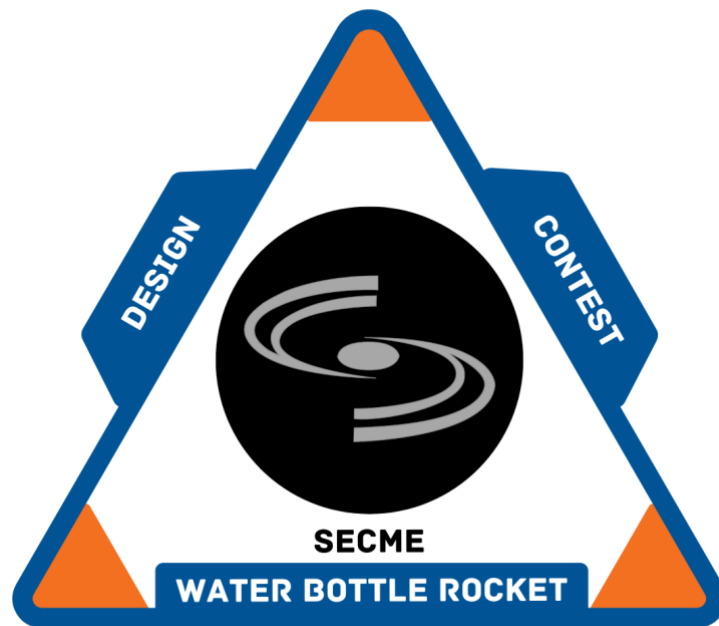


Water Bottle Rocket Design Contest Calculation Exercises



Middle/High School Division
6th – 8th Grade



Water Bottle Rocket Design Contest Calculation Exercises
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Question 1 & 2 Example:

The **distance around a circle** is known as the **circumference**. The **diameter** is the **distance across the circle**, which goes through the circle's center.

$$C = \pi \times D = \pi \times (2 \times r)$$

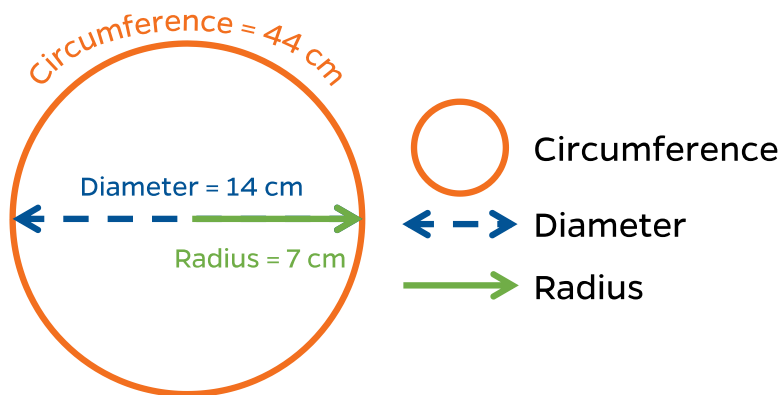
C = Circumference

π = 22/7 or 3.14, Greek Symbol pi

r = radius of a circle, $r = \frac{1}{2} D$

D = diameter of a circle, $D = (r+r) = 2 \times r = 2r$

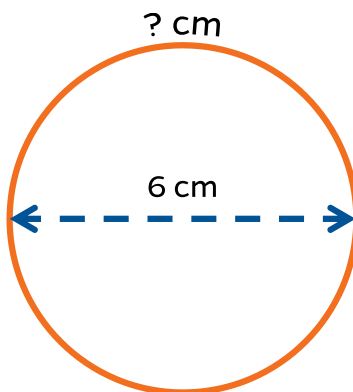
The example below shows the circle circumference is **44 cm**, the diameter is **14 cm**, and the radius is **7 cm**.





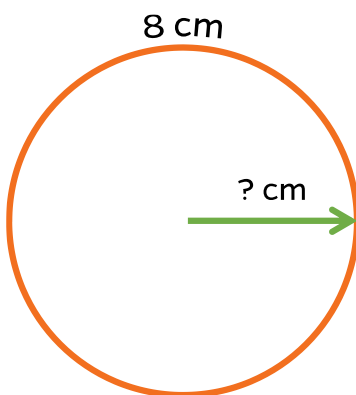
Water Bottle Rocket Design Contest Calculation Exercises
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1. What is the circumference of a circle with a diameter of 6 cm? ($\pi = 3.14$)



Circumference = ____ cm

2. What is the radius of a circle with a circumference of 8cm? ($\pi = 3.14$)



Radius = ____ cm

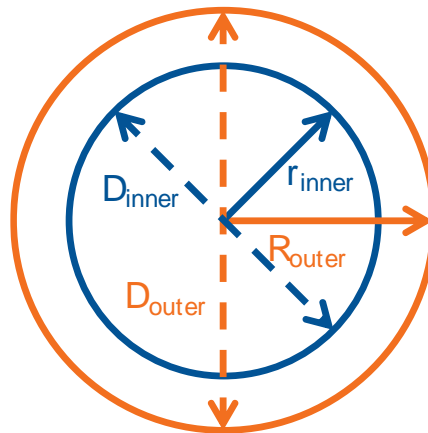


Water Bottle Rocket Design Contest Calculation Exercises
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Question 3 Example:

An annulus is a shape made of two circles, an inner circle, and an outer circle. This results in a ring shape or circle with the center removed. We see this shape when looking at the walls of the body of the water bottle rocket.

This shape consists of 2 circles, inner and out. We can measure and calculate the circumference, radius, and diameter of both inner and outer circles, shown by C_{inner} , D_{inner} and r_{inner} in blue and C_{outer} , D_{outer} , R_{outer} in orange.



The equation for the circumference of a circle is:

$$C = \pi \times D = \pi \times (2 \times r)$$

C = Circumference

$\pi = 22/7$ or 3.14, Greek Symbol pi

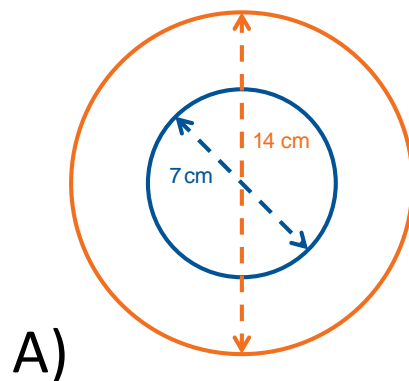
r = radius of a circle, $r = \frac{1}{2} D$

D = diameter of a circle, $D = (r+r) = 2 \times r = 2r$



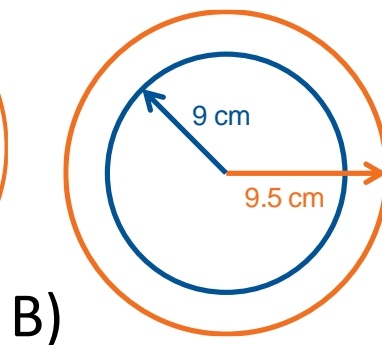
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3. What is the circumference of the inner circle and the outer circles for the annulus shapes below? (Use 3.14 for the value of π)



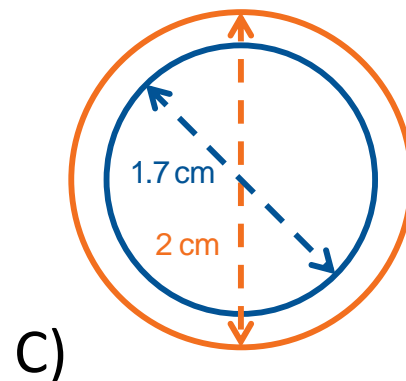
$C_{\text{inner}} = \underline{\hspace{2cm}}$ cm

$C_{\text{outer}} = \underline{\hspace{2cm}}$ cm



$C_{\text{inner}} = \underline{\hspace{2cm}}$ cm

$C_{\text{outer}} = \underline{\hspace{2cm}}$ cm



$C_{\text{inner}} = \underline{\hspace{2cm}}$ cm

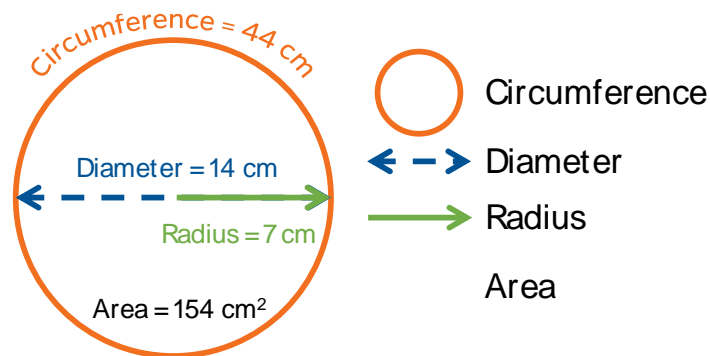
$C_{\text{outer}} = \underline{\hspace{2cm}}$ cm



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Question 4 Example:

The **area** of a shape is the space within or enclosed by the perimeter of a shape. The area of a circle is the space in the middle of the circle shaded light orange.



We can calculate the area of a circle using this formula:

$$A = \pi \times r^2$$

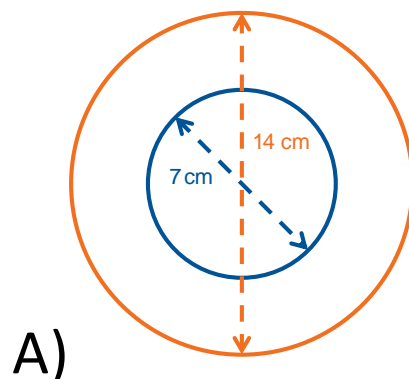
A = Area

π = 22/7 or 3.14, Greek Symbol pi

r = Radius of a circle, $r = \frac{1}{2} D$

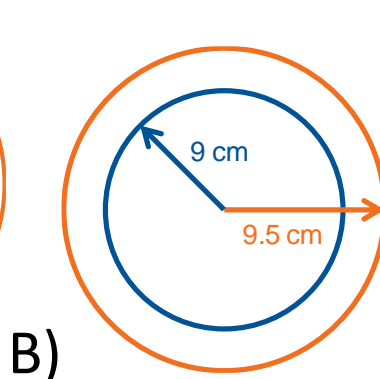
D = Diameter of a circle, $D = (r+r) = 2 \times r = 2r$

4. What is the **area** of the inner circle and the outer circles in the annulus shapes below? (Use 3.14 for the value of π)



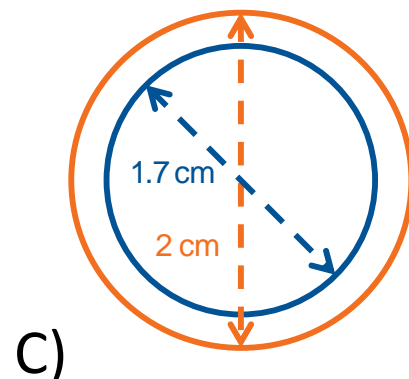
$A_{\text{inner}} = \underline{\hspace{2cm}} \text{ cm}$

$A_{\text{outer}} = \underline{\hspace{2cm}} \text{ cm}$



$A_{\text{inner}} = \underline{\hspace{2cm}} \text{ cm}$

$A_{\text{outer}} = \underline{\hspace{2cm}} \text{ cm}$



$A_{\text{inner}} = \underline{\hspace{2cm}} \text{ cm}$

$A_{\text{outer}} = \underline{\hspace{2cm}} \text{ cm}$



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Question 5 Examples:

PERIMETER

The perimeter of a figure is the total distance around a shape; in other words, it is the sum of the lengths of the sides of a shape.

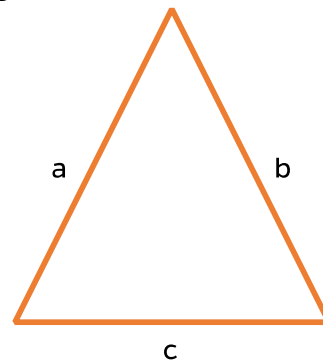
The equation for the perimeter of a triangle is the following:

$$P = a + b + c$$

a = side length

b = side length

c = side length

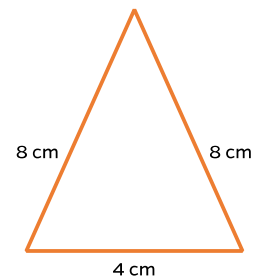


Example A: What is the perimeter of the triangle?

$$P = a + b + c$$

$$P = 8 \text{ cm} + 8 \text{ cm} + 4 \text{ cm}$$

$$P = 20 \text{ cm}$$

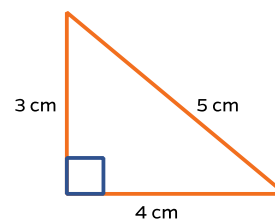


Example B: What is the perimeter of the right triangle?

$$P = a + b + c$$

$$P = 3 \text{ cm} + 5 \text{ cm} + 4 \text{ cm}$$

$$P = 12 \text{ cm}$$

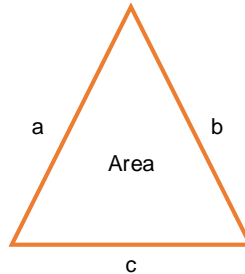




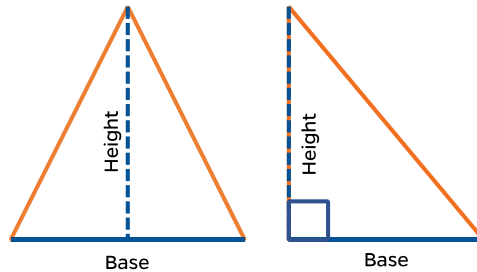
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AREA

The **area** of a shape is the space within or enclosed by the perimeter of a shape. The area of a triangle is the space between sides a, b, and c, shaded blue in the triangle below.



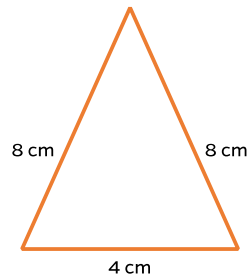
We can calculate the area of a triangle using this formula:



$$\text{Area (A)} = \frac{1}{2} b \times h$$

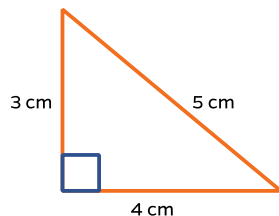
b = base of the triangle
h = height of the triangle

Example A: What is the area of the triangle fin?



$$\begin{aligned}\text{Area (A)} &= \frac{1}{2} b \times h \\ A &= \frac{1}{2} (4 \text{ cm}) \times 7.75 \text{ cm} \\ A &= 15.5 \text{ cm}^2\end{aligned}$$

Example B: What is the area of the triangle fin?



$$\begin{aligned}\text{Area (A)} &= \frac{1}{2} b \times h \\ A &= \frac{1}{2} (4 \text{ cm}) \times 3 \text{ cm} \\ A &= 6 \text{ cm}^2\end{aligned}$$



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PYTHAGOREAN THEOREM

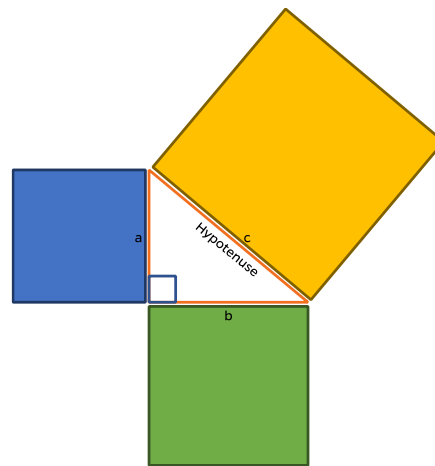
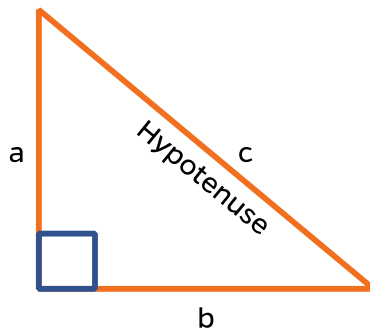
Pythagoras' Theorem can be applied when a triangle has a right angle (90°). You are able to use the equation:

$$a^2 + b^2 = c^2$$

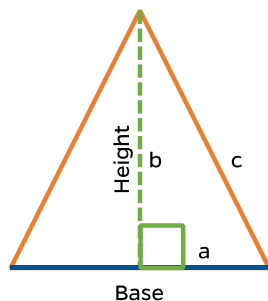
to determine the value of one side of the triangle if you have the values for the other 2. This theorem shows that, if a square was made for each side using the length, the area of the 2 sides (a & b) would equal the area of the hypotenuse (c).

("Pythagoras' Theorem," 2020)

Pythagoras' Theorem. (n.d.). Retrieved October 26, 2020, from <https://www.mathsisfun.com/pythagoras.html> "



You will need to use this equation to calculate the height of all triangles that are not right triangles. To find the height, you create a right triangle by making a center line down the middle of the triangle, this will become side b, your missing side. Side a is half of the base and side c is the hypotenuse, the long edge of the new triangle. Use the Pythagorean theorem to find side b, the height of the triangle.



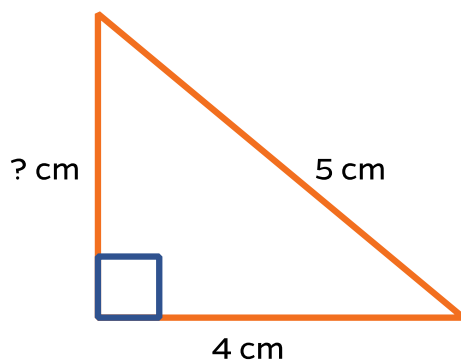
- 1) Create a line down the center, that is the height of the triangle.
- 2) Side a = ½ base, since the triangle is cut in half
- 3) Side c is the hypotenuse
- 4) Isolate b in the equation to determine the height. Solve for b.

$$a^2 + b^2 = c^2 \qquad b^2 = c^2 - a^2$$



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Example A: Calculate the a) length of the missing side, b) the perimeter, and c) the area of the right triangle rocket fin below.



Part A: Calculate the missing side of the right triangle.

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$b^2 = c^2 - a^2$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{(5cm)^2 - (4cm)^2} = \sqrt{25cm^2 - 16cm^2} = \sqrt{9cm^2}$$

$$b = 3cm$$

Part B: Calculate the perimeter of the triangle?

$$P = a + b + c$$

$$P = 4cm + 3cm + 5cm = 12cm$$

Part C: Calculate the area of the triangle.

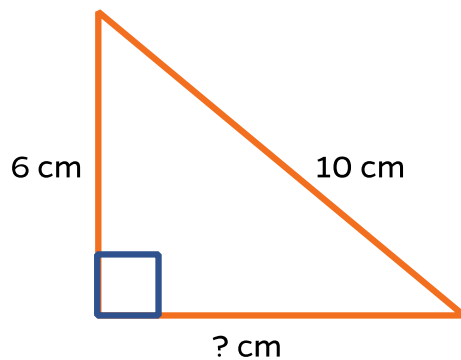
$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times (4cm) \times (3cm) = 6cm^2$$



Water Bottle Rocket Design Contest Calculation Exercises
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5. For both right triangle rocket fins, determine the a) missing side, b) perimeter and c) the area of the triangular fin. Show your work.

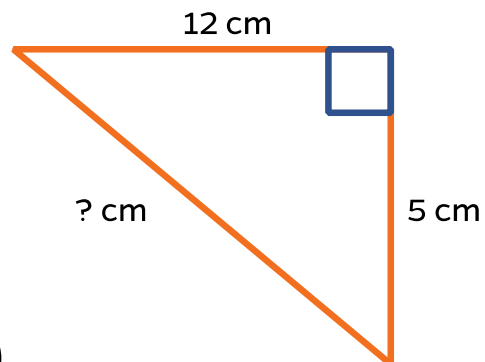


A)

Missing Side (a) = ____ cm

Perimeter = ____ cm

Area = ____ cm²



B)

Missing Side (c) = ____ cm

Perimeter = ____ cm

Area = ____ cm²



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Question 6 Examples:

MEAN

The mean is the average value of the number set, or the "central" value of the number set ("How to Find the Mean," 2020).

How to Find the Mean. (n.d.). Retrieved November 21, 2020, from <https://www.mathsisfun.com/mean.html>

To find the **mean** of set of numbers, follow these steps:

- A. Add up all the values (the numbers) in the number set to find the **sum** or **total** of the number set.
- B. **Count** the number of **values** in the number set.
- C. Divide the sum (or total) calculated in Step A by the number of values counted in Step B to calculate the **mean**.

Example A: What is the mean of this number set? 9, 13, 1, 15, 8, 4, 6

Step A: Find the sum of the number set.

$$\text{Sum} = 9 + 13 + 1 + 15 + 8 + 4 + 6$$

$$\text{Sum} = 56$$

Step B: Count the number of values in the number set. Number of Values = 7

Step C: Divide the Sum by the Number of Values. Mean = $56/7$ **Mean = 8**

MEDIAN

The median is the middle number of a set of numbers ("How to Find the Median Value," 2020).

How to Find the Median. (n.d.). Retrieved November 21, 2020, from <https://www.mathsisfun.com/median.html>

To find the **median** follow these steps:

- A. Rearrange the numbers in the number set to be in numerical order from least to greatest
- B. Find the number that is in the middle of the number set. Do this by crossing off 1 number from each end until you get to the middle of the number set.
 - a. If you have a number set with an even number of numbers, complete Step B until you get to the last 2 numbers in the middle. Calculate the B of the two middle numbers to find the median.

Example A: What is the median of this number set?

9, 13, 1, 15, 8, 4, 6

Step A: Rearrange from lowest to highest:

1, 4, 6, 8, 9, 13, 15

Step B: Cross off numbers from each end to find the middle:

~~1~~, ~~4~~, ~~6~~, **8**, **9**, ~~13~~, ~~15~~

Median = 8

Example B: What is the median of this number set?

21, 9, 13, 1, 15, 8, 4, 6

Step A: Rearrange from lowest to highest:

1, 4, 6, 8, 9, 13, 15, 21

Step B: Cross off numbers from each end to find the middle:

~~1~~, ~~4~~, ~~6~~, **8**, **9**, ~~13~~, ~~15~~, ~~21~~

Step B(a): Find the mean between the two middle numbers:

$$\text{Median} = \frac{8+9}{2} = \frac{17}{2} = 8.5$$



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MODE

The mode is the number that appears most often(frequent) in the number set ("How to Find the Mode or Modal Value," 2020). A number set having two modes is called "bimodal," and a number set having more than two modes is called "multimodal."

How to Find the Mode. (n.d.). Retrieved November 21, 2020, from <https://www.mathsisfun.com/mode.html>

To find the **mode** follow these steps:

- Put the numbers in numerical order from least to greatest.
- Count the number of times each number appears in the number set.
- The number that appears the most is the **mode**.

Example A: What is the mode of the number set? 9, 15, 13, 1, 15, 8, 4, 15, 8

Step A: Rearrange from lowest to highest: 1, 4, 8, 8, 9, 13, 15, 15, 15

Step B: Count the number of times each number appears:

1 Time: 1, 4, 9, 13 2 Times: 8 3 Times: 15

Step C: Number repeated the most is the mode: **Mode = 15**

Example 2: What is the mode of the number set? 1, 9, 15, 13, 1, 15, 8, 4, 15, 1

Step A: Rearrange from lowest to highest: 1, 1, 1, 4, 8, 9, 13, 15, 15, 15

Step B: Count the number of times each number appears:

1 Time: 4, 8, 9, 13 3 Times: 1, 15

Step C: Number repeated the most is the mode: **Modes = 1 & 15**

RANGE

The range is the difference between the lowest and highest values ("The Range (Statistics)," 2020).

The Range (Statistics). (n.d.). Retrieved November 21, 2020, from <https://www.mathsisfun.com/data/range.html>

To find the **range** follow these steps:

- Put the numbers in numerical order from least to greatest.
- Subtract the smallest number in the number set from the largest number in the data set.

Example A: What is the range of this number set? 9, 13, 1, 15, 8, 4, 15

Step A: Rearrange from lowest to highest: 1, 4, 8, 9, 13, 15, 15

Step B: Subtract the smallest number from the largest: Range = 15 – 1

Range = 14



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6. The hangtime is the amount of time the water bottle rocket stays in the air before landing. The hangtimes of eight water bottle rockets are listed below.

Rocket	Hangtime (seconds)
A	7.3
B	9.1
C	5.2
D	7.9
E	8.4
F	9.6
G	7.3
H	8.4

Determine the following using the data in the chart above. Reference the previous examples for help.

a) Longest hangtime = ____ seconds, Rocket ____

b) Shortest hangtime = ____ seconds, Rocket ____

c) Mean hangtime = ____ seconds, Rocket ____

d) Median hangtime = ____ seconds, Rocket ____

e) Mode hangtime = ____ seconds, Rocket ____

f) Range of hangtimes = ____ seconds

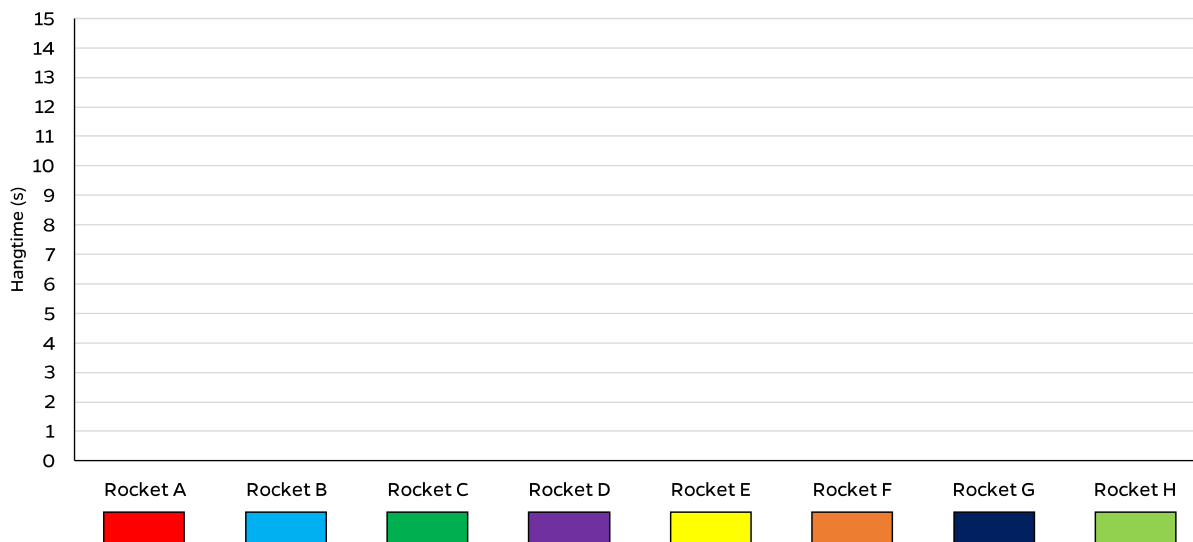


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7. Construct a bar graph to illustrate hangtimes in problem 9 with the space provided for each water bottle rocket using the same colors below the name of each Rocket.

Rocket	Hangtime (seconds)
A	7.3
B	9.1
C	5.2
D	7.9
E	8.4
F	9.6
G	7.3
H	8.4

Water Bottle Rocket Hangtime





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Question 8 Examples:

There are two systems of measurement in science and engineering, the metric system and the imperial system commonly used in the United States. As a scientist and engineer, it is important to be able to convert between the two types of systems.

Within the metric system, there are conversions within units using a series of prefixes, as shown below:

Kilo-	Hecto-	Deka-	-	Dec-	Centi-	Milli-
-10^3	-10^2	-10^1	-10^0	-10^{-1}	-10^{-2}	-10^{-3}
kg, km, kL	hg, hm, hL	dag, dam, daL	g, m, L	dg, dm, dL	cg, cm, cL	mg, mm, mL

Examples:

Measure of Mass/Weight

15 pounds (lbs) to milligrams (mg) 1lb = 453.59g

$$15\cancel{lbs} \times \frac{453.59\cancel{g}}{1\cancel{lbs}} \times \frac{1000mg}{1g} = 6,803,850mg$$

120 grams (g) to kilograms (kg) 1kg = 1000g

$$120\cancel{g} \times \frac{1kg}{1000\cancel{g}} = 0.12kg$$

Measure of Length

25 millimeters (mm) to inches (in.) 1in = 2.54cm

$$25\cancel{mm} \times \frac{1\cancel{m}}{1000\cancel{mm}} \times \frac{100cm}{1\cancel{m}} \times \frac{1in}{2.54cm} = 0.984in$$

18 kilometers (km) to meters (m) 1km = 1000m

$$18\cancel{km} \times \frac{1000m}{1\cancel{km}} = 18,000m$$

Measure of Capacity (fluids)

10 ounces (oz) to liters (L) 1L = 33.8oz

$$16\cancel{oz} \times \frac{1L}{33.8\cancel{oz}} = 0.473L$$

$$45 \text{ liters (L) to milliliters (mL)} \\ 45\cancel{L} \times \frac{1000mL}{1\cancel{L}} = 45,000mL$$

$$18 \text{ kilograms (kg) to milliliters (mL)}$$



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$$18\cancel{kg} \times \frac{1000g}{1\cancel{kg}} \times \frac{1cm^3}{1g} \times \frac{1mL}{1cm^3} = 48,000mL$$

8. Complete the following conversions:

Imperial to Metric

$2.54cm = 1 \text{ in}$

$33.81oz = 1L$

$453.59g = 1lb$

$16 \text{ oz} = \underline{\hspace{1cm}} \text{ liters}$

$14 \text{ lbs} = \underline{\hspace{1cm}} \text{ grams}$

$100 \text{ cm} = \underline{\hspace{1cm}} \text{ inches}$

$18 \text{ cm} = \underline{\hspace{1cm}} \text{ inches}$

$8 \text{ oz} = \underline{\hspace{1cm}} \text{ liters}$

$38 \text{ lbs} = \underline{\hspace{1cm}} \text{ grams}$

$50 \text{ cm} = \underline{\hspace{1cm}} \text{ inches}$

$36 \text{ oz} = \underline{\hspace{1cm}} \text{ liters}$

$4 \text{ lbs.} = \underline{\hspace{1cm}} \text{ grams}$

$6 \text{ cm} = \underline{\hspace{1cm}} \text{ inches}$

$100 \text{ lbs.} = \underline{\hspace{1cm}} \text{ grams}$

$64 \text{ oz} = \underline{\hspace{1cm}} \text{ liters}$

Metric conversions:

Kilo-	Hecto-	Deka-	-	Dec-	Centi-	Milli-
-10^3	-10^2	-10^1	-10^0	-10^{-1}	-10^{-2}	-10^{-3}
kg, km, kL	hg, hm, hL	dag, dam, daL	g, m, L	dg, dm, dL	cg, cm, cL	mg, mm, mL

$970 \text{ cg} = \underline{\hspace{1cm}} \text{ kg}$

$16.5 \text{ cL} = \underline{\hspace{1cm}} \text{ L}$

$2.45 \text{ dam} = \underline{\hspace{1cm}} \text{ hm}$

$0.2851 \text{ kg} = \underline{\hspace{1cm}} \text{ mL}$

$1000 \text{ daL} = \underline{\hspace{1cm}} \text{ kL}$

$94.52 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

$3196 \text{ dg} = \underline{\hspace{1cm}} \text{ hg}$

$14 \text{ kL} = \underline{\hspace{1cm}} \text{ mL}$

$16.41 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$



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Question 9 Examples:

The equation for the mass flow rate is the following:

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$

\dot{m} is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are *kg/s*.

A is the nozzle area (hint: it's a circle) of a two-liter bottle.

C_d is the discharge coefficient, a dimensionless constant based on nozzle shape (circle) and flow conditions.

$$C_d = 0.98$$

ρ_{H_2O} is the density of water

$$\rho_{H_2O} = 998 \frac{kg}{m^3}$$

ΔP_{Nozzle} is the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

$$\begin{aligned} \Delta P_{Nozzle} &= P_{supply} - P_{atmospheric} \\ \Delta P_{Nozzle} &= P_s - P_{atm} \end{aligned}$$

P_{supply} is the supply pressure, which ALL water bottle rockets are launched with the following pressure:

$$P_{supply} = 70 \text{ psi}$$

Below are unit conversation for pressure are the following:

$$\begin{aligned} 14.7 \text{ psi} &= 1 \text{ atm} \\ 14.7 \text{ psi} &= 1.013529 \times 10^5 \text{ Pa} \\ 14.7 \text{ psi} &= 760 \text{ mmHg} \end{aligned}$$

P_{atm} is the atmospheric pressure a constant, which is the following value:

$$14.7 \text{ psi} = 1.013529 \times 10^5 \text{ Pa}$$

Below are unit conversations for mass flow rate are the following:

$$1 \text{ Pa} = 1 \frac{N}{m^2} \qquad 1 \text{ N} = 1 \frac{kg \cdot m}{s^2}$$



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$$1 \text{ psi} = 1 \frac{\text{lb}_f}{\text{in}^2}$$

$$1 \text{ N} = 0.224809 \text{ lb}_f$$

Example: What is the mass flow rate for a water bottle rocket with a nozzle diameter of 0.84 inches with a supply pressure of 100 psi, and atmospheric pressure of 760 mmHg?

Please determine the following:

- A. Find the area of nozzle of 2-liter bottle in m^2
- B. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m^2
- C. Determine what is the mass flow in kg/s

A. Find the area of nozzle of 2-liter bottle convert to m^2

Given Information: $D_{\text{Nozzle}} = 0.84 \text{ inches}$

$$A_{\text{nozzle}} = r^2 \times \pi$$

$$r_{\text{nozzle}} = \frac{D_{\text{nozzle}}}{2} = \frac{0.84 \text{ in}}{2} = 0.42 \text{ in}$$

Convert the radius to meters from inches

$$r_{\text{nozzle}} = 0.42 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 1.2 \times 10^{-2} \text{ m}$$

$$A_{\text{nozzle}} = (1.2 \times 10^{-2} \text{ m})^2 \times \pi = 4.5 \times 10^{-4} \text{ m}^2$$

- B. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m^2

Given Information:

$$P_{\text{Supply}} = 100 \text{ psi} \quad P_{\text{atm}} = 760 \text{ mmHg} \quad 14.7 \text{ psi} = 760 \text{ mmHg} \quad 14.7 \text{ psi} = 1.013529 \times 10^5 \text{ Pa}$$

$$\Delta P_{\text{Nozzle}} = P_{\text{supply}} - P_{\text{atmospheric}}$$

$$P_{\text{supply}} = 100 \text{ psi} \times \frac{1.013529 \times 10^5 \text{ Pa}}{14.7 \text{ psi}} = 69 \times 10^3 \text{ Pa}$$



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$$P_{supply} = 69 \times 10^3 Pa \times \frac{1N}{1Pa \cdot m^2} = 6.9 \times 10^5 \frac{N}{m^2}$$

$$P_{atm} = 760mmHg \times \frac{14.7psi}{760mmHg} \times \frac{1.013529 \times 10^5 Pa}{14.7psi}$$

$$P_{atm} = 1.013529 \times 10^5 Pa$$

$$P_{atm} = 1.013529 \times 10^5 Pa \times \frac{1N}{1Pa \cdot m^2} = 1.013529 \times 10^5 \frac{N}{m^2}$$

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$

$$\Delta P_{Nozzle} = 6.9 \times 10^5 \frac{N}{m^2} - 1.013529 \times 10^5 \frac{N}{m^2}$$

$$\Delta P_{Nozzle} = 5.9 \times 10^5 \frac{N}{m^2}$$

C. Determine what is the mass flow in kg/s

Given Information: $C_d = 0.98$ $\rho_{H_2O} = 998 \text{ kg/m}^3$

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{2 \times 998 \frac{kg}{m^3} \times (5.9 \times 10^5 \frac{N}{m^2})}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{1.2 \times 10^9 \frac{kg \cdot N}{m^5} \times \frac{kg \cdot m}{1N \cdot s^2}}$$

$$\dot{m} = 4.5 \times 10^{-4} m^2 \times 0.98 \times \sqrt{1.2 \times 10^9 \frac{kg^2}{m^4 \cdot s^2}}$$



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$$\dot{m} = 4.5 \times 10^{-4} \cancel{m^2} \times 3.5 \times 10^4 \frac{kg}{\cancel{m^2} \cdot s} = 15 \frac{kg}{s}$$



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9. Determine the mass flow rate of the water bottle rocket:

Step 1: Measure the diameter of the two-liter bottle nozzle in centimeters. Figure 1 shows how to measure the two-liter bottle nozzle.



Diameter of Nozzle: _____cm

Step 2: Convert the nozzle diameter from centimeters to meters. Remember problem 8.
(Conversion Hint: 100 *cm* = 1 *m*)

Step 3: Determine the area of the nozzle of the two-liter bottle.

$$A_{nozzle} = r^2 \times \pi$$

Step 4: Determine the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

$$P_{supply} = 70psi \quad P_{atm} = 14.7psi$$

$$\Delta P_{Nozzle} = P_{supply} - P_{atmospheric}$$



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Step 5: Convert ΔP_{Nozzle} from psi to Pa.

$$14.7 \text{ psi} = 1.013529 \times 10^5 \text{ Pa}$$
$$1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}$$

Step 6: Calculate the mass flow rate for the bottle.

Demonstrate that the dimensions cancel out to give you mass units per time units.

$$\dot{m} = A_{Nozzle} \times C_d \sqrt{2 \times \rho_{H_2O} \times \Delta P_{Nozzle}}$$
$$C_d = 0.98$$
$$\rho_{H_2O} = 998 \frac{\text{kg}}{\text{m}^3}$$



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Question 10 Example

The equation for the exit velocity is flow rate is the following:

$$V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})}$$

V_{Exit} is the exit velocity of the water from the water bottle rocket, and units are m/s .

\dot{m} is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are kg/s .

A is the nozzle area (hint: it's a circle) of a two-liter bottle.

ρ_{H_2O} is the density of water

$$\rho_{H_2O} = 998 \frac{kg}{m^3}$$

Example: What is the exit velocity of the water from the rocket? The answer should be in m/s .
Given Information

$$\rho_{H_2O} = 998 \frac{kg}{m^3}$$

$$\dot{m} = 15 \frac{kg}{s}$$

$$A_{nozzle} = 4.5 \times 10^{-4} m^2$$

$$V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})} = \frac{15 \frac{kg}{s}}{(998 \frac{kg}{m^3} \times 4.5 \times 10^{-4} m^2)} = 33 \frac{m}{s}$$

10. What is the exit velocity in m/s of the water bottle rocket in problem 9?
Show your work!

$$V_{Exit} = \frac{\dot{m}}{(\rho_{H_2O} \times A_{Nozzle})}$$



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