

Risk Aversion

Risk Aversion

- Who is this guy?



Risk Aversion

- Lets watch a portion of a video...
 - 2:45 (specially)



Risk Aversion

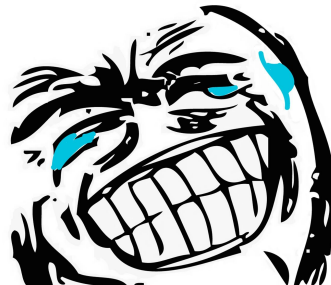
- Imagine a situation where River Plate plays against Boca Juniors and falls to 2nd division iff it loses



vs.



$u(W,D) =$

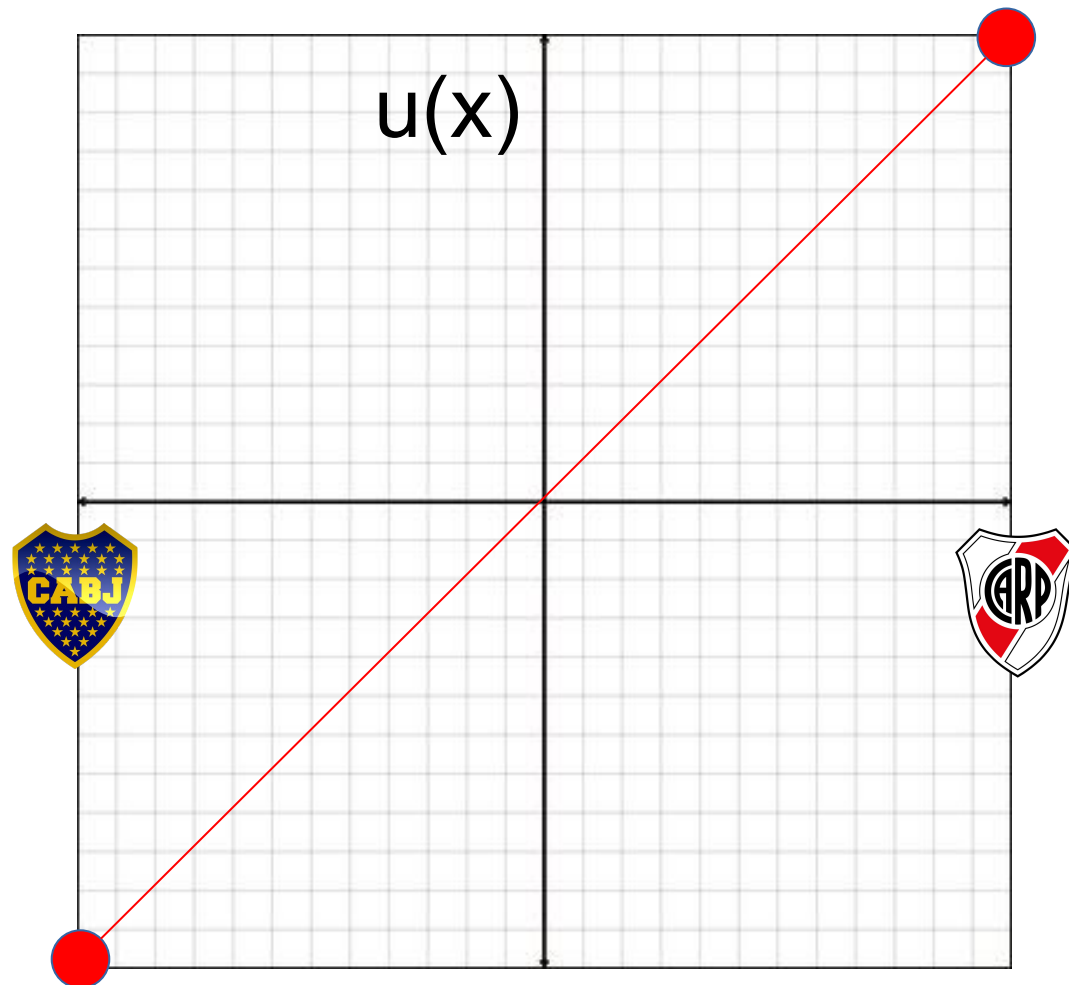


$u(L) =$



Risk Aversion

- Utilities for this lottery



Risk Aversion

- Imagine a situation where River Plate plays against Boca Juniors and falls to 2nd division iff it loses



vs.



$$u(W,D) = \$105$$

$$u(L) = -\$105$$

Risk Aversion

- What should I do if I am extremely risk averse?



vs.



$$u(W,D) = \$105$$

$$u(L) = -\$105$$

Risk Aversion

[Sportwetten](#)
[Casino](#)
[Live Casino](#)
[Games](#)
[Poker](#)
[Mobil](#)
[Club](#)
[Premium](#)

[Mein Konto](#) | [Einzahlung](#) | [Auszahlung](#) | [Boni](#) | [Hilfe und Kontakt](#)

Deutsch

Benutzername

Passwort

login

Passwort vergessen?

Jetzt anmelden

Topereignisse

- Tennis - Heutige Spiele
- 1. Bundesliga
- 2. Bundesliga
- Champions League
- Europa League
- DFB-Pokal

Sportwetten

- Livecenter (Flash 8)
- Livecenter Kalender
- Live Wetten (kein Flash)
- Fussball
- Tennis
- Basketball
- Volleyball
- Eishockey
- Am. Football
- Boxen
- Dart
- Futsal
- Golf
- Handball
- Hunderennen
- Kricket
- Motorsport
- Olympia
- Pferderennen
- Poker
- Snooker

[Sport Startseite](#) > [Fussball](#) > **1. Bundesliga - Spiele**

Fussball

Weitere Wetten für Fussball finden Sie hier

Dezimal (3.50)

[Siegwette](#) | [Half-Time Result](#) | [Handicap](#) | [Remis Geld zurück](#)

1. Bundesliga - Spiele

Siegwette

	Heim	Unentschieden	Auswärts	
VfL Wolfsburg v Freiburg 10/02/2012 20:30 CET	1.60	3.60	5.00	Alle Wetten (57)
Stuttgart v Hertha BSC Berlin 11/02/2012 15:30 CET	1.80	3.30	4.20	Alle Wetten (57)
Mainz 05 v Hannover 96 11/02/2012 15:30 CET	1.95	3.20	3.60	Alle Wetten (57)
Werder Bremen v TSG Hoffenheim 11/02/2012 15:30 CET	1.85	3.20	4.00	Alle Wetten (57)
Bayern München v Kaiserslautern 11/02/2012 15:30 CET	1.133	6.50	17.00	Alle Wetten (56)
Dortmund v Bayer Leverkusen 11/02/2012 15:30 CET	1.40	4.30	6.50	Alle Wetten (57)
Borussia Mönchengladbach v Schalke 04 11/02/2012 18:30 CET	2.30	3.20	2.85	Alle Wetten (57)
Augsburg v Nürnberg 12/02/2012 15:30 CET	2.55	3.10	2.60	Alle Wetten (57)
1. FC Köln v Hamburger SV 12/02/2012 17:30 CET	2.80	3.30	2.25	Alle Wetten (58)

Einzel und Mehrfachwetten möglich

Nur für Neukunden! 100€bonus

Ihr Wettschein hat 3 Auswahl(en)

alle Wetten auswählen	Wettschein leeren	Eintrag
<input checked="" type="checkbox"/> Stuttgart v Hertha BSC Berlin Siegwette Hertha BSC Berlin 4.20 Möglicher Gesamtgewinn 420.00 EUR	<input checked="" type="checkbox"/>	1 Wette <input type="text" value="100.00"/>
<input checked="" type="checkbox"/> Werder Bremen v TSG Hoffenheim Siegwette Unentschieden 3.20 Möglicher Gesamtgewinn 160.00 EUR	<input checked="" type="checkbox"/>	1 Wette <input type="text" value="50.00"/>
<input checked="" type="checkbox"/> Bayern München v Kaiserslautern Siegwette Bayern München 1.133 Möglicher Gesamtgewinn 113.33 EUR	<input checked="" type="checkbox"/>	1 Wette <input type="text" value="100"/>

Kombi

	Eintrag
Kombinationswette Möglicher Gesamtgewinn 0.00 EUR	1 Wette <input type="text"/>
Patent Möglicher Gesamtgewinn 0.00 EUR	7 Wetten <input type="text"/>
Trixie Möglicher Gesamtgewinn 0.00 EUR	4 Wetten <input type="text"/>
2 aus Möglicher Gesamtgewinn 0.00 EUR	3 Wetten <input type="text"/>
Gesamteinsatz:	250.00 EUR

Risk Aversion

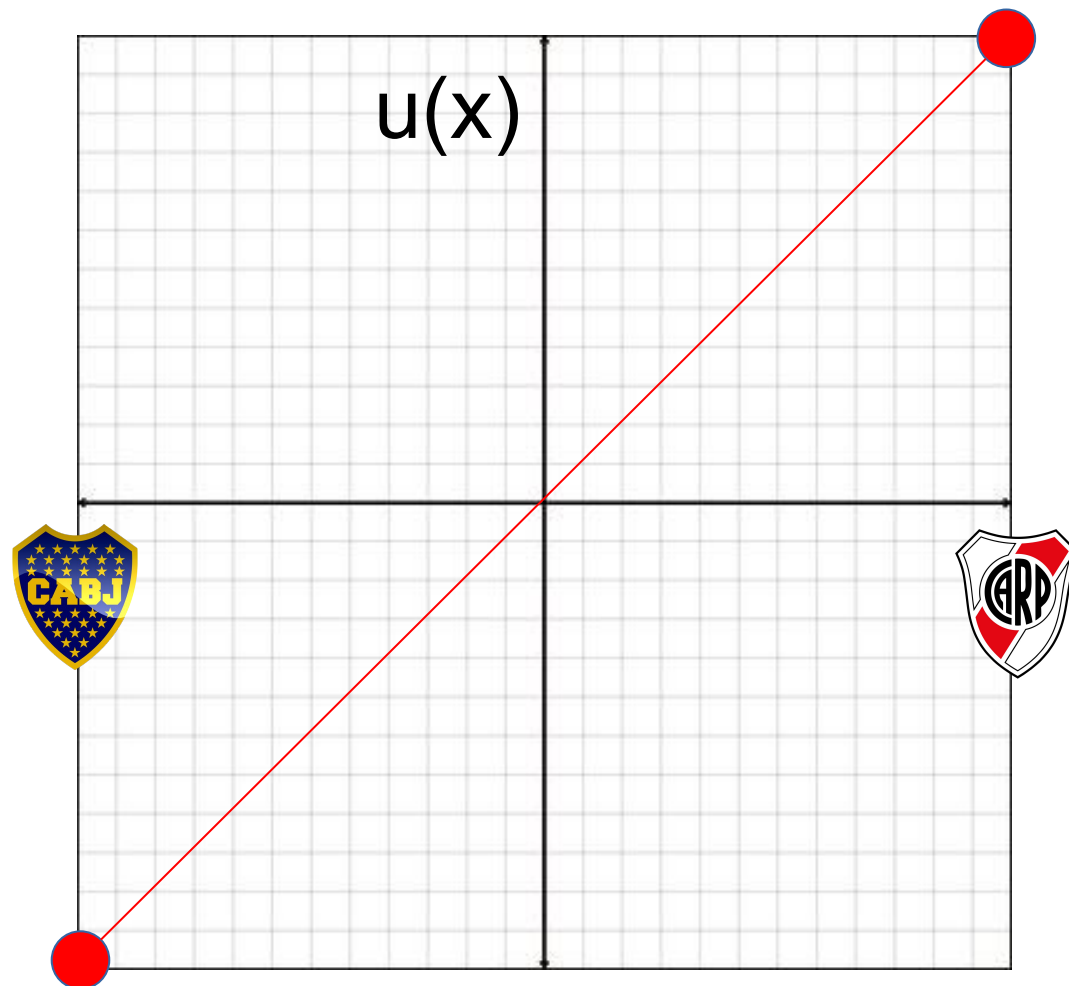
- Suppose a site is paying $r = 3$ dollars for each dollar I put on Boca Juniors
- Then...
 - let x be the amount of dollars I put on Boca Juniors
 - then, if $u(W, D) - x = u(L) + (r-1)x$, I have nothing to worry about!
 - $105 - x = -105 + 3x - x$
 - $3x = 210$
 - $x = \$70$



problem?

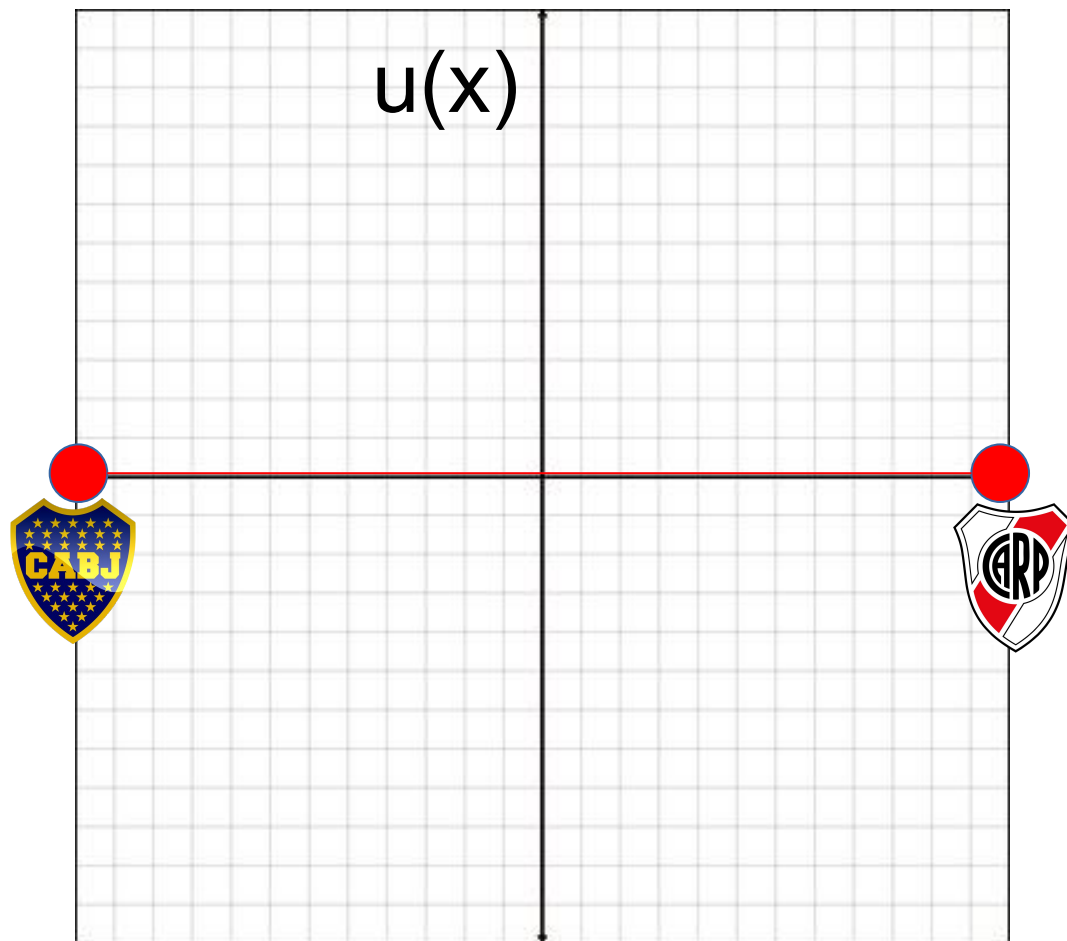
Risk Aversion

- Utilities for this lottery



Risk Aversion

- Utilities for this lottery



Risk Aversion

- ... as if I am paying \$70 for not playing this lottery

Lotteries with monetary prize

a



Smart TV Gamer LED 3D 40" Samsung
UN40J6400 - Full HD 4 HDMI 3 USB 2
Óculos

de R\$ 2.549,00

por R\$ 1.999,00

em até 10x de R\$ 199,90 sem juros
ou **R\$ 1.799,10 à vista**

disponível sob consulta

☐ Ver com outros produtos

b



Smart TV Gamer LED 4k Ultra HD 75"
Samsung - UN75JU6500 4 HDMI 3 USB
Wi-Fi

R\$ 15.999,00

em até 12x de R\$ 1.333,25 sem juros
ou **R\$ 14.399,10 à vista**

☐ Ver com outros produtos

c



Smart TV OLED Curva 3D 55" LG
55EA9850 Full HD - Conversor Integrado 4
HDMI 3 USB Wi-Fi 4 Óculos

de R\$ 11.990,00

por R\$ 7.999,00

em até 12x de R\$ 666,58 sem juros
ou **R\$ 7.199,10 à vista**

☐ Ver com outros produtos

d



Smart TV Gamer LED 75" Samsung
UN75J6300A - Full HD Conversor
Integrado 4 HDMI 3 USB Wi-Fi

R\$ 11.499,00

em até 12x de R\$ 958,25 sem juros
ou **R\$ 10.349,10 à vista**

☐ Ver com outros produtos

$$u(c) = \$7.999,00$$

Lotteries with monetary prize

a



b



$$u(a) = \$5,00$$

$$u(b) = \$1.299,00$$

Lotteries with monetary prize

- Preference relations \succeq over the space of lotteries for which there is a continuous function u , such that \succeq is represented by



- The function Eu assigns to the lottery p the expectation of the random variable that receives the value $u(x)$ with a probability $p(x)$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$30	\$50	\$70
p	50%	0%	50%
q	0%	100%	0%

$$Eu(p) = \$50,00$$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$50	\$100
p	50%	0%	50%
q	0%	100%	0%

$$Eu(p) = \$50,00$$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30	\$100
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30M	\$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Let's play a game...

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30M	\$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Is there an x value that would make you choose p ?

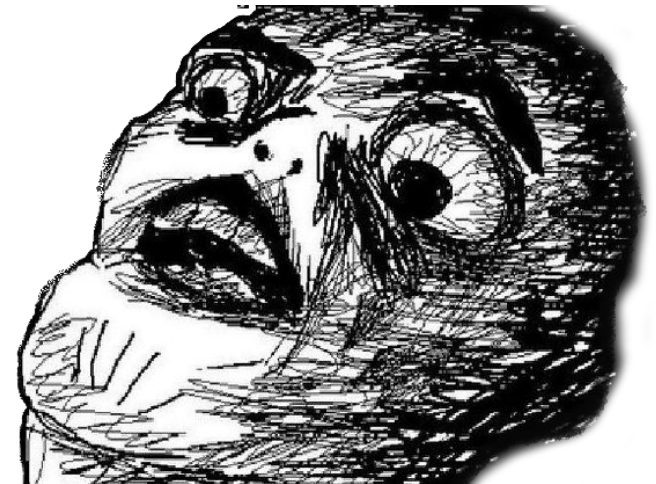
lottery	\$0	\$30M	[\$x]M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_1 that will make you choose p

lottery	\$0	\$30M	$[\mathbf{x}_1]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_1 that will make you choose p

lottery	\$0	\$30M	$[\mathbf{x}_1]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- YOU WON!

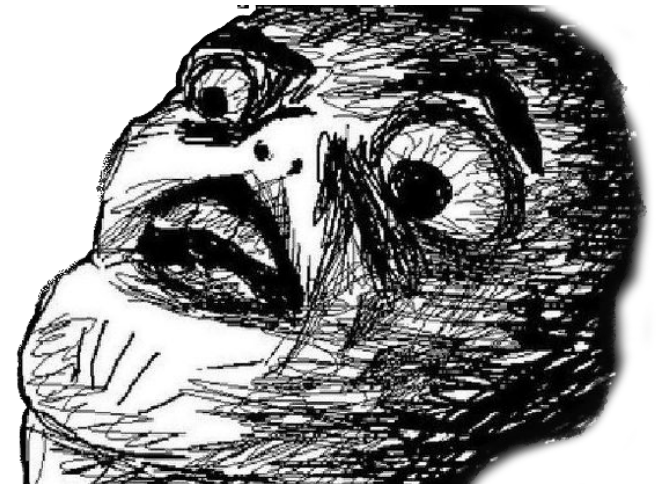


Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_2 that will make you choose p

lottery	\$0	$[\mathbf{x}_1]\text{M}$	$[\mathbf{x}_2]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose now a x_2 that will make you choose p

lottery	\$0	$[\mathbf{x}_1]$ M	$[\mathbf{x}_2]$ M
p	50%	0%	50%
q	0%	100%	0%

- YOU WON!

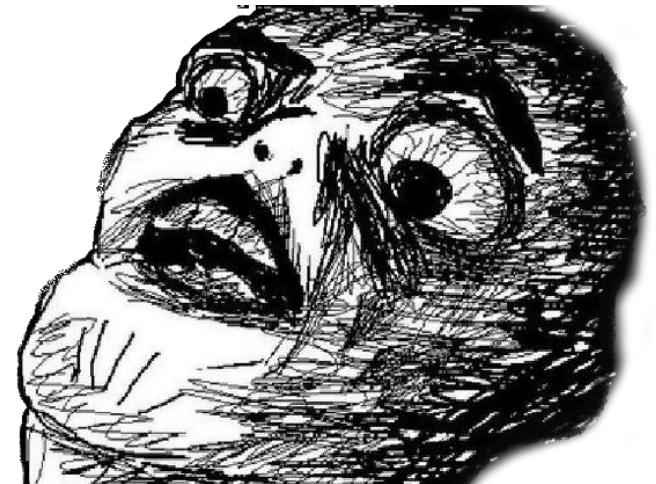


Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_3 that will make you choose p

lottery	\$0	$[\mathbf{x}_2]\text{M}$	$[\mathbf{x}_3]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If utility is not bounded, this game goes on forever
- More important:
 - What is the probability of our decision maker being left with no money?
 - 1!
- St. Petersburg paradox



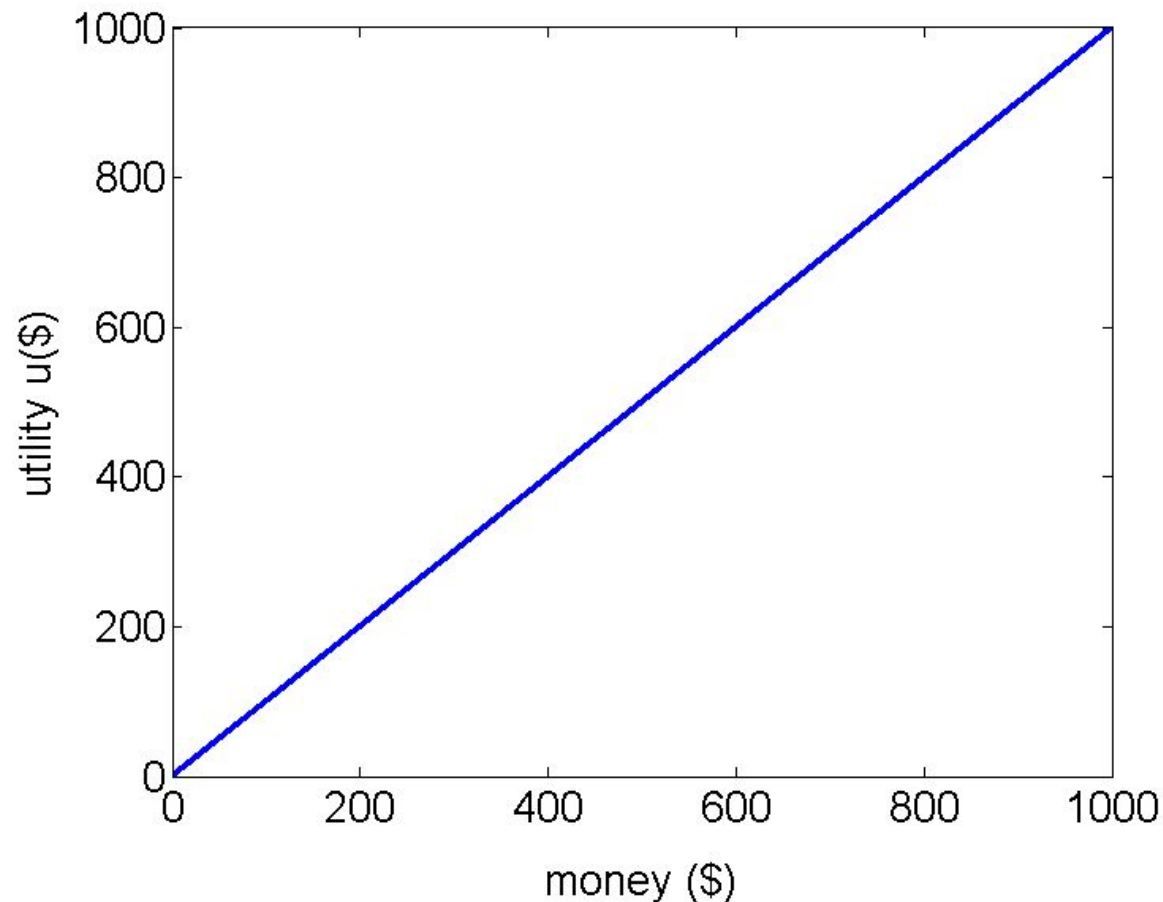
Lotteries with monetary prize

- St. Petersburg paradox, proposed by Nicolaus Bernoulli more than 300 years ago



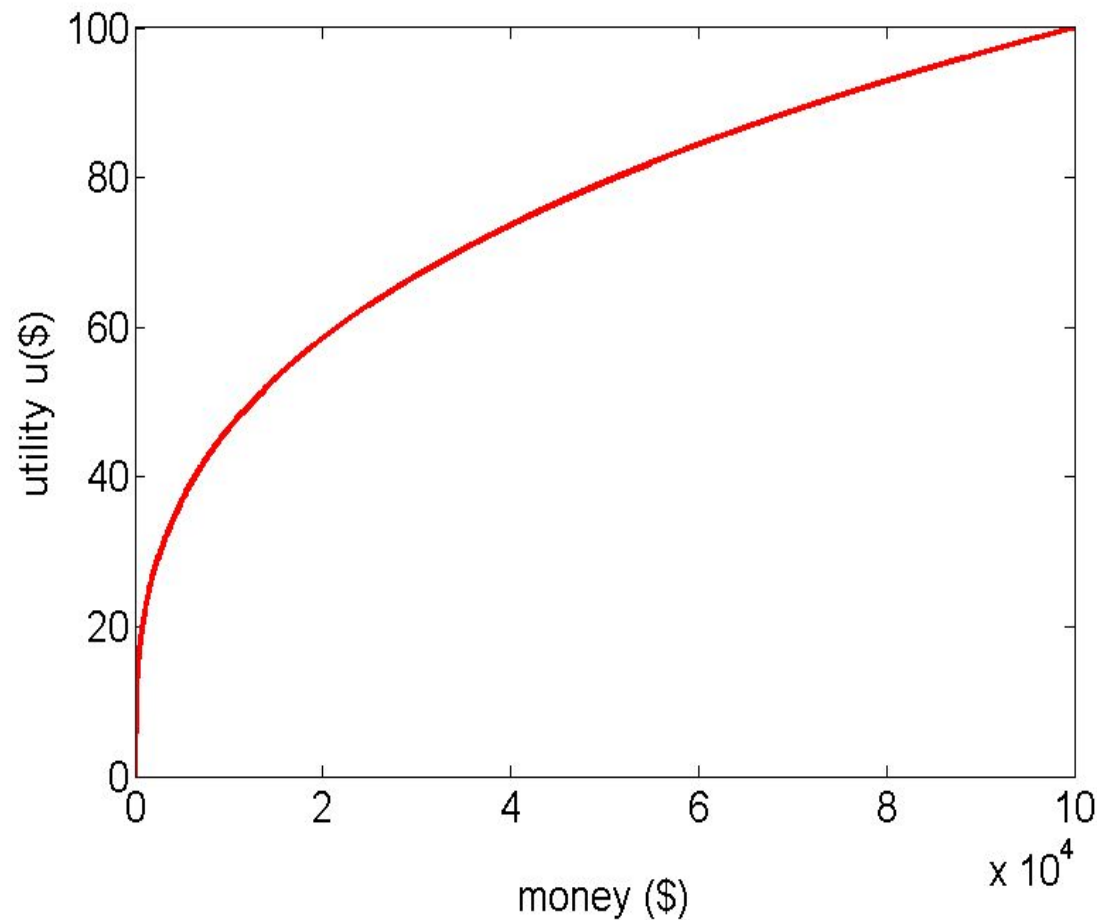
Lotteries with monetary prize

- Money vs. Utility over money



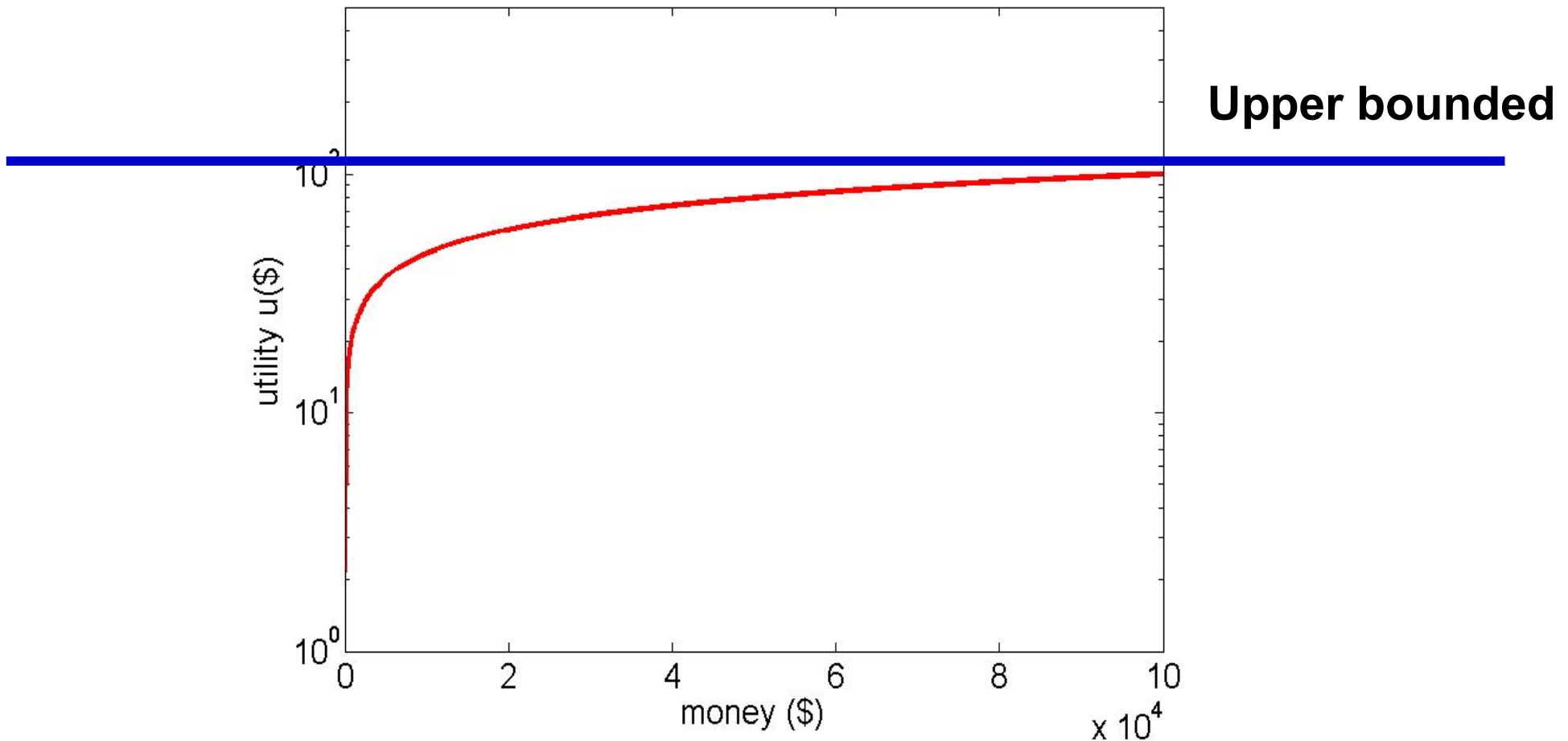
Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Daniel Bernoulli (cousin of Nicolaus) explains :
 - “The value of an item must not be based upon its price, but rather on the utility it yields.
 - The price of the item is dependent only on the thing itself and is equal for everyone.
 - The utility, however, is dependent on the particular circumstances of the person making the estimate.”



Lotteries with monetary prize

- Two insights about the St. Petersburg paradox
 - The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others
 - The utility from gaining an additional dollar would decrease with wealth
 - Utility increases as wealth increases and at a declining rate

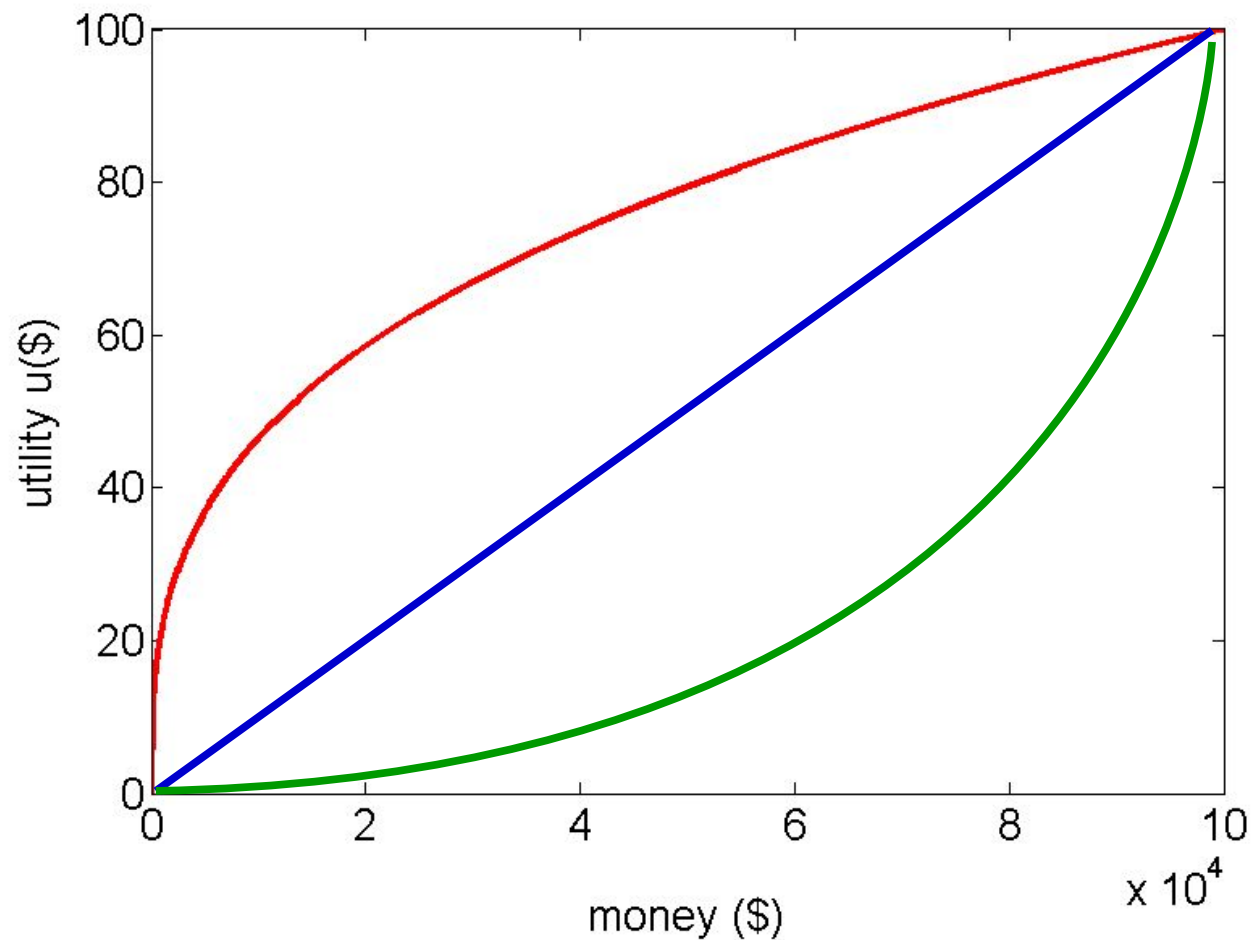
Lotteries with monetary prize

- The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others



Risk Aversion

- How to measure?



Risk Aversion

- How to measure?
 - Every prize should be converted to money
 - Define a utility function $u(\$)$ over money
 - For a lottery p over prizes, calculate:
 - $E(p)$: the expected amount of money of lottery p
 - $Eu(p)$: the expected utility of lottery p
 - $u(E(p))$: the utility of the expected amount of money of p
 - $CE(p)$: the certainty equivalent
 - the amount of money I am willing to pay to play lottery p

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk neutral
 - $u(E(p)) = Eu(p)$ and $E(p) = CE(p) = \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk averse
 - $u(E(p)) > Eu(p)$ and $E(p) > CE(p) < \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk seeking
 - $u(E(p)) < Eu(p)$ and $E(p) < CE(p) > \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- Claim:
 - Let \succsim be a preference on $L(\mathbf{Z})$ represented by the vNM utility function u ,
the preference relation \succsim is **risk averse** iff u is strictly concave
- Proof
 - In the book

Reminder: strictly concave functions

$$f((1-\alpha)x + \alpha y) > (1-\alpha)f(x) + \alpha f(y)$$

consider a lottery with two prizes x and y

and $p(y) = \alpha$ and $p(x) = 1-\alpha$

$$E(p) = (1-\alpha)x + \alpha y$$

if the function f is strictly concave

$$f(E(p)) > (1-\alpha)f(x) + \alpha f(y)$$

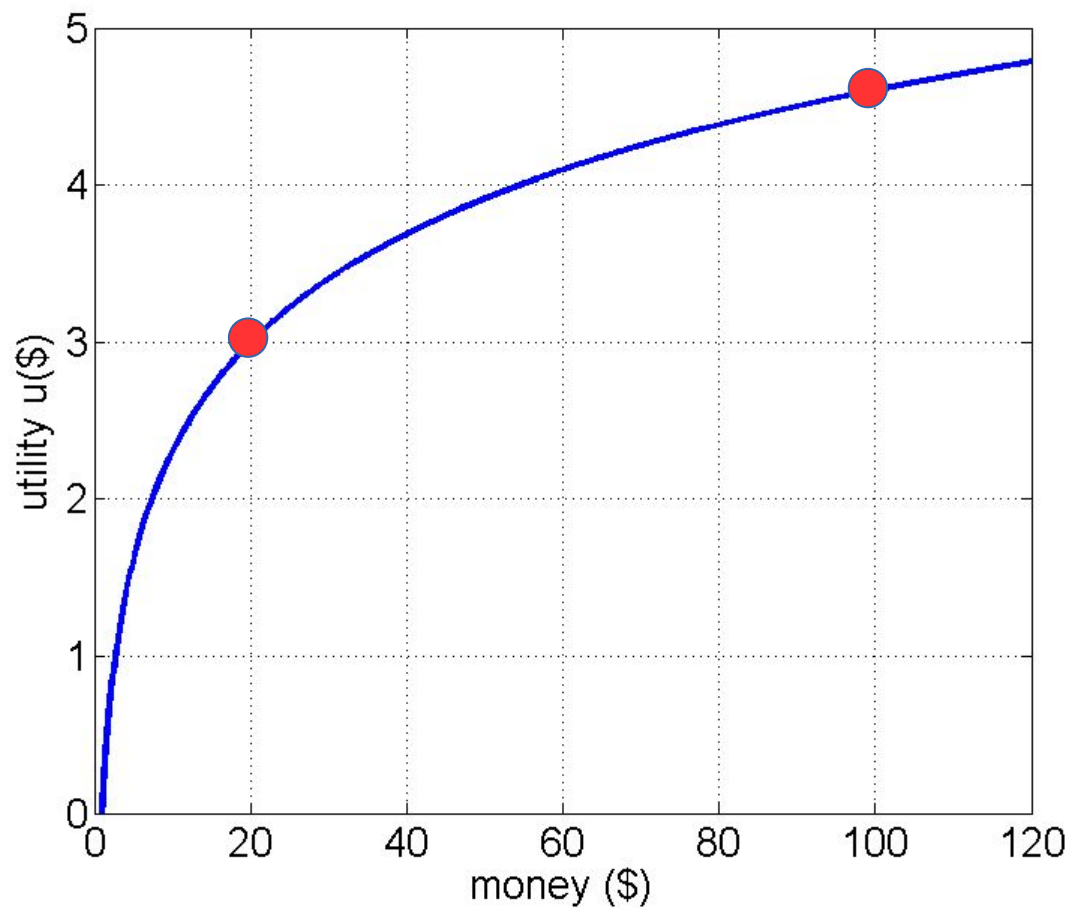
if f is the utility function

$$u(E(p)) > (1-\alpha)u(x) + \alpha u(y)$$

$$u(E(p)) > Eu(p) \text{ (definition of risk aversion)}$$

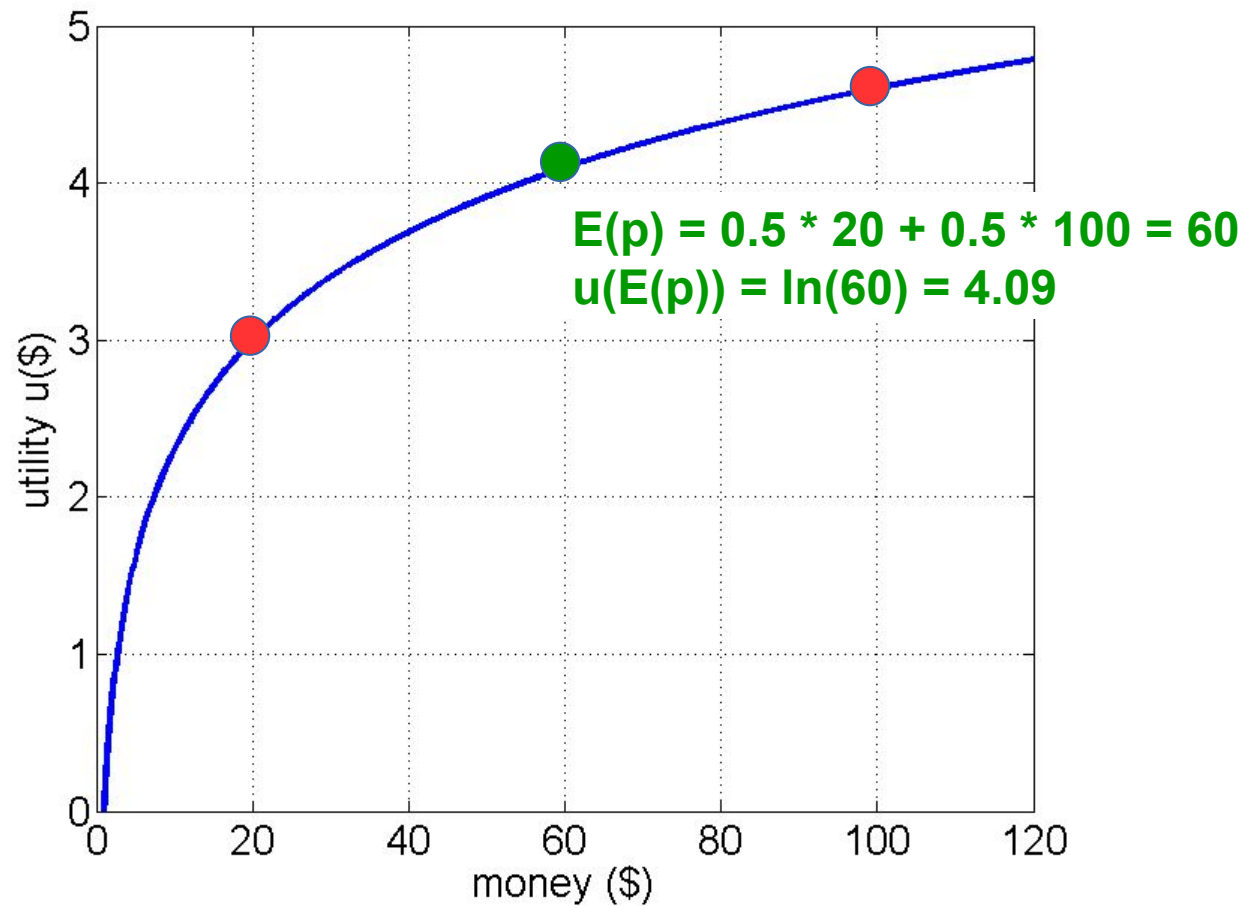
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \text{ \$20 } \oplus (0.5) \text{ \$100}$



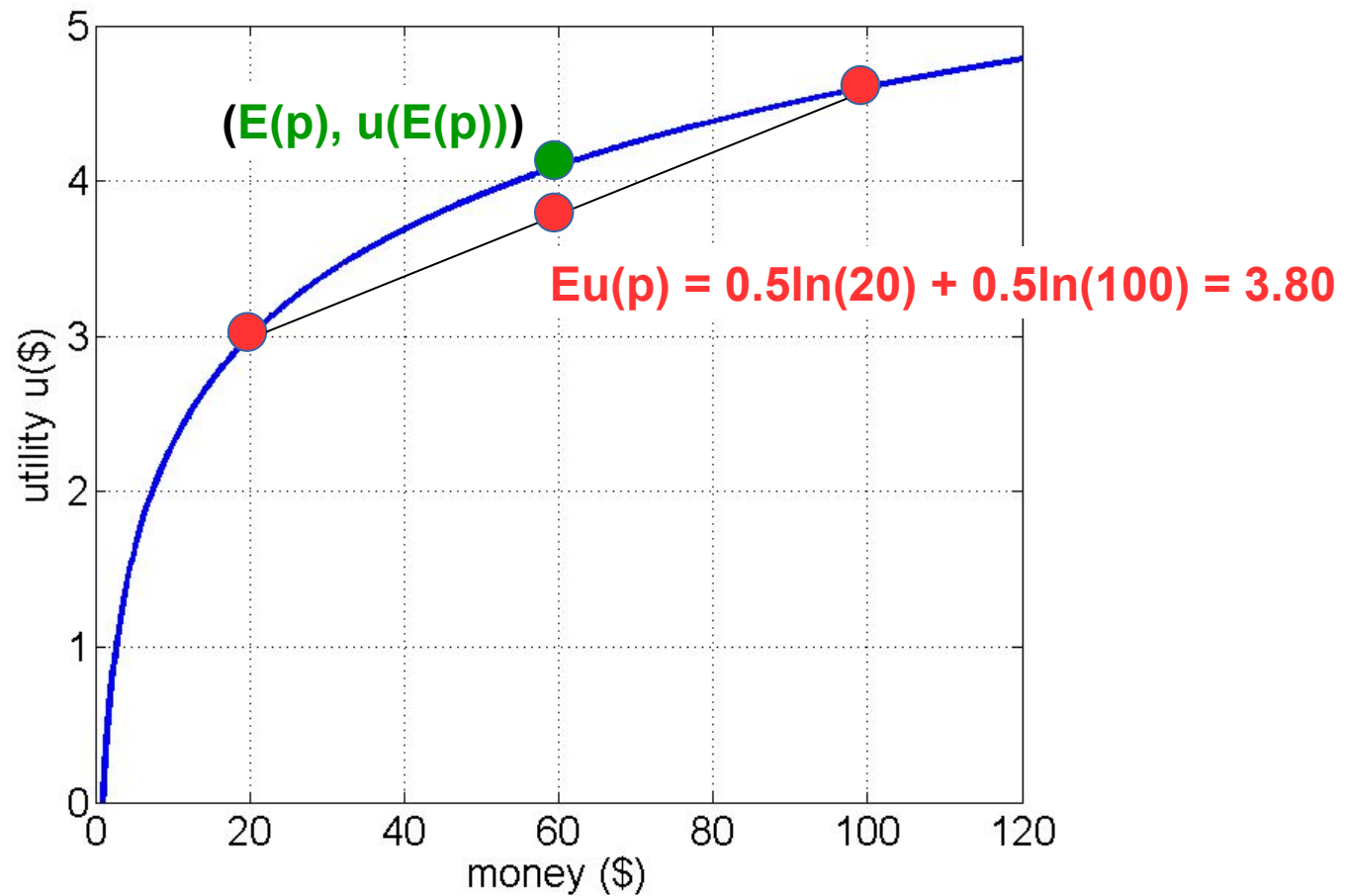
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \text{ \$20 } \oplus (0.5) \text{ \$100}$



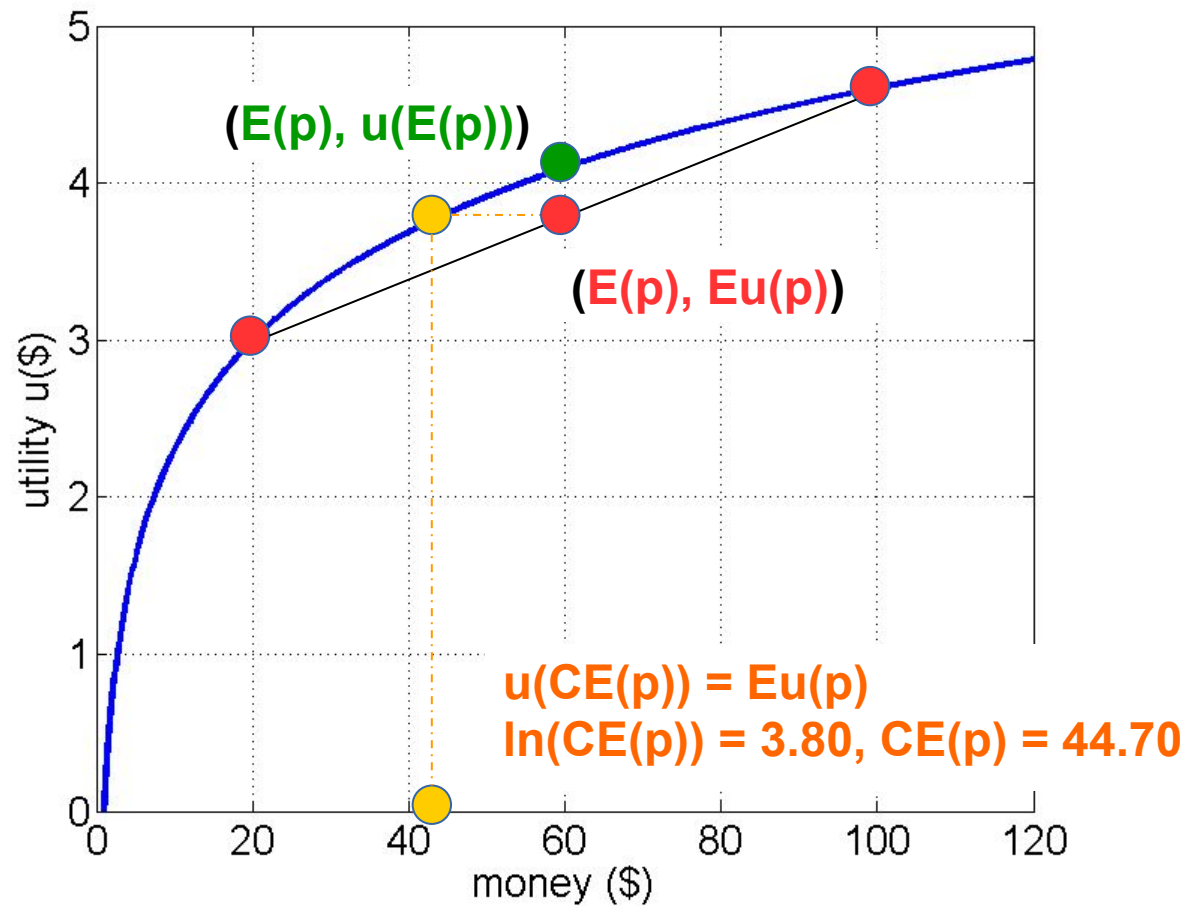
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \$20 \oplus (0.5) \100



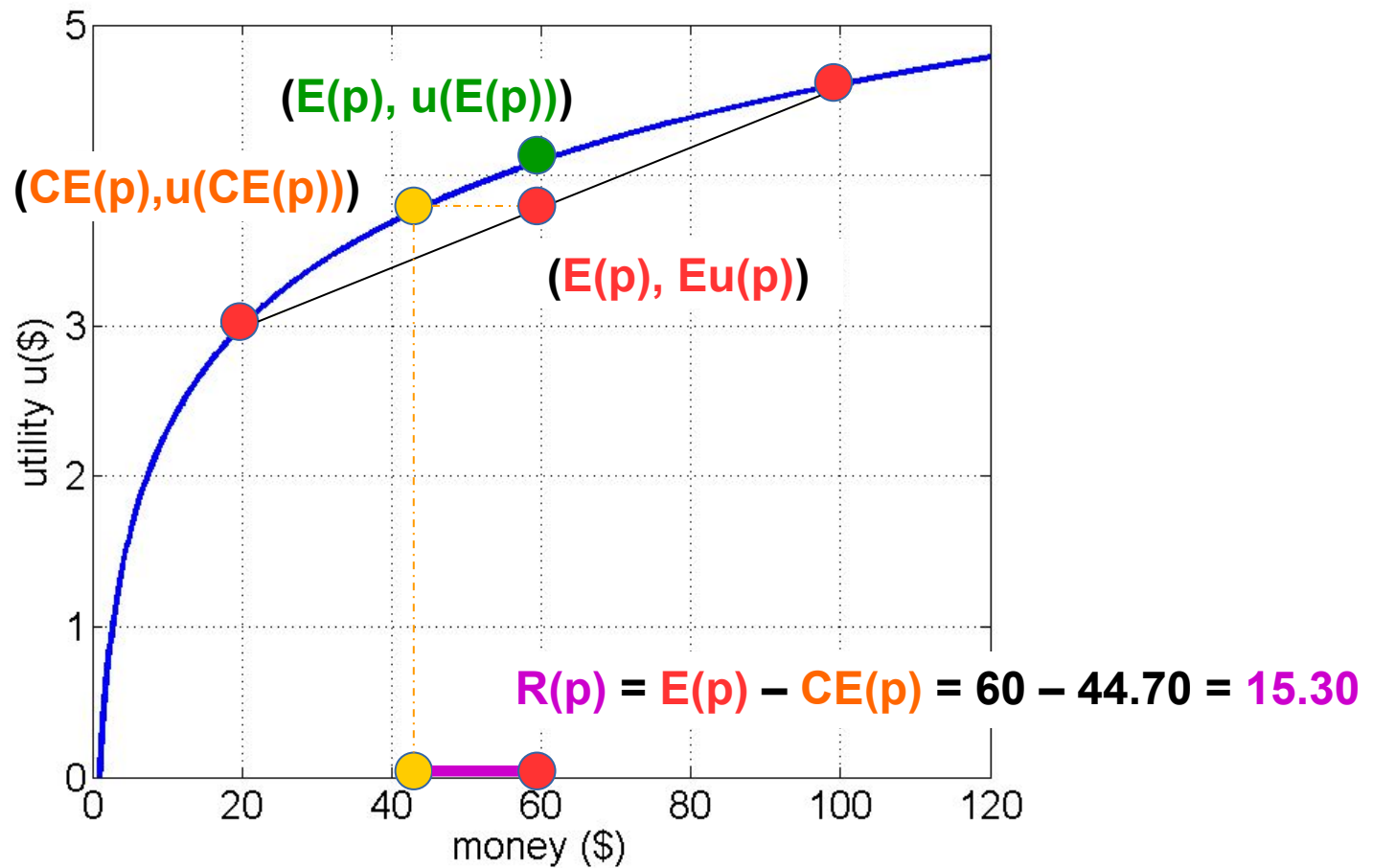
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \$20 \oplus (0.5) \100



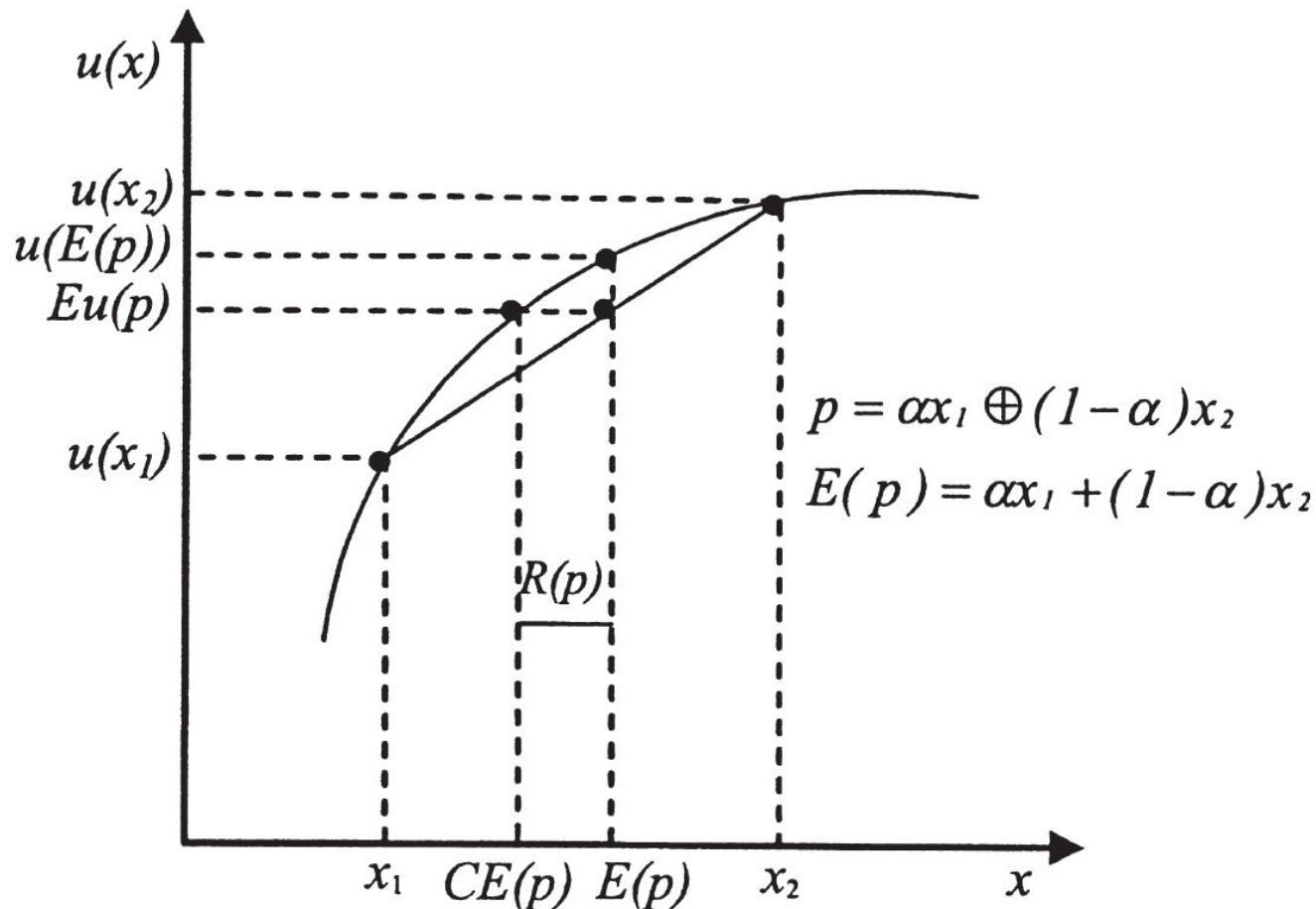
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \$20 \oplus (0.5) \100



Risk Aversion

- Risk premium $R(p)$
 - $R(p) = E(p) - CE(p)$

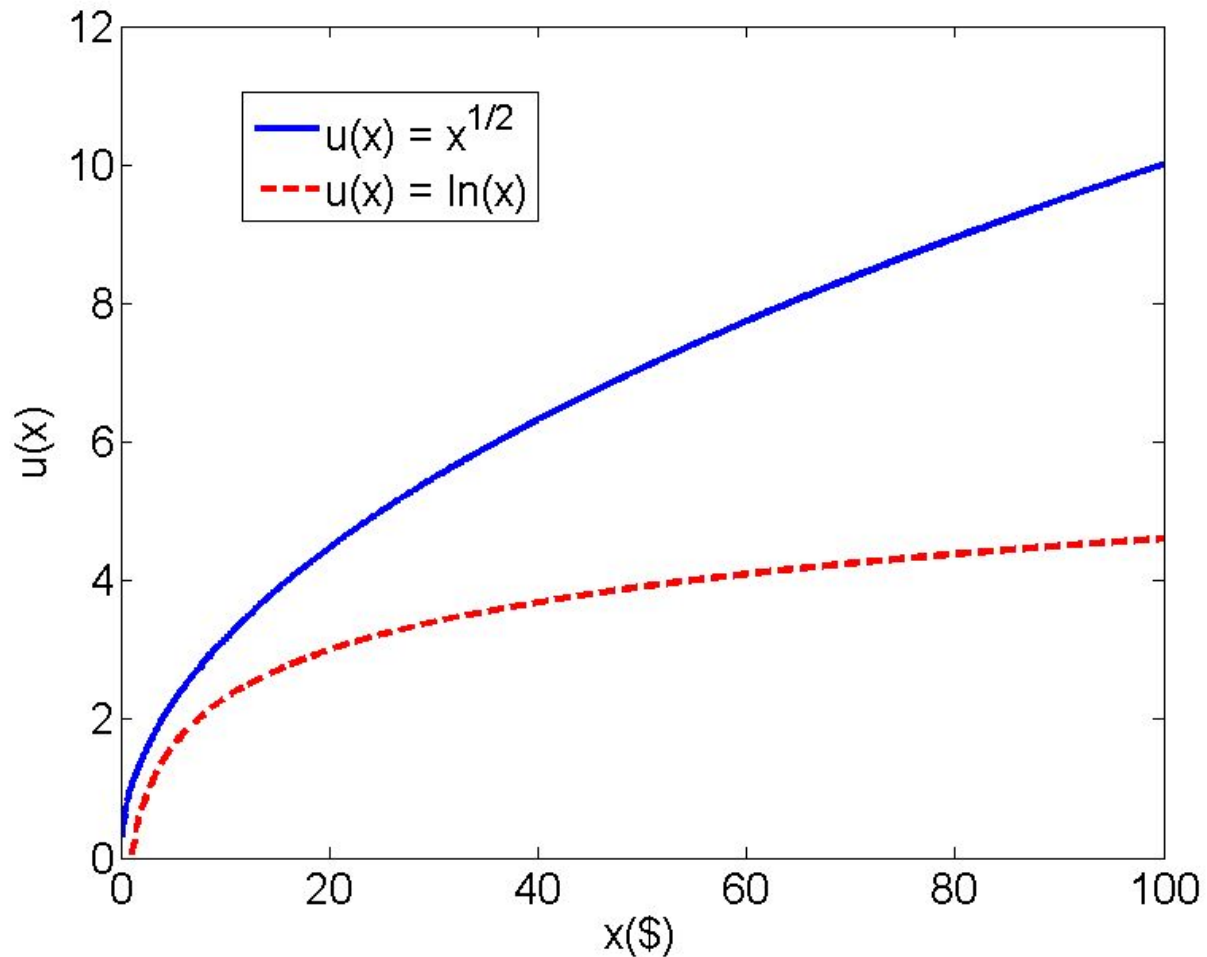


Risk Aversion

- Claim:
 - The preference relation \succsim_1 is more risk averse than \succsim_2 if $\mathbf{CE}_1(p) \leq \mathbf{CE}_2(p)$ for all p

Risk Aversion

- Which individual is more risk averse?

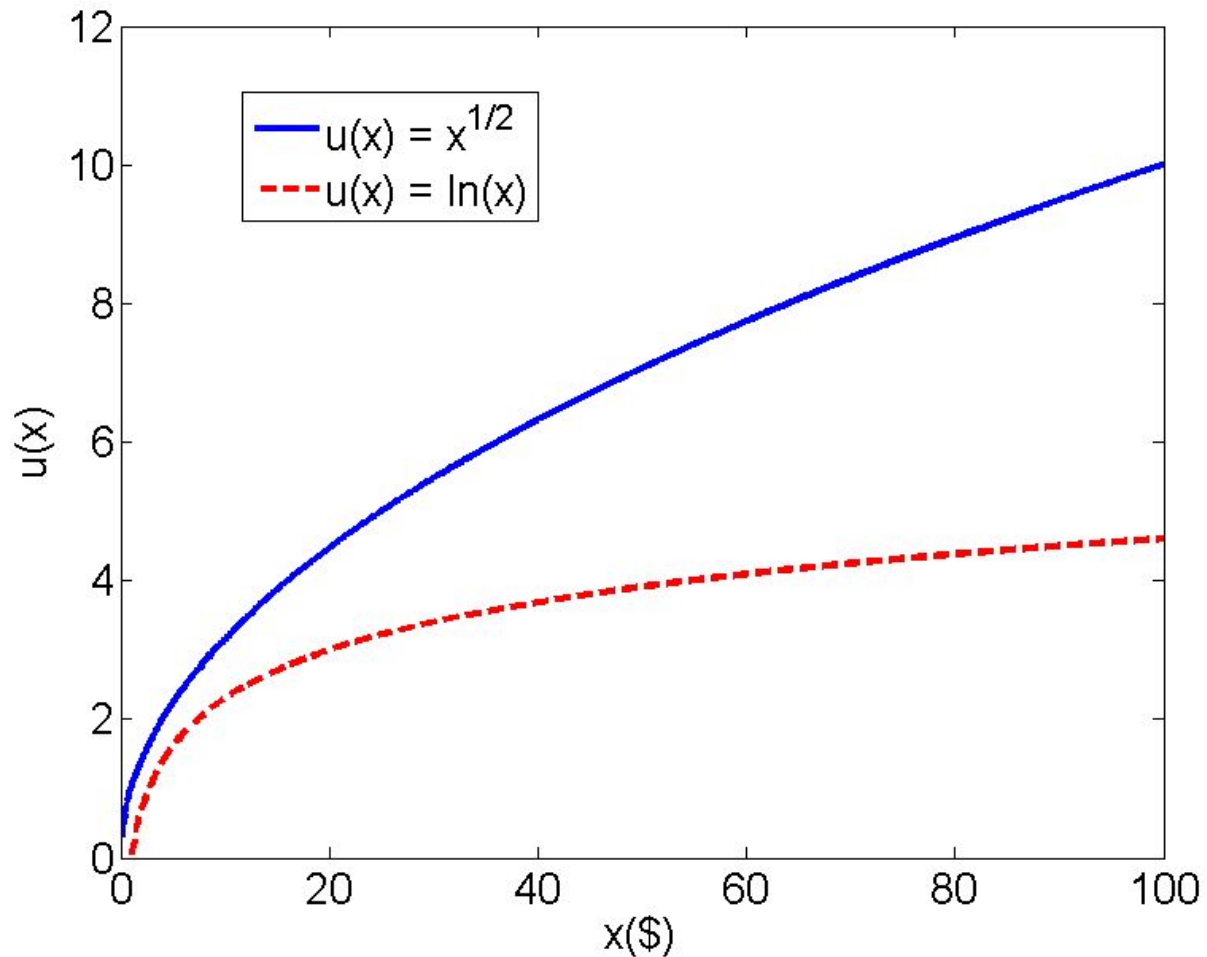


Risk Aversion

- Another definition of the relation “more risk averse” exists when vNM utility functions are twice differentiable:
 - Let u_1 and u_2 be twice differentiable vNM utility functions representing \succeq_1 and \succeq_2 , respectively
 - The preference relation \succeq_1 is more risk averse than \succeq_2 if $r_1(x) \geq r_2(x)$ for all x , where
 - $r_i(x) = -u''_i(x)/u'_i(x)$
- The number $r(x)$ is called the coefficient of absolute risk aversion of u at x
- A higher coefficient of absolute risk aversion means a more risk-averse decision maker

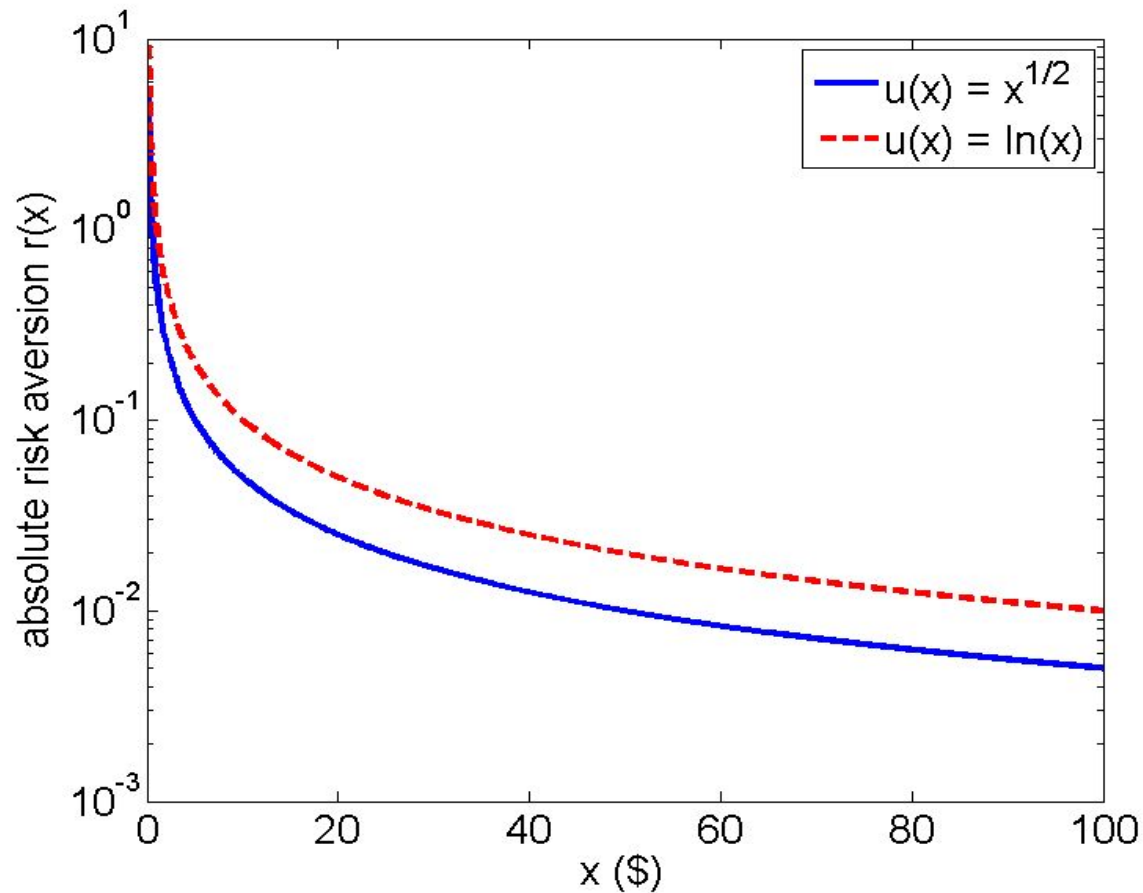
Risk Aversion

- Which individual is more risk averse?



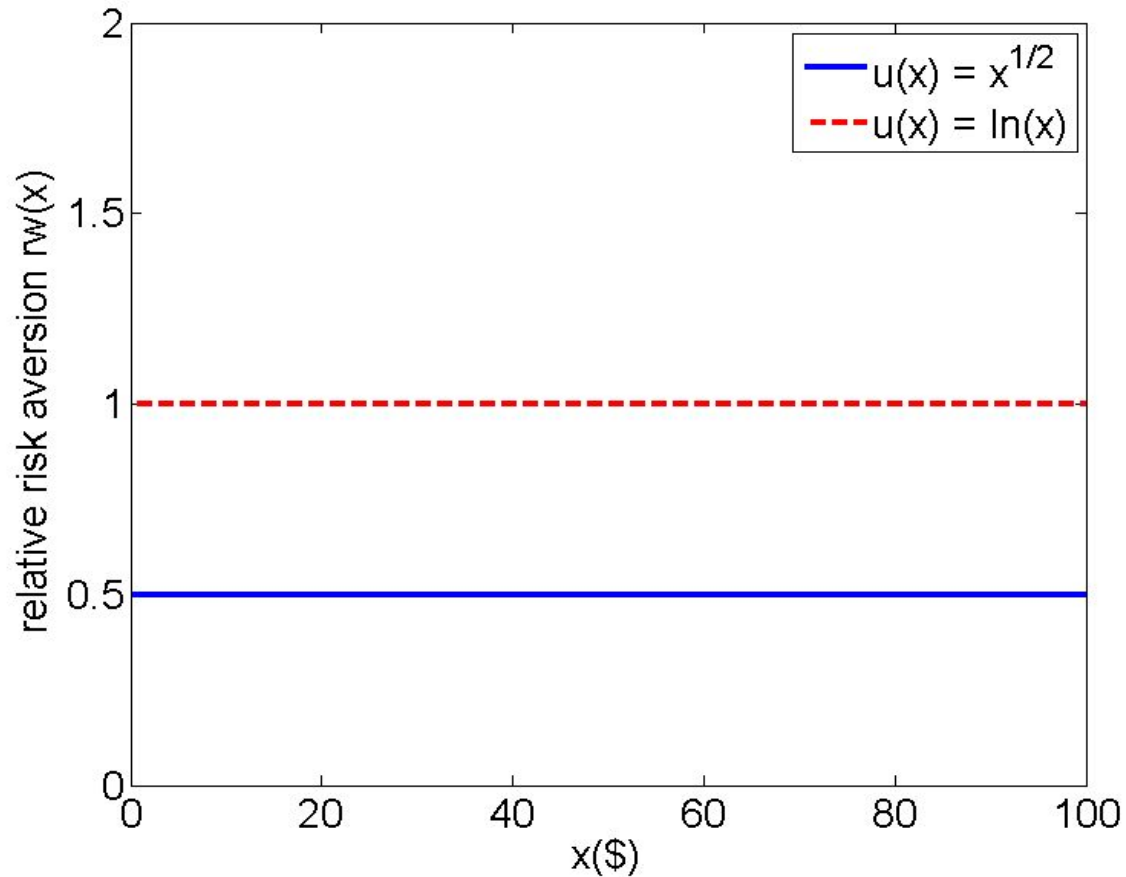
Risk Aversion

- Absolute risk aversion coefficient r
 - $r(x) = -u''(x) / u'(x)$



Risk Aversion

- Relative risk aversion coefficient ***rw***
 - ***$rw(x) = x r(x)$***

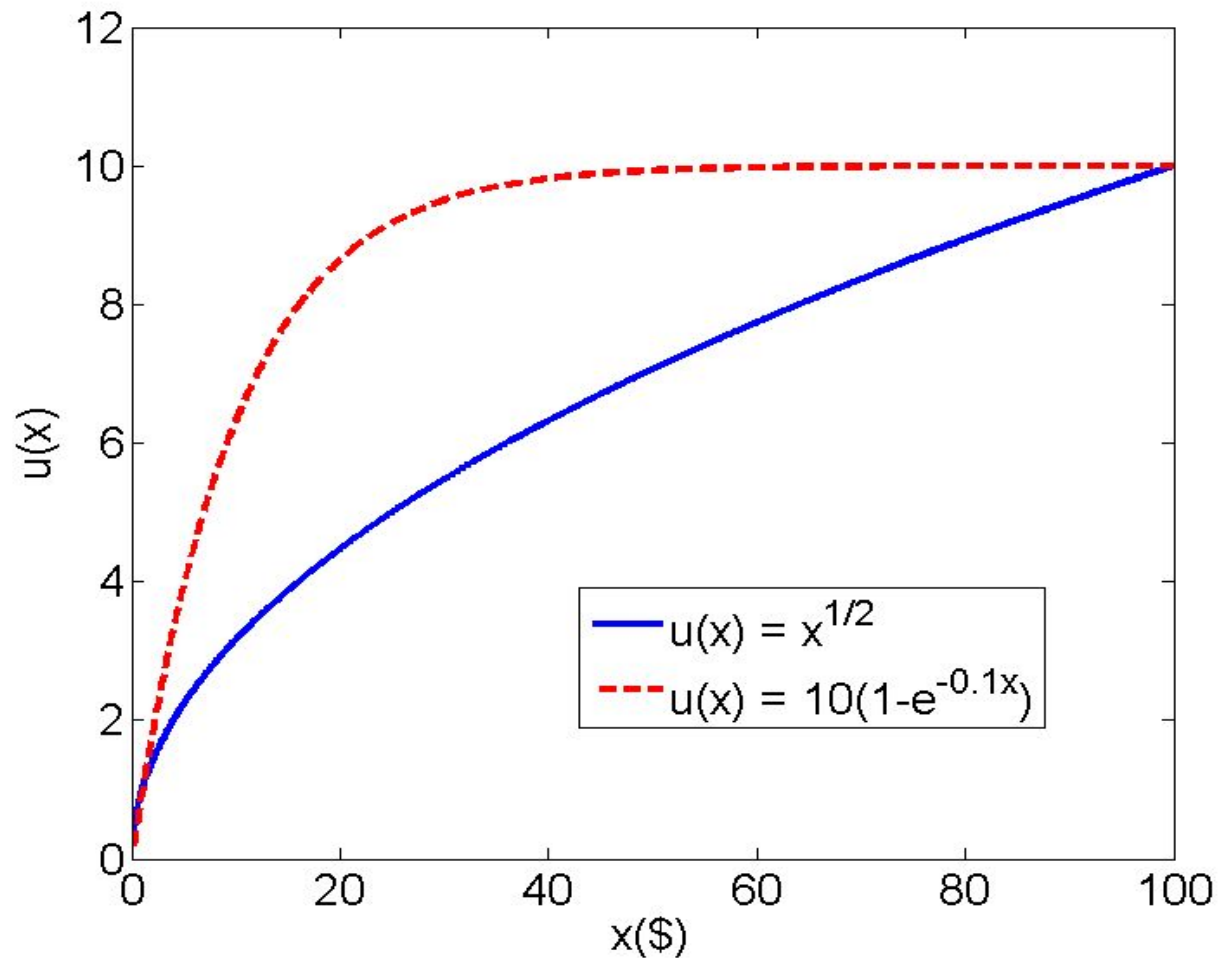


Risk Aversion

- Absolute risk aversion coefficient r
 - If $r(x)$ decreases with x , then individuals will invest larger money amounts in risky assets as they get wealthier
- Relative risk aversion coefficient rw
 - If $rw(x)$ is constant with x , individuals will invest the same percentage of their wealth in risky assets as they get wealthier

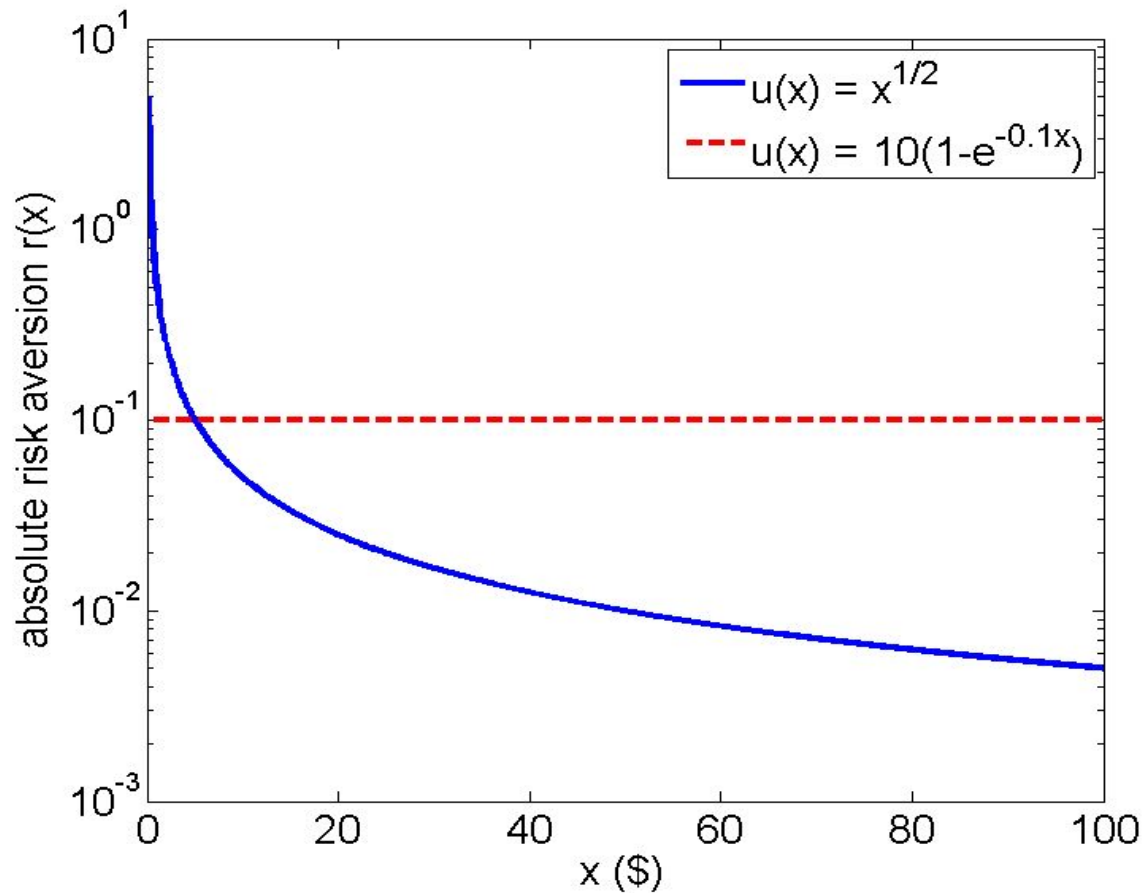
Risk Aversion

- Which individual is more risk averse?



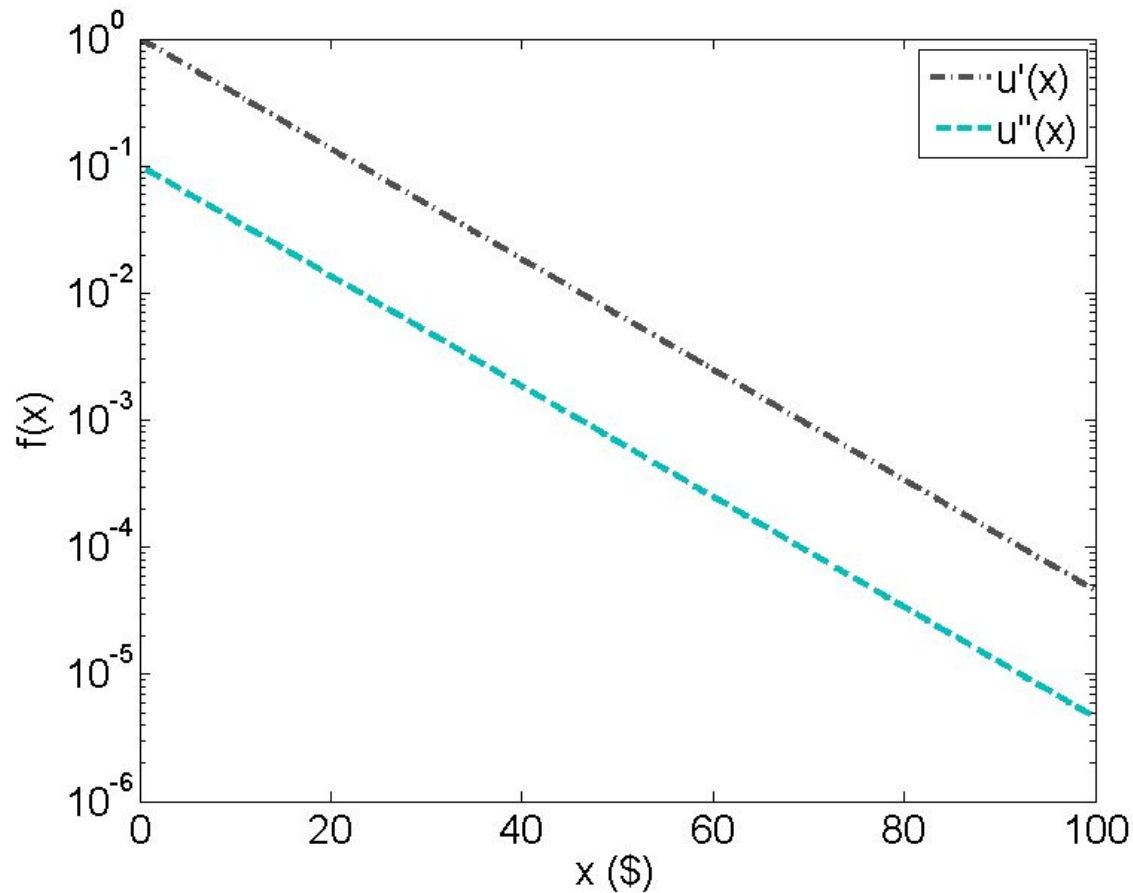
Risk Aversion

- Absolute risk aversion coefficient r
 - $r(x) = -u''(x) / u'(x)$



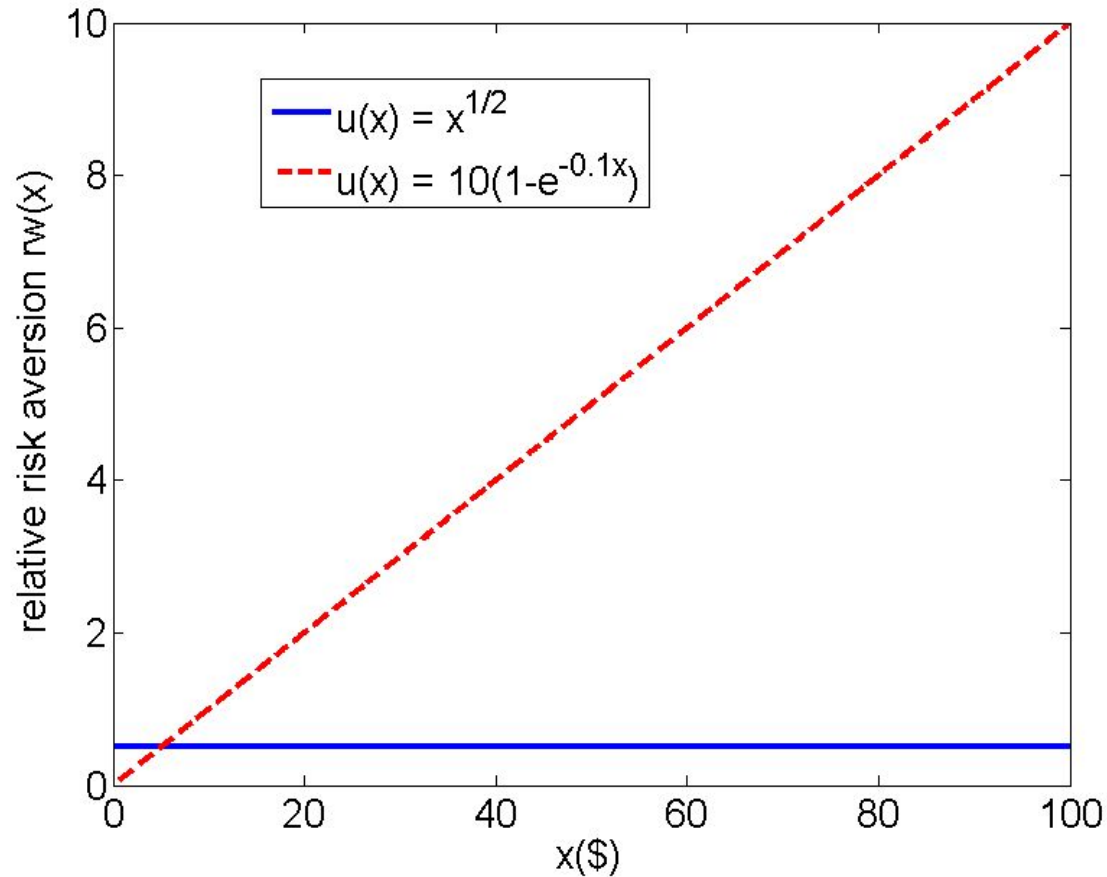
Risk Aversion

- Absolute risk aversion coefficient **r**
 - **$u(x) = 10(1 - e^{-0.1x})$**



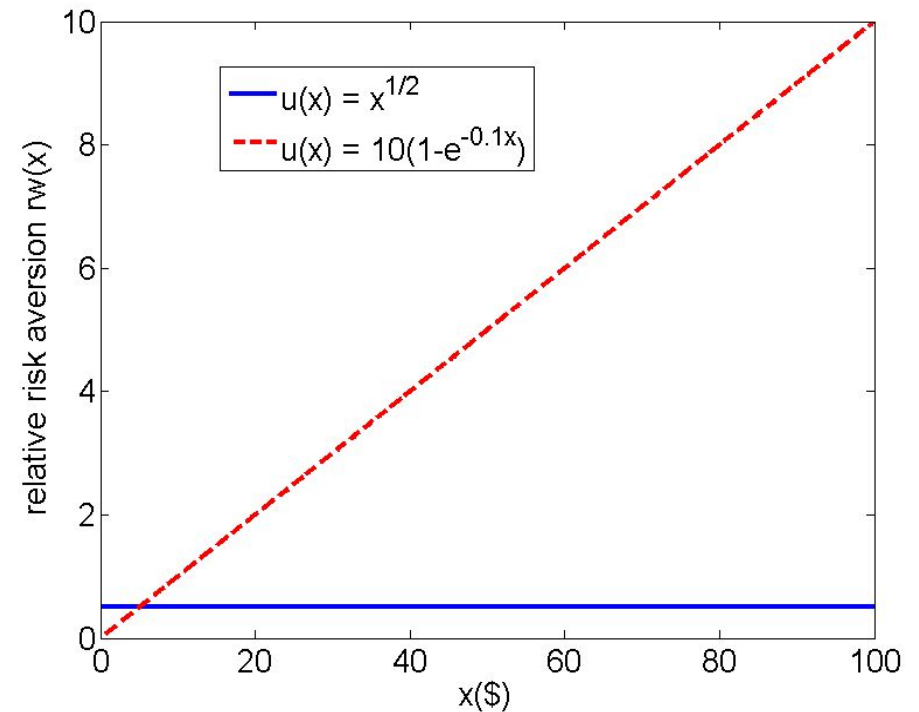
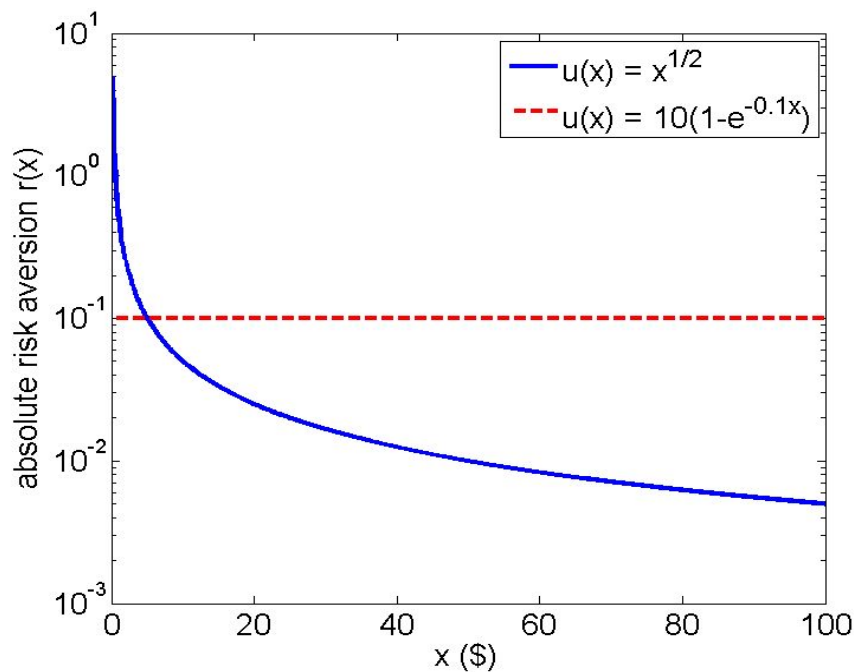
Risk Aversion

- Relative risk aversion coefficient ***rw***
 - ***$rw(x) = x r(x)$***



Risk Aversion

- What happens with an agent represented by the red curve?



Risk Aversion

- Absolute risk aversion coefficient r
 - With constant $r(x)$, the amount of wealth that we expose to risk remains constant as wealth increases
 - Invariance to wealth
- Relative risk aversion coefficient rw
 - If $rw(x)$ is **increasing** with x , individuals will invest **less percentage** of their wealth in risky assets as they get wealthier

Invariance to Wealth

- Claim:
 - Assume that u is a vNM utility function representing preferences \succeq , which are monotonic and exhibit invariance to wealth
 - Then u must be exponential or linear
- Proof in the book

First-Order Stochastic Domination

- Is there other ways to compare lotteries besides using the expected utility Eu ?
- Now we see when a lottery p first-order stochastically dominates a lottery q
 - Or $pD_1 q$

First-Order Stochastic Domination

- Which lottery do you prefer?

	\$0	\$20	\$50	\$100	\$200
<i>p</i>	0.1	0.1	0.2	0.3	0.3
<i>q</i>	0.15	0.05	0.25	0.35	0.20

First-Order Stochastic Domination

- Which lottery do you prefer?
 - $G(p, x) = \sum_{z \geq x} p(z)$

	\$0	\$20	\$50	\$100	\$200
$G(p, x)$	1 (0.1)	0.9 (0.1)	0.8 (0.2)	0.6 (0.3)	0.3 (0.3)
$G(q, x)$	1 (0.15)	0.85 (0.05)	0.8 (0.25)	0.55 (0.35)	0.2 (0.20)

First-Order Stochastic Domination

- Which lottery do you prefer?
 - $F(p, x) = \sum_{z \leq x} p(z)$

	\$0	\$20	\$50	\$100	\$200
$F(p, x)$	0.1 (0.1)	0.2 (0.1)	0.4 (0.2)	0.7 (0.3)	1 (0.3)
$F(q, x)$	0.15 (0.15)	0.2 (0.05)	0.45 (0.25)	0.80 (0.35)	1 (0.20)

First-Order Stochastic Domination

- Claim:
 - pD_1q iff for all x , $G(p, x) \geq G(q, x)$ or
 - pD_1q iff for all x , $F(p, x) \leq F(q, x)$

