LISTA DE EXERCÍCIOS 2 - PAA

1) Determine e prove uma equivalência assintótica para as recorrências abaixo:

a) $T(n) = T(n-3) + \boxed{1}$

Achando o n:

$n - 3K = -3K + n$

$-3K = -n$

$K = n/3$

$T(n-3-3) + 1 + \boxed{1} = T(n-6) + \boxed{2}$

$T(n-6-3) + 1 + \boxed{2} = T(n-9) + \boxed{3}$

$T(n-9-3) + 1 + \boxed{3} = T(n-12) + \boxed{4}$

$= \boxed{T(n-3K) + K}$

$T(n - \cancel{3} \cdot \frac{n}{\cancel{3}}) + \frac{n}{3}$

$T(0) + n/3 \quad O(n/3)$

b) $T(n) = \boxed{2}T(n-2) + \log n$ *Interessante esse caso, quando multiplica por 2

$2 \cdot [2T(n-4) + \log(n-2)] + \log n = \boxed{4T(n-4) + 2\log(n-2) + \log(n)}$

$2 \cdot [4T(n-6) + \log(n-4)] + 2\log(n-2) + \log n =$

$\boxed{8T(n-6) + 4\log(n-4) + 2\log(n-2) + \log n}$

$2^K T(n-2K) + \sum_{i=0}^{K-1} 2^i \cdot \log(n-2i)$

Achando o n: $n - 2K = 0 \rightarrow -2K = -n$ (

$2^{\frac{n}{2}} T(0)$

c) $T(n) = T(n-1) + n$

$T(n-1-1) + (n-1) + n = T(n-2) + (n-1) + n = T(n-2) + 2n - 1$

$T(n-3) + (n-2) + (n-1) + n = T(n-3) + 3n - 2 - 1$

$T(n-K) - nK + \sum_{i=0}^{K-1} i$

$-K + n = 0$

$K = n$

$T(0) + n^2 - \frac{(n-1)n}{2} = O(n^2)$

ATALHO $\rightarrow \sum_{i=0}^{K-1} i = \frac{(K-1) \cdot K}{2}$

d) $2T(n-1)+n^2+1$

$2.[2T(n-2)+(n-1)^2+1]+n^2+1 \Rightarrow 4T(n-2)+2(n-1)^2+2+n^2+1$

$2[4T(n-3)+2(n-2)^2+2+(n-1)^2+1]+n^2+1 \Rightarrow 8T(n-3)+4(n-2)^2+4+2(n-1)^2$

$2^k T(n-k)+\sum_{i=0}^{k-1} 2^i((n-i)^2+1) \leftarrow 2^n$ cresce + rápido que Σ

$O(2^n)$

2) Determine e prove uma equivalência assintótica para as recorrências abaixo:

a) $T(n)=2T\left(\frac{n}{2}\right)+1$ $\qquad \frac{n}{2^k}=1 \quad 2^k=n$

$4T\left(\frac{n}{4}\right)+2+1$ $\qquad k=\log(n) \quad \sum_{i=0}^{k-1} 2^i = 2^k-1$

$8T\left(\frac{n}{8}\right)+4+2+1$

$2^k T\left(\frac{n}{2^k}\right)+\sum_{i=0}^{k-1} 2^i \to 2^{\log n} \cdot T(1)+\sum 2^i$ $\qquad 2^{\log_2 n}=n$

$n \cdot T(1)+2^{\log n}-1$

$n \cdot T(1)+n-1 = O(n)$

b) $T(n)=4T\left(\frac{n}{2}\right)+\log(n)$

$=4\left(4T\left(\frac{n}{4}\right)+\log\left(\frac{n}{2}\right)\right)+\log(n) = 16T\left(\frac{n}{4}\right)+4.(\log(n)-\log(2))+\log(n)$

$=16T\left(\frac{n}{4}\right)+5.\log(n)-4.\log(2)$

$=4.\left(16T\left(\frac{n}{8}\right)+\log\left(\frac{n}{4}\right)\right)+5\log(n)-4\log(2)$

$=64T\left(\frac{n}{8}\right)+16\log\left(\frac{n}{4}\right)+5\log(n)-4\log(2)$

$=64T\left(\frac{n}{8}\right)+16.(\log(n)-\log(4))+5\log(n)-4\log(2)$

$=64T\left(\frac{n}{8}\right)+16\log(n)-16\log_2(4)+5\log(n)-4\log(2)$

$=64T\left(\frac{n}{8}\right)+21\log(n)-32-4\log(2)$

$T(n)=4^k T\left(\frac{n}{2^k}\right)+\sum_{i=0}^{k-1} 4^i \log(n/2^i)$ $\qquad k=\frac{n}{2^k}=2^k=n$

$\log_2 n$

$T(n)=4^{\log_2 n}T(1)+\sum 4^i(\log(n)-i\log(2))$

$n^2+\log(n)\sum 4^i-\log(2)\sum i4^i \to O(n^2\log(n))$

c) $T(n) = 7T\left(\frac{n}{3}\right) + n$

$7\left(7T\left(\frac{n}{9}\right) + \frac{n}{3}\right) + n = 49T\left(\frac{n}{9}\right) + 7 \cdot \frac{n}{3} + n$

$7\left(49T\left(\frac{n}{27}\right) + 7 \cdot \frac{n}{9} + \frac{n}{3}\right) + n = 7.7.7T\left(\frac{n}{27}\right) + 7.7\left(\frac{n}{9}\right) + \frac{n}{3} + n$

$T(K) = 7^K\, T\left(\frac{n}{3^K}\right) + \sum_{i=0}^{K-1} 7^i \left(\frac{n}{3^i}\right)$

Achando o K ← Propriedade de log

$\boxed{\frac{n}{3^K} = 1 \quad n = 3^K \Rightarrow \log_3(n)}$

$\boxed{7^{\log_3 n} = n^{\log_3 7}}$

$T(n) = 7^{\log_3(n)} \cdot T(1) + \sum_{i=0}^{\log_3(n)-1} \frac{7^i}{3^i} \cdot n$

$= n^{\log_3(7)} \cdot T(1) + n \cdot \sum_{i=0}^{\log_3 n - 1} \frac{7^i}{3^i}$ $\quad (7/3)^i$

P.G. $\Rightarrow \sum r^i = \frac{r^K - 1}{r - 1}$

$= n^{\log_3(7)} \cdot T(1) + n \cdot \frac{(7/3)^{\log_3(n)} - 1}{(7/3) - 1} = \frac{n^{\log_3 \frac{7}{3}} - 1}{\frac{4}{3}} = \frac{3}{4} \cdot \left(n^{\log_3 7/3} - 1\right)$

Como $n^{\log_3(7)}$ domina...

$\Theta\left(n^{\log_3 7}\right)$

③ Utilizando o Teorema Mestre determine uma equivalência assintótica para

i) $T(n) = 2T(n/4) + 1$

$a = 2$ $\quad \log_b a = \log_4 2 = \boxed{1/2}$

$b = 4$

$f(n) = \boxed{1}$ $\quad 1 < n^{1/2} \quad \Theta(n^{1/2})$ caso 1

$f(n) < n^{\log_b a}$ (CASO 1)

$f(n) = n^{\log_b a}$ (CASO 2)

$f(n) > n^{\log_b a}$ (CASO 3)

$n^{1/2} = \sqrt{n}$

ii) $T(n) = 2T(n/4) + n$

$a = 2$ $\quad f(n) = n$ $\quad n > n^{1/2}$

$b = 4$ $\quad \log_b a = \log_4 2 = 1/2$ $\quad \Theta(n)$ caso 3

iii) $T(n) = 2T\left(\frac{n}{4}\right) + \log(n)$

$a = 2$ $\quad f(n) = \log(n)$ $\quad \log(n) < n^{1/2} \quad \Theta(n^{1/2})$

$b = 4$ $\quad \log_b a = \log_4 2 = 1/2$

$\log_{10}(100) = 2 \quad \sqrt{100} = 10$

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iv) $T(n) = 4T(n/2) + 1$

$a = 4$ $\quad f(n) = 1$ $\quad 1 < n^2$ $\quad \Theta(n^2)$ CASO 1

$b = 2$ $\quad \log_b a = \log_2 4 = 2$

v) $T(n) = 4T(n/2) + n$

$a = 4$ $\quad f(n) = n$ $\quad n < n^2$ $\quad \Theta(n^2)$ CASO 1

$b = 2$ $\quad \log_b a = \log_2 4 = 2$

vi) $T(n) = 4T(n/2) + \log(n)$

$a = 4$ $\quad f(n) = \log(n)$ $\quad \log(n) < n^{\log_2 4}$ $\quad \Theta(n^2)$ CASO 1

$b = 2$ $\quad \log_b a = \log_2 4 = 2$

vii) $T(n) = 4T(n/2) + 1$

$a = 4$ $\quad f(n) = 1$ $\quad 1 < n^2$ $\quad \Theta(n^2)$ CASO 1

$b = 2$ $\quad \log_b a = \log_2 4 = 2$

viii) $T(n) = 2T(n/2) + \log(n)$

$a = 2$ $\quad f(n) = \log(n)$ $\quad \log(n) < n$ $\quad \Theta(n)$ CASO 1

$b = 2$ $\quad \log_b a = \log_2 2 = 1$

ix) $T(n) = 2T(n/2) + n$

$a = 2$ $\quad f(n) = n$ $\quad n = n$ $\quad T(n) = \Theta(n \log n)$ CASO 2

$b = 2$ $\quad \log_2 2 = 1$

x) $T(n) = 2T(n/3) + 1$

$a = 2$ $\quad f(n) = 1$ $\quad 1 < n^{\log_3 2}$

$b = 3$ $\quad \log_b a = \log_3 2$

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xi) $T(n) = 2T(n/3) + \log(n)$

$a = 2$ $\quad f(n) = \log(n)$ $\quad \log(n) \neq n^{\log_3 2}$ $\quad \Theta(n^{\log_3 2})$

$b = 3$ $\quad \log_b a = \log_3 2$

xii) $T(n) = 2T(n/3) + n$ $\quad \log_3 2 < 1$

$a = 2$ $\quad f(n) = n$ $\quad n^1 > n^{\log_3 2}$ $\quad \Theta(n)$

$b = 3$ $\quad \log_b a = \log_3 2$

Exercício 4 Determine um limite assintótico para $T(n) = 2T(\sqrt{n})$. Dica: Faça uma substituição de variável. Faça $m = \log(n)$

Se $m = \log(n)$, então $n = 2^m$ (pois $\log n = m \Rightarrow n = 2^m$) Propriedade do log

Substituindo na função original: $T(2^m) = 2T(2^{m/2})$

Seja $S(m) = T(2^m)$

$S(m) = 2S(\frac{m}{2})$

$a = 2$ $\quad \log_2 2 = 1$ $\quad 0 \leq 1$ caso 1

$b = 2$ $\quad S(m) = \Theta(m)$

$f(m) = 0$ $\quad S(m) = T(2^m) = \log(2^m) = \log(n)$

Exercício 5 Determine um limite assintótico para $T(n) = 2T(\sqrt{n}) + \log(n)$. Mesma dica do ex. 4.

Se $m = \log_2(n)$, então $n = 2^m$ $\quad$ Exemplo: $\log_2(8) = 3 \Rightarrow 2^3 = 8$

$T(2^m) = 2T(2^{\frac{m}{2}}) + \log(2^m)$ $\quad$ POTÊNCIA: $(a^m)^n = a^{m \cdot n}$

$= 2T(2^{\frac{m}{2}}) + m$ $\quad (2^m)^{\frac{1}{2}} = 2^{m \cdot \frac{1}{2}} \Rightarrow 2^{m/2}$

Se $S(m) = T(2^m)$ $\quad$ Como $S(m) = T(2^m)$ e $m = \log(n)$

$S(m) = 2S(m/2) + m$ $\quad T(n) = \log(n)$

$a = 2$ $\quad m \geq 1$ $\quad \Theta(\log n)$

$b = 2$ $\quad \Theta(m)$

$f(m) = m$