

Risk Aversion

Risk Aversion

- Who is this guy?



Risk Aversion

- Lets watch a portion of a video...
 - 2:45 (specially)



Risk Aversion

- Imagine a situation where River Plate plays against Boca Juniors and falls to 2nd division iff it loses



vs.



$u(W,D) =$

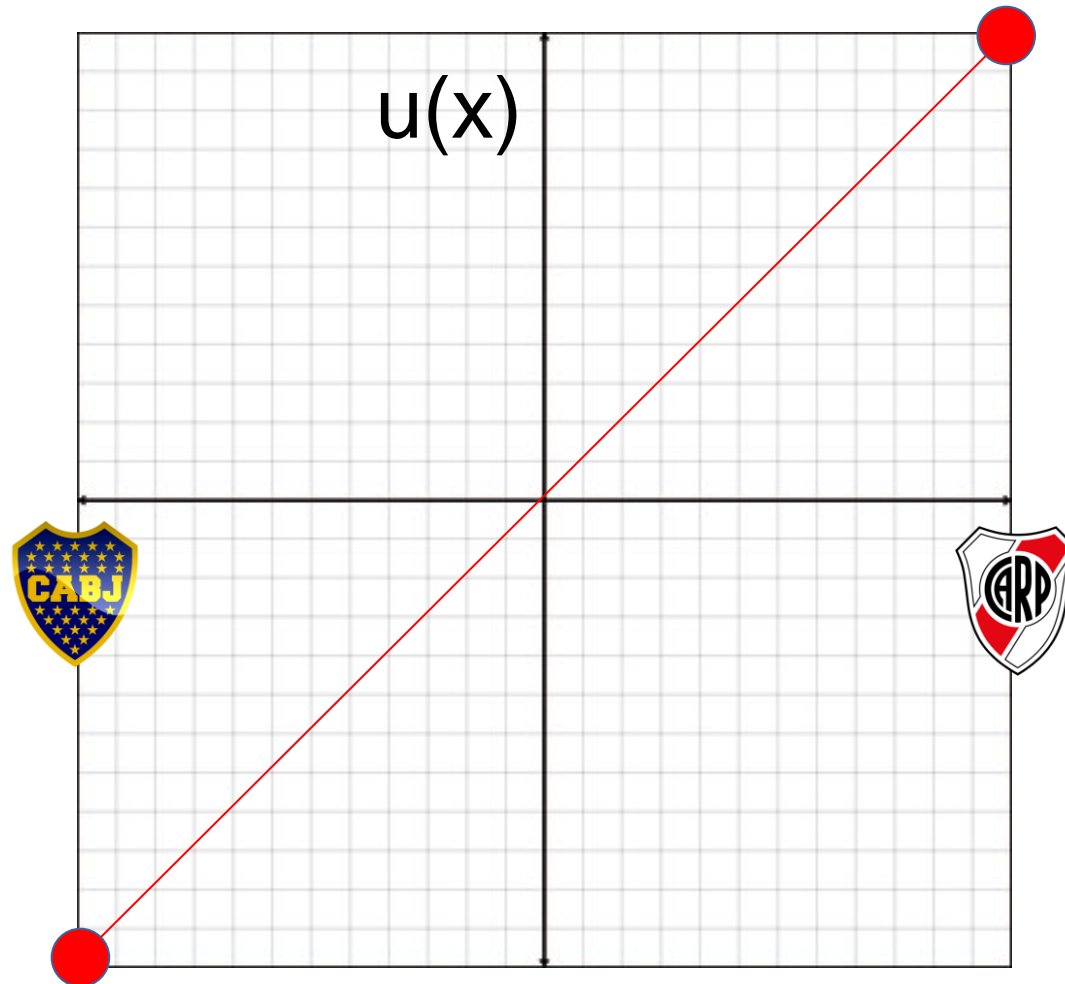


$u(L) =$



Risk Aversion

- Utilities for this lottery



Risk Aversion

- Imagine a situation where River Plate plays against Boca Juniors and falls to 2nd division iff it loses



vs.



$$u(W,D) = \$105$$

$$u(L) = -\$105$$

Risk Aversion

- What should I do if I am extremely risk averse?



vs.



$$u(W,D) = \$105$$

$$u(L) = -\$105$$

Risk Aversion

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Topereignisse
Tennis - Heutige Spiele
1. Bundesliga
2. Bundesliga
Champions League
Europa League
DFB-Pokal
Sportwetten
Livecenter (Flash 8)
Livecenter Kalender
Live Wetten (kein Flash)
Fussball
Tennis
Basketball
Volleyball
Eishockey
Am. Football
Boxen
Dart
Futsal
Golf
Handball
Hunderennen
Krieket
Motorsport
Olympia
Pferderennen
Poker
Quota

Sport Startseite Fussball 1. Bundesliga - Spiele
Fussball Weitere Wetten für Fussball finden Sie hier Dezimal (3.50)
Siegwette | Half-Time Result | Handicap | Remis Geld zurück
1. Bundesliga - Spiele
Siegwette

	Heim	Unentschieden	Auswärts	
VfL Wolfsburg v Freiburg 10/02/2012 20:30 CET	1.60	3.60	5.00	Alle Wetten (57)
Stuttgart v Hertha BSC Berlin 11/02/2012 15:30 CET	1.80	3.30	4.20	Alle Wetten (57)
Mainz 05 v Hannover 96 11/02/2012 15:30 CET	1.95	3.20	3.60	Alle Wetten (57)
Werder Bremen v TSG Hoffenheim 11/02/2012 15:30 CET	1.85	3.20	4.00	Alle Wetten (57)
Bayern München v Kaiserslautern 11/02/2012 15:30 CET	1.133	6.50	17.00	Alle Wetten (56)
Dortmund v Bayer Leverkusen 11/02/2012 15:30 CET	1.40	4.30	6.50	Alle Wetten (57)
Borussia Mönchengladbach v Schalke 04 11/02/2012 18:30 CET	2.30	3.20	2.85	Alle Wetten (57)
Augsburg v Nürnberg 12/02/2012 15:30 CET	2.55	3.10	2.60	Alle Wetten (57)
1. FC Köln v Hamburger SV 12/02/2012 17:30 CET	2.80	3.30	2.25	Alle Wetten (58)

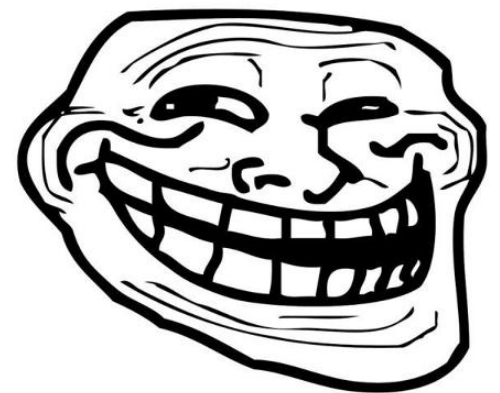
Einzel und Mehrfachwetten möglich

Nur für Neukunden! 100€bonus
Ihr Wettschein hat 3 Auswahl(en)

alle Wetten auswählen	Wettschein leeren	Einsatz
<input checked="" type="checkbox"/> Stuttgart v Hertha BSC Berlin Siegwette Hertha BSC Berlin Möglicher Gesamtgewinn 420.00 EUR	<input checked="" type="checkbox"/>	1 Wette 100.00
<input checked="" type="checkbox"/> Werder Bremen v TSG Hoffenheim Siegwette Unentschieden Möglicher Gesamtgewinn 160.00 EUR	<input checked="" type="checkbox"/>	1 Wette 50.00
<input checked="" type="checkbox"/> Bayern München v Kaiserslautern Siegwette Bayern München Möglicher Gesamtgewinn 113.33 EUR	<input checked="" type="checkbox"/>	1 Wette 100
Kombi		
Kombinationswette		1 Wette Möglicher Gesamtgewinn 0.00 EUR
Patent		7 Wetten Möglicher Gesamtgewinn 0.00 EUR
Trixie		4 Wetten Möglicher Gesamtgewinn 0.00 EUR
2 aus		3 Wetten Möglicher Gesamtgewinn 0.00 EUR
Gesamteinsatz:		250.00 EUR

Risk Aversion

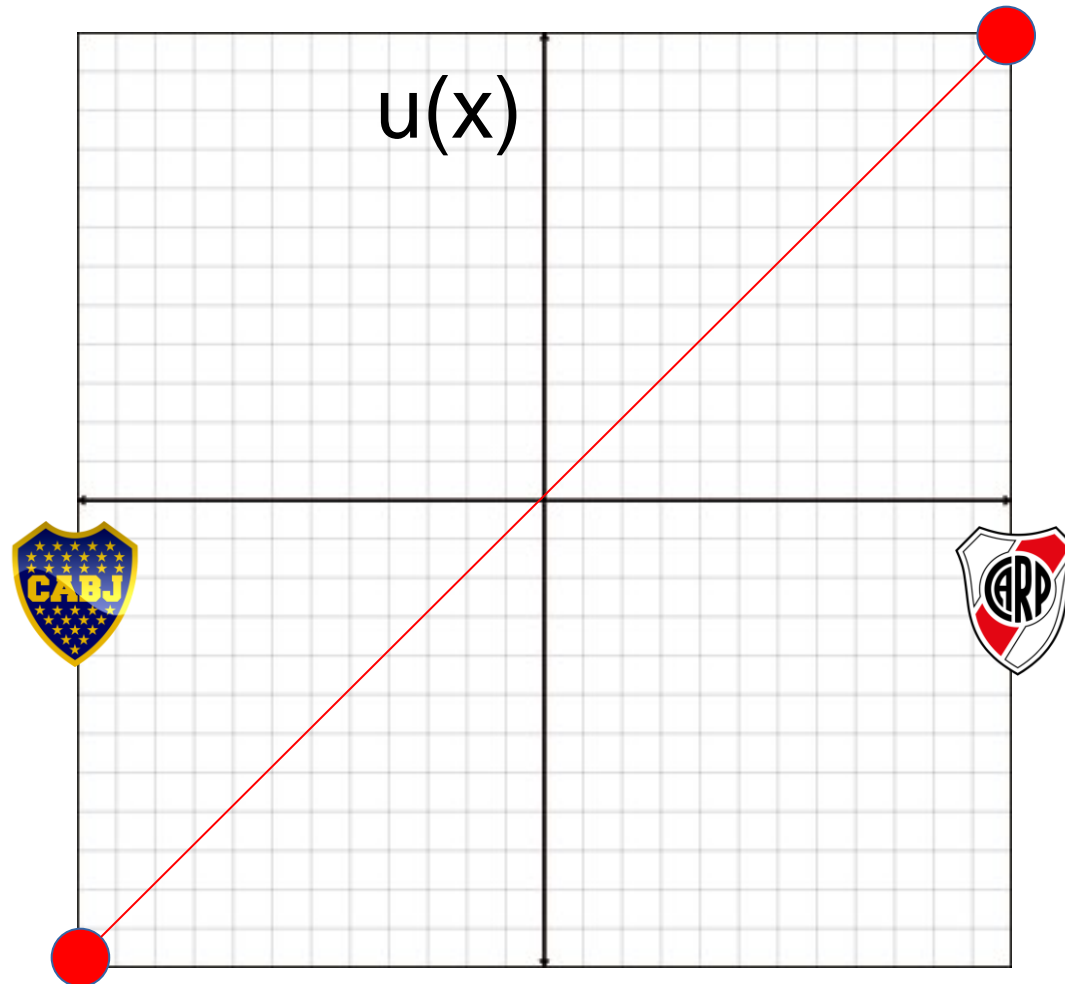
- Suppose a site is paying $r = 3$ dollars for each dollar I put on Boca Juniors
- Then...
 - let x be the amount of dollars I put on Boca Juniors
 - then, if $u(W, D) - x = u(L) + (r-1)x$, I have nothing to worry about!
 - $105 - x = -105 + 3x - x$
 - $3x = 210$
 - $x = \$70$



problem?

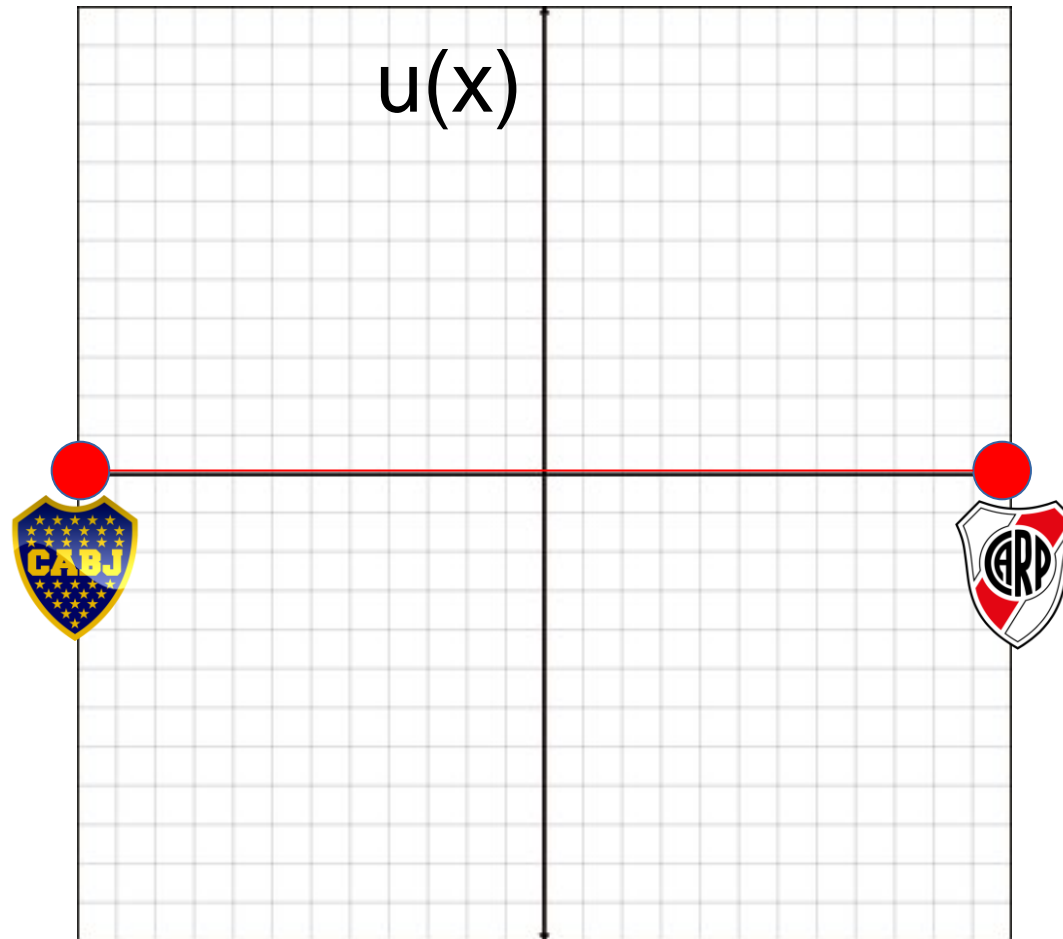
Risk Aversion

- Utilities for this lottery



Risk Aversion

- Utilities for this lottery



Risk Aversion

- ... as if I am paying \$70 for not playing this lottery

Lotteries with monetary prize

a



Smart TV Gamer LED 3D 40" Samsung
UN40J6400 - Full HD 4 HDMI 3 USB 2
Óculos
de R\$ 2.549,00
por R\$ 1.999,00
em até 10x de R\$ 199,90 sem juros
ou **R\$ 1.799,10 à vista**
disponível sob consulta

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b



Smart TV Gamer LED 4k Ultra HD 75"
Samsung - UN75JU6500 4 HDMI 3 USB
Wi-Fi
R\$ 15.999,00
em até 12x de R\$ 1.333,25 sem juros
ou **R\$ 14.399,10 à vista**

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c



Smart TV OLED Curva 3D 55" LG
55EA9850 Full HD - Conversor Integrado 4
HDMI 3 USB Wi-Fi 4 Óculos
de R\$ 11.990,00
por R\$ 7.999,00
em até 12x de R\$ 666,58 sem juros
ou **R\$ 7.199,10 à vista**

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d



Smart TV Gamer LED 75" Samsung
UN75J6300A - Full HD Conversor
Integrado 4 HDMI 3 USB Wi-Fi
R\$ 11.499,00
em até 12x de R\$ 958,25 sem juros
ou **R\$ 10.349,10 à vista**

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$$u(c) = \$7.999,00$$

Lotteries with monetary prize

a



b



$$u(a) = \$5,00$$

$$u(b) = \$1.299,00$$

Lotteries with monetary prize

- Preference relations \succsim over the space of lotteries for which there is a continuous function u , such that \succsim is represented by

$$Eu(p) = \sum_{z \in Z} p(z)u(z)$$

- The function Eu assigns to the lottery p the expectation of the random variable that receives the value $u(x)$ with a probability $p(x)$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$30	\$50	\$70
p	50%	0%	50%
q	0%	100%	0%

$$Eu(p) = \$50,00$$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$50	\$100
p	50%	0%	50%
q	0%	100%	0%

$$Eu(p) = \$50,00$$

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30	\$100
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30M	\$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Let's play a game...

Lotteries with monetary prize

- Which lottery do you prefer?

lottery	\$0	\$30M	\$100M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- Is there an x value that would make you choose p ?

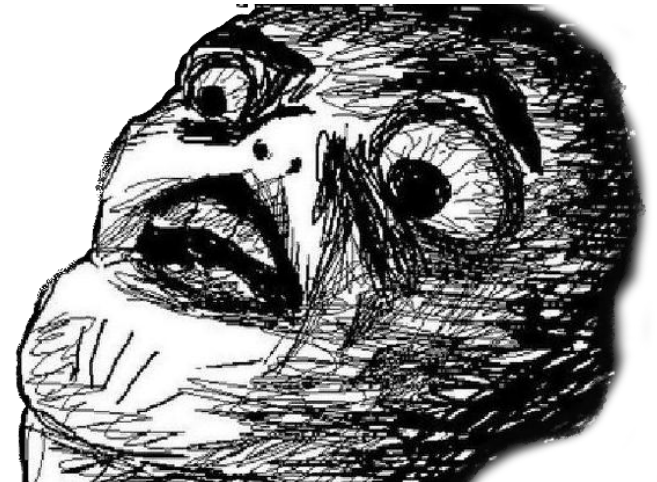
lottery	\$0	\$30M	[\$x]M
p	50%	0%	50%
q	0%	100%	0%

Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_1 that will make you choose p

lottery	\$0	\$30M	$[\mathbf{x}_1]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_1 that will make you choose p

lottery	\$0	\$30M	$[\mathbf{x}_1]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- YOU WON!

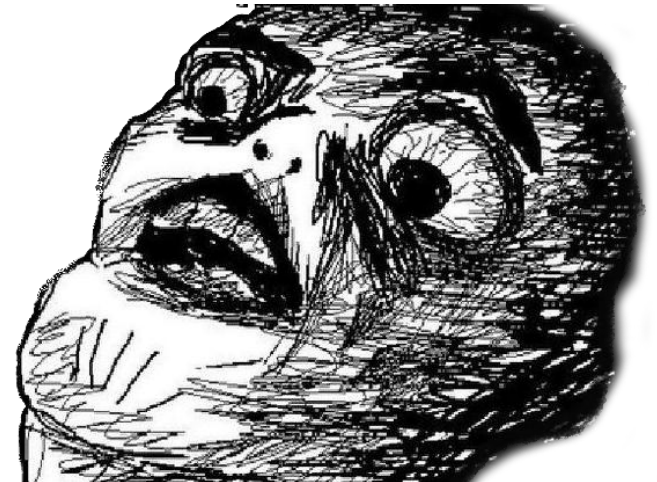


Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_2 that will make you choose p

lottery	\$0	\$ x_1 M	\$ x_2 M
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose now a x_2 that will make you choose p

lottery	\$0	\$[x_1]M	\$[x_2]M
p	50%	0%	50%
q	0%	100%	0%

- YOU WON!

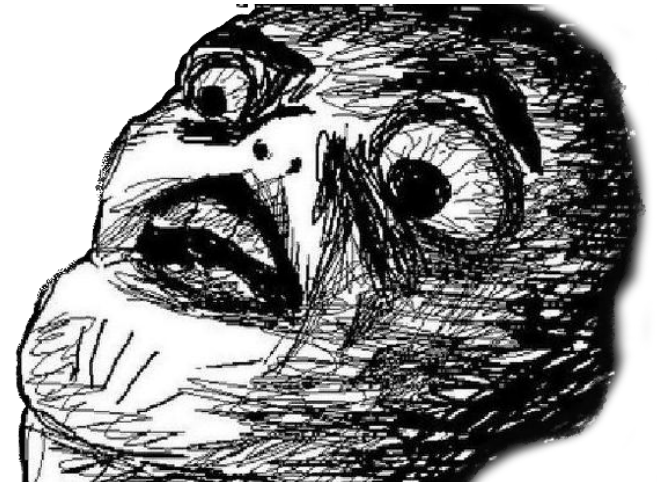


Lotteries with monetary prize

- If yes, let's imagine the following situation...
 - Choose x_3 that will make you choose p

lottery	\$0	$[\mathbf{x}_2]\text{M}$	$[\mathbf{x}_3]\text{M}$
p	50%	0%	50%
q	0%	100%	0%

- Now flip a coin



Lotteries with monetary prize

- If utility is not bounded, this game goes on forever
- More important:
 - What is the probability of our decision maker being left with no money?
 - 1!
- St. Petersburg paradox



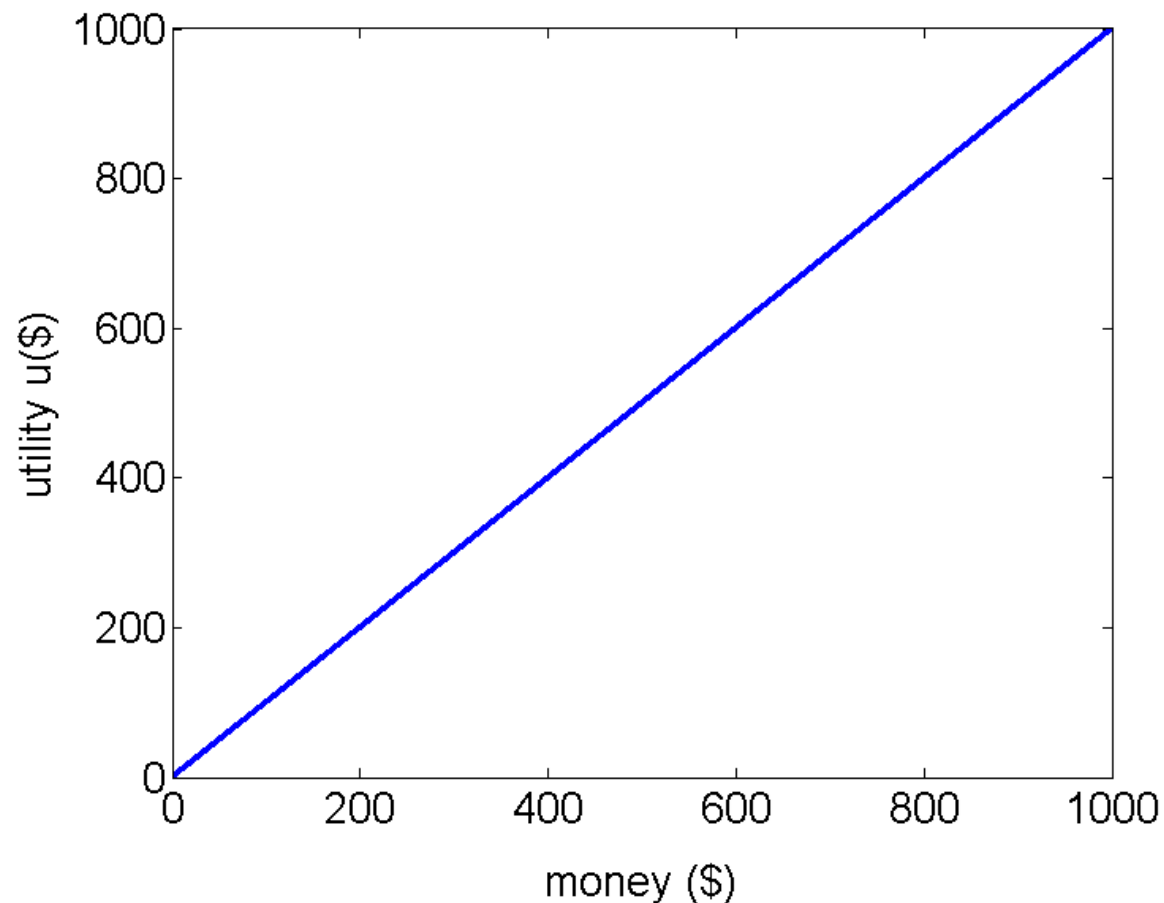
Lotteries with monetary prize

- St. Petersburg paradox, proposed by Nicolaus Bernoulli more than 300 years ago



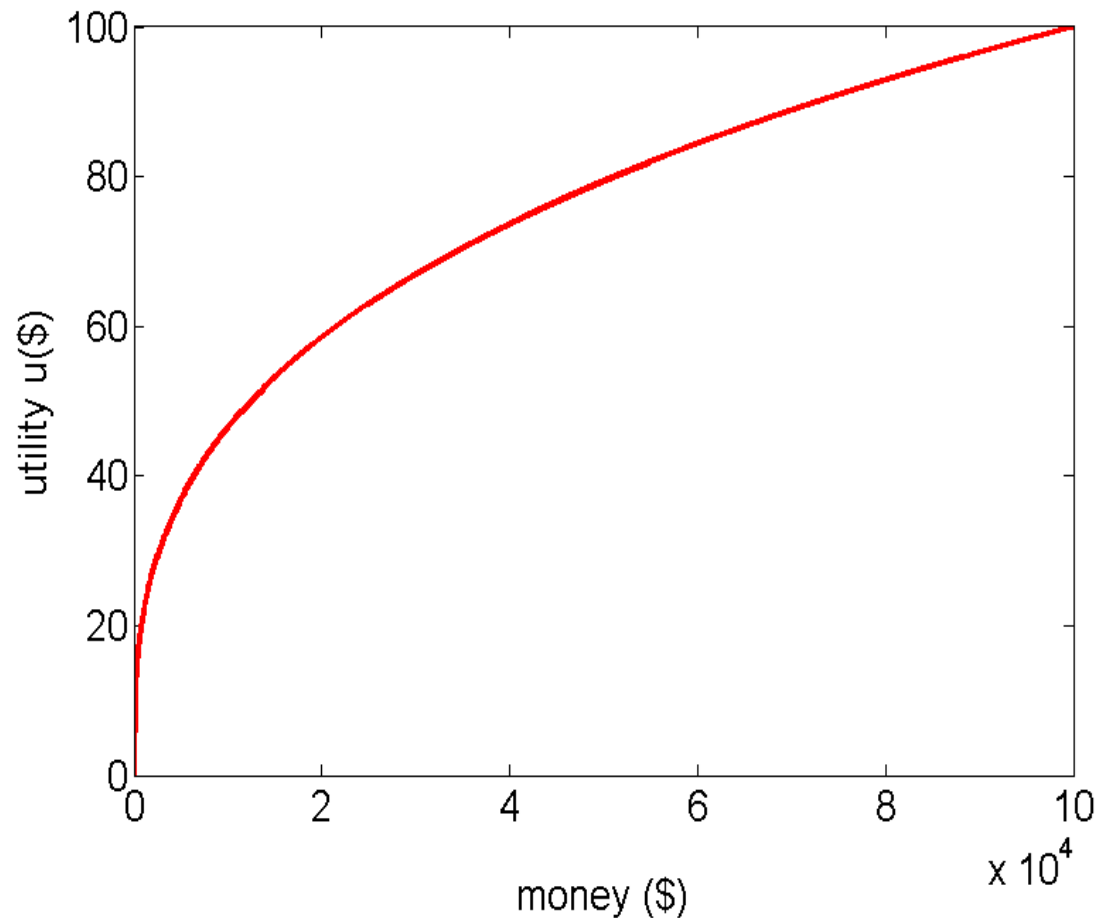
Lotteries with monetary prize

- Money vs. Utility over money



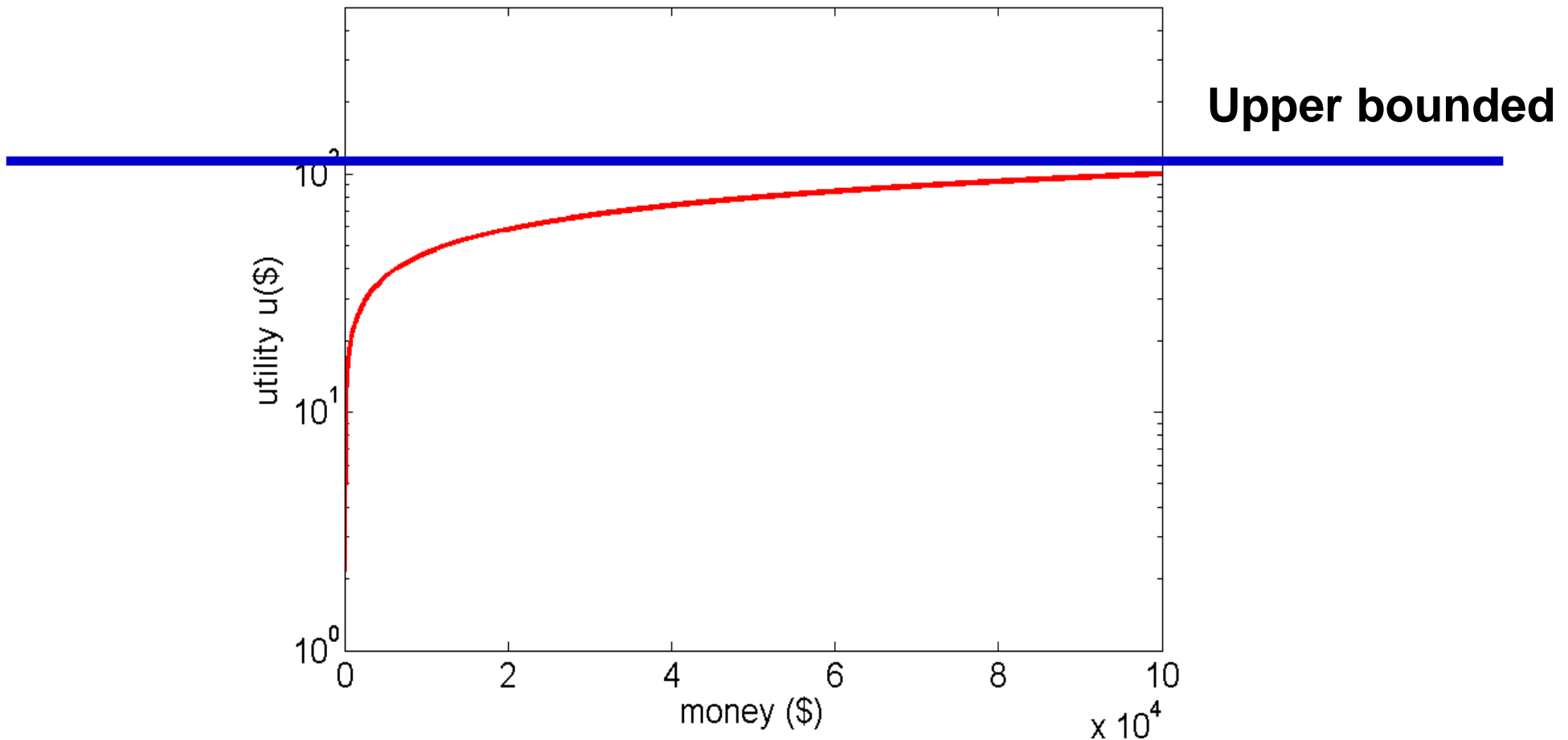
Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Money vs. Utility over money



Lotteries with monetary prize

- Daniel Bernoulli (cousin of Nicolaus) explains :
 - “The value of an item must not be based upon its price, but rather on the utility it yields.
 - The price of the item is dependent only on the thing itself and is equal for everyone.
 - The utility, however, is dependent on the particular circumstances of the person making the estimate.”



Lotteries with monetary prize

- Two insights about the St. Petersburg paradox
 - The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others
 - The utility from gaining an additional dollar would decrease with wealth
 - Utility increases as wealth increases and at a declining rate

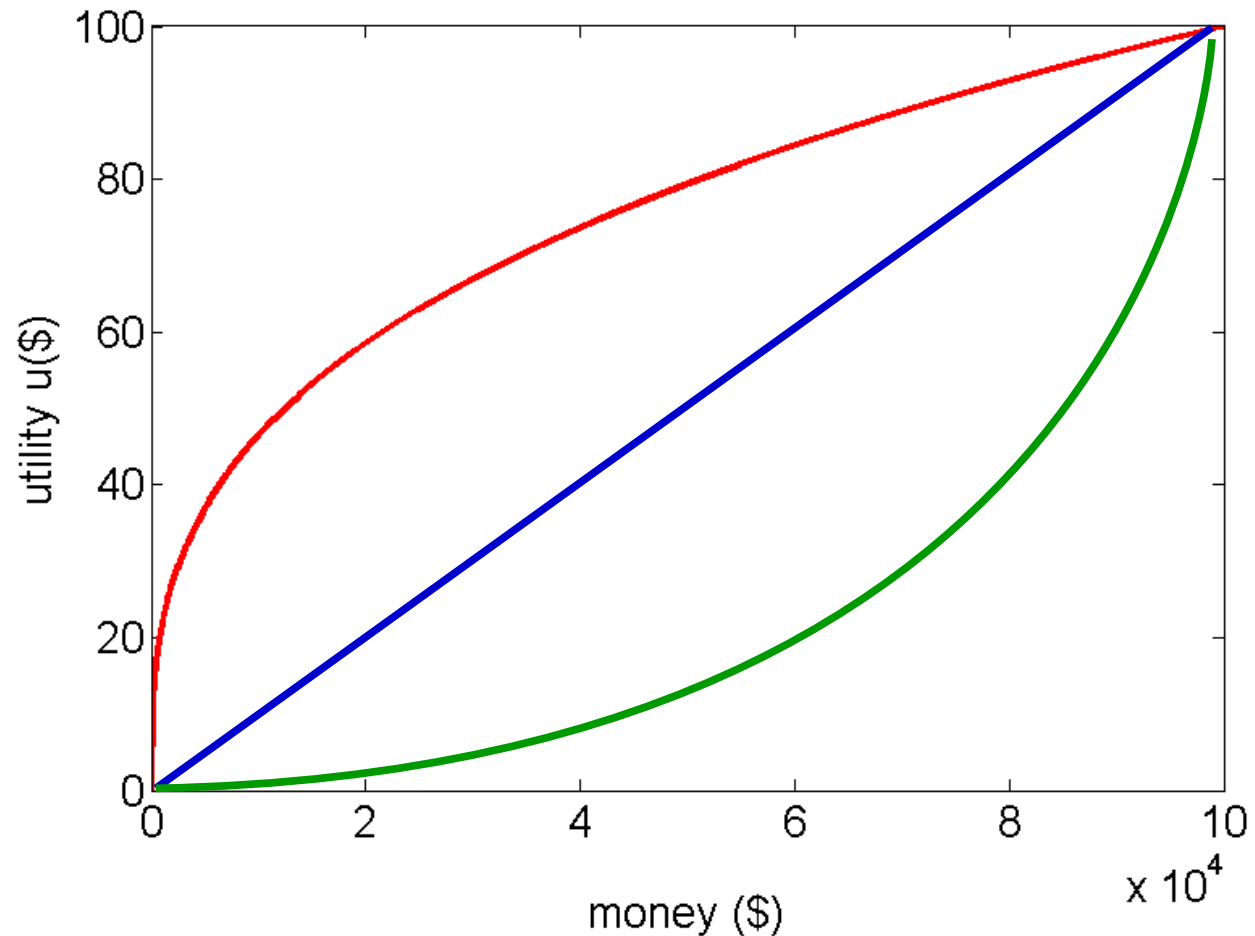
Lotteries with monetary prize

- The value attached to this gamble would vary across individuals, with some individuals willing to pay more than others



Risk Aversion

- How to measure?



Risk Aversion

- How to measure?
 - Every prize should be converted to money
 - Define a utility function $u(\$)$ over money
 - For a lottery p over prizes, calculate:
 - $E(p)$: the expected amount of money of lottery p
 - $Eu(p)$: the expected utility of lottery p
 - $u(E(p))$: the utility of the expected amount of money of p
 - $CE(p)$: the certainty equivalent
 - the amount of money I am willing to pay to play lottery p

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk neutral
 - $u(E(p)) = Eu(p)$ and $E(p) = CE(p) = \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk averse
 - $u(E(p)) > Eu(p)$ and $E(p) > CE(p) < \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- How much would you pay to play lottery p ?
 - Let $CE(p)$, the certainty equivalent, be the amount of money I am willing to pay to play lottery p
- How much A and B value to you?
 - $u(A)$ and $u(B)$ (or $u(\$20)$ and $u(\$80)$)
 - $Eu(p) = p(A)u(A) + p(B)u(B) = 0.25 u(\$20) + 0.75 u(\$80)$
 - $E(p) = p(A)A + p(B)B = 0.25 \$20 + 0.75 \$80 = \65
- If I am risk seeking
 - $u(E(p)) < Eu(p)$ and $E(p) < CE(p) > \$65$

lottery	A(\$20)	B(\$80)
p	25%	75%

Risk Aversion

- Claim:
 - Let \succsim be a preference on $L(\mathbf{Z})$ represented by the vNM utility function u ,
the preference relation \succsim is **risk averse** iff u is strictly concave
- Proof
 - In the book

Reminder: strictly concave functions

$$f((1-\alpha)x + \alpha y) > (1-\alpha)f(x) + \alpha f(y)$$

consider a lottery with two prizes x and y

and $p(y) = \alpha$ and $p(x) = 1-\alpha$

$$E(p) = (1-\alpha)x + \alpha y$$

if the function f is strictly concave

$$f(E(p)) > (1-\alpha)f(x) + \alpha f(y)$$

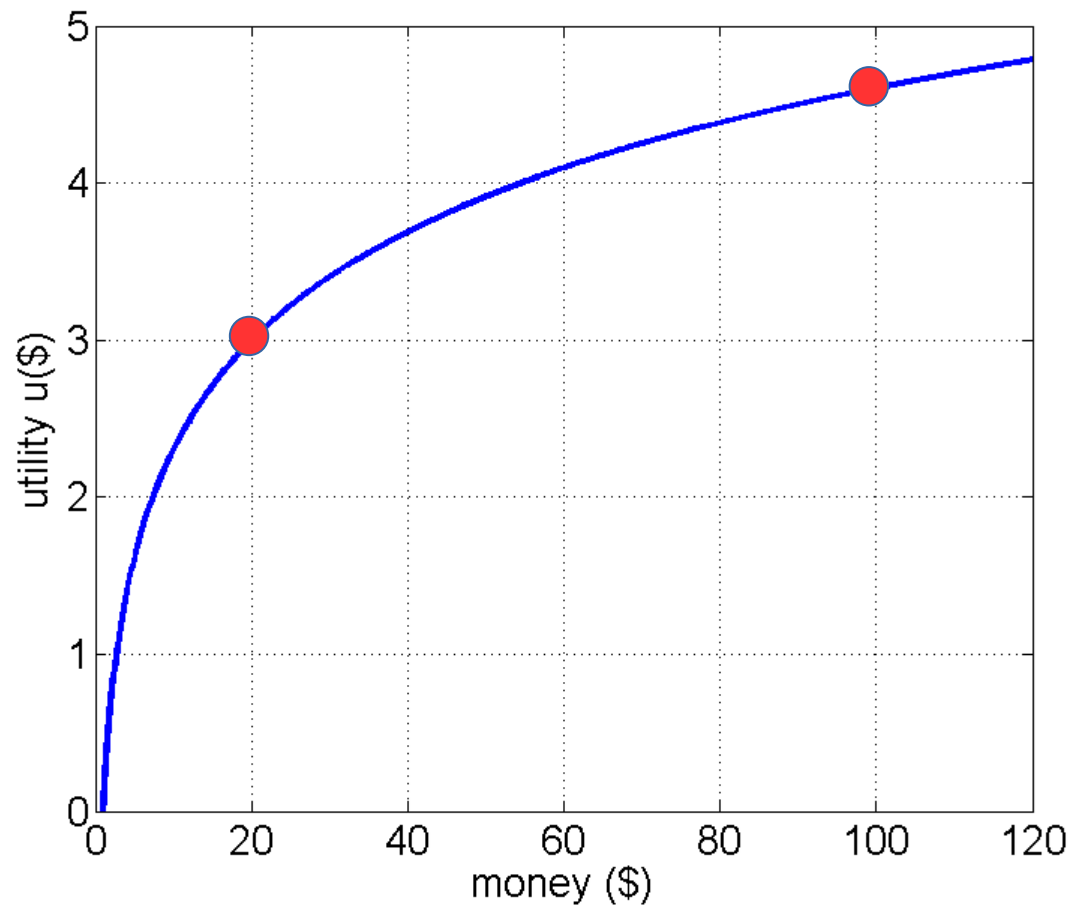
if f is the utility function

$$u(E(p)) > (1-\alpha)u(x) + \alpha u(y)$$

$$u(E(p)) > Eu(p) \text{ (definition of risk aversion)}$$

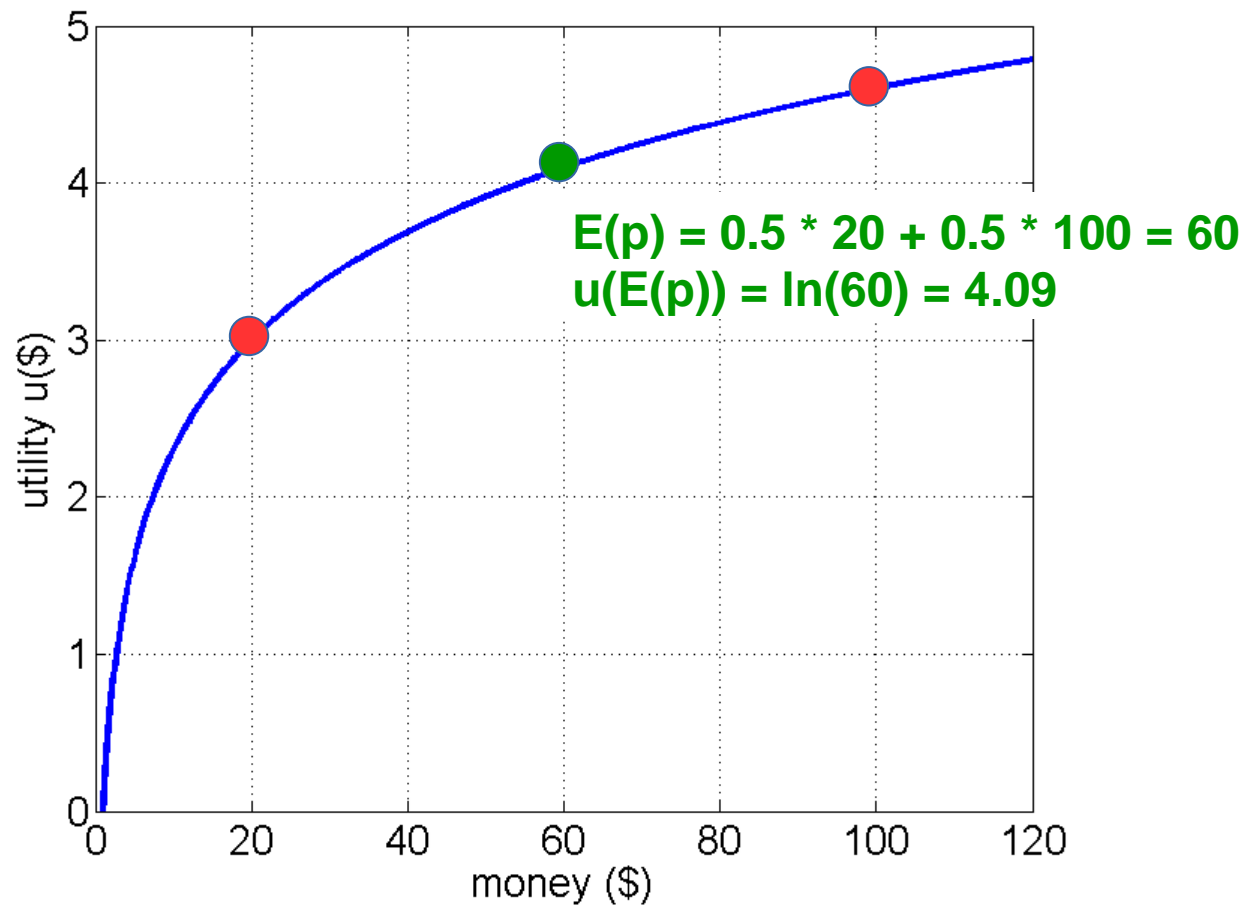
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \text{ \$20 } \oplus (0.5) \text{ \$100}$



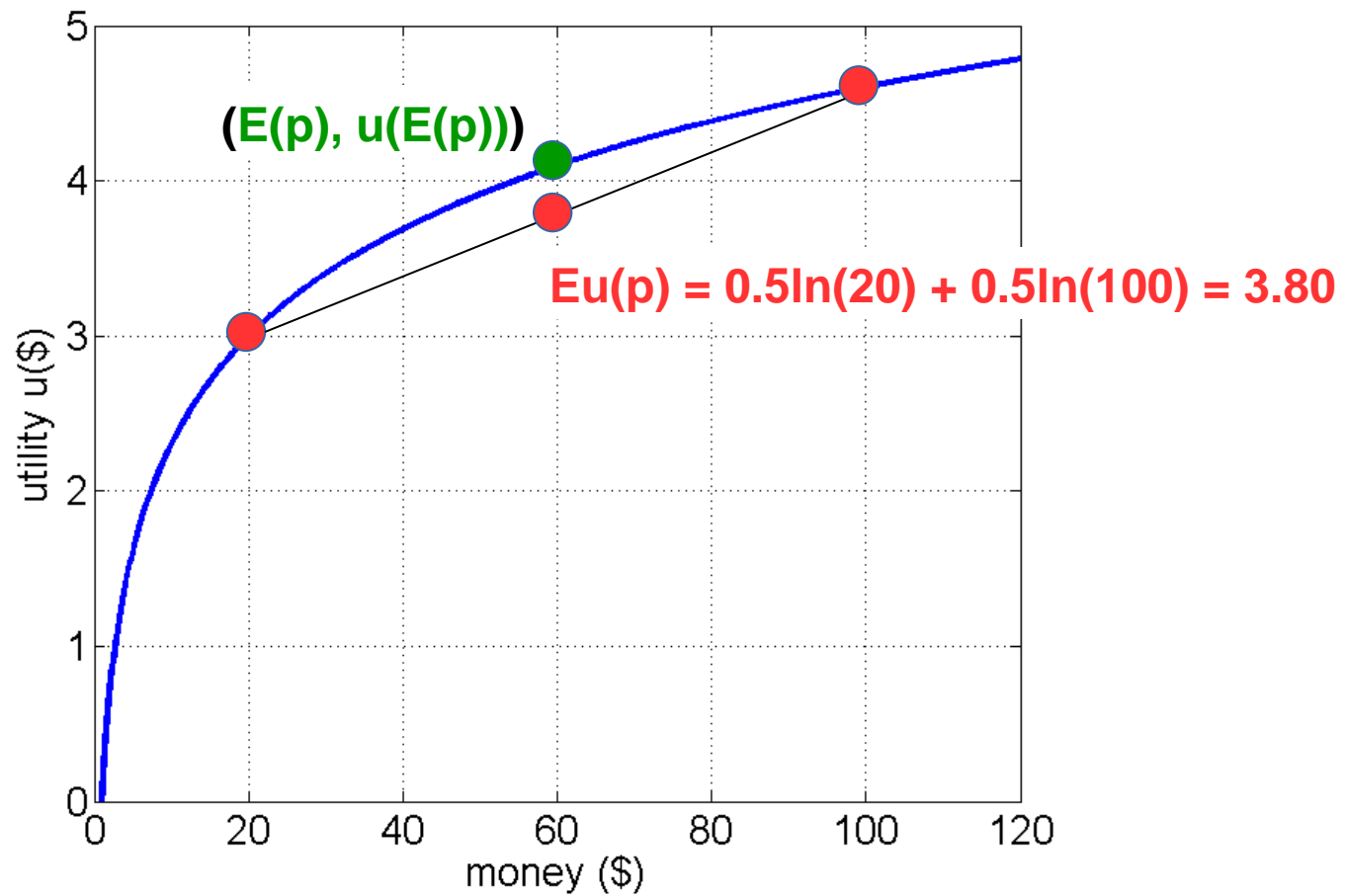
Risk Aversion

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- $p = (0.5) \text{ \$20 } \oplus (0.5) \text{ \$100}$



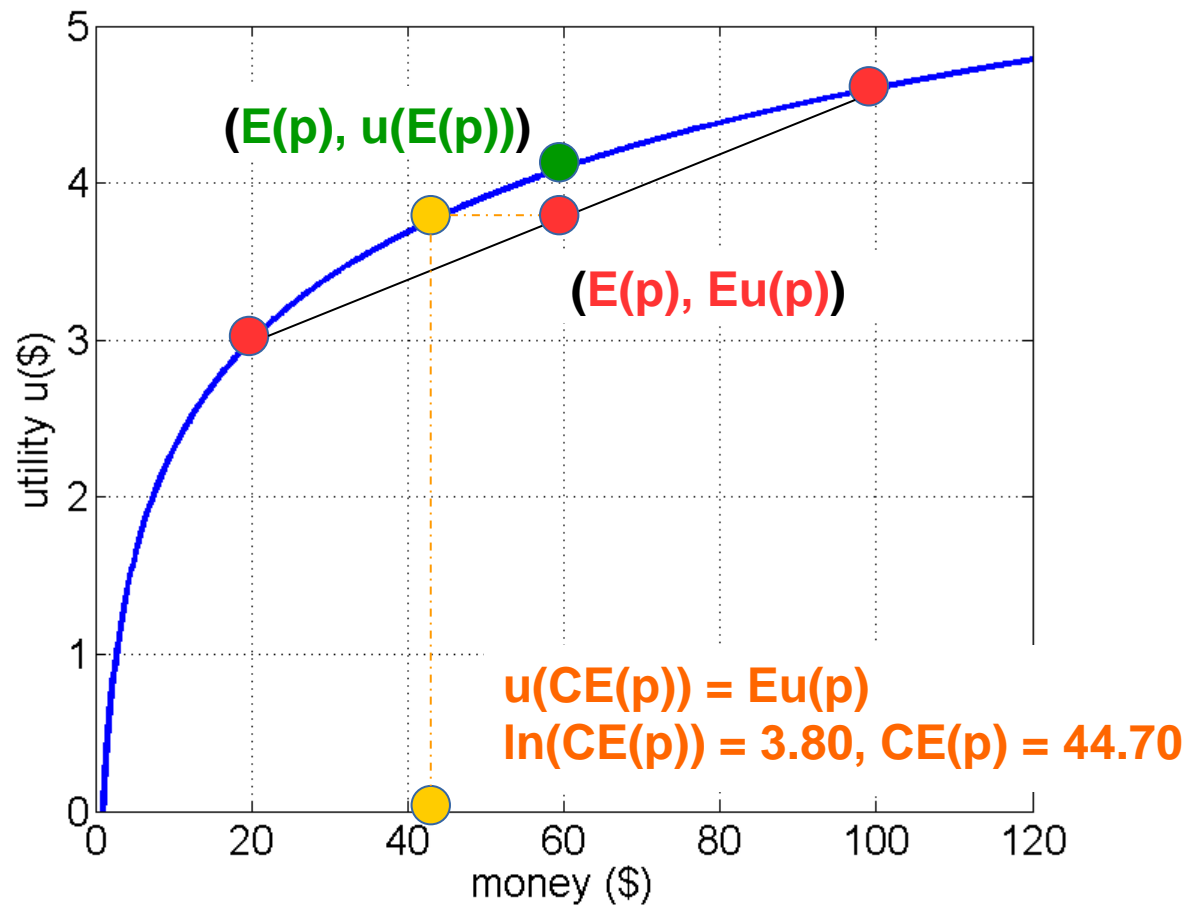
Risk Aversion

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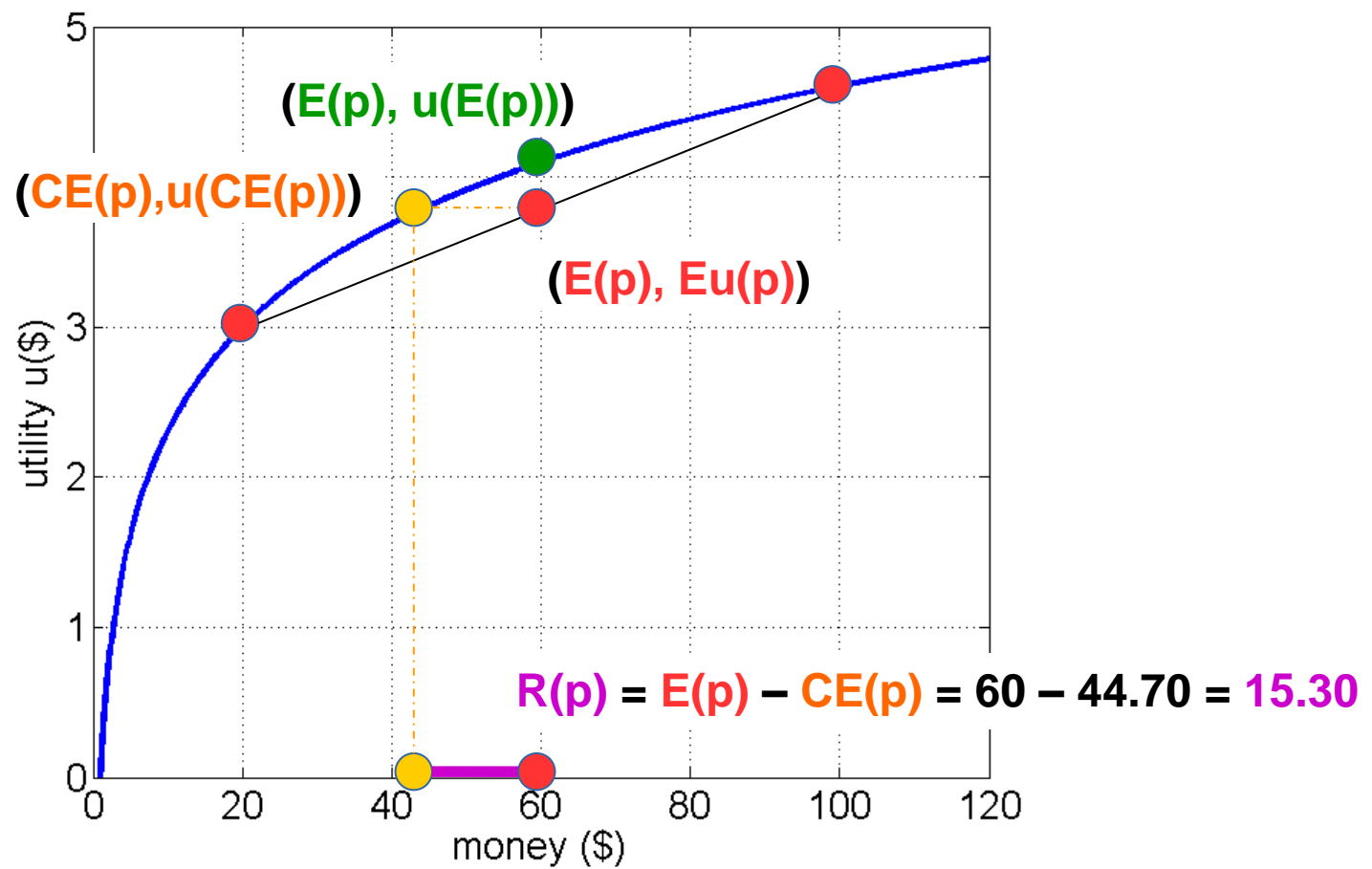
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \$20 \oplus (0.5) \100



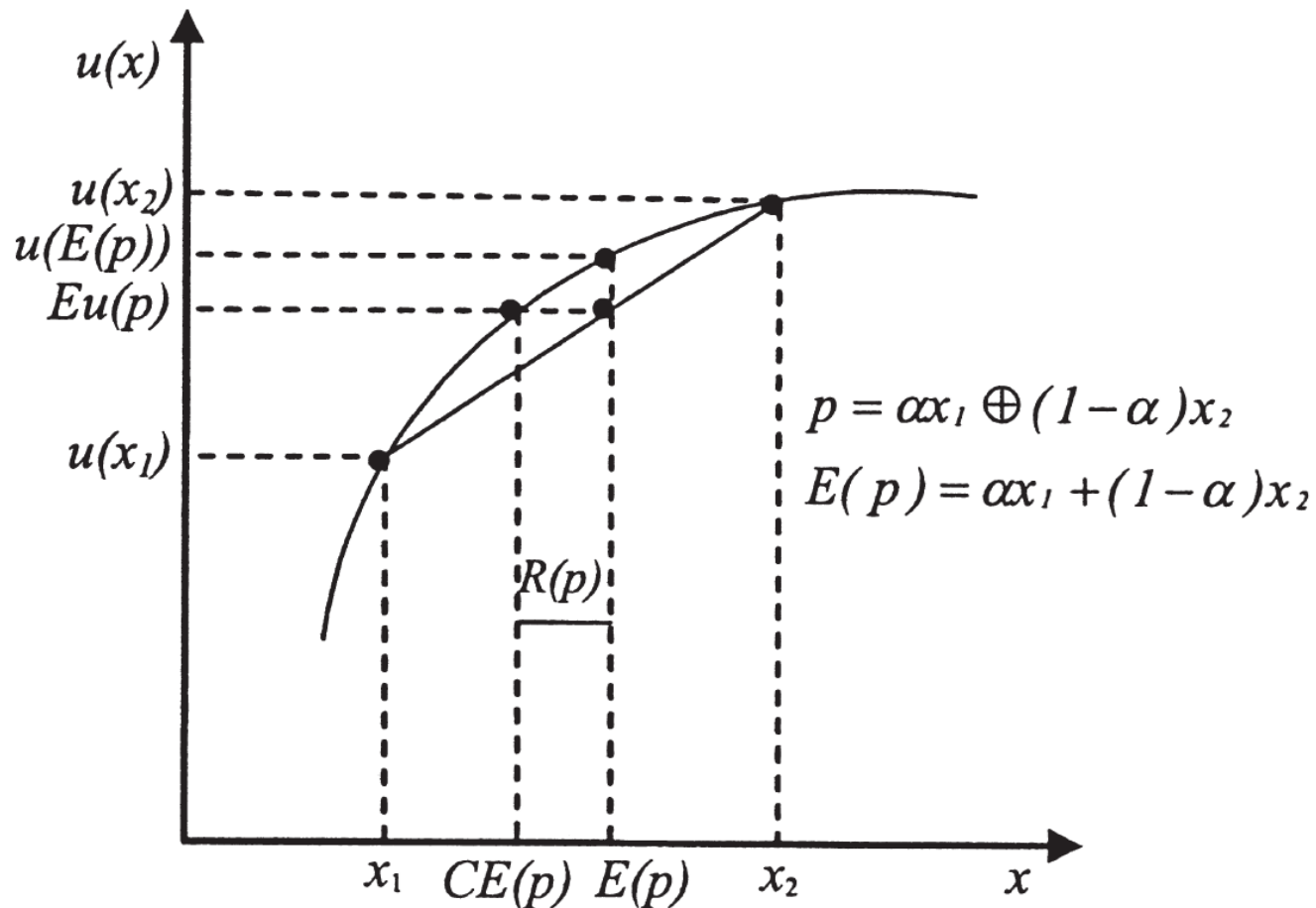
Risk Aversion

- $u(x) = \ln(x)$
- $p = (0.5) \$20 \oplus (0.5) \100



Risk Aversion

- Risk premium $R(p)$
 - $R(p) = E(p) - CE(p)$

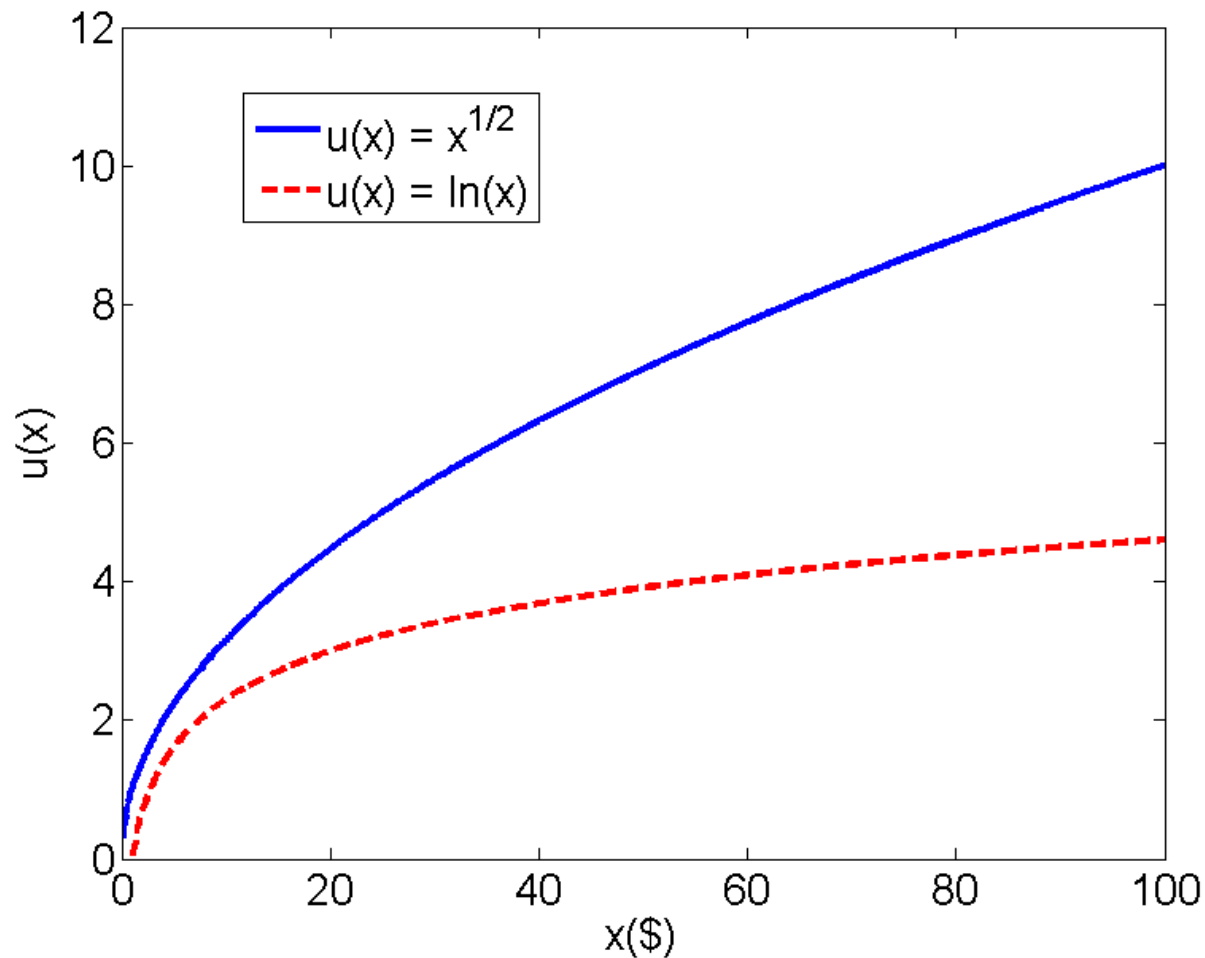


Risk Aversion

- Claim:
 - The preference relation \succsim_1 is more risk averse than \succsim_2 if $\mathbf{CE}_1(p) \leq \mathbf{CE}_2(p)$ for all p

Risk Aversion

- Which individual is more risk averse?

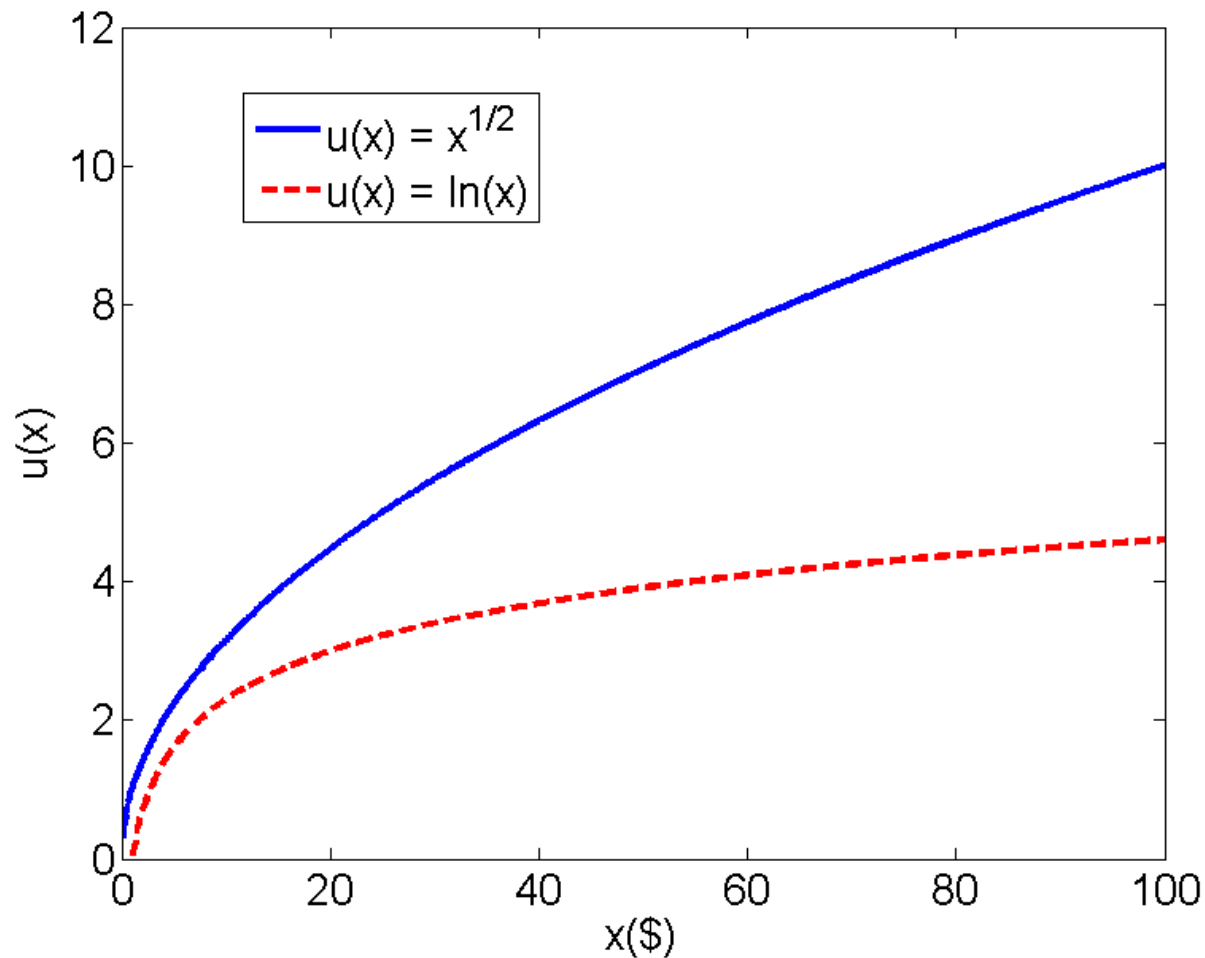


Risk Aversion

- Another definition of the relation “more risk averse” exists when vNM utility functions are twice differentiable:
 - Let u_1 and u_2 be twice differentiable vNM utility functions representing \succeq_1 and \succeq_2 , respectively
 - The preference relation \succeq_1 is more risk averse than \succeq_2 if $r_1(x) \geq r_2(x)$ for all x , where
 - $r_i(x) = -u''_i(x)/u'_i(x)$
- The number $r(x)$ is called the coefficient of absolute risk aversion of u at x
- A higher coefficient of absolute risk aversion means a more risk-averse decision maker

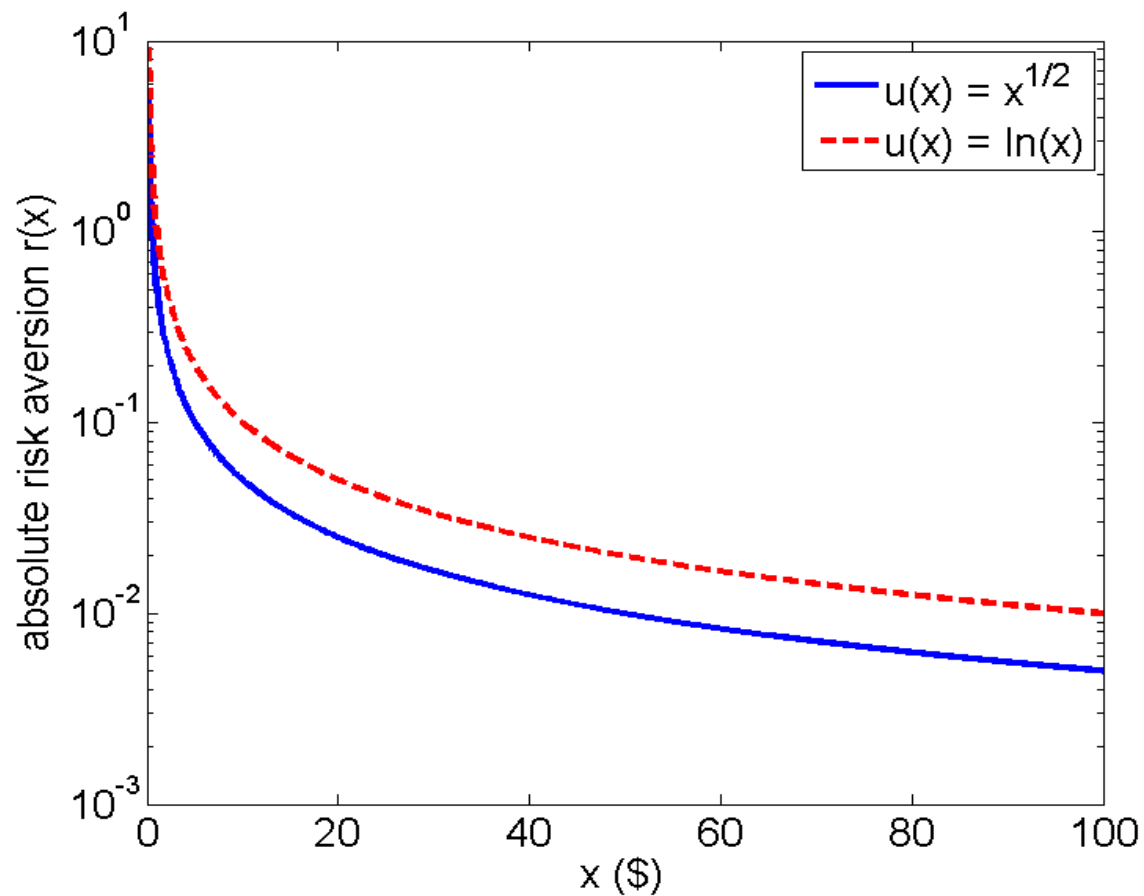
Risk Aversion

- Which individual is more risk averse?



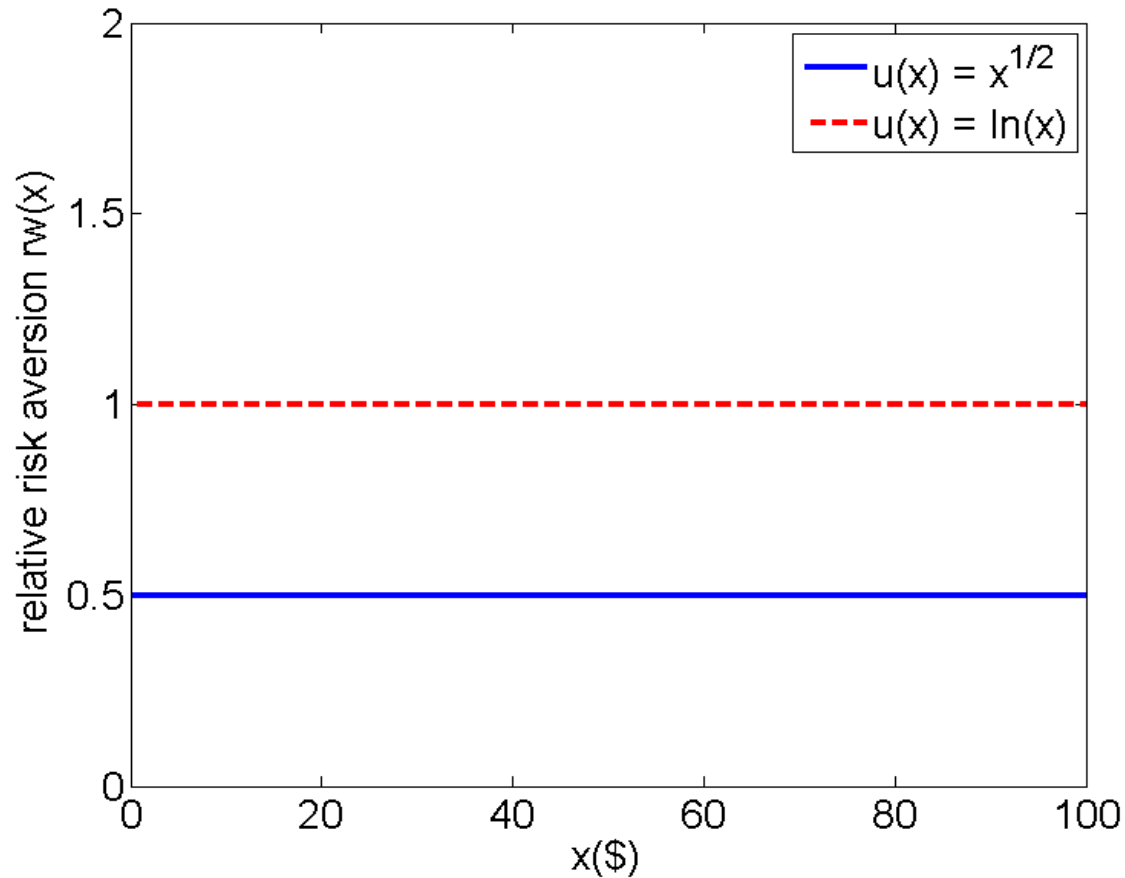
Risk Aversion

- Absolute risk aversion coefficient r
 - $r(x) = -u''(x) / u'(x)$



Risk Aversion

- Relative risk aversion coefficient ***rw***
 - ***rw(x) = x r(x)***

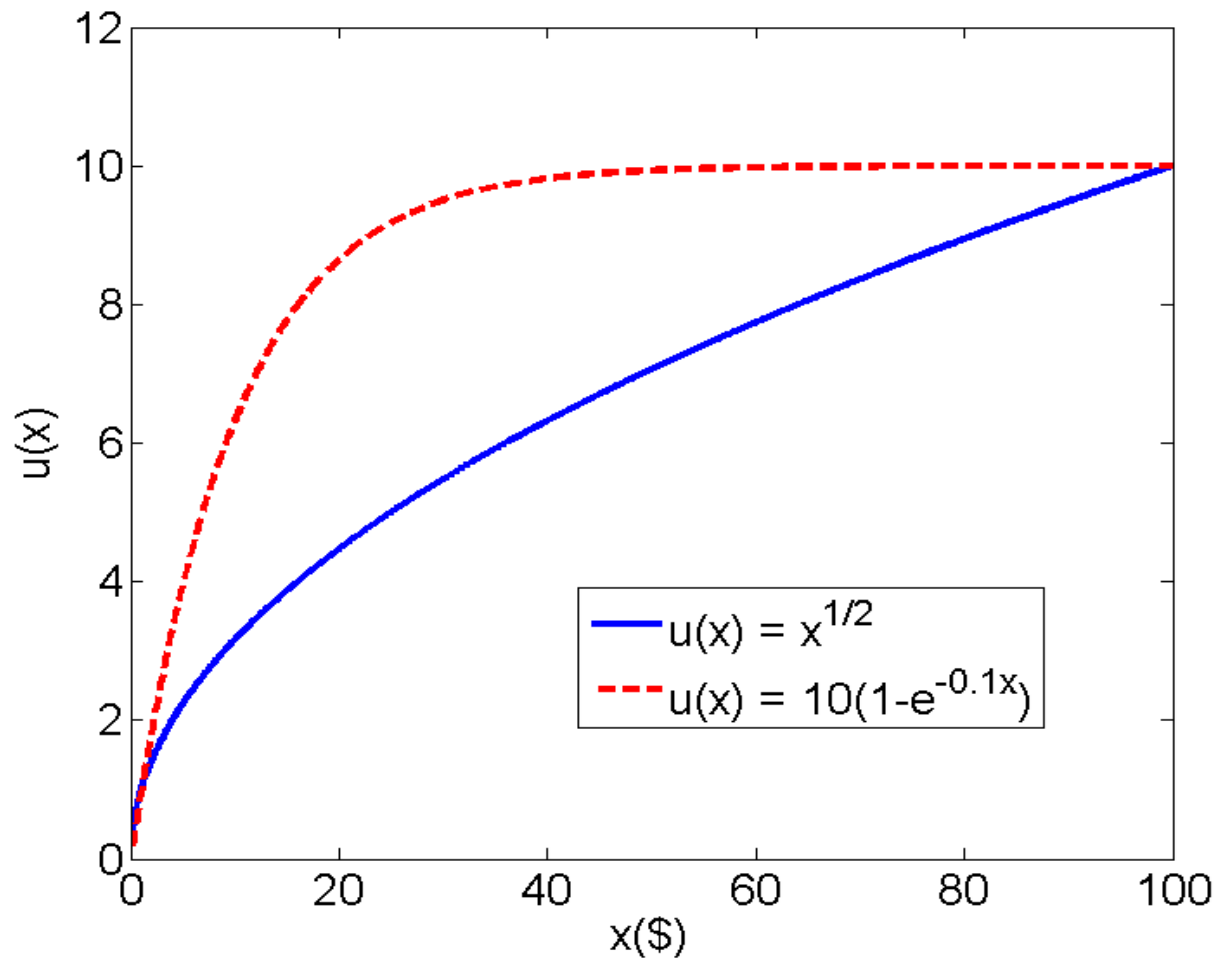


Risk Aversion

- Absolute risk aversion coefficient r
 - If $r(x)$ decreases with x , then individuals will invest larger money amounts in risky assets as they get wealthier
- Relative risk aversion coefficient rw
 - If $rw(x)$ is constant with x , individuals will invest the same percentage of their wealth in risky assets as they get wealthier

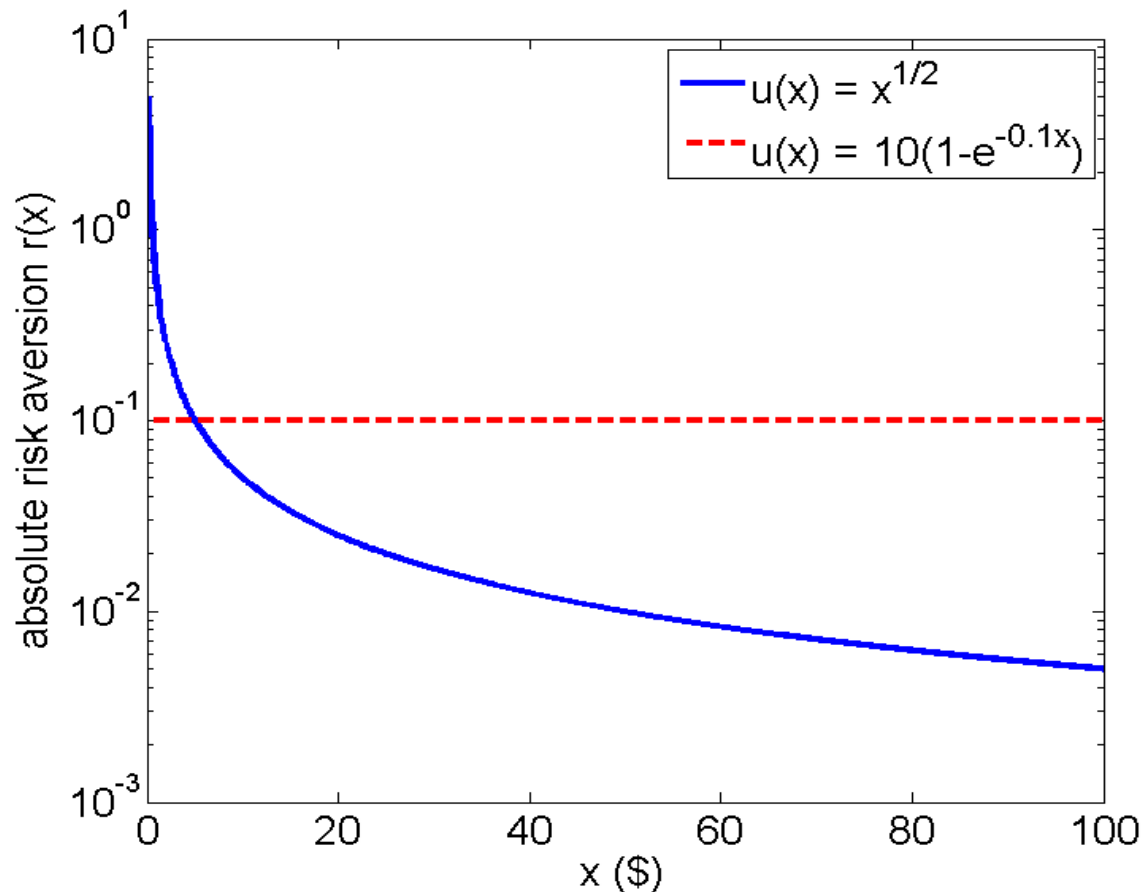
Risk Aversion

- Which individual is more risk averse?



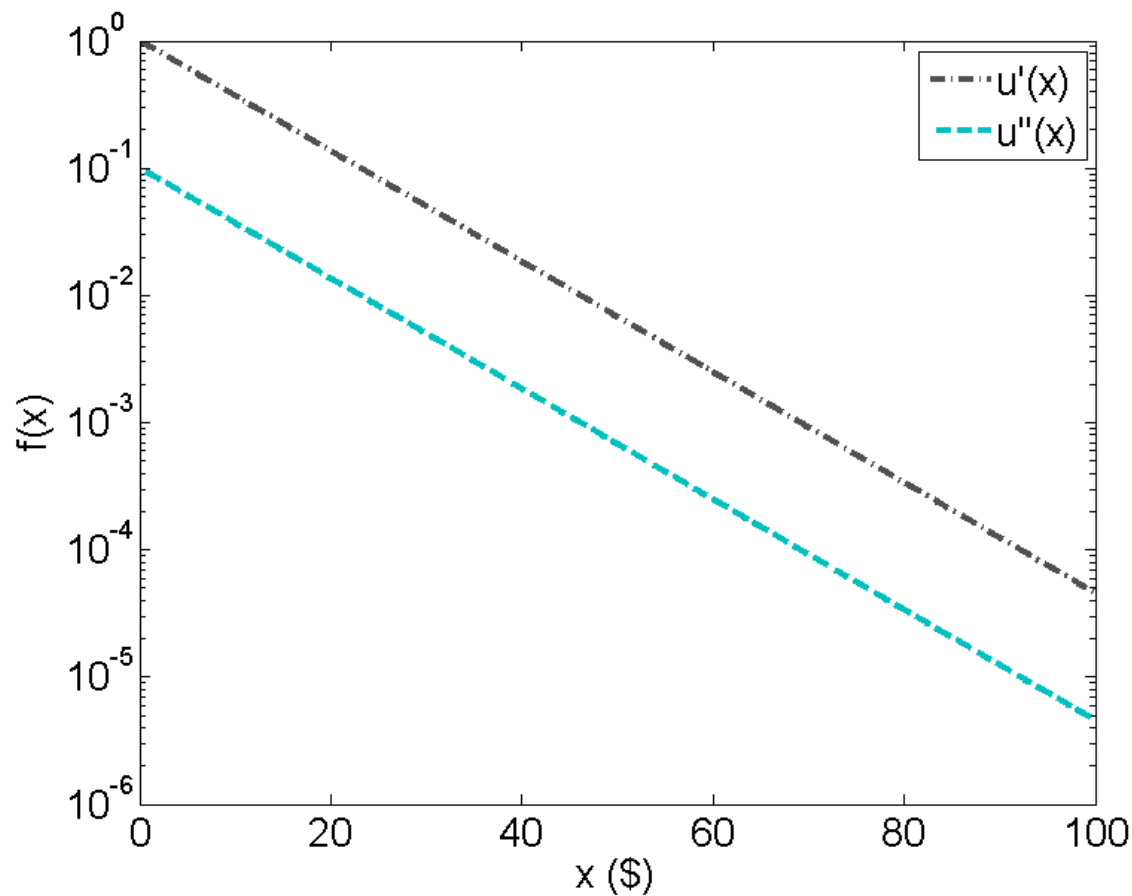
Risk Aversion

- Absolute risk aversion coefficient r
 - $r(x) = -u''(x) / u'(x)$



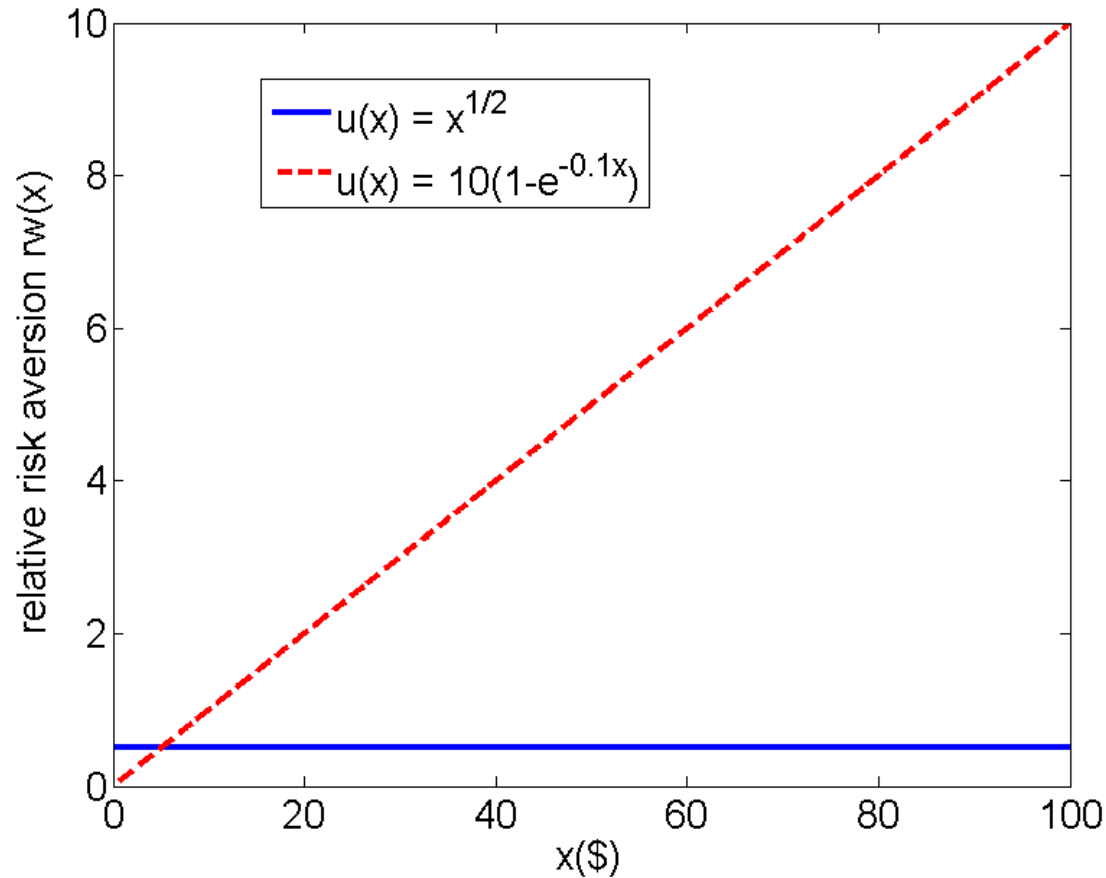
Risk Aversion

- Absolute risk aversion coefficient r
 - $u(x) = 10(1 - e^{-0.1x})$



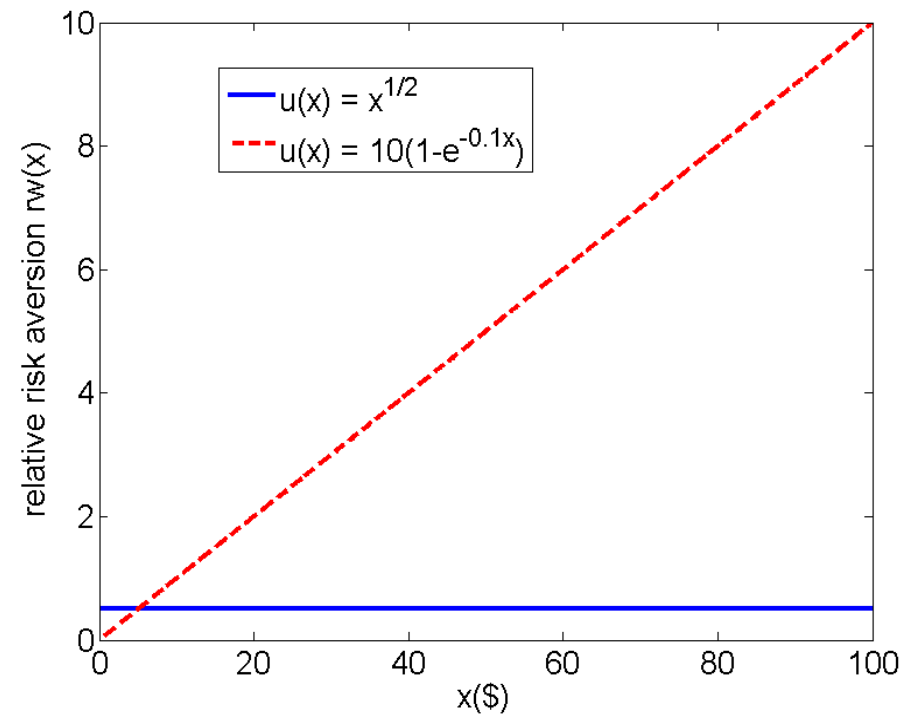
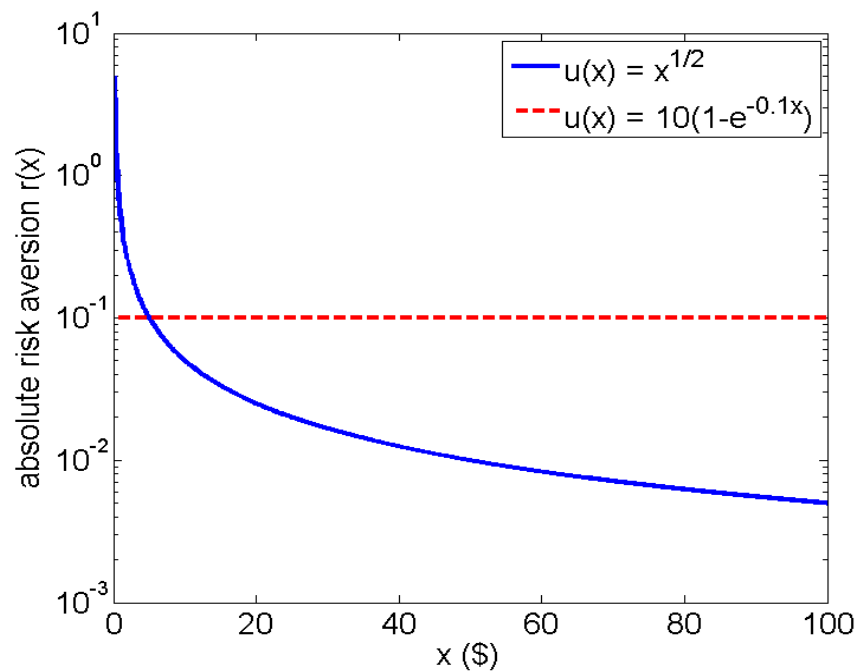
Risk Aversion

- Relative risk aversion coefficient ***rw***
 - ***rw(x) = x r(x)***



Risk Aversion

- What happens with an agent represented by the red curve?



Risk Aversion

- Absolute risk aversion coefficient r
 - With constant $r(x)$, the amount of wealth that we expose to risk remains constant as wealth increases
 - Invariance to wealth
- Relative risk aversion coefficient rw
 - If $rw(x)$ is increasing with x , individuals will invest less percentage of their wealth in risky assets as they get wealthier

Invariance to Wealth

- Claim:
 - Assume that u is a vNM utility function representing preferences \succeq , which are monotonic and exhibit invariance to wealth
 - Then u must be exponential or linear
- Proof in the book

First-Order Stochastic Domination

- Is there other ways to compare lotteries besides using the expected utility **Eu** ?
- Now we see when a lottery **p** first-order stochastically dominates a lottery **q**
 - Or **$pD_1 q$**

First-Order Stochastic Domination

- Which lottery do you prefer?

	\$0	\$20	\$50	\$100	\$200
<i>p</i>	0.1	0.1	0.2	0.3	0.3
<i>q</i>	0.15	0.05	0.25	0.35	0.20

First-Order Stochastic Domination

- Which lottery do you prefer?
 - $G(p, x) = \sum_{z \geq x} p(z)$

	\$0	\$20	\$50	\$100	\$200
$G(p, x)$	1 (0.1)	0.9 (0.1)	0.8 (0.2)	0.6 (0.3)	0.3 (0.3)
$G(q, x)$	1 (0.15)	0.85 (0.05)	0.8 (0.25)	0.55 (0.35)	0.2 (0.20)

First-Order Stochastic Domination

- Which lottery do you prefer?
 - $F(p, x) = \sum_{z \leq x} p(z)$

	\$0	\$20	\$50	\$100	\$200
$F(p, x)$	0.1 (0.1)	0.2 (0.1)	0.4 (0.2)	0.7 (0.3)	1 (0.3)
$F(q, x)$	0.15 (0.15)	0.2 (0.05)	0.45 (0.25)	0.80 (0.35)	1 (0.20)

First-Order Stochastic Domination

- Claim:
 - pD_1q iff for all x , $G(p, x) \geq G(q, x)$ or
 - pD_1q iff for all x , $F(p, x) \leq F(q, x)$

