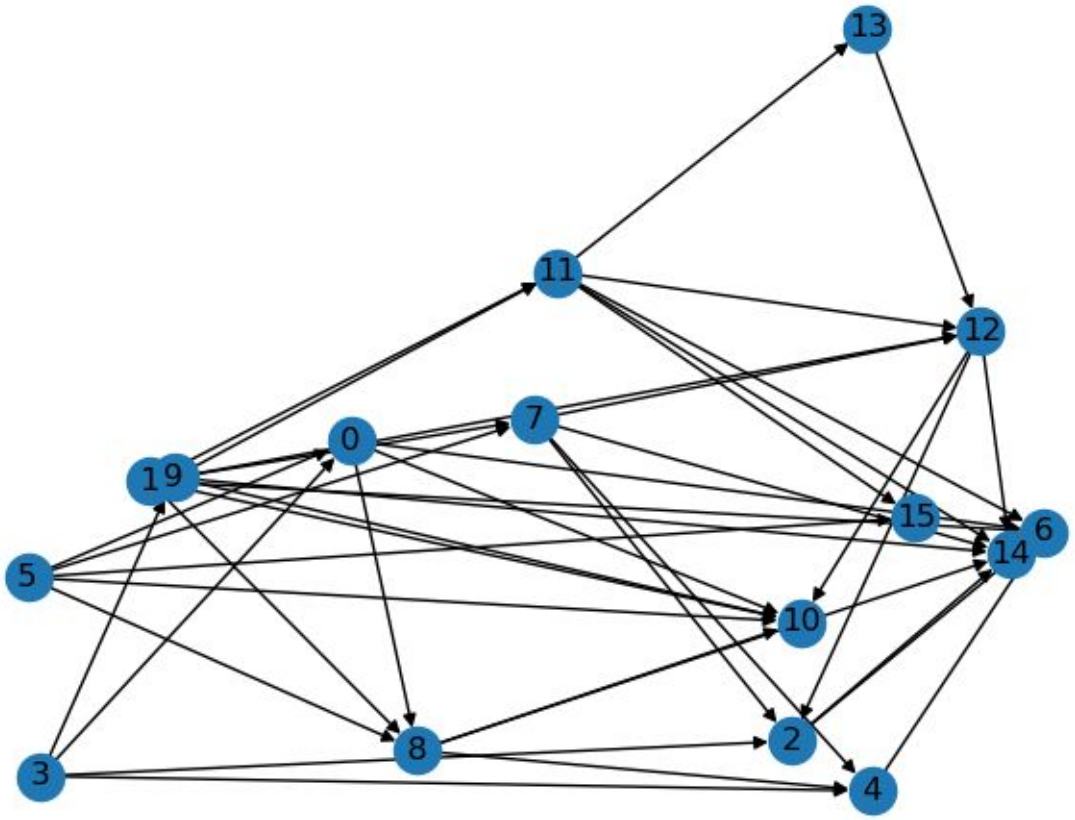
A discussion of transitivity

- # of intransitivities: 0 (LH)



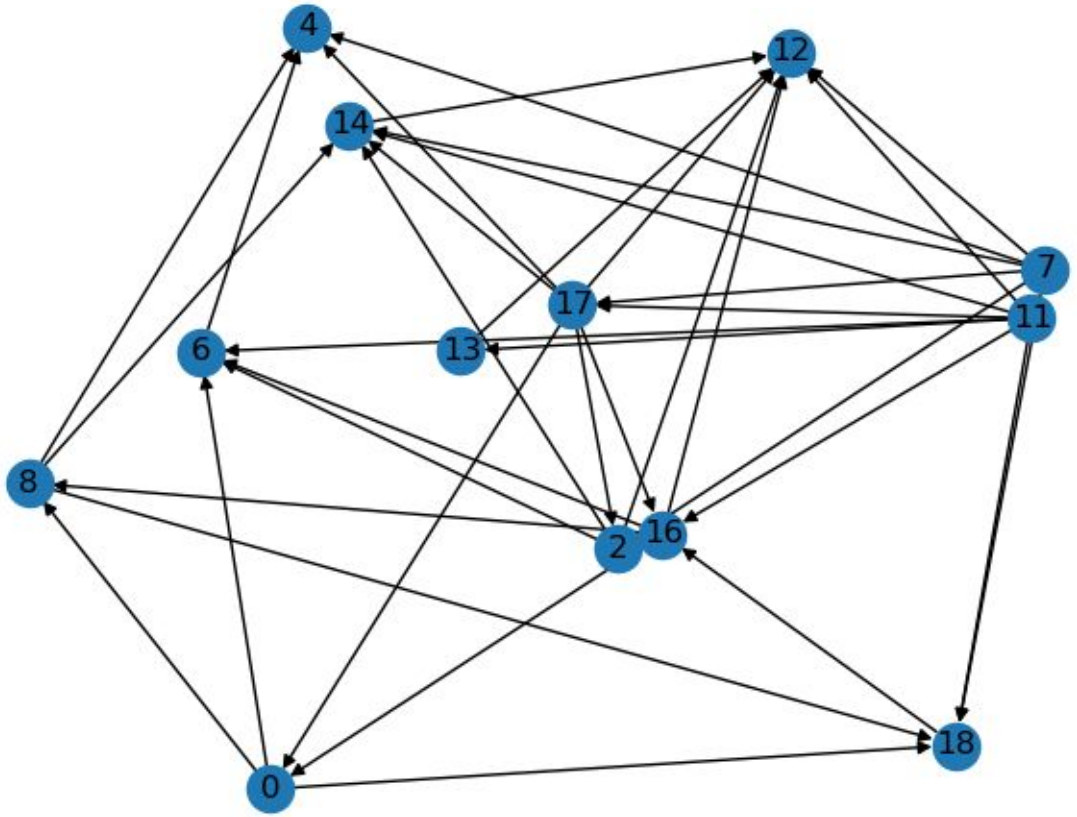
A discussion of transitivity

- # of intransitivities: 0 (PS)



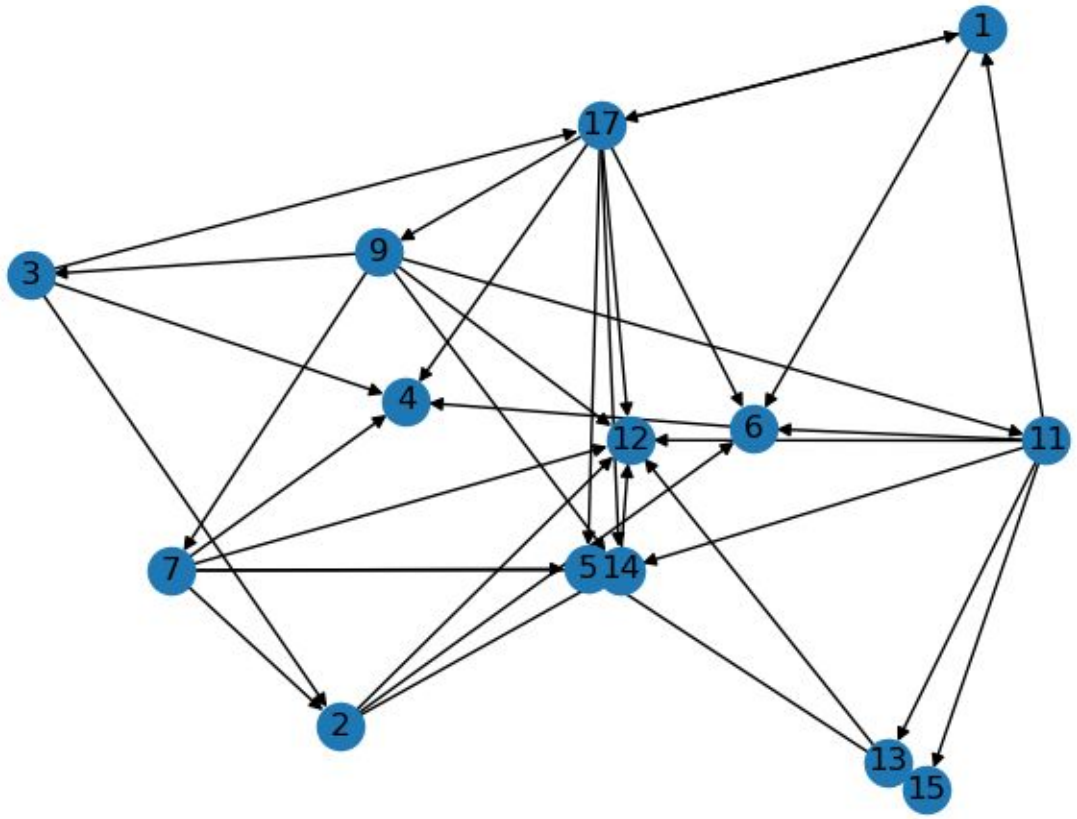
A discussion of transitivity

- # of intransitivities: 3 (RS)



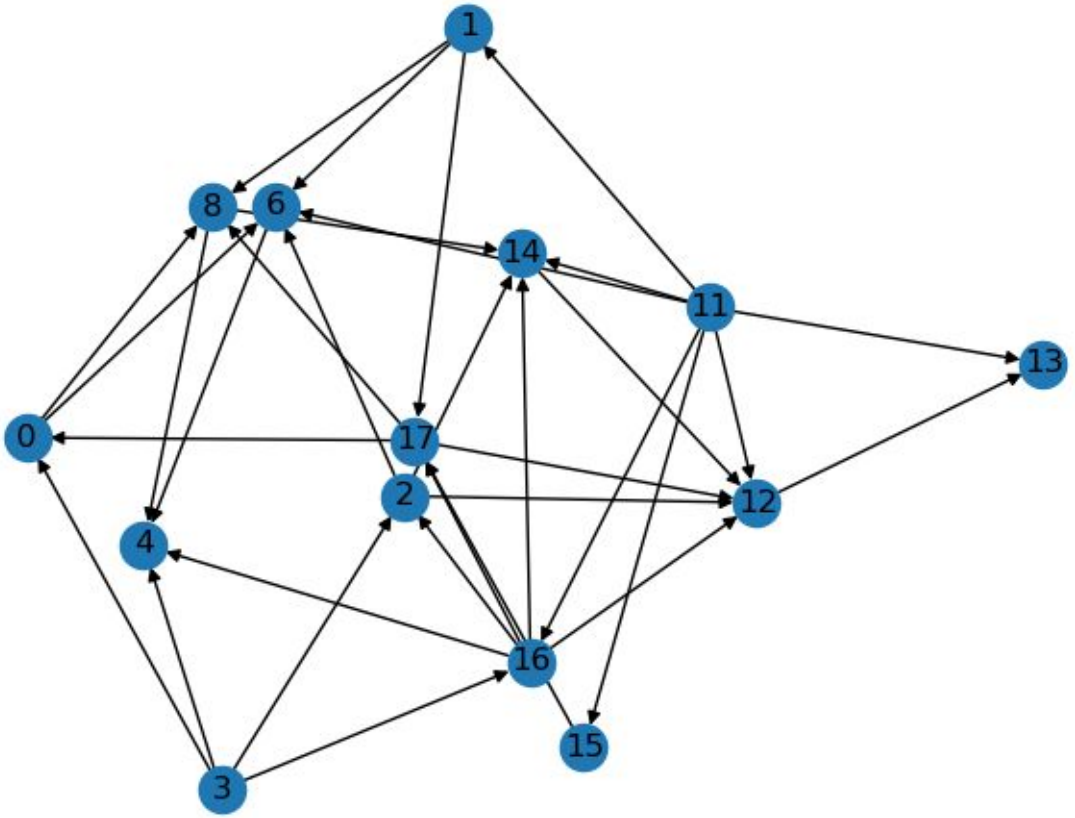
A discussion of transitivity

- # of intransitivities: 4 (SS)



A discussion of transitivity

- # of intransitivities: 0 (TP)
  - 1/6 *unreasonable* answers



A discussion of transitivity

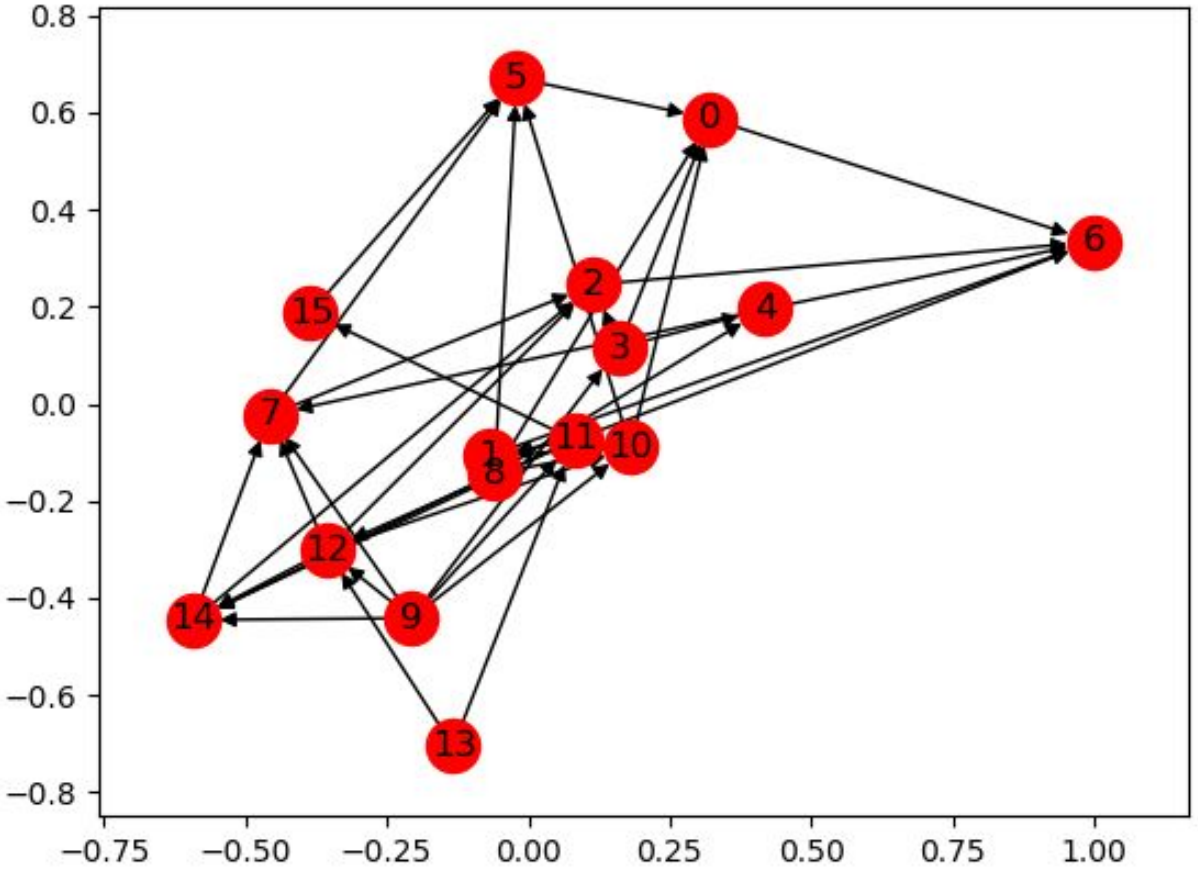
- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 63 students who responded questionnaire Q1 in 2019/02…
- ...31 (49%) had no intransitivities
- The median number intransitivities per student was 1
- The mean was 6.68 (!)

Easy questions?

- How would you react if somebody fails to answer those easy questions?
- Out of 63 students who responded questionnaire Q1 in 2019/02…
- ...13 (21%) gave unreasonable answers to the easy questions

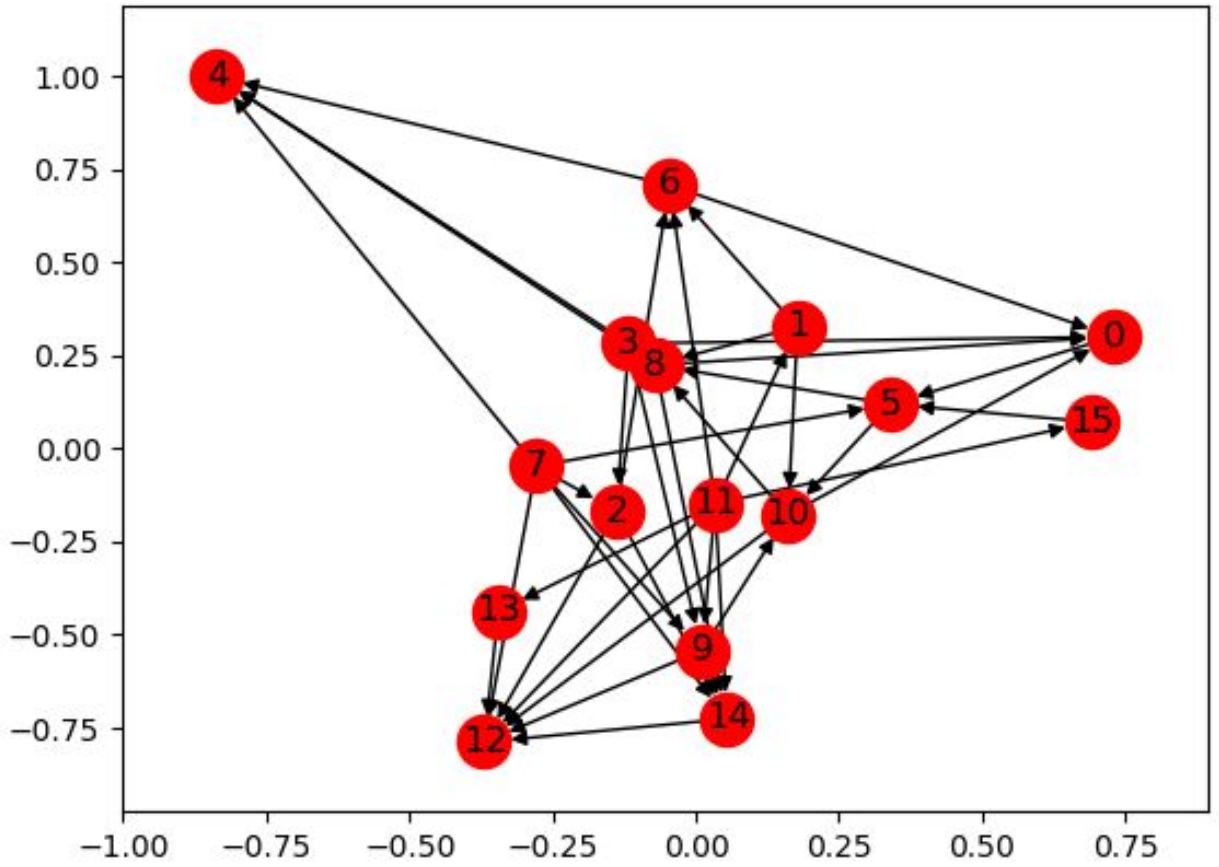
A discussion of transitivity

- # of intransitivities: 0 (I. E.)



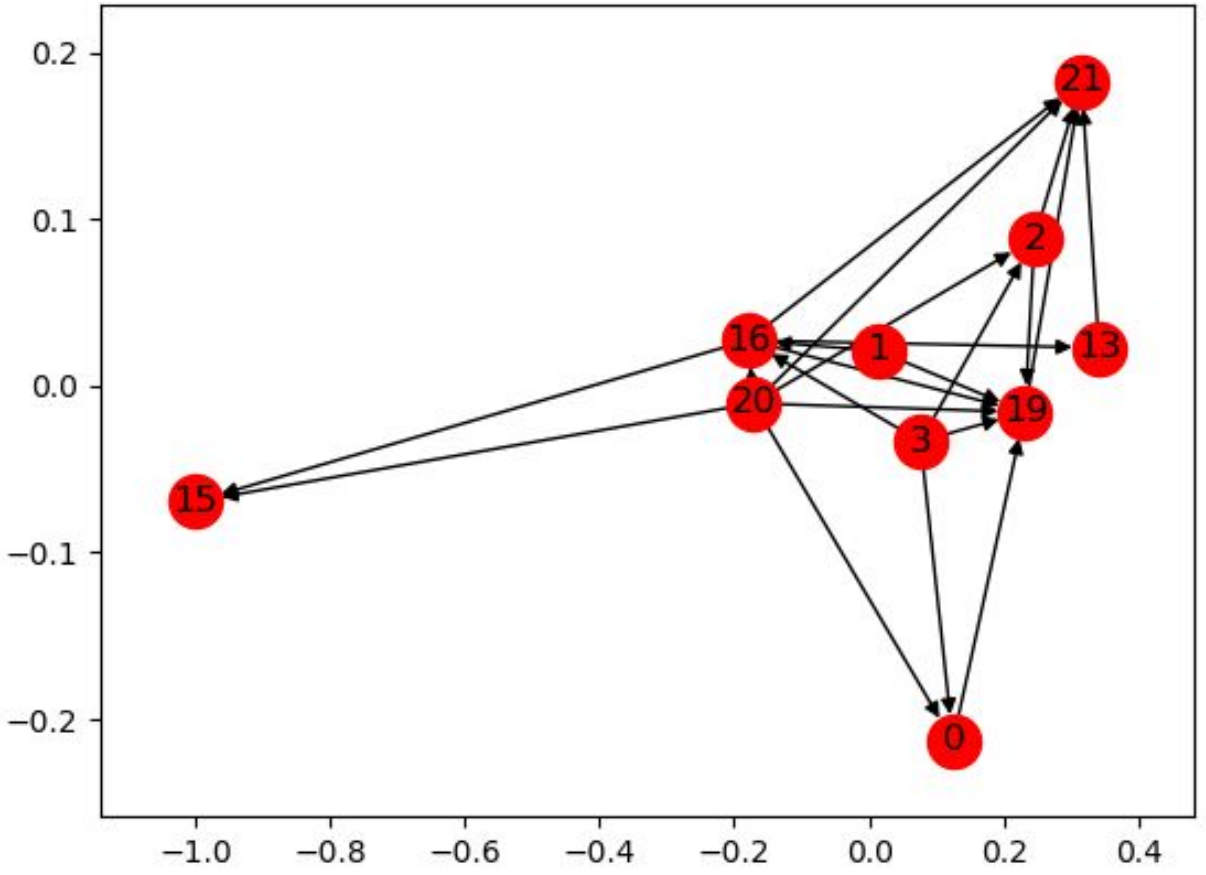
A discussion of transitivity

- # of intransitivities: 3 (A. M.)



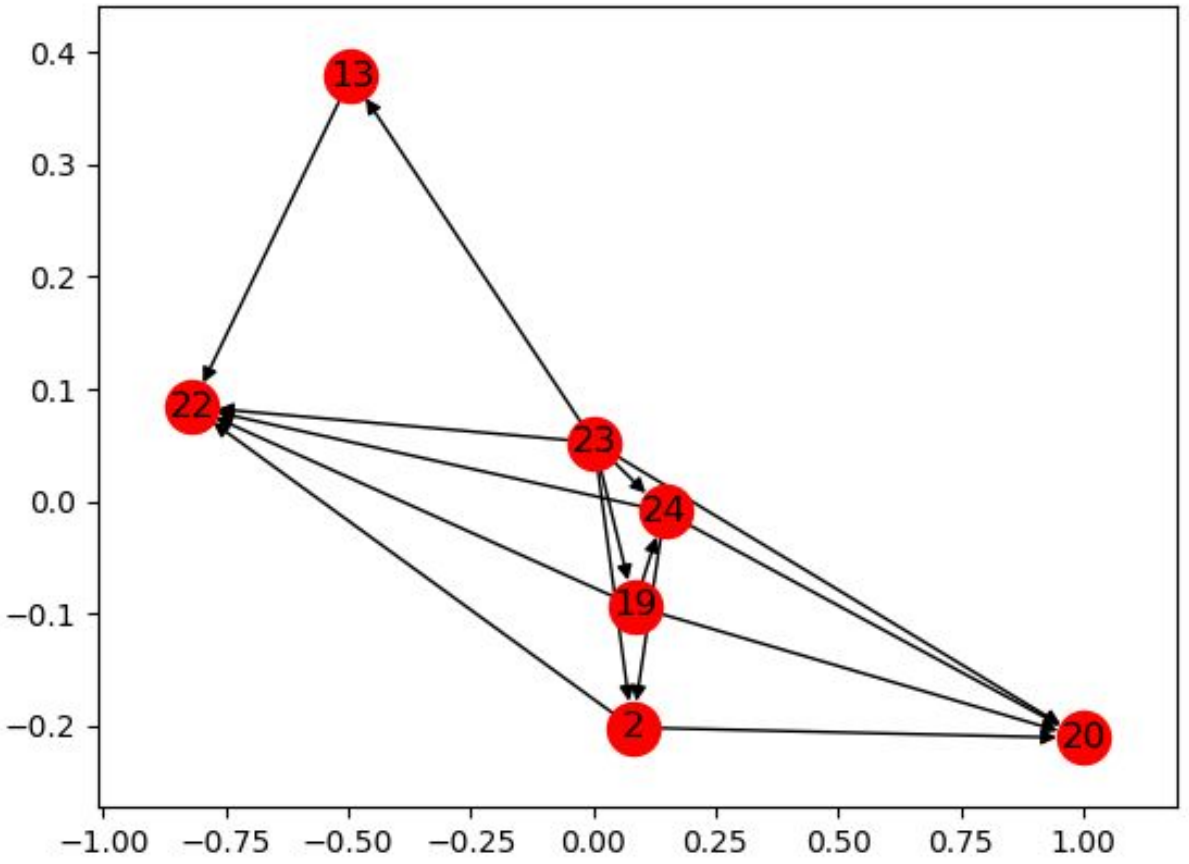
A discussion of transitivity

- # of intransitivities: 0 (K. C.)



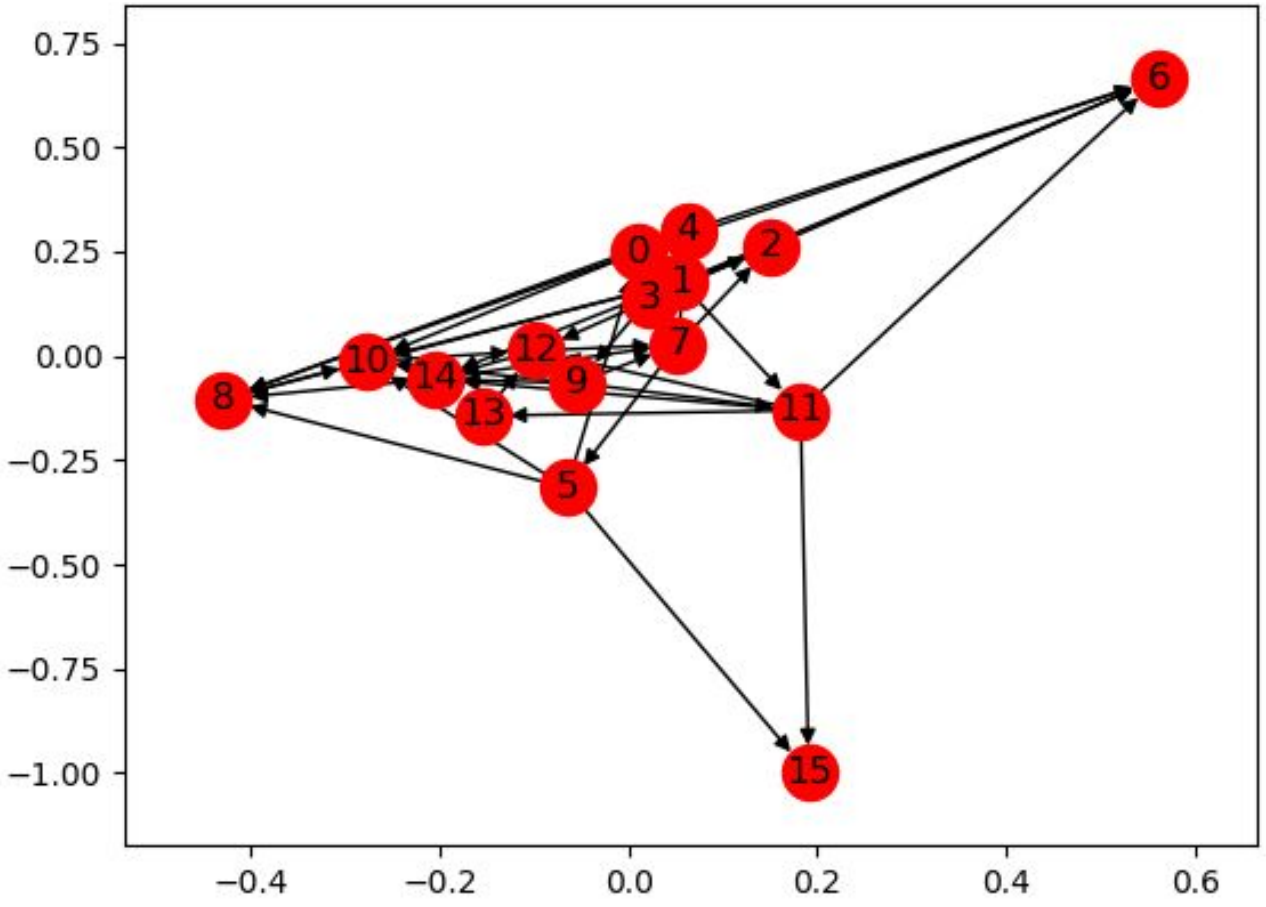
A discussion of transitivity

- # of intransitivities: 0 (L. C.)



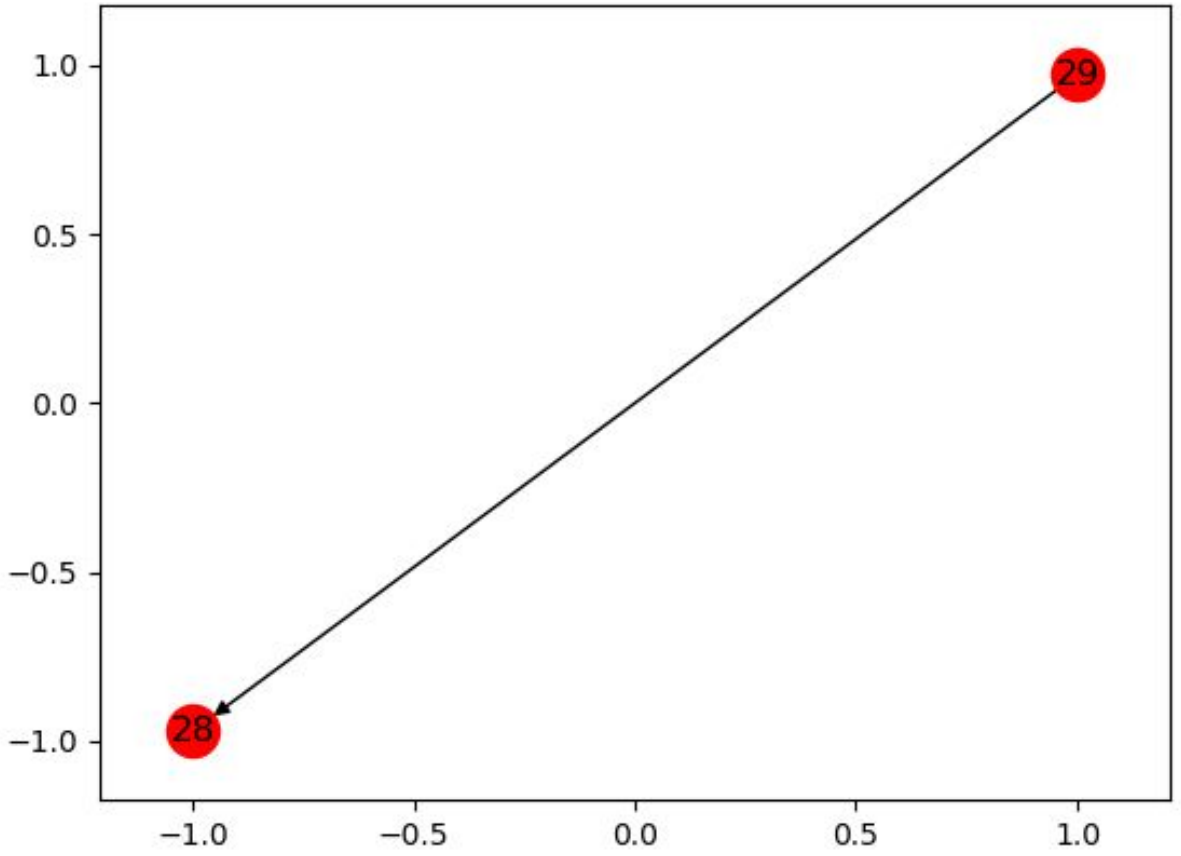
A discussion of transitivity

- # of intransitivities: 15 (S. A.)



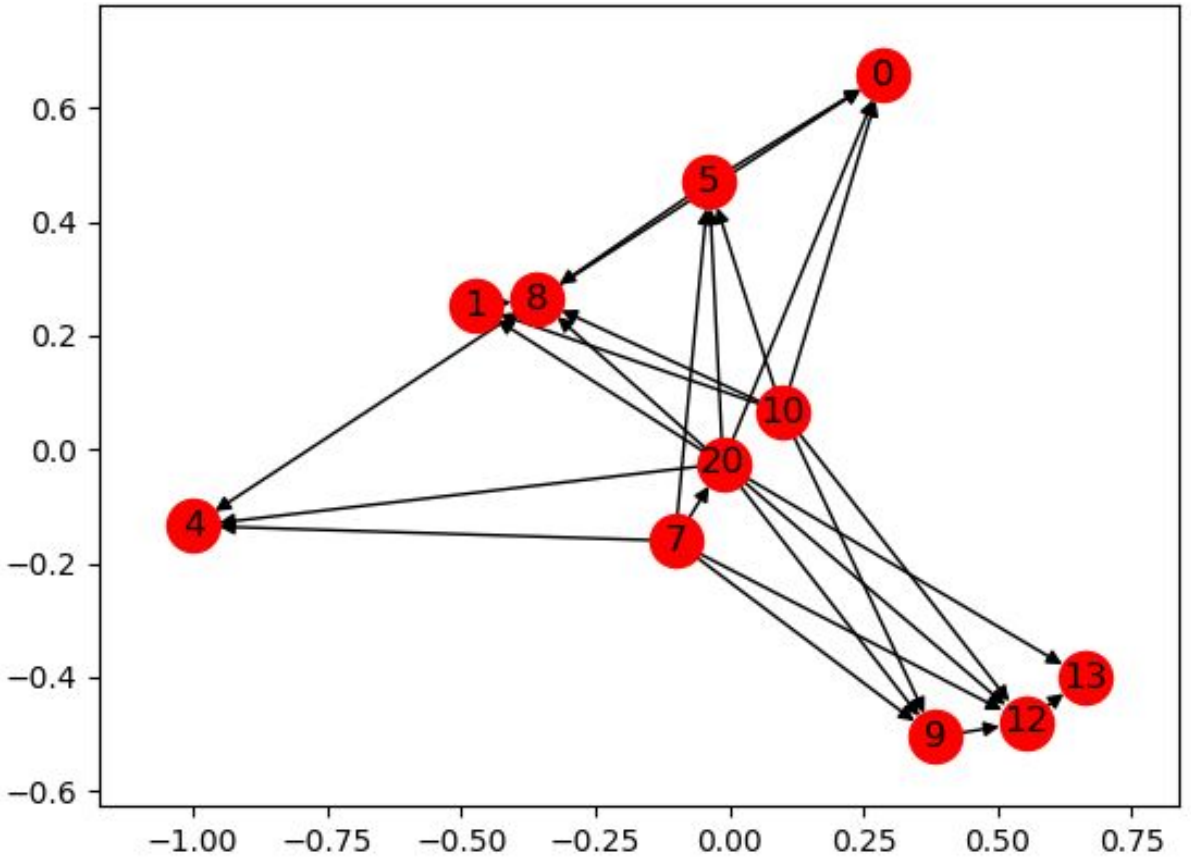
A discussion of transitivity

- # of intransitivities: 0 (I. R.)



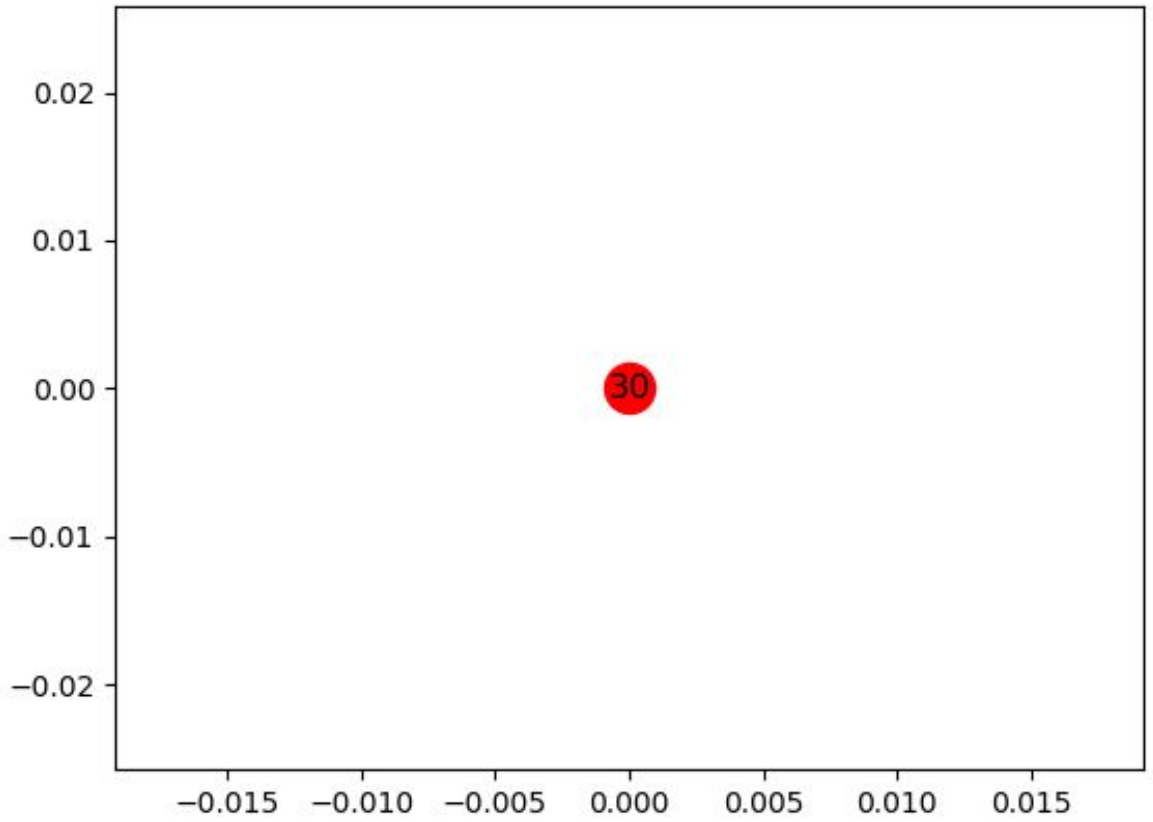
A discussion of transitivity

- # of intransitivities: 0 (R. C.)
  - 5/6 unreasonable answers



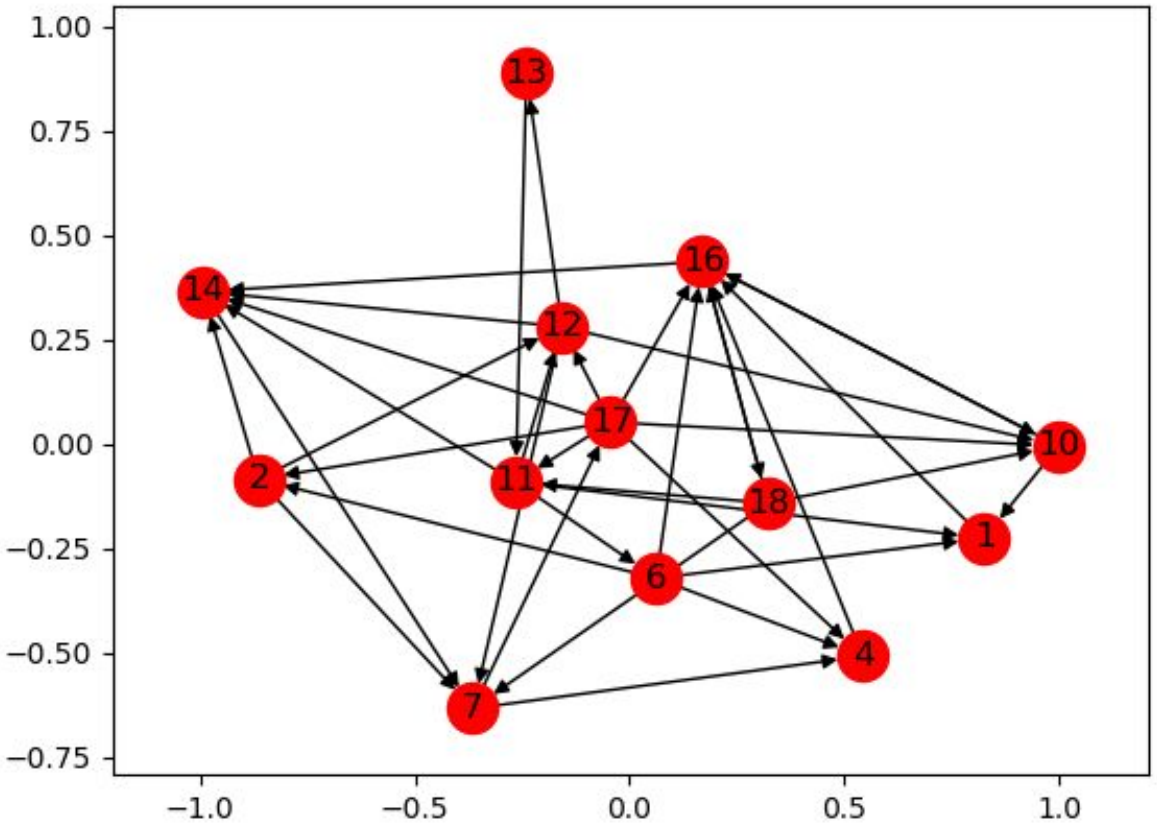
A discussion of transitivity

- # of intransitivities: 1 (T. S.)
  - 2/6 unreasonable answers



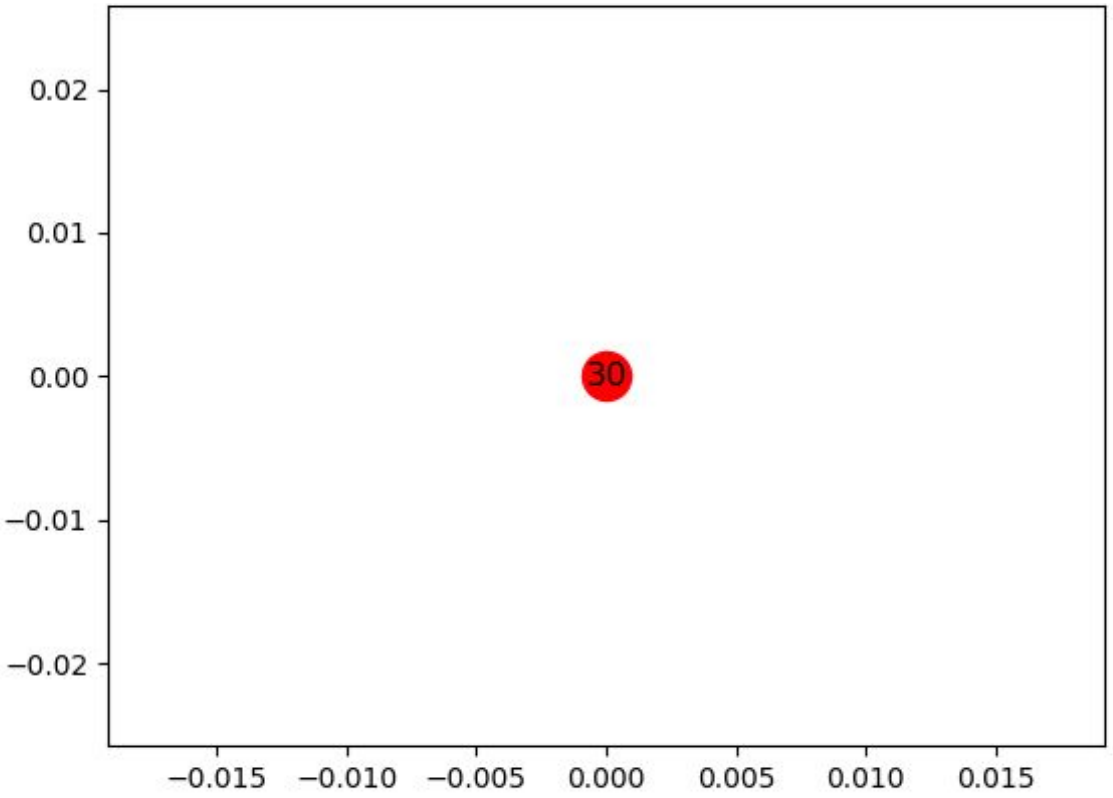
A discussion of transitivity

- # of intransitivities: 126 (A. F.)
  - 3/6 unreasonable answers



A discussion of transitivity

- # of intransitivities: 0 (G. F.)
  - 6/6 unreasonable answers



A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 38 students who responded questionnaire Q1 in 2017/02…
- ...only 4 (10.5%) had no intransitivities
- The median number intransitivities per student was 2.5
- The mean was 13.18 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 39 students who responded questionnaire Q1 in 2016/02…
- ...only 3 (7.7%) had no intransitivities
- The median number intransitivities per student was 4
- The mean was 12.1 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 27 students who responded questionnaire Q1 in 2015/02…
- ...only 4 (14.8%) had no intransitivities
- The median number intransitivities per student was 3
- The mean was 15.4 (!)

A discussion of transitivity

- How would you react if somebody told you he prefers ***x*** to ***y***, ***y*** to ***z***, and ***z*** to ***x***?
- Out of 458 students who responded a simple preference questionnaire (details in the book),
- only 57 (12.44%) had no intransitivities in their answers,
- The median number intransitivities per student was 7

Procedures that cause violations of transitivity

- Any ideas?

Procedures that cause violations of transitivity

- Being crazy?

Procedures that cause violations of transitivity

- Being lazy?

Procedures that cause violations of transitivity

- Aggregation of considerations as a source of intransitivity
  - ***X = {a, b, c}*** and the individual has three primitive considerations in mind (eg: price, taste, quality)
  - The individual finds an alternative ***x*** better than an alternative ***y*** if a majority of considerations supports ***x***
  - If the three considerations rank the alternatives as $a \succ_1 b \succ_1 c$, $b \succ_2 c \succ_2 a$, and $c \succ_3 a \succ_3 b$, then…
  - … the individual determines ***a*** to be preferred over ***b***, ***b*** over ***c***, and ***c*** over ***a***, thus violating transitivity

Procedures that cause violations of transitivity

- The use of similarities as an obstacle to transitivity
  - In some cases, an individual may express ***indifference*** in a comparison between two elements that are ***too "close"*** to be distinguishable
  - Let ***X*** be the set of real numbers
  - Consider an individual whose attitude toward the alternatives is ***"the larger the better"***, but he cannot determine whether ***a*** is greater than ***b*** unless the difference is at least ***1***
  - He will assign $f(x, y) = x \succ y$ if $x \geq y + 1$ and $f(x, y) = I$ if $|x - y| < 1$
  - Is this function always transitive?
  - This is not a preference relation because $1.5 \sim 0.8$ and $0.8 \sim 0.3$, but it is not true that $1.5 \sim 0.3$

Preferences

- **Definition 1**
  - Preferences on a set ***X*** are a function ***f***
  - that assigns to any pair ***(x, y)*** of distinct elements in ***X*** exactly one of the three "values"
  - ***x*** $\succ$ ***y***, ***y*** $\succ$ ***x***, or ***I***
  - so that for any three different elements ***x***, ***y***, and ***z*** in ***X***, the following two properties hold:
    - No order effect: ***f(x, y) = f(y, x)***
    - Transitivity:
      - if ***f(x, y) = x*** $\succ$ ***y*** and ***f(y, z) = y*** $\succ$ ***z***, then ***f(x, z) = x*** $\succ$ ***z*** and
      - if ***f(x, y) = I*** and ***f(y, z) = I***, then ***f(x, z) = I***

A discussion of transitivity

- Is this definition weak?
- For example, if $f(x, y) = x \succ y$ and $f(y, z) = I$, can $f(x, z)$ be different than $x \succ z$?
- No! Proof in the book

Questionnaire R

R(x,y) (for all $x, y \in X$, not necessarily distinct):

Is x at least as preferred as y? Tick one and only one of the following two options:

- ☐ Yes
- ☐ No

Questionnaire R

- By a “legal” response we mean that the respondent ticks exactly one of the boxes in each question
- To qualify as preferences, a legal response must also satisfy two conditions:
  - The answer to at least one of the questions ***R(x, y)*** and ***R(y, x)*** must be ***Yes***
  - For every ***x, y, z*** $\in$ ***X***, if the answers to the questions ***R(x, y)*** and ***R(y, z)*** are ***Yes***, then so is the answer to the question ***R(x, z)***

The equivalence of the two definitions

- If I get a questionnaire Q from “Smith”, can I fill questionnaire R for “Smith”?

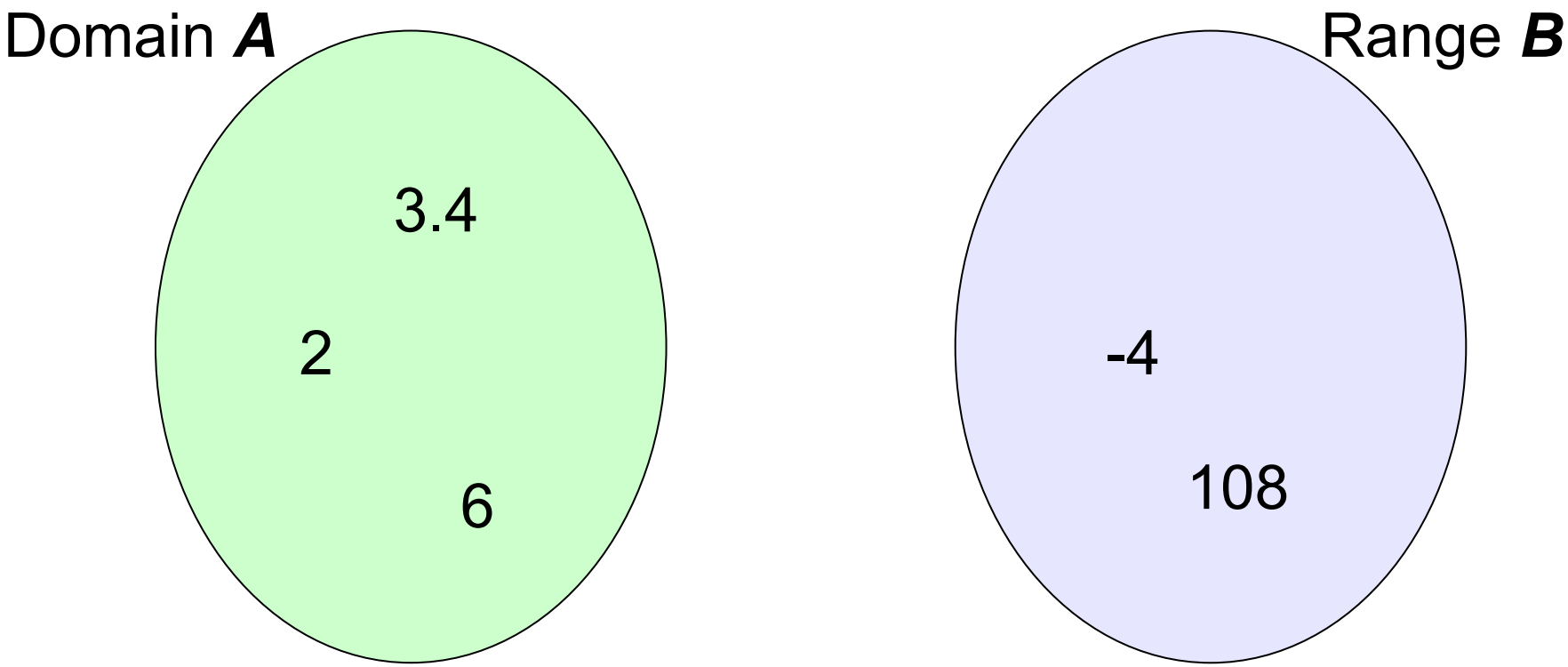
Q(x,y) (for all distinct x and y in X):
How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

R(x,y) (for all $x, y \in X$, not necessarily distinct):
Is x at least as preferred as y? Tick one and only one of the following two options:

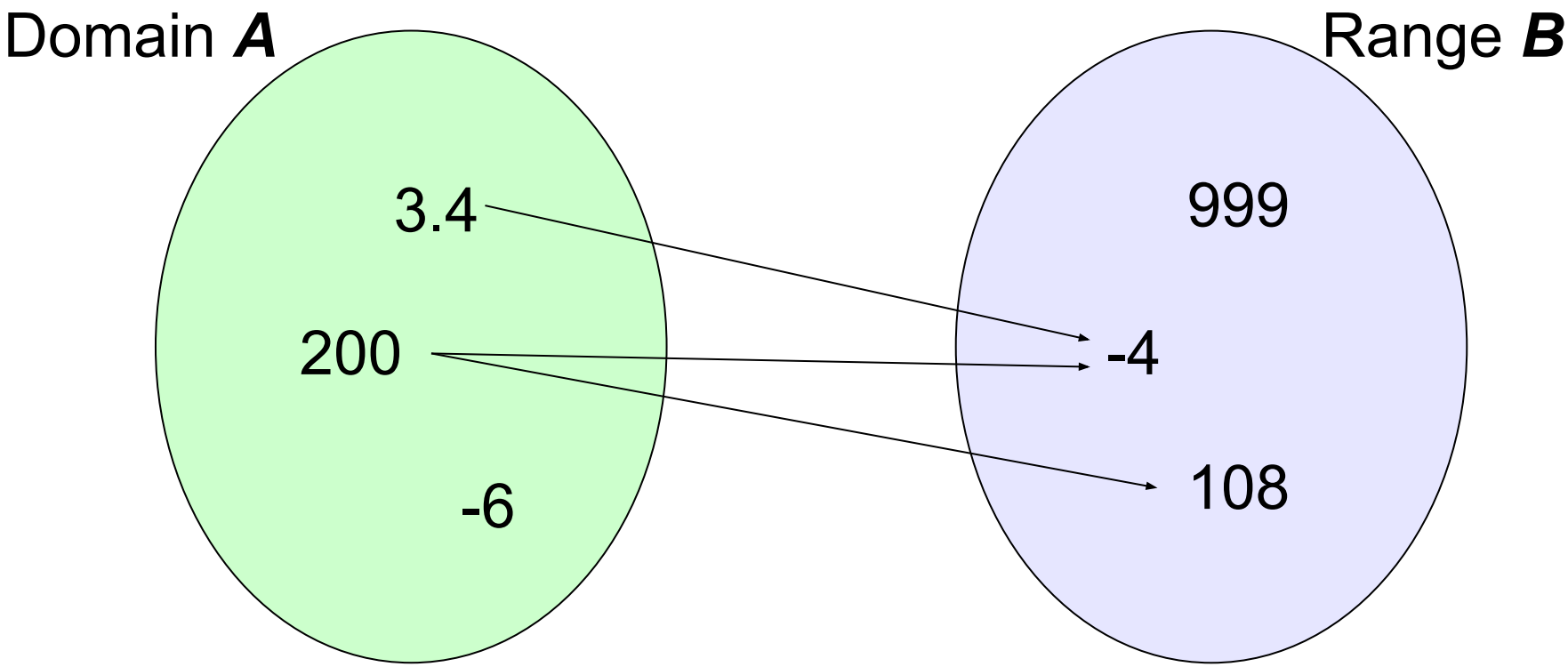
☐ Yes
☐ No

Reminder: relation



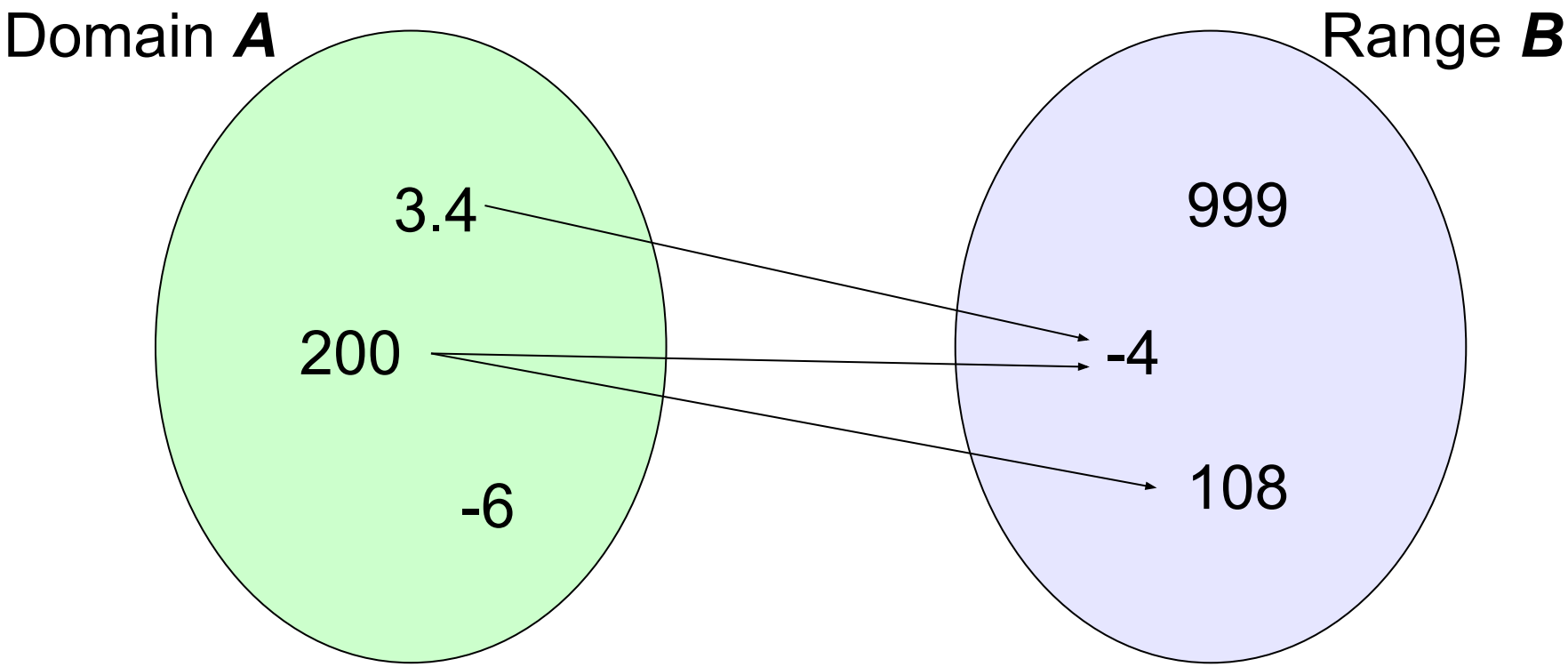
A relation *R* is a set of ordered pairs

Reminder: relation



$$R \subseteq A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Reminder: relation



R = {(3.4,-4), (200,-4), (200,108)}

Questionnaire R

- We identify a response to this questionnaire with the **binary relation** $\succsim$ on the set ***X*** defined by $x \succsim y$ if the answer to the question ***R(x, y)*** is ***Yes***
- Ex: If ***x*** is at least as preferred as ***y***, then $x \succsim y$

Reminder

- An n-ary relation on X is a subset of X^n
- Examples:
  - "Being a parent of" is a binary relation on the set of human beings
  - "being a hat" is an unary relation on the set of objects
  - "$x + y = z$" is a 3-ary relation on the set of numbers
  - " x is better than y more than x' is better than y' " is 4-ary relation on a set of alternatives

Reminder

- An n-ary relation on $\boldsymbol{X}$ can be thought of as a response to a questionnaire regarding all n-tuples of elements of $\boldsymbol{X}$ where each question can get only a ***Yes*** answer
- Ex: is $\boldsymbol{a_1 \precsim a_2 \precsim a_3 \precsim \ldots \precsim a_n}$? (Yes)

Preferences

- Definition 2
  - Preferences on a set X is a binary relation $\succsim$ on X satisfying:
    - ***Completeness***: For any x, $y \in X$, $x \succsim y$, or $y \succsim x$
    - ***Transitivity***: For any x, y, $z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$

The equivalence of the two definitions

- If I get a questionnaire Q from “Smith”, can I fill questionnaire R for “Smith”?

Q(x,y) (for all distinct x and y in X):
How do you compare x and y? Tick one and only one of the following three options:

☐ I prefer x to y (this answer is denoted as $x \succ y$).
☐ I prefer y to x (this answer is denoted by $y \succ x$).
☐ I am indifferent (this answer is denoted by I).

R(x,y) (for all $x, y \in X$, not necessarily distinct):
Is x at least as preferred as y? Tick one and only one of the following two options:

☐ Yes
☐ No

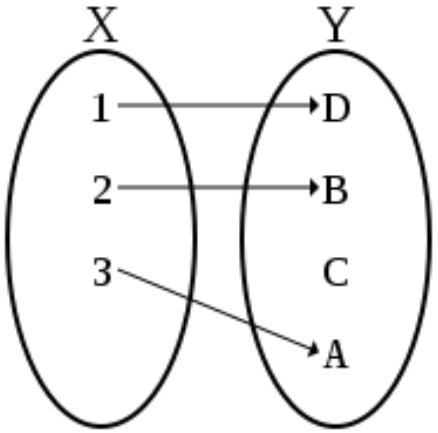
Reminder

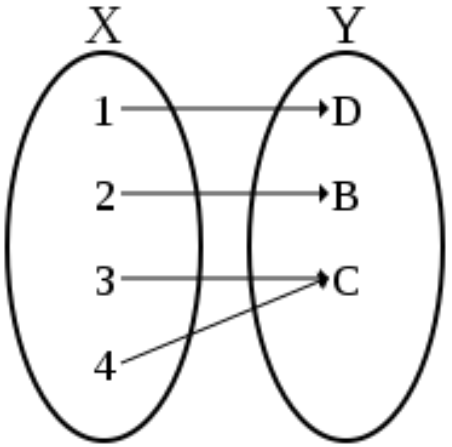
- The function ***f : X → Y*** is a ***one-to-one*** function (or injection) if ***f(x) = f(y)*** implies that ***x = y***
  - Ex: Brazilians → CPF number

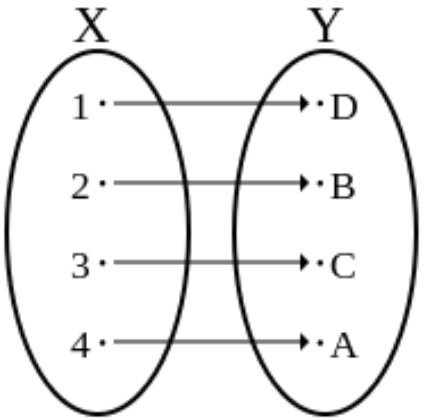
- The function ***f : X → Y*** is an ***onto*** function (or surjection) if for every ***y ∈ Y*** there is an ***x ∈ X*** such that ***f(x) = y***
  - Ex: people → country of birth

- The function ***f : X → Y*** is a ***one-to-one and onto*** function (or bijection, or one-to-one correspondence) if for every ***y ∈ Y*** there is a unique ***x ∈ X*** such that ***f(x) = y***
  - Ex: Brazilians→ Passport

Reminder







The equivalence of the two definitions

- If I get a questionnaire Q from “Smith”, can I fill questionnaire R for “Smith”?
- We need to construct a ***one-to-one and onto*** function answers to Q and answers to R, such that the correspondence preserves the meaning of the responses to the two questionnaires

The equivalence of the two definitions

A response to:	A response to:	
	$Q(x,y)$ and $Q(y,x)$	$R(x,y)$ and $R(y,x)$
$x \succ y$	Yes	No
I	Yes	Yes
$y \succ x$	No	Yes

Summary

- Preferences on $\boldsymbol{X}$ are a binary relation $\succsim$ on a set $\boldsymbol{X}$ satisfying completeness and transitivity
- Notate $\boldsymbol{x} \succ \boldsymbol{y}$ when both $\boldsymbol{x} \succsim \boldsymbol{y}$ and <u>not</u> $\boldsymbol{y} \succsim \boldsymbol{x}$, and $\boldsymbol{x} \sim \boldsymbol{y}$ when $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{x}$

Summary

- Now, with one single relation ($\succsim$), we can describe the full preference relation towards the items in ***X***
  - With questionnaire ***Q***, we needed two relations: $\succ$ and ***I***

Summary

- Modeling exercise with two methodological points
  - 1. When we introduce two formalizations of the same verbal concept, we have to make sure that they indeed carry the same meaning
  - 2. When we construct a formal concept, we make assumptions beyond those explicitly mentioned. Being aware of the implicit assumptions is important for understanding the concept and is useful in coming up with ideas for alternative formalizations

Fun problem

- Let's listen to the Shepard tone
  - https://www.youtube.com/watch?v=BzNzgsAE4F0
- Can you think of any economic analogies?

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800
L3	$800	$900	$1000	$500	$600	$700

Fun problem

- Roll a die and get a prize! Which lottery do you prefer?

	1	2	3	4	5	6
L1	$1000	$500	$600	$700	$800	$900
L2	$900	$1000	$500	$600	$700	$800
L3	$800	$900	$1000	$500	$600	$700
L4	$700	$800	$900	$1000	$500	$600
L5	$600	$700	$800	$900	$1000	$500
L6	$500	$600	$700	$800	$900	$1000