Lista 1

Teoria dos Jogos em Computação

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Utility

1. Give an example of preferences over a countable set in which the preferences cannot be represented by a utility function that returns only integers as values.

2. A farmer wants to dig a well in a square field. The preferences of the farmer on the possible locations are lexicographic, i.e:

- If $x_1 < x_2$ then $(x_1, y_1) \succ (x_2, y_2)$ for all y_1, y_2.

- If $x_1 = x_2 = x$, then $(x, y_1) \succ (x, y_2)$ iff $y_1 < y_2$.

First, asssume that the field has dimensions $[0, 1000] \times [0, 1000]$ and construct a linear utility function that represents this relation. Second, construct a utility function assuming that the field has dimensions $[0, \infty] \times [0, \infty]$. For both cases, assume that the well location must have **integer** coordinates.

3. Is the statement "if both U and V represent $\succsim$, then there is a strictly monotonic function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x) = f(U(x))$" correct?

Tip: consider $V(x) = x$ and $U(x) = \begin{cases} x & if \quad x \leq 0 \\ x+1 & if \quad x > 0 \end{cases}$

4. Can a continuous preference relation be represented by a discontinuous utility function?

5. Show that in the case of $X = \mathbb{R}$, the preference relation that is represented by the discontinuous utility function $u(x) = floor(x)$ is not a continuous relation.
$floor(x)$: the largest integer n such that $x \geq n$

Choice

6. The following are descriptions of decision–making procedures. Discuss whether the procedures can be described in the framework of the choice model discussed in this course and whether they are compatible with the "rational man" paradigm. In other words, can I construct a utility function $u(x)$ based solely on the set of alternatives $x \in X$ according with these procedures? Explain why (e.g. with an example).

a. The decision maker chooses an alternative in order to maximize another person's suffering.

b. The decision maker asks his two children to rank the alternatives and then chooses the alternative that is the best on average (you can use your own definition of "best on average").

c. The decision maker has an ideal point in mind and chooses the alternative that is closest to it.

d. The decision maker looks for the alternative that appears most often in a list of alternatives.

e. The decision maker has an ordering in mind and always chooses the median element.

7. Consider the following choice procedure: a decision maker has a strict ordering $\succsim$ over the set X and assigns to each $x \in X$ a natural number $class(x)$ to be interpreted as the "class" of x. Given a choice problem A, he chooses the best element in A from those belonging to the most common class in A (i.e., the class that appears in A most often). If there is more than one most common class, he picks the best element from the members of A that belong to a most common class with the highest class number.

a. Is this procedure consistent with the "rational man" paradigm?

b. Define the relation: xPy if x is chosen from $\{x, y\}$. Show that the relation P is a strict ordering (complete, asymmetric, and transitive).

Expected Utility

8. Which lottery do you prefer?

- $L = (0.25z_1, 0.25z_2, 0.25z_3, 0.25z_4)$ OR

- $L' = (0.15z_1, 0.50z_2, 0.15z_3, 0.20z_4)$

Suppose, by continuity: $z_2 \sim z_2' = (0.6z_1, 0.4z_4)$ and $z_3 \sim z_3' = (0.2z_1, 0.8z_4)$

9. T or **F**. Justify or give a counterexample.

a. A lottery p is preferred to q because the expected utility $U(p)$ is greater than the expected utility $U(q)$.

b. Suppose that $A \succ B \succ C \succ D$ and that the vNM utilities of these prizes satisfy $v(A) + v(D) = v(B) + v(C)$, then $(\frac{1}{2}B, \frac{1}{2}C)$ should be preferred to $(\frac{1}{2}A, \frac{1}{2}D)$ because, although they have the same expected utility, the former has the smaller utility variance.

c. Suppose that $A \succ B \succ C \succ D$ and that the vNM utility function has the property that $v(A) - v(B) \succ v(C) - v(D)$, then the change from B to A is more preferred than the change from D to C.

10. Verify whether each of the following preference relations over lotteries satisfy (or not) von Neumann and Morgenstern axioms (I and C). Consult the book "Lecture Notes in Microeconomic Theory by Ariel Rubinstein", Pages 95 and 96, for more details.

a. The worst case (the decision maker evaluates lotteries by the worst possible case).

b. Increasing the probability of a "good" consequence.

Risk Aversion

11. Adam lives in the Garden of Eden and eats only apples. Time in the garden is discrete ($t = 1, 2, ...$) and apples are eaten only in discrete units. Adam possesses preferences over the set of streams of apple consumption. Assume that:

a. Adam likes to eat up to 2 apples a day and cannot bear to eat 3 apples a day.

b. Adam is **impatient**. He would be delighted to increase his consumption on day t from 0 to 1 or from 1 to 2 apples at the expense of an apple he is promised a day later $t + 1$.

c. In any day in which he does not have an apple, he prefers to get 1 apple immediately in exchange for 2 apples tomorrow.

d. Adam expects to live for 120 years.

Show that if (poor) Adam is offered a stream of 2 apples starting in day 4 for the rest of his expected life, he would be willing to exchange that offer for 1 apple right away.
Tips:

- (b) means that <u>one single apple</u> is promised to Adam on day $t + 1$

- initial stream offered to Adam can be represented by (0,0,0,2,2,...,2,2)

- evolve it due to Adam's preferences

12. Given the pairs of lotteries in tables 1 and 2, in each case, which one do you prefer? Explain considering the *First-Order Stochastic Domination* concept.

Table 1: (a) or (b)?

	chance %	90	6	1	3
(a)	prize $	0	45	30	-15
(b)	chance %	90	7	1	2
	prize $	0	45	-10	-15

Table 2: (c) or (d)?

	chance %	40	35	15	10
(c)	prize $	0	10	50	200
(d)	chance %	40	35	15	10
	prize $	0	25	40	180

13. A gambling house charges $15 for the lottery below.

lottery	$0	$36	$64
p	0.50	0.30	0.20

Will a person, whose whose utility function over money is $u(x) = \frac{5}{4}\sqrt{x}$, pay to play p? Justify your answer.