Water Bottle Rocket Design Contest
Calculation Exercises

Middle/High School Division
9th – 12th Grade

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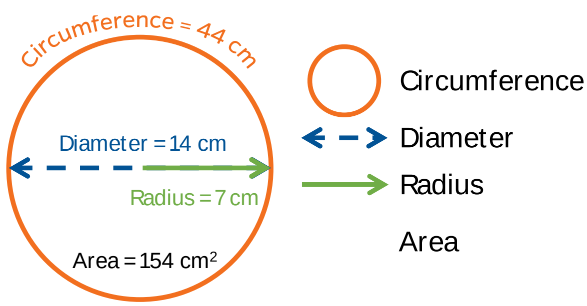
Question 1 Equations:

CIRCLE

$$C = \pi \times D = \pi \times (2 \times r)$$
$$A = \pi r^2$$

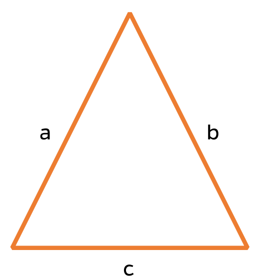
C = Circumference
D = Diameter = (r+r) = 2r
π = 22/7 or 3.14, pi
A = Area
r = Radius = ½ D

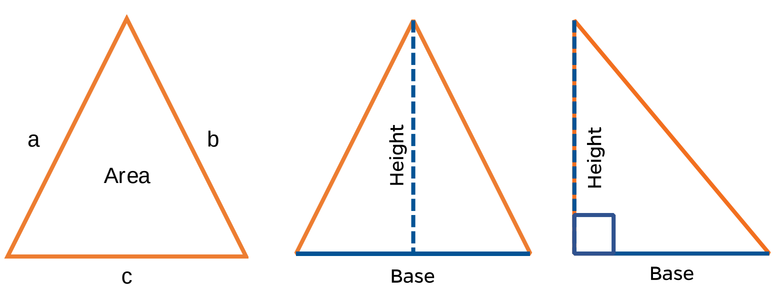


TRIANGLE

$$P = a + b + c$$

P = Perimeter
a = side length
b = side length
c = side length





$$A = \frac{1}{2} b \times h$$

A = Area
b = base of the triangle
h = height of the triangle

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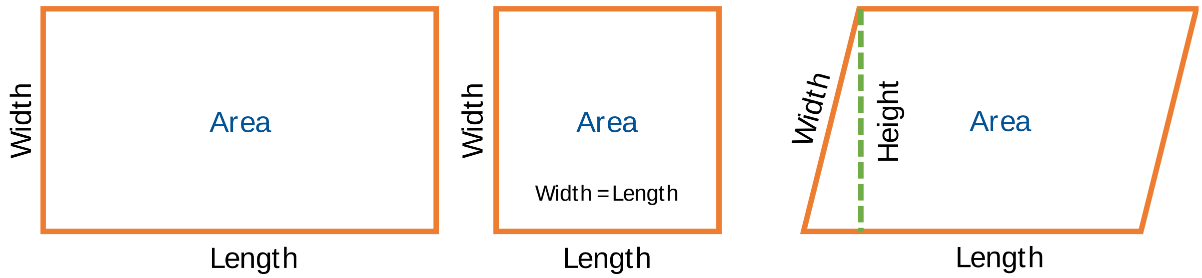
RECTANGLE/SQUARE/PARALLELOGRAM



$$P = 2(l + w)$$

P = Perimeter
l = length
w = width

Rectangle/Square/ Parallelogram

$$A = l \times w$$

A = Area
l = length
w = width

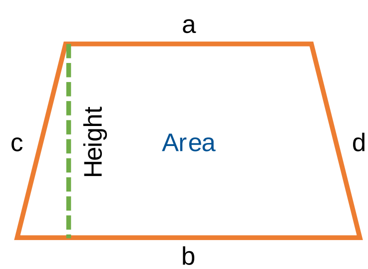
Rectangle/Square

$$A = l \times h$$

A = Area
l = length
h = height

Parallelogram

TRAPEZOID



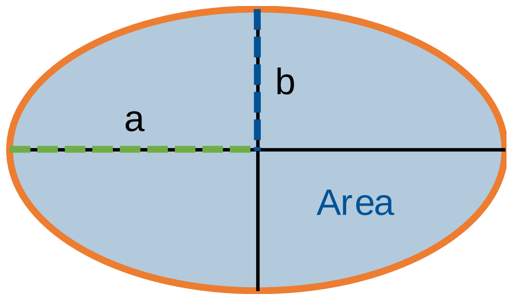
$$P = a + b + c + d$$

P = Perimeter
a = top side length
b = bottom side length
c = left side length
d = right side length

$$A = \frac{1}{2} \times h(a + b)$$

A = Area
h = height
a = top side length
b = bottom side length

ELLIPSE



$$\boldsymbol{P = \pi(a+b)(1 + \frac{3h}{10+\sqrt{4-3h}}}$$

$$\boldsymbol{h = \frac{(a-b)^2}{(a+b)^2}}$$

P = Perimeter
a = major axis
b = minor axis

$$\boldsymbol{A = \pi \times a \times b}$$

A = Area
a = major axis
b = minor axis

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1. Skywalker's class constructed ten water bottle rockets.

   A. Calculate the circumference of the two-liter bottle nozzle.
   Given information: the diameter of a two-liter bottle nozzle D_{Nozzle} = 22.225 mm

   B. Determine the ratio of the circumference of the two-liter bottle to the diameter.
   Given Information: D_{Nozzle} = 22.225 mm, $C = 2\pi r = \pi D$

   C. Determine the area of the two-liter bottle nozzle. $A_{nozzle} = \pi r^2$

   D. Determine the area and perimeter of each fin from all ten water bottle rockets in the chart below. Complete the chart below

Table 1.1: Ten Water Bottle

Rocket #	**Fin Shape**	**Base (cm)**	**Height (cm)**	**Major Axis (cm)**	**Minor Axis (cm)**	**Perimeter (cm)**	**Area (cm^2)**
1	Right Triangle	6	10	-	-		
2	Right Triangle	6	8	-	-		
3	Rectangle	6	10	-	-		
4	Rectangle	6	8	-	-		
5	Right Trapezoid	10	3	-	-		
6	Right Trapezoid	10	4	-	-		
7	Trapezoid	12	3	-	-		
8	Trapezoid	12	4	-	-		
9	Ellipse	-	-	10	6		
10	Ellipse	-	-	10	4		

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2. Calculate the maximum height of the projectile motion of the water bottle rocket.
Use this online calculator to determine the maximum height. You will need velocity (m/s), launch angle, initial height (m). Check your units!

Rocket #	Fin Shape	Velocity (m/s)	Launch Angle	Initial Rocket Height H_o (cm)	Maximum Height H_{Max} (m)
1	Right Triangle	6	90°	76	
2	Rectangle	9	90°	76	
3	Right Trapezoid	10	90°	76	
4	Trapezoid	12	90°	76	
5	Ellipse	17	90°	76	

3. A water bottle rocket is launched straight up from the top of a 24 ft tall building with an initial speed of 92 ft/s. The function $h(t)$ can model the water bottle rocket's height as a function of time $h(t)$ = – 16t2 + 92t + 24. How long will it take for the water bottle rocket to hit the ground if the height was equal to 95 ft?

$$\boldsymbol{h(t) = -16t^2 + vt + h}$$

h is the height
t is the hangtime in seconds
v is the velocity in ft/s

Use the quadratic equation to calculate the time. Use a graphing calculator to verify your results. Calculate the hangtime of a water bottle rocket using the function. (Hint: $h(t)$ = 0 is when the water bottle rocket hits the ground).

Question 4 & 5 Example: A water bottle rocket is launched at an angle of θ = 90° above the horizontal. Initial time t = 0 and initial velocity of v_0=36 ft/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.

Use Figure 1 to answer the questions below.

- Find the water bottle rocket maximum height attained in after launch in feet.
- Find the water bottle rocket hangtime in seconds.

When vertical velocity = 0

$$y_{max} = -\frac{1}{2}gt^2 + (v_0 sin\theta)t$$

Vertical velocity is:

$$\frac{dy}{dt} = -gt + (v_0 sin\theta) = 0$$

Gravity is:

$$g = 32\frac{ft}{s^2} = 9.8\frac{m}{s^2}$$

Click to see an example problem.

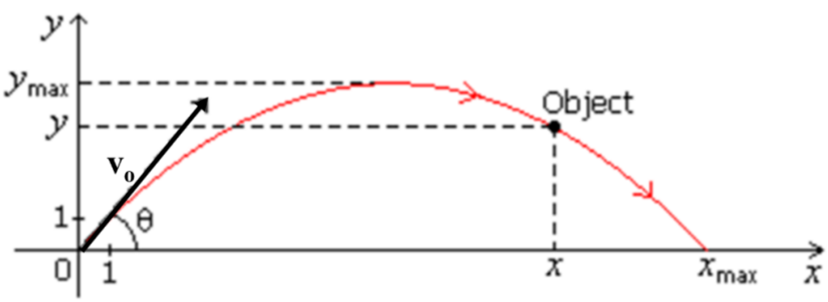

Figure 1: Trajectory of Projectile Motion
The arrow head indicated the direction of motion

Example A: Determine the maximum height of the water bottle rocket.

Equation 1:

$$y_{max} = -\frac{1}{2}gt^2 + (v_0 sin\theta)t$$

Equation 2:

$$\frac{dy}{dt} = -gt + (v_0 sin\theta)$$

Step 1 – Set Equation 2 equal to 0.

$$\frac{dy}{dt} = -gt + (v_0 sin\theta) = 0$$

$$gt - gt + v_0 sin\theta = (0 + gt)$$

$$\frac{v_0 sin\theta}{g} = \frac{\cancel{g}t}{\cancel{g}}$$

Equation 3:

$$t = \frac{v_0 sin\theta}{g}$$

Step 2 – Substitute Equation 3 into Equation 1.

$$y_{max} = \frac{1}{2}gt^2 + (v_0 sin\theta)t$$

$$y_{max} = -\frac{1}{2}g\left(\frac{v_0 sin\theta}{g}\right)^2 + (v_0 sin\theta)(\frac{v_0 sin\theta}{g})$$

$$y_{max} = -\frac{1}{2}\frac{\cancel{g}}{g^{\cancel{2}}}(v_0 sin\theta)^2 + \frac{(v_0 sin\theta)^2}{g}$$

$$y_{max} = -\frac{1}{2}\frac{(v_0 sin\theta)^2}{g} + \frac{(v_0 sin\theta)^2}{g}$$

Equation 4:

$$y_{max} = \frac{1}{2}\frac{(v_0 sin\theta)^2}{g}$$

Step 3 – Substitute the given values into Equation 4 and solve for max height.

$$y_{max} = \frac{1}{2}\frac{(v_0 sin\theta)^2}{g} = \frac{1}{2}\frac{\left(36\frac{ft}{s}sin(90°)\right)^2}{32\frac{ft}{s^2}}$$

$$= \frac{1}{2}\frac{\left(36\frac{ft}{s}(1)\right)^2}{32\frac{ft}{s^2}} = 20.3\ ft$$

Example B: Determine the maximum hangtime of the water bottle rocket.
Equation 1:

$$y_{max} = -\frac{1}{2}gt^2 + (v_0 sin\theta)t$$

Step 1 – Substitute zero into Equation 1 and solve for t.

$$y_{max} = -\frac{1}{2}gt^2 + (v_0 sin\theta)t = 0$$

$$0 = -\frac{1}{2}gt^2 + (v_0 sin\theta)t$$

$$0 = t\left(-\frac{1}{2}gt + (v_0 sin\theta)\right)$$

$$\frac{1}{2}gt = (v_0 sin\theta)$$

$$\left(\frac{2}{g}\right)\frac{1}{2}gt = (v_0 sin\theta)\left(\frac{2}{g}\right)$$

$$t_{max} = \frac{2(v_0 sin\theta)}{g}$$

Step 2 – Substitute in given values.

$$t_{max} = \frac{2(v_0 sin\theta)}{g} = \frac{2\left(36\frac{ft}{s}sin(90°)\right)}{32\frac{ft}{s^2}}$$

$$t_{max} = 2.3s$$

4. A water bottle rocket is launched at an angle of θ = 90° above the horizontal. Initial time t = 0 and initial velocity of v_0=92 ft/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.
Use the figure below to answer the questions

- Find the water bottle rocket maximum height attained in after launch in feet.
- Find the water bottle rocket hangtime in seconds.

5. A water bottle rocket is launched at an angle of θ = 90° above the horizontal. Initial time t = 0 and initial velocity of v_0=28m/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.
Use the figure below to answer the questions

- Find the water bottle rocket maximum height attained in after launch in feet.
- Find the water bottle rocket hangtime in seconds.

For questions 4 & 5:
When vertical velocity = 0

$$\boldsymbol{y_{max} = -\frac{1}{2}gt^2 + (v_0 sin\theta)t}$$

Vertical velocity is:

$$\boldsymbol{\frac{dy}{dt} = -gt + (v_0 sin\theta) = 0}$$

Gravity is:

$$\boldsymbol{g = 32\frac{ft}{s^2} = 9.8\frac{m}{s^2}}$$

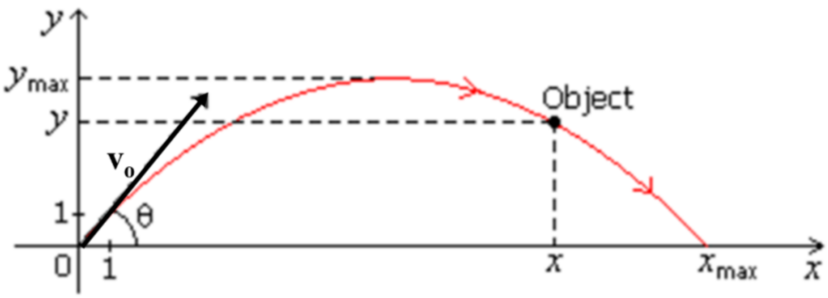

Figure 1: Trajectory of Projectile Motion

The arrow head indicated the direction of motion

Question 6 Examples:

The equation for the mass flow rate is the following:

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$

$\dot{m}$ is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are kg/s.

A is the nozzle area (hint: it's a circle) of a two-liter bottle.

$\mathbf{C_d}$ is the discharge coefficient, a dimensionless constant based on nozzle shape (circle) and flow conditions.

$$C_d = 0.98$$

ρ_{H_2O} is the density of water

$$\rho_{H_2O} = 998\,\frac{kg}{m^3}$$

ΔP_{Nozzle} is the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$
$$\Delta P_{Nozzle} = P_s - P_{atm}$$

P_{supply} is the supply pressure, which ALL water bottle rockets are launched with the following pressure:

$$P_{supply} = 70\ psi$$

Below are unit conversation for pressure are the following:

$$14.7\ psi = 1\ atm$$
$$14.7\ psi = 1.013529 \times 10^5\ Pa$$
$$14.7\ psi = 760\ mmHg$$

P_{atm} is the atmospheric pressure a constant, which is the following value:

$$14.7\ psi = 1.013529 \times 10^5\ Pa$$

Below are unit conversations for mass flow rate are the following:

$$1\ Pa = 1\ \frac{N}{m^2} \qquad 1\ N = 1\ \frac{kg \cdot m}{s^2}$$

$$1\ psi = 1\ \frac{lb_f}{in^2} \qquad 1\ N = 0.224809\ lb_f$$

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Example: What is the mass flow rate for a water bottle rocket with a nozzle diameter of 0.84 inches with a supply pressure of 100 psi, and atmospheric pressure of 760 mmHg?

Please determine the following:

A. Find the area of nozzle of 2-liter bottle in m^2
B. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m^2
C. Determine what is the mass flow in kg/s

A. Find the area of nozzle of 2-liter bottle convert to m^2

Given Information: D_{Nozzle} = 0.84 $inches$

$$A_{nozzle} = r^2 \times \pi$$

$$r_{nozzle} = \frac{D_{nozzle}}{2} = \frac{0.84in}{2} = 0.42in$$

Convert the radius to meters from inches

$$r_{nozzle} = 0.42\cancel{in} \times \frac{2.54\cancel{cm}}{1\cancel{in}} \times \frac{1m}{100\cancel{cm}} = 1.2 \times 10^{-2}m$$

$$A_{nozzle} = (1.2 \times 10^{-2}m)^2 \times \pi = 4.5 \times 10^{-4}m^2$$

B. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m2 Given Information:

P_{Supply}=100psi $Patm$=760$mmHg$ 14.7psi=760$mmHg$ 14.7psi=1.013529 × $10^5$$Pa$

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$

$$P_{supply} = 100psi \times \frac{1.013529 \times 10^5 Pa}{14.7psi} = 69 \times 10^3 Pa$$

$$P_{supply} = 69 \times 10^3 \cancel{Pa} \times \frac{1N}{1\cancel{Pa} \cdot m^2} = 6.9 \times 10^5 \frac{N}{m^2}$$

$$P_{atm} = \cancel{760mmHg} \times \frac{\cancel{14.7psi}}{\cancel{760mmHg}} \times \frac{1.013529 \times 10^5 Pa}{\cancel{14.7psi}}$$

$$P_{atm} = 1.013529 \times 10^5 Pa$$

$$P_{atm} = 1.013529 \times 10^5 \cancel{Pa} \times \frac{1N}{1\cancel{Pa} \cdot m^2} = 1.013529 \times 10^5 \frac{N}{m^2}$$

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$

$$\Delta P_{Nozzle} = 6.9 \times 10^5 \frac{N}{m^2} - 1.013529 \times 10^5 \frac{N}{m^2}$$

$$\Delta P_{Nozzle} = 5.9 \times 10^5 \frac{N}{m^2}$$

C. Determine what is the mass flow in kg/s

Given Information: C_d = 0.98 ρ_{H2O} = 998 kg/m^3

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{2 \times 998 \frac{kg}{m^3} \times (5.9 \times 10^5 \frac{N}{m^2})}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{1.2 \times 10^9 \frac{kg \cdot \cancel{N}}{m^5} \times \frac{kg \cdot m}{1\cancel{N} \cdot s^2}}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{1.2 \times 10^9 \frac{kg^2}{m^4 \cdot s^2}}$$

$$\dot{m} = 4.5 \times 10^{-4} \cancel{m^2} \times 3.5 \times 10^4 \frac{kg}{\cancel{m^2} \cdot s} = 15 \frac{kg}{s}$$

6. Determine the mass flow rate of the water bottle rocket:

Step 1: Convert the nozzle diameter from centimeters to meters. (1000 mm = 1 cm)
D_{Nozzle}= 22.225 mm

Step 2: Determine the area of the nozzle of the two-liter bottle.

$$A_{nozzle} = r^2 \times \pi$$

Step 4: Determine the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

$$P_{supply} = 70psi \qquad P_{atm} = 14.7psi$$

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$

Step 5: Convert $\Delta PNozzle$ from psi to Pa.

$$14.7\ psi = 1.013529 \times 10^5\ Pa$$

$$1\ Pa = 1\ \frac{N}{m^2}$$

Step 6: Calculate the mass flow rate for the bottle.
Demonstrate that the dimensions cancel out to give you mass units per time units.

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$

$$C_d = 0.98 \qquad \rho_{H_2O} = 998 \frac{kg}{m^3}$$

Question 7 Example

The equation for the exit velocity is flow rate is the following:

$$\boldsymbol{V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})}}$$

$\boldsymbol{V_{Exit}}$ is the exit velocity of the water from the water bottle rocket, and units are m/s.
$\boldsymbol{\dot{m}}$ is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are kg/s.
A is the nozzle area (hint: it's a circle) of a two-liter bottle.
$\boldsymbol{\rho_{H_2O}}$ is the density of water

$$\boldsymbol{\rho_{H_2O} = 998\frac{kg}{m^3}}$$

Example: What is the exit velocity of the water from the rocket? The answer should be in m/s.
Given Information

$$\boldsymbol{\rho_{H_2O} = 998\frac{kg}{m^3}} \qquad \boldsymbol{\dot{m} = 15\frac{kg}{s}} \qquad \boldsymbol{A_{nozzle} = 4.5 \times 10^{-4} m^2}$$

$$\boldsymbol{V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})} = \frac{15\frac{\cancel{kg}}{s}}{(998\frac{\cancel{kg}}{\cancel{m^3}} \times 4.5 \times 10^{-4}\cancel{m^2})} = 33\frac{m}{s}}$$

7. What is the exit velocity in m/s of the water bottle rocket in problem 6?
Show your work!

$$\boldsymbol{V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})}}$$

$$\boldsymbol{\rho_{H_2O} = 998\frac{kg}{m^3}}$$

8. Calculate the following using the measurements from your SECME team's water bottle:

   A. The diameter of the water bottle rocket nozzle used to construct your rocket convert from centimeters to inches.

$$\boldsymbol{D_{nozzle}} =$$

   B. The area of the water bottle rocket nozzle used to construct your rocket convert from meters to feet.

$$\boldsymbol{A_{nozzle}} =$$

   C. Determine the pressure drop across the water bottle rocket nozzle or throat in Pa, $mm\ Hg$, and $torr$.

$$\boldsymbol{\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}}$$

   D. Determine the mass rate of the water bottle rocket used to construct your rocket in both kg/s and lbf.

$$\boldsymbol{\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}}$$

$$\boldsymbol{\dot{m}} =$$

   E. Determine the exit velocity of the water bottle rocket used to construct your rocket in m/s to mph = $miles/hour$.

$$\boldsymbol{V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})}}$$

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CALCULATION SHEET

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