MAT282 - Laboratorio de Modelación I

Optimization and visualization of the planning of a three-node electrical network during a day

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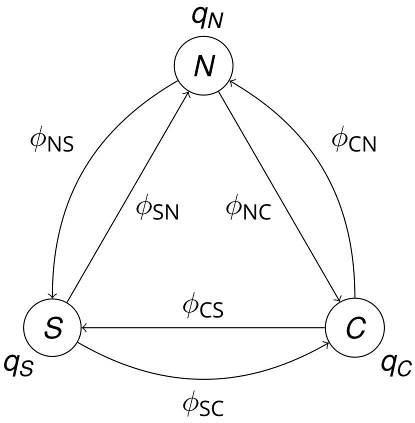
Presentation Overview

Introduction

- The main objective of our work is to model the Chilean electrical network throughout a day, to obtain a planning to be carried out to satisfy the demands at the lowest possible cost.
- To achieve the objective, we developed a code in Python that solves this problem and provides relevant information for the optimal planning of the network during a day through data and graphs.

A graph of the chilean electricity network

- We can model the chilean electricity network as a directed connected graph.



General form of the electrical network problem

$$
\begin{aligned}
\min_{(q,\phi)} \quad & \sum_{i \in I} \mathrm{p}_i(q_i) \\
\text{s.a.} \quad & q_i + \sum_{e \in E_i} (\phi_e - \mathrm{r}(\phi_e)) - \sum_{s \in S_i} \phi_s = D_i, && \forall i \in I \\
& q_i \in [0, Q_i], && \forall i \in I \\
& \phi_j \geq 0, && \forall j \in E
\end{aligned}
$$

Main equation to solve

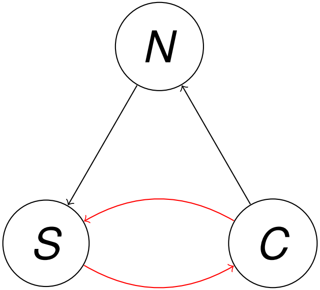
$$
\begin{aligned}
\min_{(q,\phi)} \quad & \mathrm{p}_N(q_N) + \mathrm{p}_S(q_S) + \mathrm{p}_C(q_C) \\
\text{s.a.} \quad & q_N + (1-r)(\phi_{SN} + \phi_{CN}) - (\phi_{NS} + \phi_{NC}) = D_N, \\
& q_S + (1-r)(\phi_{NS} + \phi_{CS}) - (\phi_{SN} + \phi_{SC}) = D_S, \\
& q_C + (1-r)(\phi_{SC} + \phi_{NC}) - (\phi_{CS} + \phi_{CN}) = D_C, \\
& q_N \in [0, Q_N], \\
& q_S \in [0, Q_S], \\
& q_C \in [0, Q_C], \\
& \phi_{NS}, \phi_{SN}, \phi_{SC}, \phi_{CS}, \phi_{NC}, \phi_{CN} \geq 0.
\end{aligned}
$$

Feasible Solutions v/s Optimal Solutions

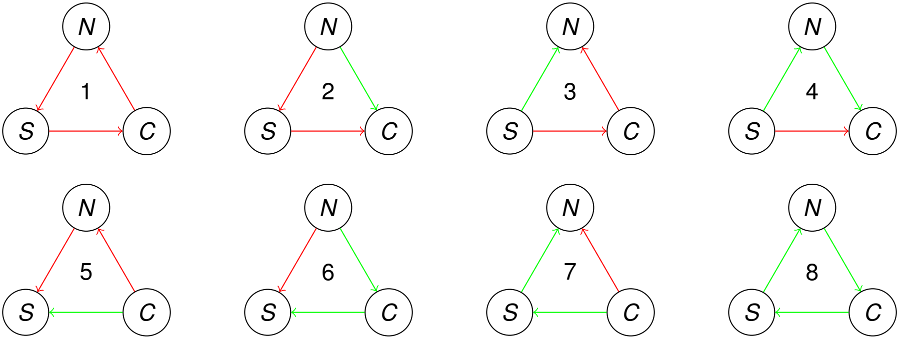
- The function to optimize only depends on the electricity production variables.
- If A and B are two feasible solutions such that $q_i^A \leq q_i^B, \forall i \in I$, then B cannot be an optimal solution.

Mutual flux

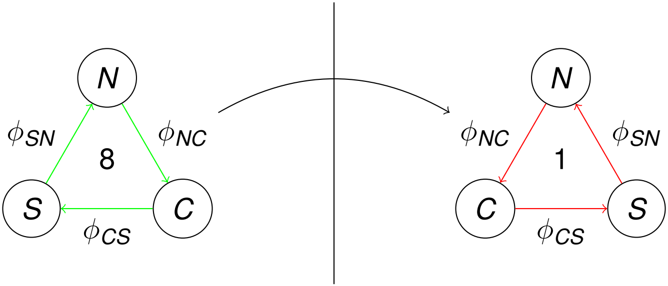
- South and Center nodes are using both directional fluxes at the same time (net energy loss).
- Lowest flux can be subtracted to both fluxes.



Remaining graphs



Graph symmetry



- If an algorithm could solve Graph 1, we could solve Graph 8 via permutations of variables! (keep in mind...)

Splitting the problem

Subproblem 1

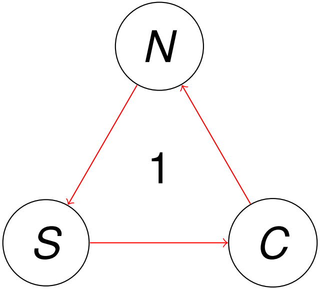
$$
\begin{aligned}
\min_{(q,\phi)} \quad & \mathrm{p}_N(q_N) + \mathrm{p}_S(q_S) + \mathrm{p}_C(q_C) \\
\text{s.a.} \quad & q_N + (1-r)\phi_{CN} - \phi_{NS} = D_N \\
& q_S + (1-r)\phi_{NS} - \phi_{SC} = D_S \\
& q_C + (1-r)\phi_{SC} - \phi_{CN} = D_C \\
& q_N \in [0, Q_N] \\
& q_S \in [0, Q_S] \\
& q_C \in [0, Q_C] \\
& \phi_{NS}, \phi_{SC}, \phi_{CN} \geq 0
\end{aligned}
$$

Subproblem 2

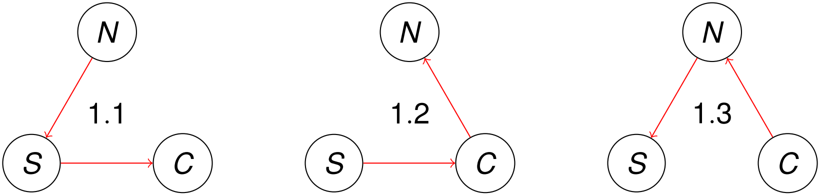
$$
\begin{aligned}
\min_{(q,\phi)} \quad & \mathrm{p}_N(q_N) + \mathrm{p}_S(q_S) + \mathrm{p}_C(q_C) \\
\text{s.a.} \quad & q_N - \phi_{NS} - \phi_{NC} = D_N \\
& q_S + (1-r)\phi_{NS} - \phi_{SC} = D_S \\
& q_C + (1-r)(\phi_{SC} + \phi_{NC}) = D_C \\
& q_N \in [0, Q_N] \\
& q_S \in [0, Q_S] \\
& q_C \in [0, Q_C] \\
& \phi_{NS}, \phi_{SC}, \phi_{NC} \geq 0
\end{aligned}
$$

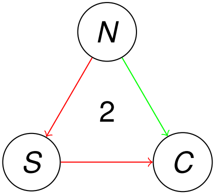
Subproblem 1 - Circular flux

- Very similar to a Mutual Flux, but with a longer chain of fluxes (net energy loss on the whole chain).
- Lowest flux in the chain can be subtracted to all fluxes



Remaining graphs



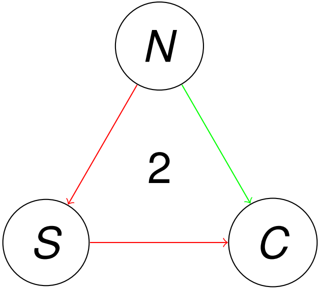


All similar to graph 2!
(with $\phi_{NC} = 0$)

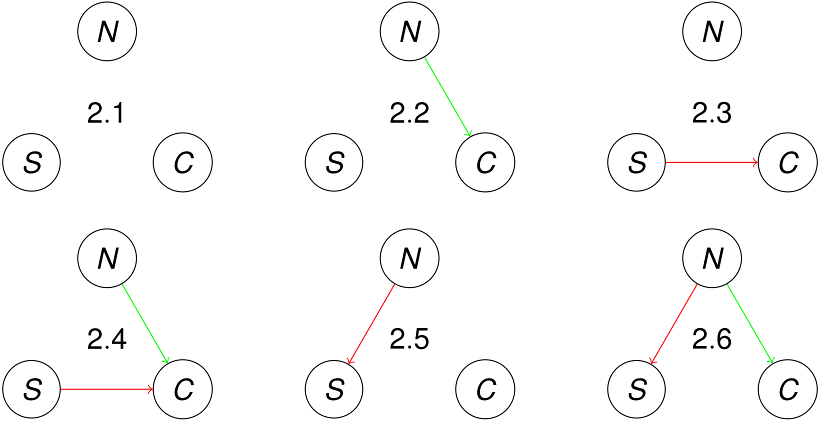
- Subproblem 1 is **contained** in Subproblem 2.

Subproblem 2 - Multiple routes

- There exist two routes to get from the North to Center node!
- Both ϕ_{NS} and ϕ_{SC} cannot be used at the same time



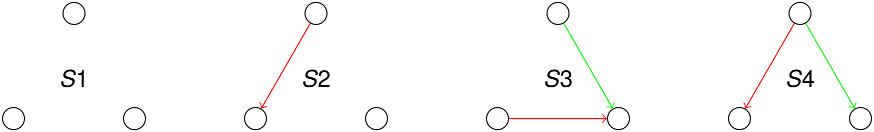
Possible graphs after simplification



- Graphs 2.2, 2.3 and 2.5 are similar.

Possible graphs for a solution

- Since Subproblem 2 can solve the main problem, these are the only configurations an optimal solution can have!



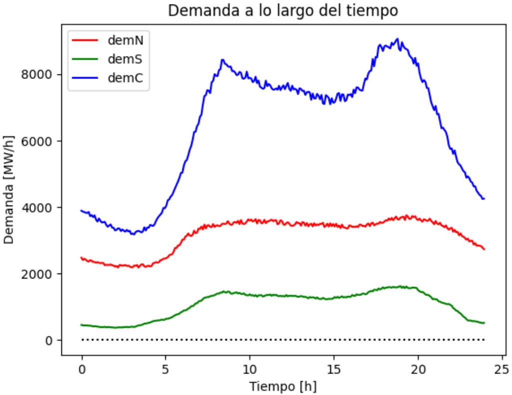
- At maximum, there can only be two active fluxes.

Modeling of the chilean electrical network

- There exists an algorithm that can find an optimal solution to Subproblem 2.
- We created a Python code that applies the algorithm to permutations to find the optimal solution for the main problem.
- This code can be iterated throughout a day to model the production plan of the whole network on a full day!

Demands

- Linear interpolations with added variance that follow electricity demand curve shapes (heaps, lows, proportionality).
- Overall demand: $10500[MW/h]$
- North 30%, South 10%, Center 60%.



Production and Technology

- $Q_N = 6000[MW/h]$, $Q_S = 2000[MW/h]$, $Q_N = 12000[MW/h]$.
- Contributions of each type of technology to the total production on each node:

	Solar	Coal	Gas
North	30%	30%	40%
South	10%	40%	50%
Center	20%	10%	70%

- 0.5% energy loss on flux.

Production cost functions

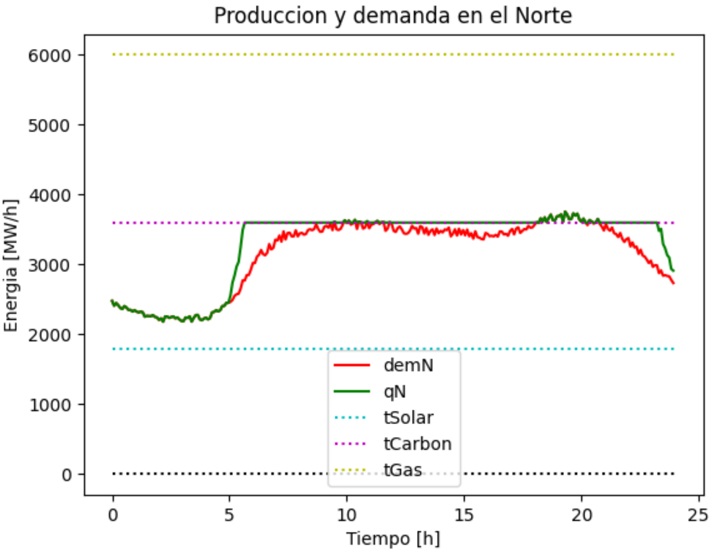
- Solar, Carbon and Gas cost per megawatt: 0\$, 40\$, 80\$.
- With technology contributions, production cost is:

$$p_N(q) = \begin{cases} 0 & , \text{si } 0 \leq q < 1800 \\ 40 \cdot (q - 1800) & , \text{si } 1800 \leq q < 3600 \\ 80 \cdot (q - 3600) + 72000 & , \text{si } 3600 \leq q \leq 6000 \end{cases}$$

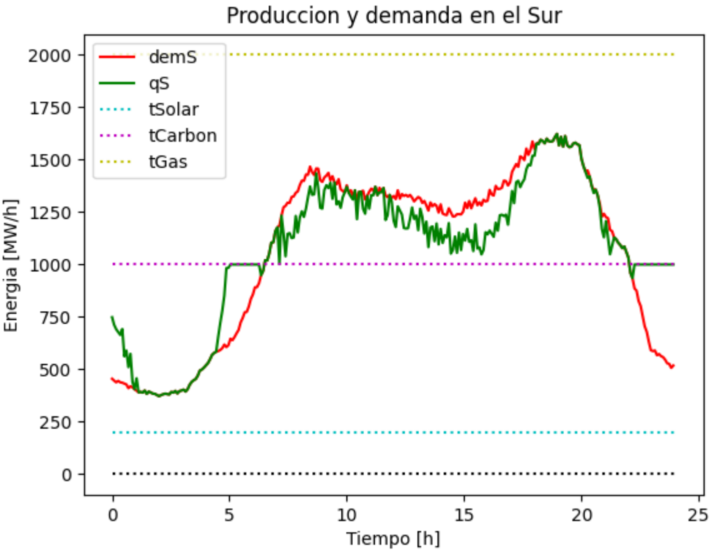
$$p_S(q) = \begin{cases} 0 & , \text{si } 0 \leq q < 200 \\ 40 \cdot (q - 200) & , \text{si } 200 \leq q < 1000 \\ 80 \cdot (q - 1000) + 32000 & , \text{si } 1000 \leq q \leq 2000 \end{cases}$$

$$p_C(q) = \begin{cases} 0 & , \text{si } 0 \leq q < 2400 \\ 40 \cdot (q - 2400) & , \text{si } 2400 \leq q < 3600 \\ 80 \cdot (q - 3600) + 48000 & , \text{si } 3600 \leq q \leq 12000 \end{cases}$$

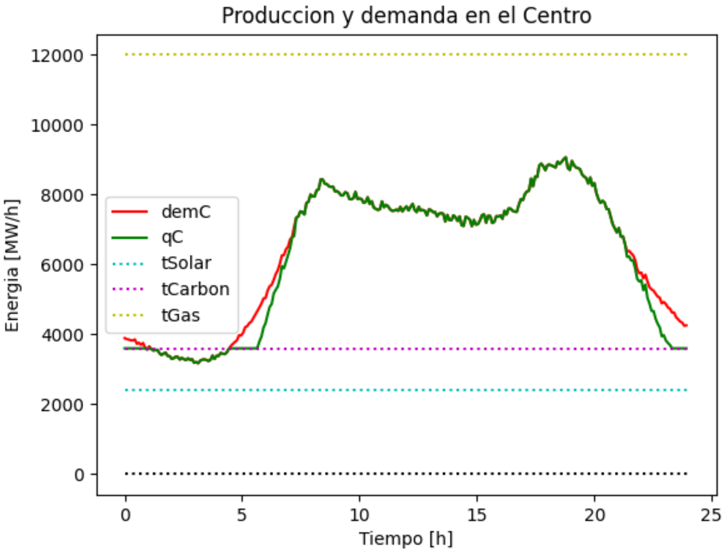
Results: Plots



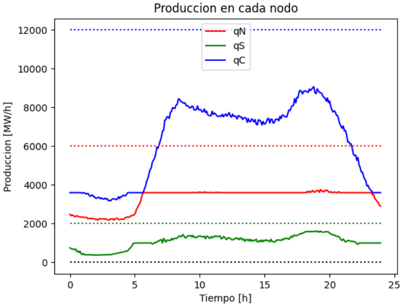
Results: Plots

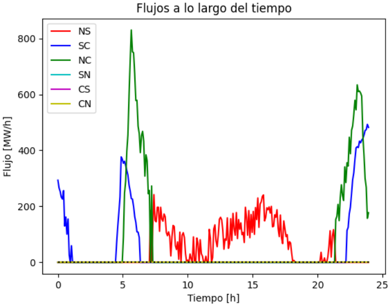


Results: Plots

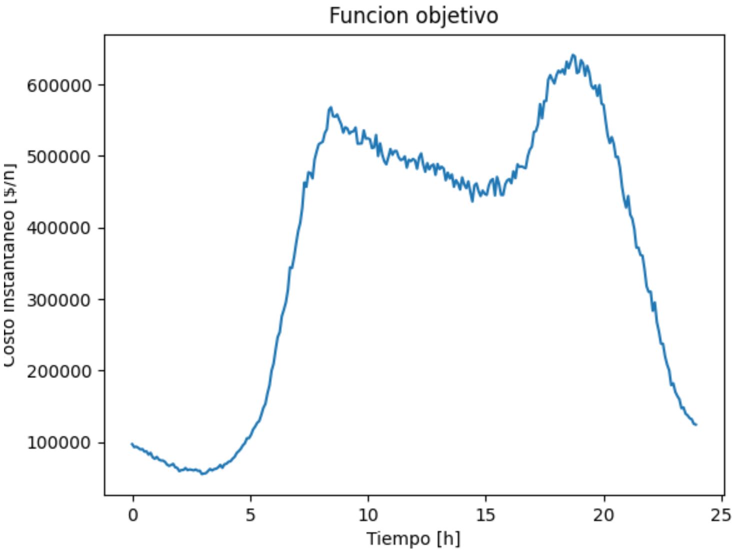


Results: Plots





Results: Plots



Results: Data

- Total cost: 8767565.29 $
- Total production: 255527.73 [MW]
  - North: 79326.11 [MW]
  - South: 25791.57 [MW]
  - Center: 150410.05 [MW]
- Total flux: 4433.55 [MW]
  - North-South: 1225.3 [MW]
  - South-North: 0.0 [MW]
  - South-Center: 1235.01 [MW]
  - Center-South: 0.0 [MW]
  - North-Center: 1973.24 [MW]
  - Center-North: 0.0 [MW]
- Total energy loss: 22.17 [MW]

Conclusions

- We managed to make a simple model the chilean network.
- The model captures trends that happen in real life (North, South and Center dynamic).
- Optimal solution behavior can be understood with plots.
- Fast to execute and apply to similar electrical networking modeling scenarios.

Future work ahead

- Consider the enviromental damage of solutions.
- Variable productions and technology contributions.
- Add more nodes to the network.
- Non-linear cost functions.

References

- Universidad de Chile, Facultad de Ciencias Físicas y Matemáticas, Mariano Vazquez, Estudio de Problemas de Optimización y Equilibrio sobre una Red de Producción Eléctrica, 2023.
- Comisión Nacional de Energía - Reporte Energético Financiero - Vol. Nº10 - 2019.
- U.S. Energy Information Administration, Demand for electricity changes through the day, APRIL 6, 2011.
- © 2023 Society for Industrial and Applied Mathematics, Pollution Regulation for Electricity Generators in a Transmission Network, Nicolás Hernández-Santibáñez, Alejandro Jofré and Dylan Posamma.

Thank you for your attention :)

Questions? Comments?