Lista 2

1\. a) $T(n) = T(n-3) + 1$

$T(n-3-3) + 1 + 1 = T(n-6) + 2$

$T(n-6-3) + 1 + 2 = T(n-9) + 3$

$= T(n-3k) + k$

$T(n - \cancel{3} \cdot \frac{n}{\cancel{3}}) + n/3$

$T(0) + n/3$ $\quad O(n/3)$

$n - 3k = -3k + n$

$-3k = -n$

$k = n/3$

b) $T(n) = 2T(n-2) + \log n$

$T(n-2) = 2(2T(n-2-2) + \log n - 2) + \log n$

$= 2(2T(n-4) + \log n - 2) + \log n$

$= 2^2 T(n-4) + 2\log n - 2 + \log n$

$= 2^2(2T(n-6) + \log n - 4)\ 2\log n - 2 + \log n$

$= 2^3 T(n-6) + 2^2 \log n - 4 + 2\log n - 2 + \log n \longrightarrow 2^0 \log n - 0$

$= 2^i T(n-2i) + \sum_{j=0}^{i-1} 2^j \log n - 2j$

$n - 2i = 0$

$n = 2i$

$i = \frac{n}{2}$

$= 2^{n/2} T\left(n - \frac{2n}{2}\right) + \sum_{j=0}^{n/2 - 1} 2^j \log n - 2j$

$T(n) = \underline{2^{n/2}}\, T(0) + \sum_{j=0}^{n/2 - 1} 2^j \log n - 2j \quad \rightarrow O(2^n)$

ou $O(2^{n/2})$

c) $T(n) = T(n-1) + n$

$T(n-1) = T(n-1-1) + (n-1) + n$

$= T(n-2) + n + n - 1$

$= T(n-2-1) + (n-2) + 2n - 1$

$= T(n-3) + 3n - 3$

$= T(n-3-1) + (n-4) + 3n - 3 = T(n-4) + 4n - 7$

$n - i = 0$ $\quad = T(n) = T(n-i) + in - ?$

$n = i$

$T(n) = T(n-n) + n \cdot n - ?$

$T(n) = T(0) + n^2 - ? \longrightarrow O(n^2)$

d) $2T(n-1) + n^2 + 1$

$$2[2T(n-2) + (n-1)^2 + 1] + n^2 + 1$$

$$2[4T(n-3) + 2(n-2)^2 + 2 + (n-1)^2 + 1] + n^2 + 1$$

$$2^k T(n-k) + \sum_{i=0}^{k-1} 2^i ((n-i)^2 + 1)$$

$$O(2^n)$$

2. a) $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$$T(n/2) = 2\left(2T\left(\frac{n/2}{2}\right) + 1\right) + 1$$

$$= 2^2 T\left(\frac{n}{2}\cdot\frac{1}{2}\right) + 2 + 1$$

$$= 2^2 T\left(\frac{n}{4}\right) + 2 + 1$$

$$T(n/4) = 2T\left(\frac{n}{8}\right) + 1$$

$$= 2^2\left(2T\left(\frac{n}{8}\right) + 1\right) + 2 + 1$$

$$= 2^3 T\left(\frac{n}{8}\right) + 2^2 + 2 + 1$$

$$= 2^3 T\left(\frac{n}{8}\right) + 2^2 + 2^1 + 2^0$$

$$= 2^i T\left(\frac{n}{2^i}\right) + \sum_{j=0}^{i-1} 2^j$$

$\frac{n}{2^i} = 1$

$n = 2^i$

$\log_2 n = \log_2 2^i$

$\log n = i$

$$T(n) = 2^{\log_2 n} T\left(\frac{n}{2^{\log n}}\right) + \sum_{j=0}^{\log n - 1} 2^j$$

$$T(n) = nT(1) + \sum_{j=0}^{\log n - 1} 2^j$$ → Tem que resolver o somatório ?

$$T(n) = n + \frac{2^{\log n - 1 + 1} - 1}{2 - 1}$$

$$T(n) = n + 2^{\log_2 n} - 1$$

$$T(n) = n + n - 1$$

$$T(n) = 2n - 1$$ → $O(n)$

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2.b) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$4\left(4T\left(\frac{n}{4}\right) + \log\left(\frac{n}{2}\right) + \log n\right.$

$4\left(16T\left(\frac{n}{8}\right) + \log\left(\frac{n}{4}\right)\right) + 5\log(n) - 4\log(2)$

$64T\left(\frac{n}{8}\right) + 16\log(n/4) + 5\log n - 4\log(2)$

$T(n) = 4^k T(2^{n/k}) + \sum_{i=0}^{k-1} 4^i \log(n/2^i)$

$n^2 + \log(n)\sum 4^i - \log(2)\sum i\, 4^i$

$O n^2 \log(n)$

2. c) $T(n) = 7T(n/3) + n$

$$7(7T(n/9) + n/3) + n$$

$$7\left(49T\left(\frac{n}{27}\right) + 7 \cdot \frac{n}{9} + \frac{n}{3}\right) + n$$

$$T(k) = 7^k T\left(\frac{n}{3^k}\right) + \sum_{i=0}^{k-1} 7^i \left(\frac{n}{3}\right)$$

$$T(n) = 7^{\log_3(n)} \cdot T(1) \sum_{i=0}^{\log_3 n-1} \frac{7^i}{3} n \quad \ldots$$

$\hookrightarrow O(n^{\log_3 7})$

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3. i) $T(n) = 2T\left(\frac{n}{4}\right) + 1$

$a = 2$

$b = 4$

$\log_4 2 = \frac{1}{2}$

$f(n) = 1$

$T(n) = \Theta(n^{1/2})$

p/ $\epsilon = 1/2$

Caso 1: $f(n) = 1 = O(n^{\frac{1}{2} - \frac{1}{2}}) = O(n^0) = O(1)$ Classe das constantes

$\boxed{n^{\frac{1}{2}} = \sqrt{n}}$

ii) $T(n) = 2T\left(\frac{n}{4}\right) + n$

$a = 2 \quad b = 4$

$\log_4 2 = \frac{1}{2}$

$f(n) = n$

para $\epsilon = 1/2$

Caso 3:

$f(n) = n = \Omega(n^{1/2 + 1/2})$

$= \Omega(n)$

$c = 3$

$2f\left(\frac{n}{4}\right) \leq 3f(n)$

$\frac{n}{2} \leq 3n$

$T(n) = \Theta(f(n)) = \Theta(n)$

iii) $T(n) = 2T\left(\frac{n}{4}\right) + \log n$

$a = 2 \quad b = 4 \quad f(n) = \log n$

$\log_4 2 = 1/2$

para $\epsilon =$

Caso 1:

$f(n) = O(n^{1/2 - 1/4}) = O(n^{1/4})$

$T(n) = \Theta(n^{1/2})$

iv) $T(n) = 4T\left(\frac{n}{2}\right) + 1$

$a = 4 \quad b = 2 \quad f(n) = 1$

$\log_2 4 = 2$

Caso 1:

para $\epsilon = 2$

$f(n) = O(n^{2-2}) = O(n^0) = O(1)$

$T(n) = \Theta(n^2)$

v) $T(n) = 4T\left(\frac{n}{2}\right) + n$ → às vezes uma função pode [illegible] em [illegible] de um caso [illegible]

$a=4 \quad b=2 \quad f(n)=n$

$\log_2 4 = 2$ Caso 1:

para $\epsilon = 1$

$f(n) = O(n^{2-1}) = O(n^1) = O(n)$

$T(n) = \Theta(n^2)$

vi) $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a=4 \quad b=2 \quad f(n) = \log n$

$\log_2 4 = 2$

$\epsilon = 1$

Caso 1:

$f(n) = O(n^{2-1}) = O(n^1) = O(n)$

$T(n) = \Theta(n^2)$

vii) $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$a=2 \quad b=2 \quad f(n)=1$

$\log_2 2 = 1$

$\epsilon = 1$

Caso 1:

$f(n) = O(n^{1-1}) = O(n^0) = O(1)$

$T(n) = \Theta(n)$

viii) $T(n) = 2T\left(\frac{n}{2}\right) + \log n$

$a=2 \quad b=2 \quad f(n) = \log n$

$\log_2 2 = 1$

$\epsilon = 1/4$

Caso 1:

$f(n) = O(n^{1-\frac{1}{4}}) = O(n^{3/4})$

$T(n) = \Theta(n^1)$

ix) $T(n) = 2T\left(\frac{n}{2}\right) + n$

$a=2 \quad b=2 \quad f(n)=n$

$\log_2 2 = 1$

Caso 2:

$f(n) = \Theta(n)$

$T(n) = \Theta(n \log n)$

x) $T(n) = 2T\left(\frac{n}{3}\right) + 1$

$a = 2 \quad b = 3 \quad f(n) = 1$

$\log_3 2 = \log_b^a \qquad 1 \leq n^{\log_3 2}$

xi) $T(n) = 2T\left(\frac{n}{3}\right) + \log n$

$a = 2 \quad b = 3 \quad f(n) = \log n$

$\log(n) \to n^{\log_3 2} \qquad O(n^{\log_3 2})$

xii) $T(n) = 2T\left(\frac{n}{3}\right) + n$

$a = 2 \quad b = 3 \quad f(n) = n \qquad n^1 > n^{\log_3 2}$

$\log_b^a = \log_3^2 \qquad O(n)$

$$4.\ T(n) = 2T(\sqrt{n})$$

$$m = \log_2 n$$

$$2^m = \cancel{2}^{\log_{\cancel{2}} n}$$

$$2^m = n$$

$$T_1(2^m) = 2T_1((2^m)^{1/2})$$

$$= 2T_1(2^{m/2})$$

$$T_2(m) = T_1(2^m)$$

$$T(n) = T_1(2^m) = T_2(m)$$

$$\sqrt{n} = 2^{m/2}$$

$$T(\sqrt{n}) = T_1(2^{m/2}) = T_2(m/2)$$

$$T_2(m) = 2T_2(m/2)$$

$$= 2(2T_2(\tfrac{m/2}{2}))$$

$$= 2(2T_2(m/4)$$

$$= 2^2 T_2(m/4)$$

$$= 2(2^2 T_2(\tfrac{m/4}{2})$$

$$= 2^3 T(m/8)$$

$$T_2(m) = 2^i T_2(m/2^i)$$

$$\frac{m}{2^i} = 1 \qquad \log_2 m = \log_{\cancel{2}} \cancel{2}^i \qquad \hookrightarrow T_2(m) = 2^{\log m} T_2(m/2^{\log m})$$

$$m = 2^i \qquad \log m = i \qquad T_2(m) = m T_2(m/m)$$

$$T_2(m) = m T_2(1)$$

Caso base = 1

$T_2(1) = 1$

$T_2(m) = m = O(m)$

$\downarrow$

$\longrightarrow O(\log n)$

5. $T(n) = 2T(\sqrt{n}) + \log n$

$m = \log_2 n$

$2^m = \cancel{2}^{\log_2 n}$

$2^m = n$

$T_a(2^m) = 2T((2^m)^{1/2}) + m$

$= 2T(2^{m/2}) + m$

$T_b(m) = T_a(2^m) = T(n)$

$\sqrt{n} = 2^{m/2}$

$T(\sqrt{n}) = T_a((2^m)^{1/2}) = T_b(m/2)$

$T_b(m) = 2T_2(m/2) + m$

$a = 2 \quad b = 2 \quad f(m) = m$

$\log_2 2 = 1$

Caso 2:

$f(m) = \Theta(m)$

$T_b(m) = \Theta(m \log m)$

$T(n) = \Theta(\log(n) \log(\log(n)))$

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6. $T(n) = T(n/4) + T(n/5) + T(n/6) + n$

$\Omega(T(n))$

$T(n) \geq T(n/6) + n$

$\Omega(n) \geq \Omega(n/6) + n$

$\Omega(n)$

Resolvida com outro teorema

$$T(n) = \overset{1}{3}\,T(n/4) + n$$

Teorema mestre

$a = 3$

$b = 4$

$\log_4 b = 0$