Block 1.3

Modelling age-effects in the dynamics of infections

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Measles-rubella modelling workshop,
Mumbai 5-8th February 2024

UK Health Security Agency

LONDON SCHOOL of HYGIENE &TROPICAL MEDICINE

Objectives

By the end of this session you should:

- Know why we might need to use age-structured models
- How age-structured models might be set up
- Know how different levels of vaccination coverage may affect:
    a) the age-specific proportion susceptible in the population
    b) the age-specific number of new infections per unit time
    c) the average age at infection.
- Be aware of the effect of mixing patterns on the impact of vaccination

Motives for using age-structured models

Example - Rubella and Congenital Rubella Syndrome

- Infection with rubella during pregnancy may result in the child being born with Congenital Rubella Syndrome (CRS)

- Where the rubella infection incidence is high, the burden of CRS is very low:
    - Most women were infected and became immune in childhood so few are first infected when pregnant

Motives for using age-structured models (1)

Example - Rubella and Congenital Rubella Syndrome

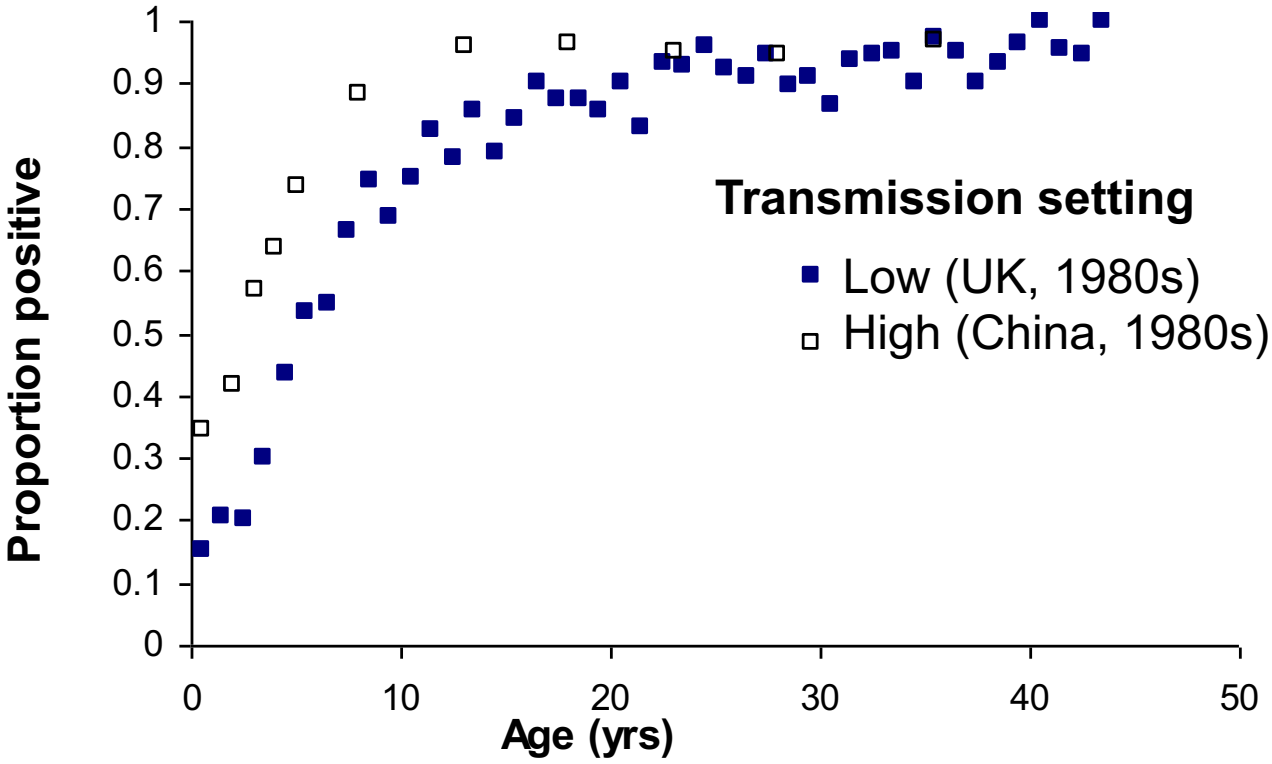
- Introducing vaccination among young children can lead to an increased CRS burden because:
  - vaccination -> ↓ prevalence of infectious people

    => ↓ opportunity for infection (i.e. ↓ force of infection)
    => a large proportion of unvaccinated people may reach child-bearing age still susceptible and may then be infected

    => ↑ burden of CRS.

Note that the increase in the CRS burden depends on the setting and the vaccination coverage

Age-specific proportion seropositive to rubella antibodies in a high and a low transmission setting

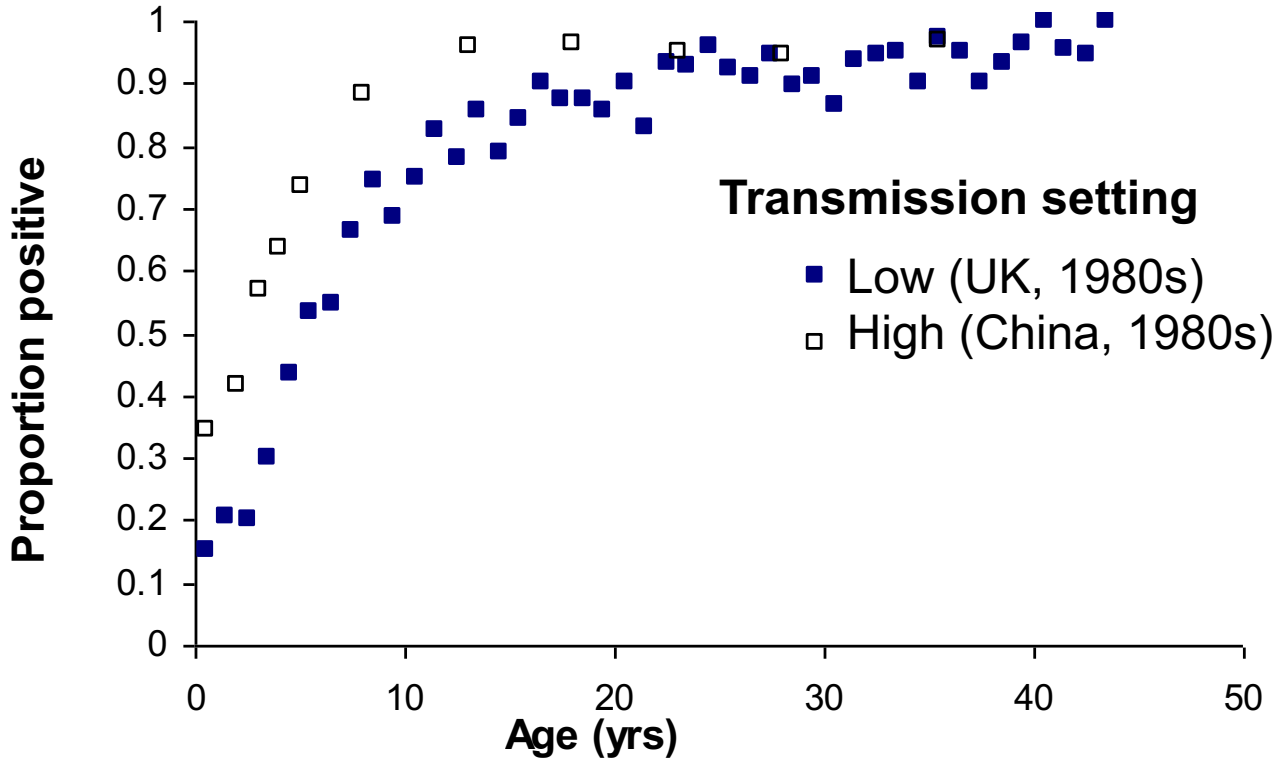


Data sources: Wannian (1985), Farrington (1990))

Question: Which setting is the high transmission setting?

Answer – China, because most are seropositive by about age 15 years

Age-specific proportion seropositive to rubella antibodies in a high and a low transmission setting



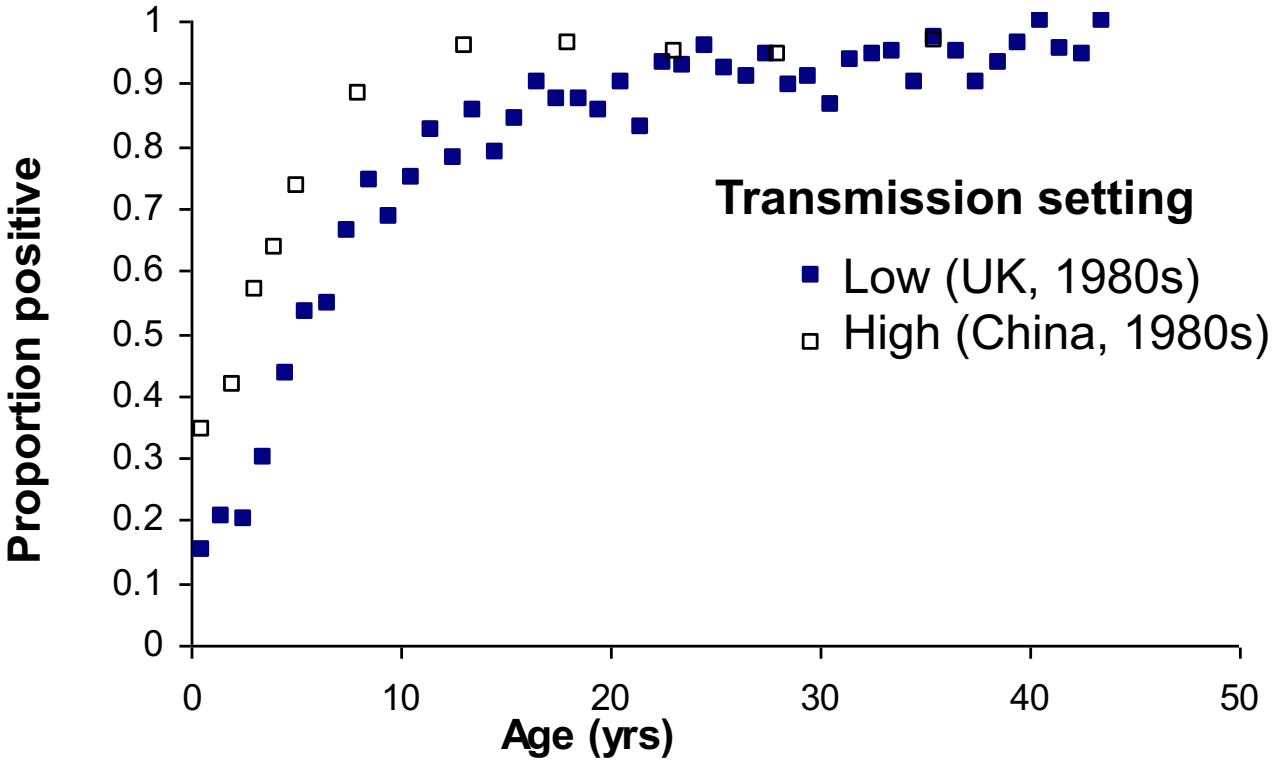
Data sources: Wannian (1985), Farrington (1990))

Question: In which setting might you be more cautious when introducing rubella vaccination in young children?

Answer – high transmission. The CRS burden is very low pre-vaccine introduction, so caution is needed to avoid greatly worsening a small problem

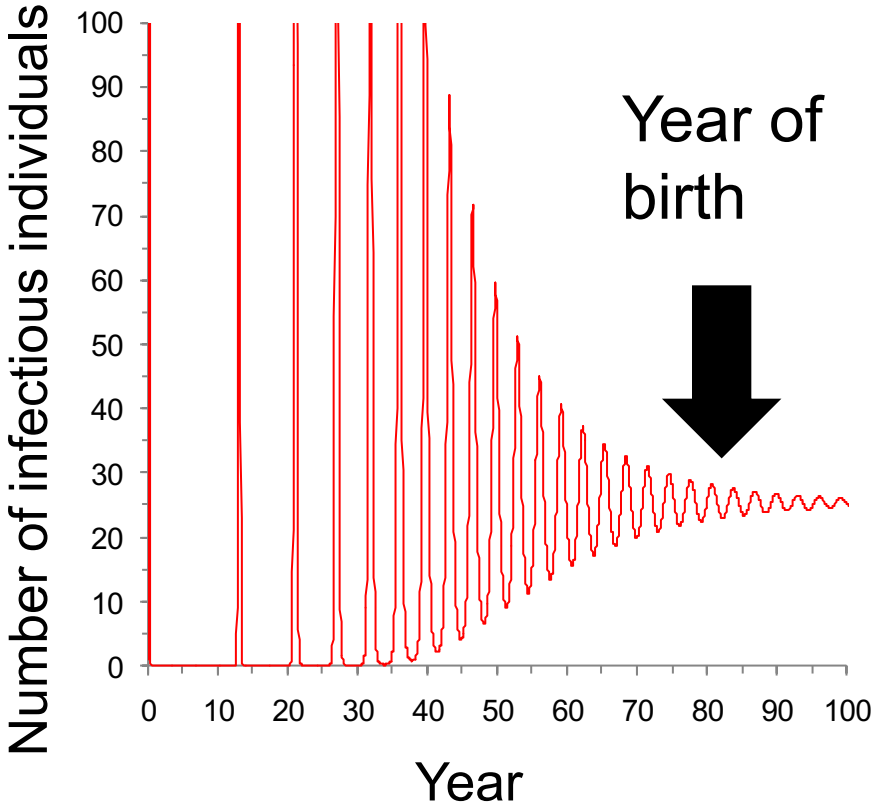
Age-specific proportion seropositive to rubella antibodies in a high and a low transmission setting

Data sources: Wannian (1985), Farrington (1990))



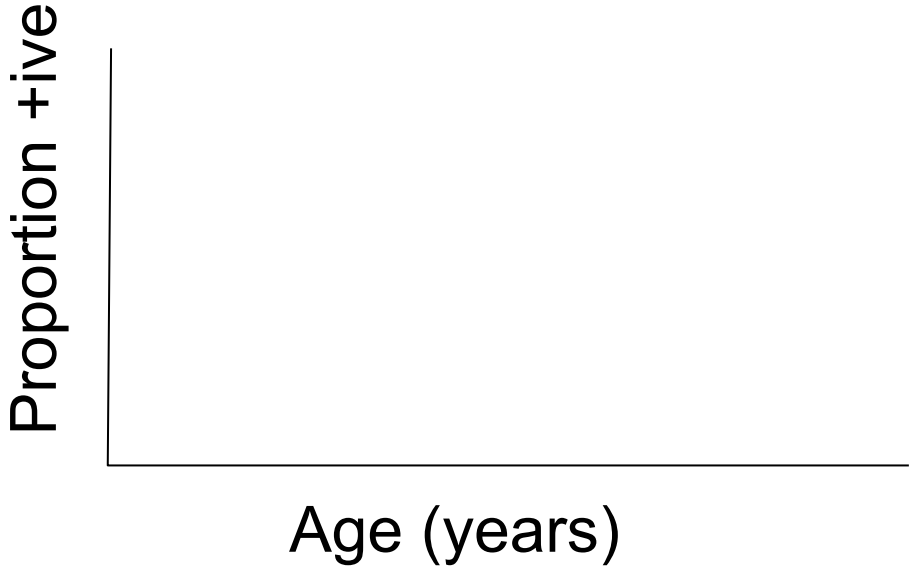
Question: What is the relationship between these age-specific patterns, R_0 and the dynamic predictions from the models seen previously?

Predictions of the number of infectious individuals using the measles model used so far

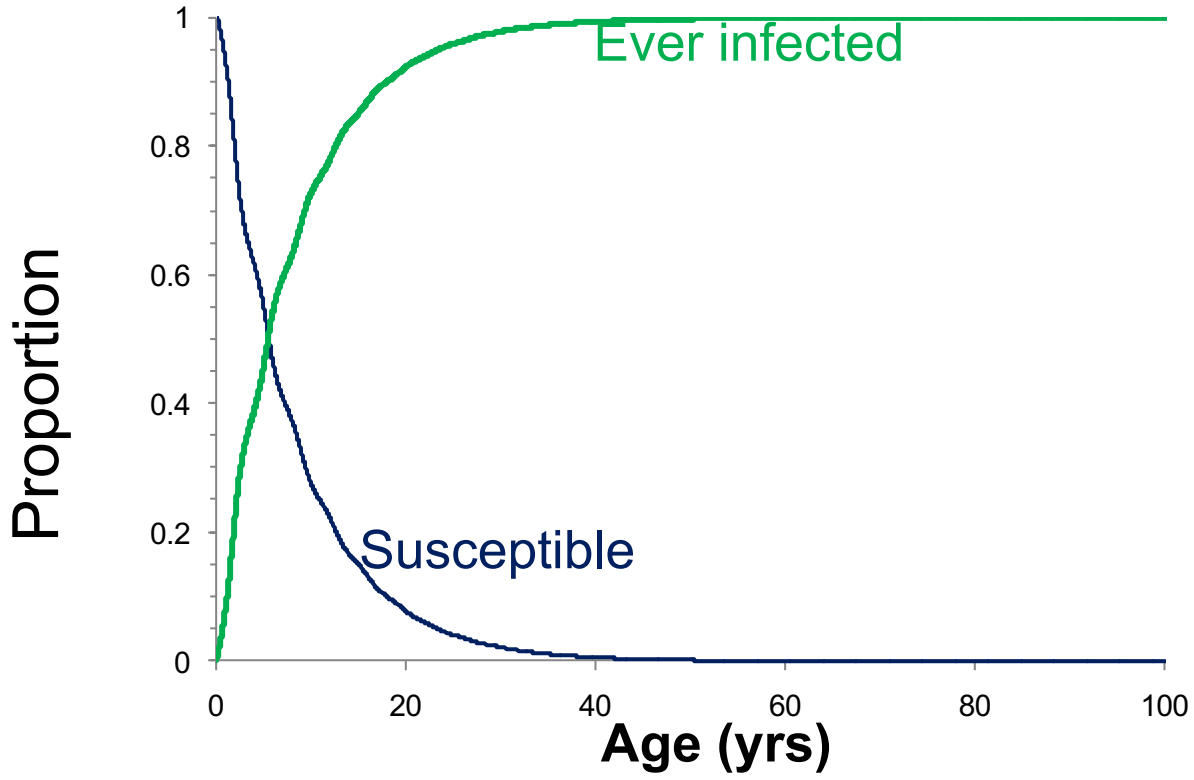


Experiment...

Suppose we track people born in year 80...
How might the proportion who have ever experienced infection (and ∴ seropositive) change as the cohort ages?

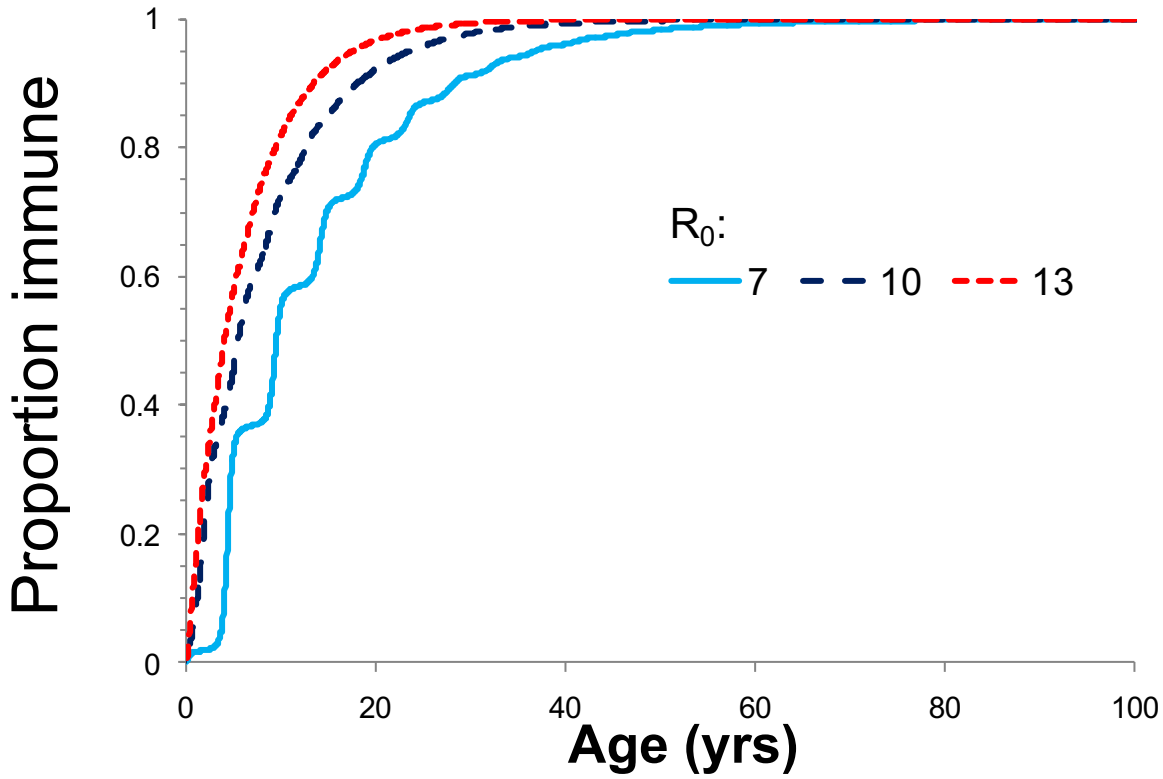


Predictions from the dynamic transmission model of the proportion of individuals born in year 80 that should have been infected by different ages



NB if an infection is endemic, these patterns should be similar to those seen in cross-sectional data

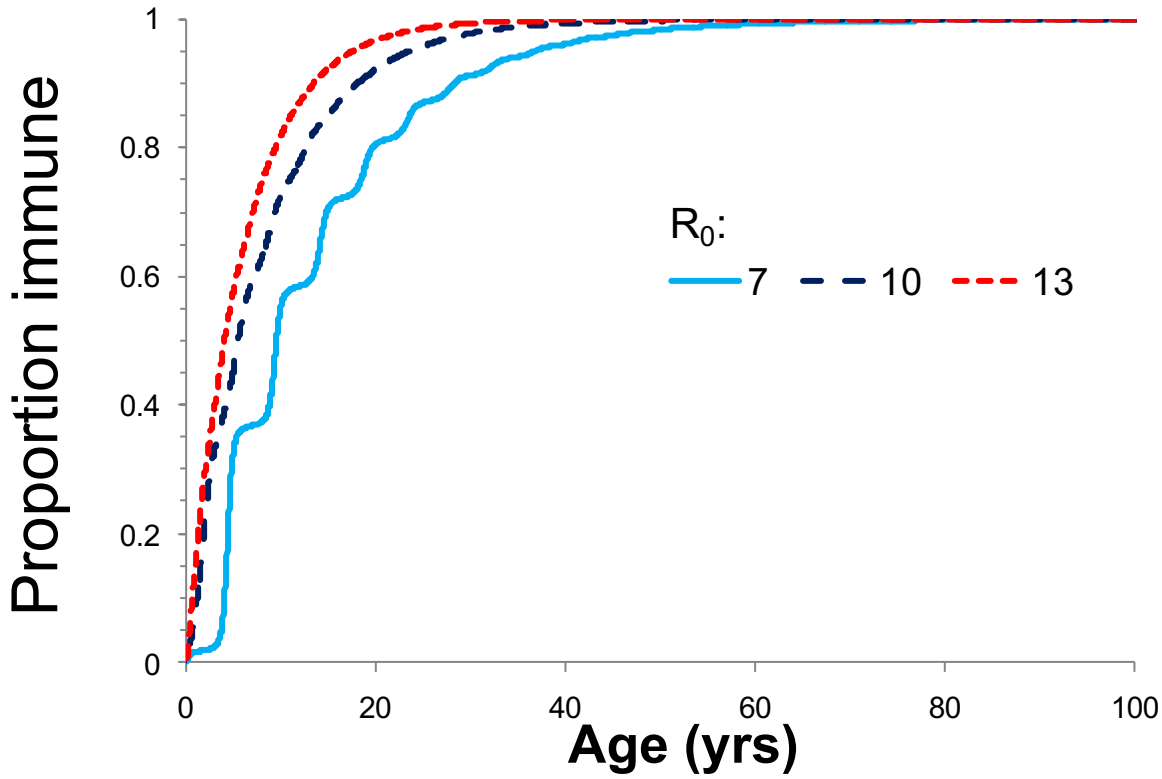
R_0 influences the proportion of individuals that have ever been infected (and who should be seropositive) by given ages:



In principle, the effect of vaccination on the age-specific proportion susceptible and the number of new infections could be studied with this model, by including (many!) additional equations

Easier to do this with an age-structured model, with an appropriate R_0

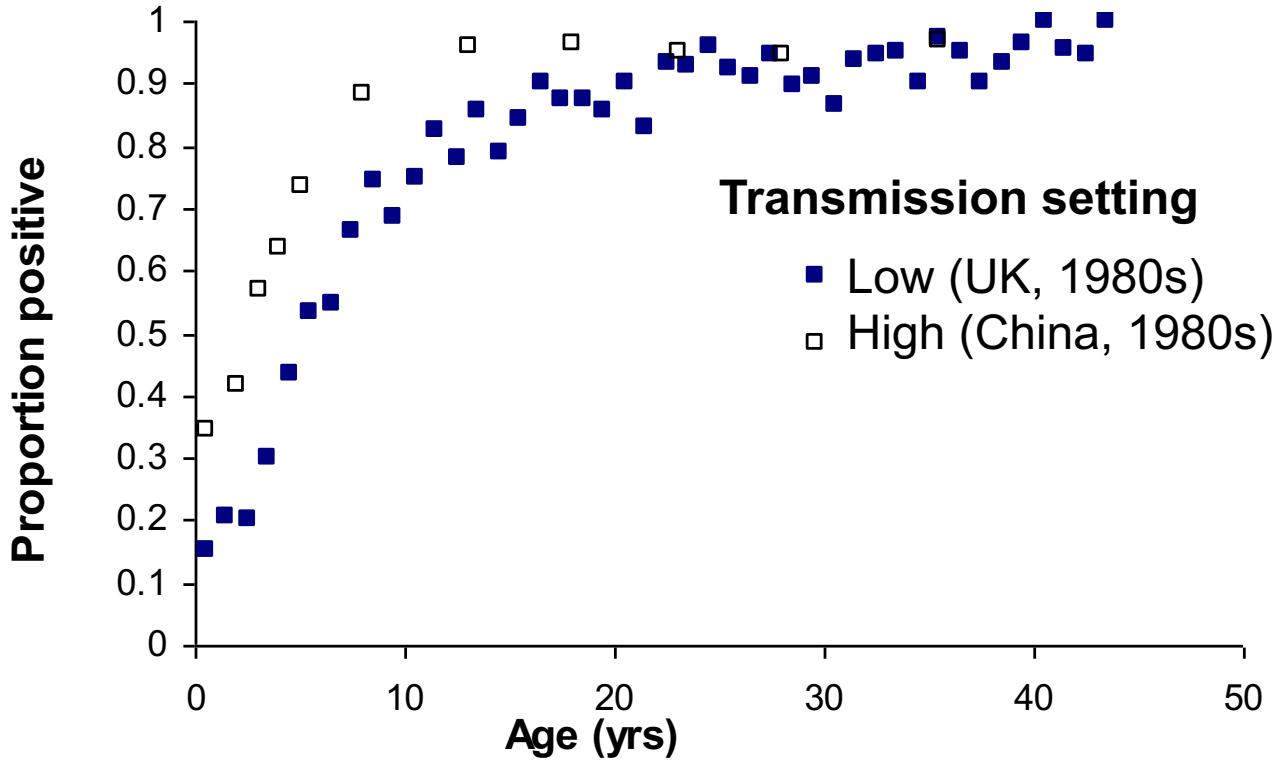
R_0 influences the proportion of individuals that have ever been infected (and who should be seropositive) by given ages:



We can calculate what R_0 is for an infection by analysing seroprevalence data using appropriate models – see later sessions…

Age-specific proportion seropositive to rubella antibodies in a high and a low transmission setting

Data sources: Wannian (1985), Farrington (1990))



What was R_0 for these settings, assuming random mixing?

Transmission setting	R_0 (=L/A)	Force of infection (% pa) (λ)	Average age at infection (yrs) (A=1/λ)	Herd immunity threshold (=1-1/R_0)
Low	7	11.6	8.6	86%
High	12	20.3	4.9	92%

Objectives

By the end of this session you should know:

- Why we might need to use age-structured models
- How age-structured models might be set up
- How different levels of vaccination coverage may affect:
    a) the age-specific proportion susceptible in the population
    b) the age-specific number of new infections per unit time
    c) the average age at infection.
- Be aware of the effect of mixing patterns on the impact of vaccination

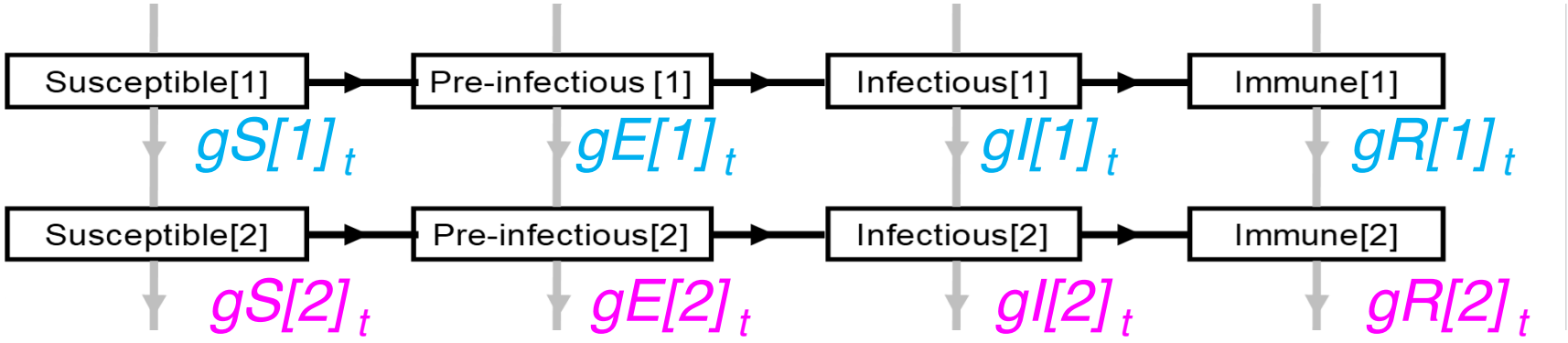
Key steps in developing an age structured model

- Decide on the size of the age band that you want to model, e.g. single year age bands, 6 monthly, monthly?

- Set up separate compartments for each age band

- Decide how people move from one age band to the next
  - Constant ageing rate (=1/average duration of stay in the age band)
  - Only move people from one age group to the next at prescribed time points, e.g. at the end of the year, if using single year age bands (Schenzle or Realistic Age Structure (RAS) approach)

Moving people to the next age band – continuous ageing



$$S[2]_{t+1} = S[2]_t - \lambda_t S[2]_t - gS[2]_t + gS[1]_t$$

$$E[2]_{t+1} = E[2]_t + \lambda_t S[2]_t - fE[2]_t - gE[2]_t + gE[1]_t$$

$$I[2]_{t+1} = I[2]_t + fE[2]_t - r\,I[2]_t - gI[2]_t + gI[1]_t$$

$$R[2]_{t+1} = R[2]_t + rI[2]_t - gR[2]_t + gR[1]_t$$

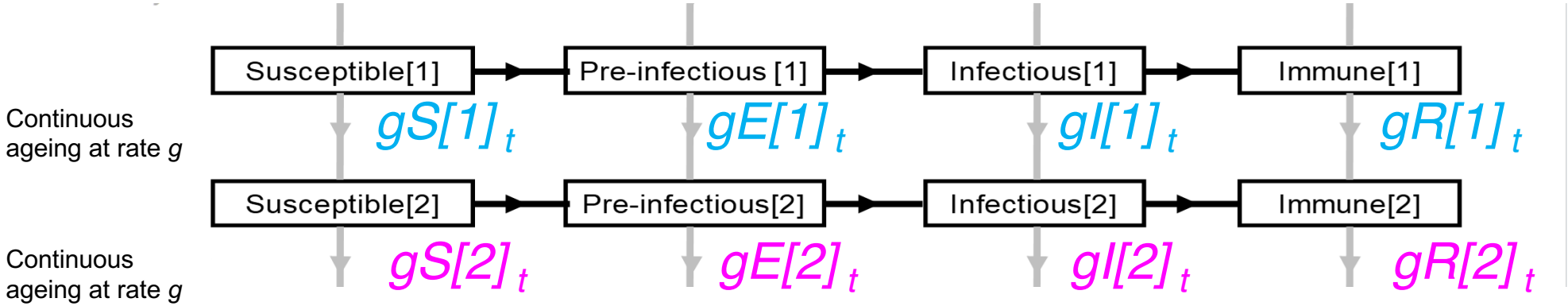
Ageing rate (g) =1/average duration of stay in the age band

Example

Average duration of stay in each band = 1 year or 365 days

g =1/365 = 0.0027 per day

Moving people to the next age band – continuous ageing



Advantages

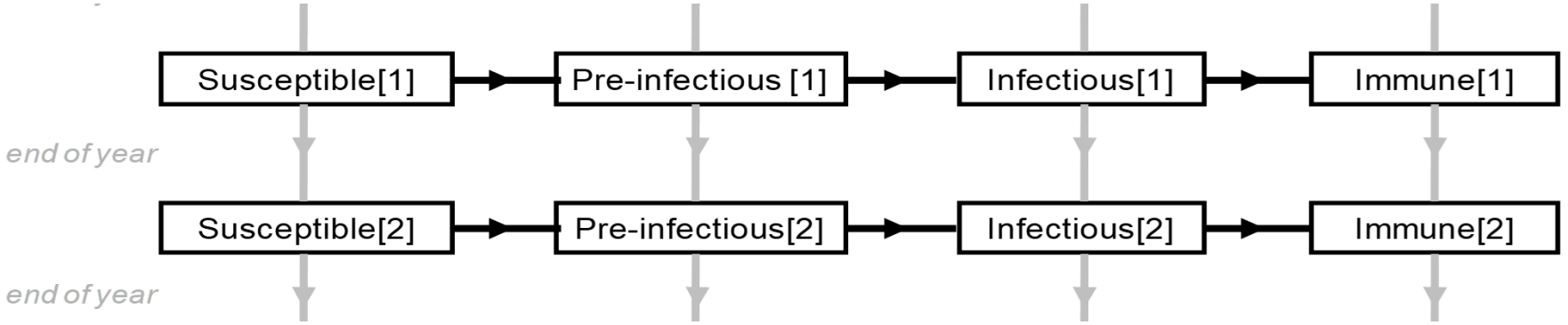
Quick and fairly straightforward to set up

Disadvantages

The same rate is applied to all people in the same age band, irrespective of how long they've been there:

- => someone in a young age band may enter an older age band very quickly

Moving people to the next age band – ageing at fixed time steps



Example – single year age bands

If t is any time other than the end of the year:

$$S[2]_{t+1} = S[2]_t - \lambda_t S[2]_t$$

$$E[2]_{t+1} = E[2]_t + \lambda_t S[2]_t - fE[2]_t$$

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If t is the end of the year:

$$S[2]_{t+1} = S[1]_t - \lambda_t S[1]_t$$

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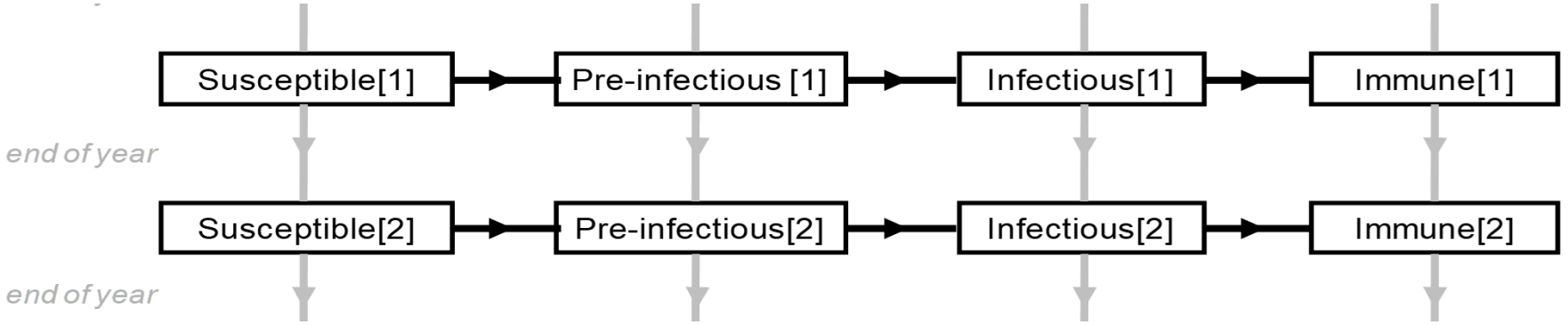
Moving people to the next age band – ageing at fixed time steps



Advantages

People move between age bands at the same time, so the age band contains people who have been in the model for the same time

Disadvantages?

Model equations depend on whether they relate to the end of the time step or all other times

Key steps in developing an age structured model

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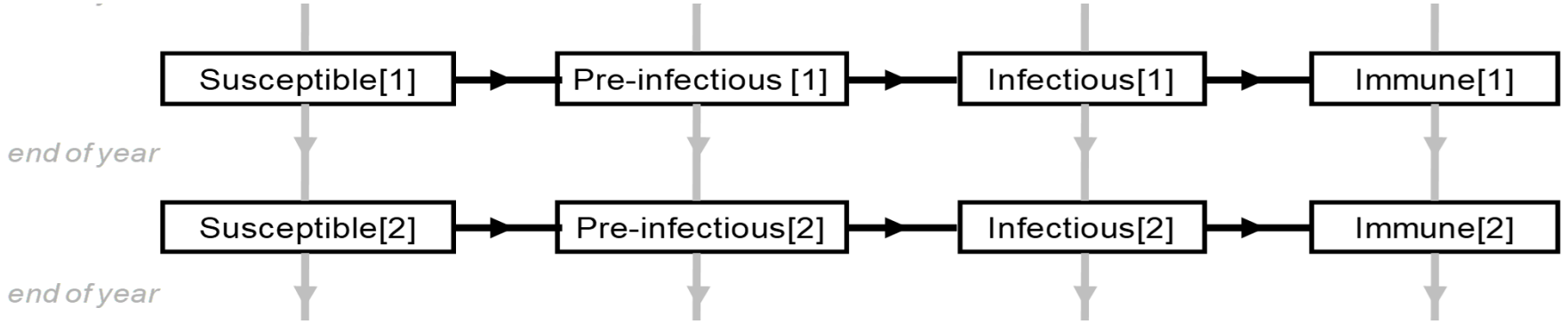
  - Only move people from one age group to the next at prescribed time points, e.g. at the end of the year, if using single year age bands (Schenzle or Realistic Age Structure (RAS) approach)
- Decide on whether people mix randomly or if there's age dependent mixing
- Ensure that infectious people in all age bands can infect others

Expressions for the force of infection



For a model with no age structure (just one age group):

$$\text{Force of infection} = \beta I_t$$

For a model with two age groups and assuming random mixing:

$$\text{Force of infection} = \beta(I_t[1] + I_t[2])$$

For a model with n age groups and assuming random mixing:

$$\text{Force of infection} = \beta(I_t[1] + I_t[2] + .. I_t[n])$$

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Software for this session

For this session, we will use Berkeley Madonna, available from:
https://berkeley-madonna.myshopify.com/pages/download

The free version is sufficient for this session. it will allow you to run the models but not save changes

Start with Rubella model 1.mmd.
You may need to open Berkeley Madonna first and then open the model from File -> open

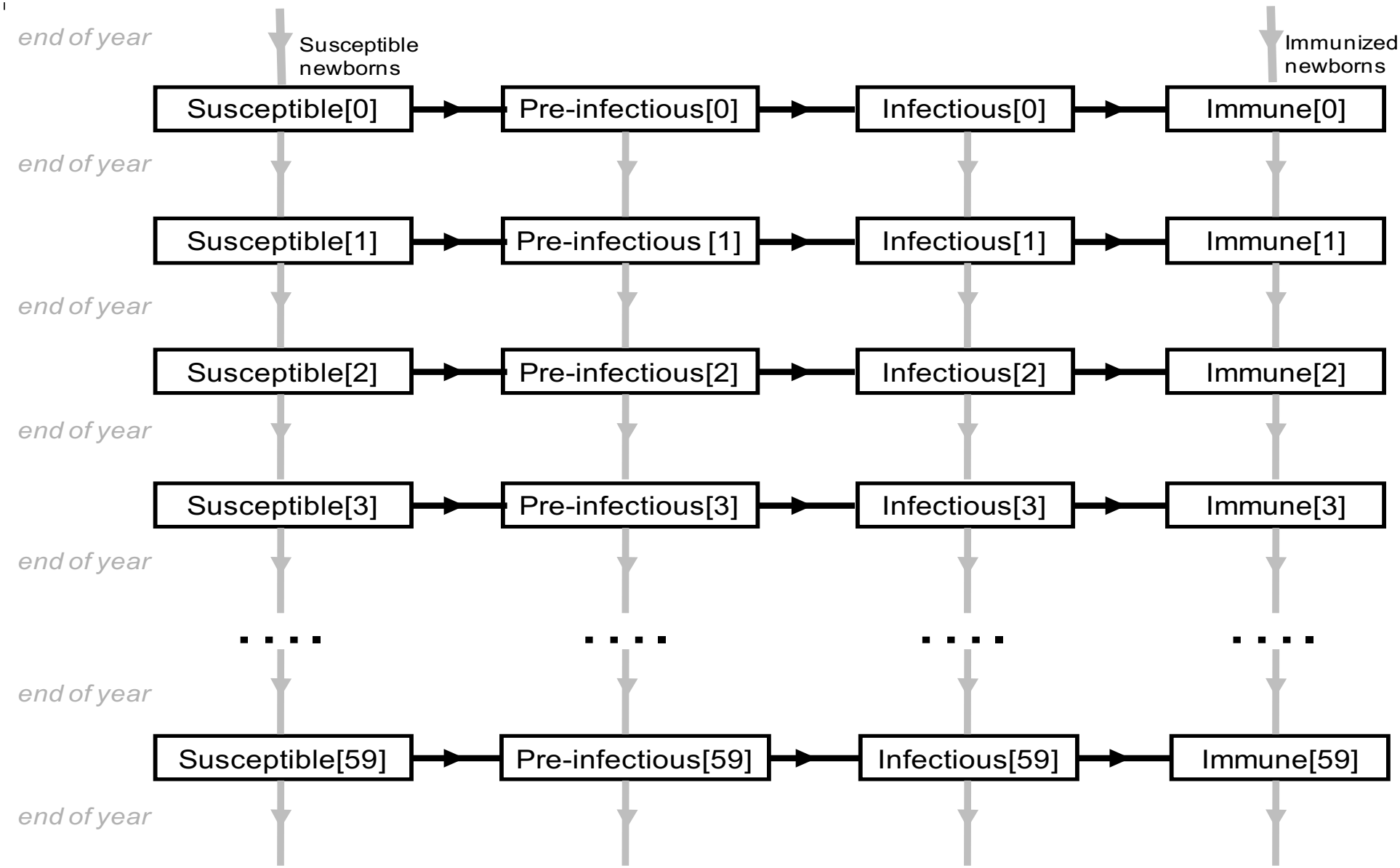
Key features of the model (1)

- 1 year age strata for the ages 0-59 years

  => separate equations for each age stratum

- Births/vaccinees added into Sus[0] and Imm[0]

Diagram of the model in this session



Key features of the model (1)

- 1 year age strata for the ages 0-59 years

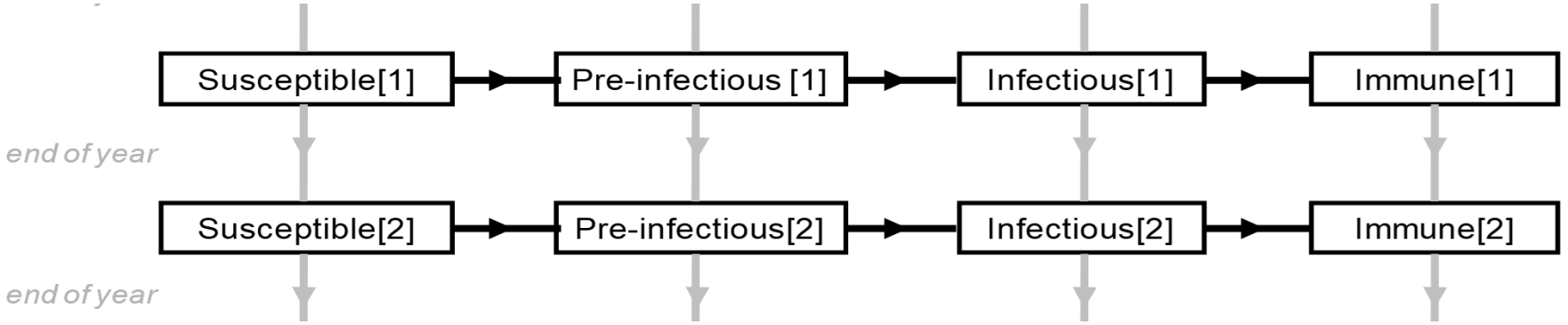
  => separate equations for each age stratum

- Births/vaccinees added into Sus[0] and Imm[0]

- All individuals change their age stratum at the end of the year

  => (Difference) equations for time t+1 depend on whether t is at the start or end of each year

Moving people to the next age band – ageing at fixed time steps



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Key features of the model (1)

- Rectangular age distribution: 1000 individuals per age stratum
    - See graph 1 after running the model
- Mixing is random and force_of_infn=*beta * (Array_sum(Infectious[*])*

  i.e. equivalent to:

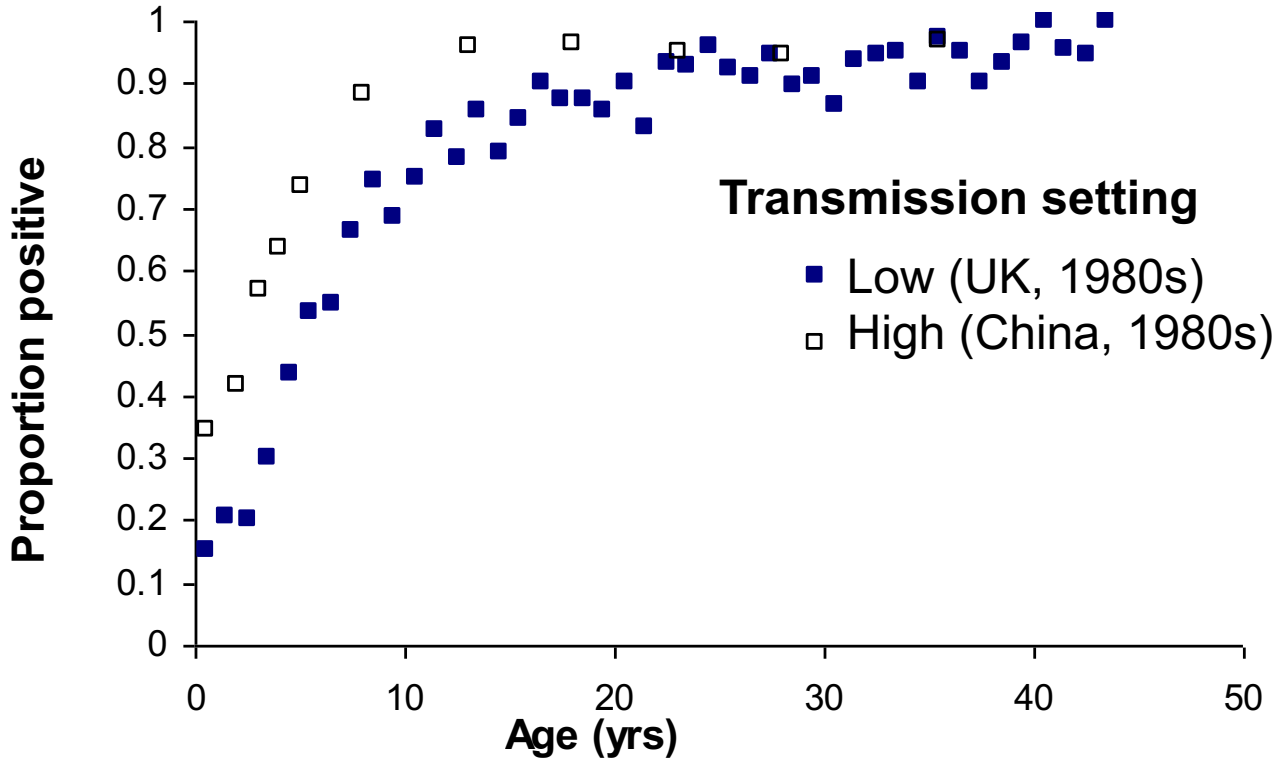
  beta*(Infectious[0] + Infectious[1] +Infectious[2]+…+Infectious[59])

$$\beta = \frac{R_0}{Total\ population\ size \times Duration\ of\ infeciusness}$$

R_0 = 12 - see parameters panel on right-hand side

Age-specific proportion seropositive to rubella antibodies in a high and a low transmission setting

Data sources: Wannian (1985), Farrington (1990))



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Low	7	11.6	8.6	86%
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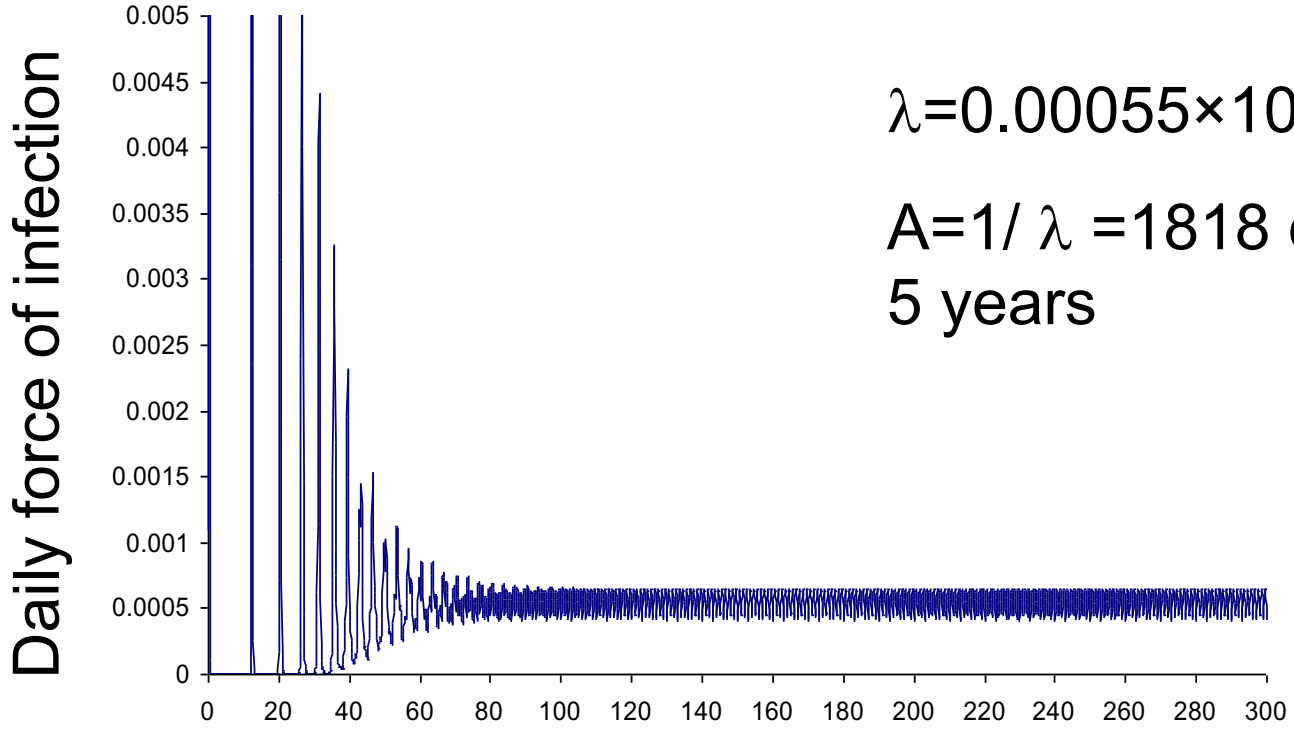
Q.1 The average annual force of infection in the high transmission setting is roughly 20%/year.

a) What be should the long-term average daily force of infection in the model if it reflects this setting with no vaccination?

b) Check that the model reproduces this force of infection by running the model

a) A force of infection of 20%/year is approximately equivalent to a daily force of infection of 0.2/365 ≈ 0.00055 per day

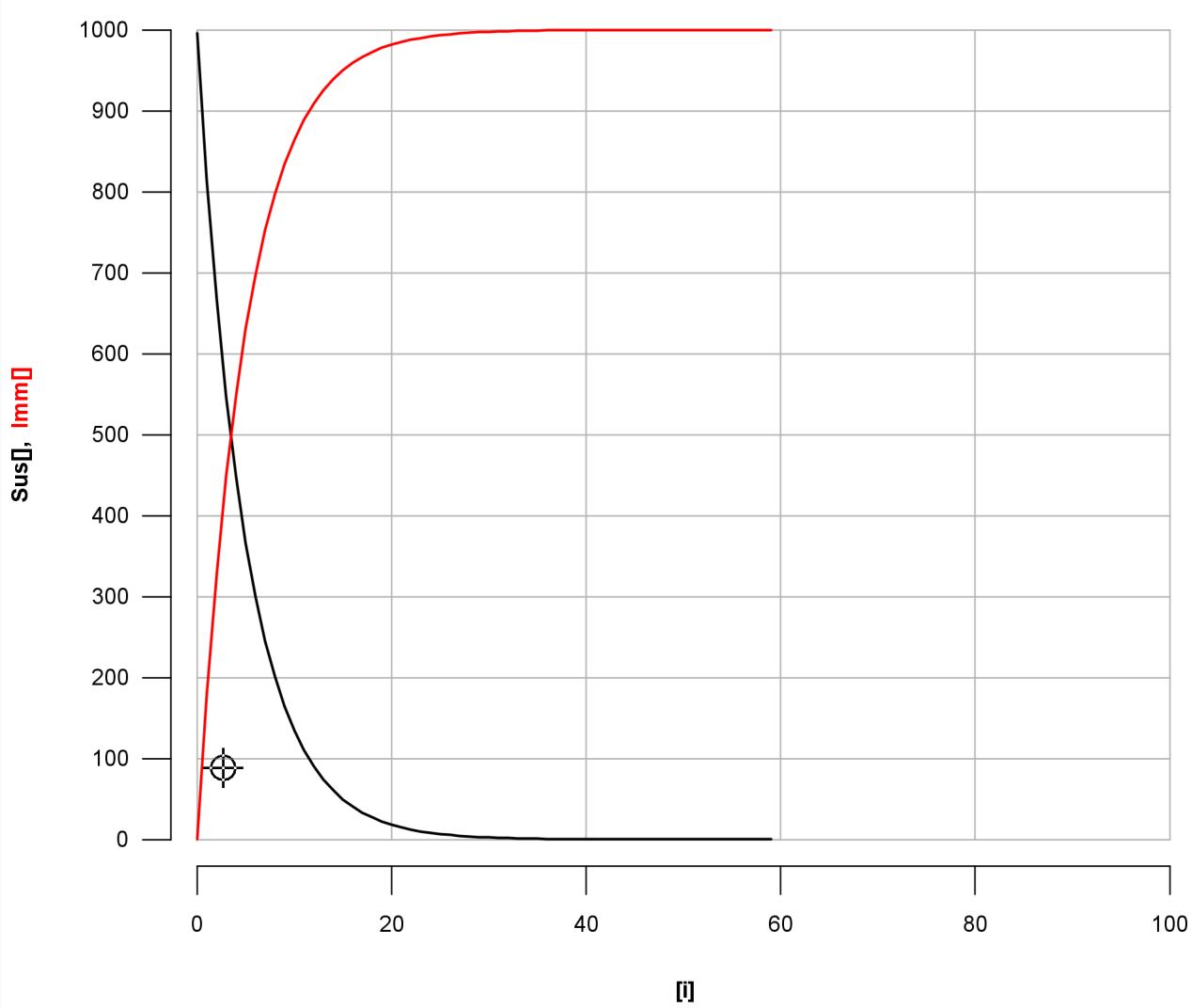
b) See Graph 2



$\lambda = 0.00055 \times 10^{-4}$/day

A=1/ λ =1818 days or 5 years

Check that the age-specific number susceptible predicted in the model is consistent with what you might expect based on the seroprevalence data – see graph 3



Simulation exercises:

I Exploring the effect of vaccinating very young children (“routine” vaccination)

Key features of the model (cont)

•Proportion vaccinated is currently zero. Changing the value for prop vacc to a value above 0 will result in vaccination among newborns occurring from year 100

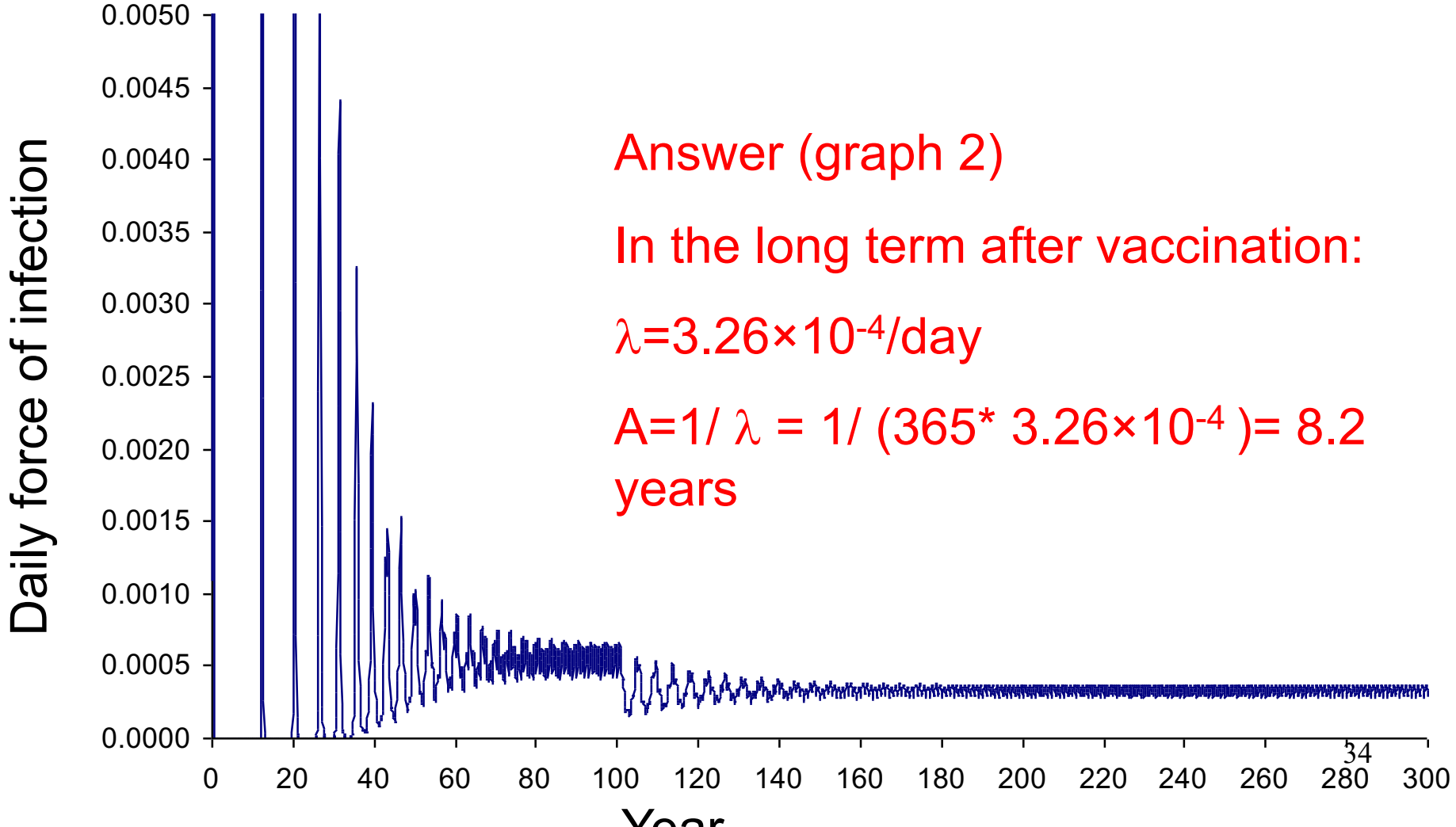
Berkeley Madonna code

```
prop_vacc = 0.0
yr_start_vacc = 100
birth_cov  = IF (year>yr_start_vacc) THEN prop_vacc ELSE 0
effectively vaccinated
```

Q1.2 Change the value for prop_vacc to 0.4.

a) *According to graph 2, how does the introduction of infant MMR vaccination at this level affect the long-term average force of infection?*

b) *According to the formula A=1/λ, what is the long-term average age at infection following the introduction of infant MMR vaccination?*



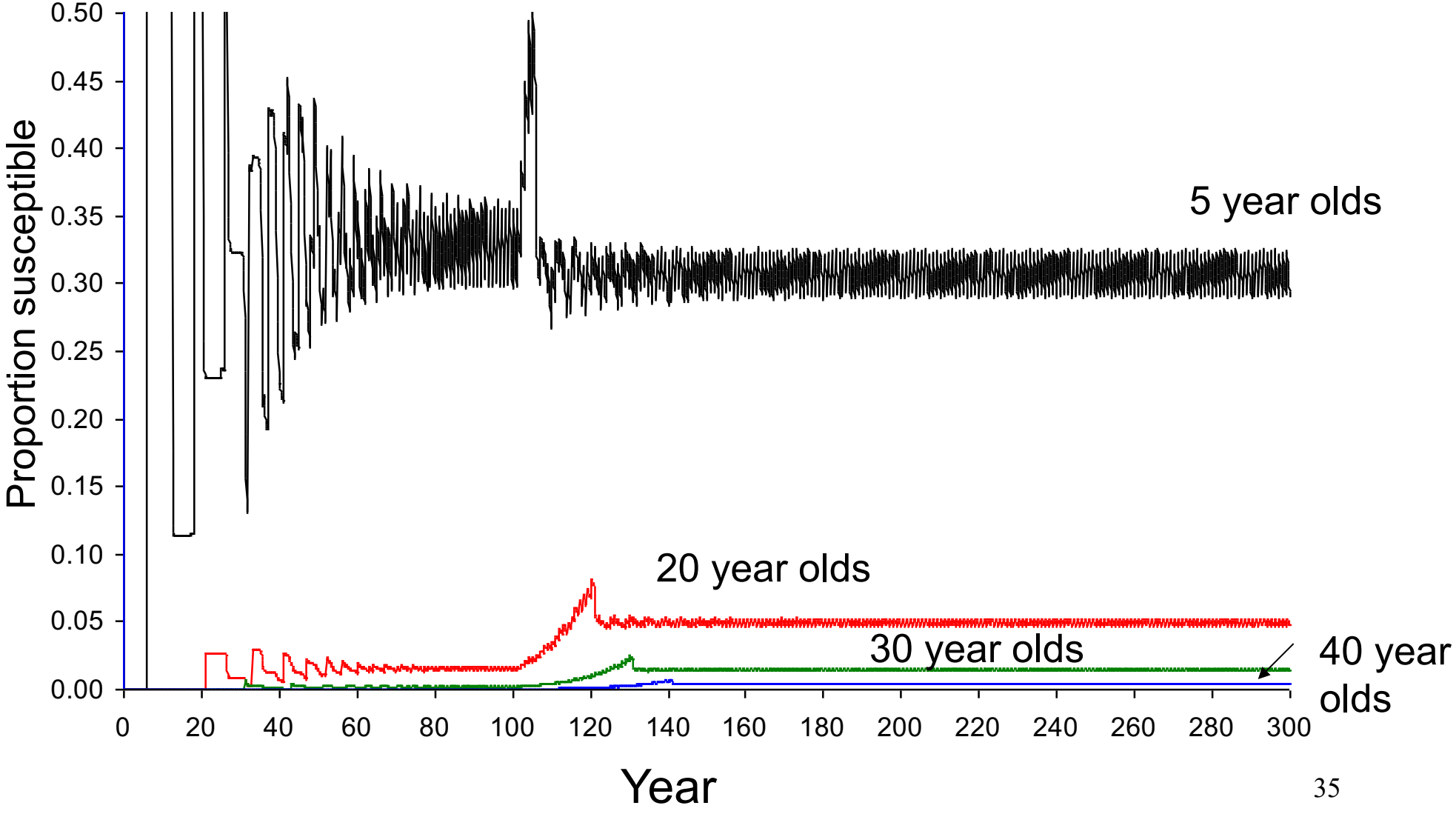
Answer (graph 2)

In the long term after vaccination:

$\lambda = 3.26 \times 10^{-4}$/day

$A = 1/\lambda = 1/(365 * 3.26 \times 10^{-4}) = 8.2$ years

Q1.3, Q1.4 Model predictions of impact of 40% effective vaccination coverage among newborns on the proportion susceptible in the high transmission setting (graph 4)



Q1.3

a) *Why does the average proportion of 5, 20, 30 and 40 year olds who are susceptible to infection increase in the short-term?*
   - *Vaccination leads to a reduction in the prevalence of infectious cases and hence a reduction in the force of infection.*
   - *In the short-term, the reduced opportunity for exposure leads to an increase in the proportion of individuals of a given age who are still susceptible.*

b) *How soon after the introduction of MMR vaccination does the proportion of 5, 20, 30 and 40 year olds who are susceptible peak? Is this what you would expect? Why?*

   Decreases occur 5, 20, 30 and 40 years after the introduction of vaccination - . once the first cohorts of vaccinees reach these ages.

Q1.4a) Why do you think the proportion of 5 year olds who are susceptible in the long-term after the introduction of vaccination of newborns is less than that before the introduction of vaccination?

Q1.4b) Why do you think the proportion of 20 year olds who are susceptible in the long-term after the introduction of vaccination of newborns is higher than that before the introduction of vaccination?

Ans to Q1.4a) & b) The long-term proportion of individuals who are susceptible in any given age group depends on:

Direct effect of vaccination

=> ↓ proportion susceptible

Indirect effect of vaccination

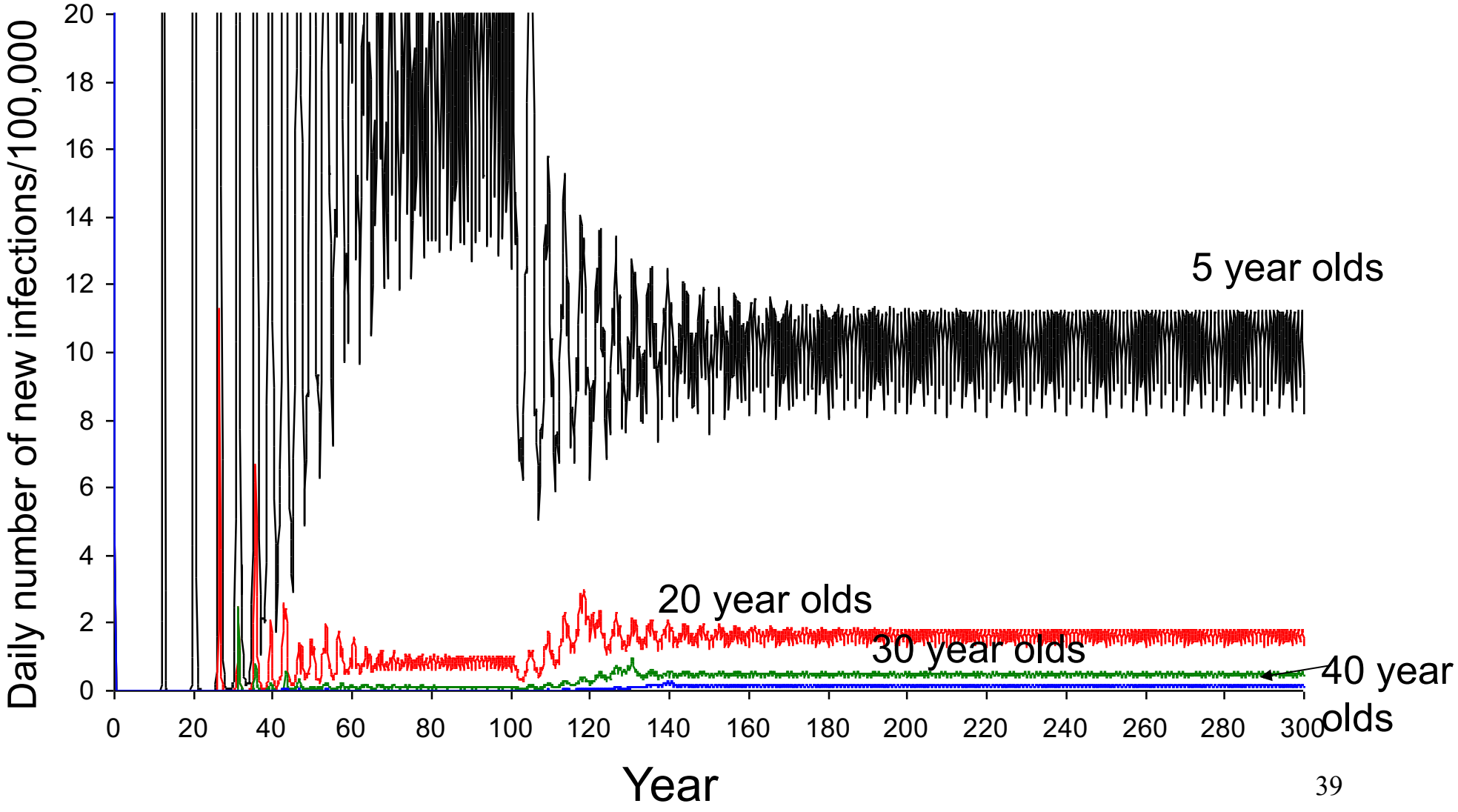
=> ↓ force of infection

=> ↑ proportion still susceptible by a given age

In the long-term:

- direct effect is the same for adults and children (the proportion vaccinated is identical)
- indirect effect is smaller for children than adults since children have fewer person years of exposure to the reduced force of infection than adults

Q1.5 Model predictions of the impact of 40% effective vaccination coverage among newborns on the daily number of new infections/100,000 in the high transmission setting



Why does the daily no. of new infections decrease among 5 year olds following the introduction of vaccination of newborns? Why does it increase for 20 year olds?

Number of new infections/unit time at age a = $\lambda(t)S(a)$

Following vaccination: ↓ ↓ 5 yr olds

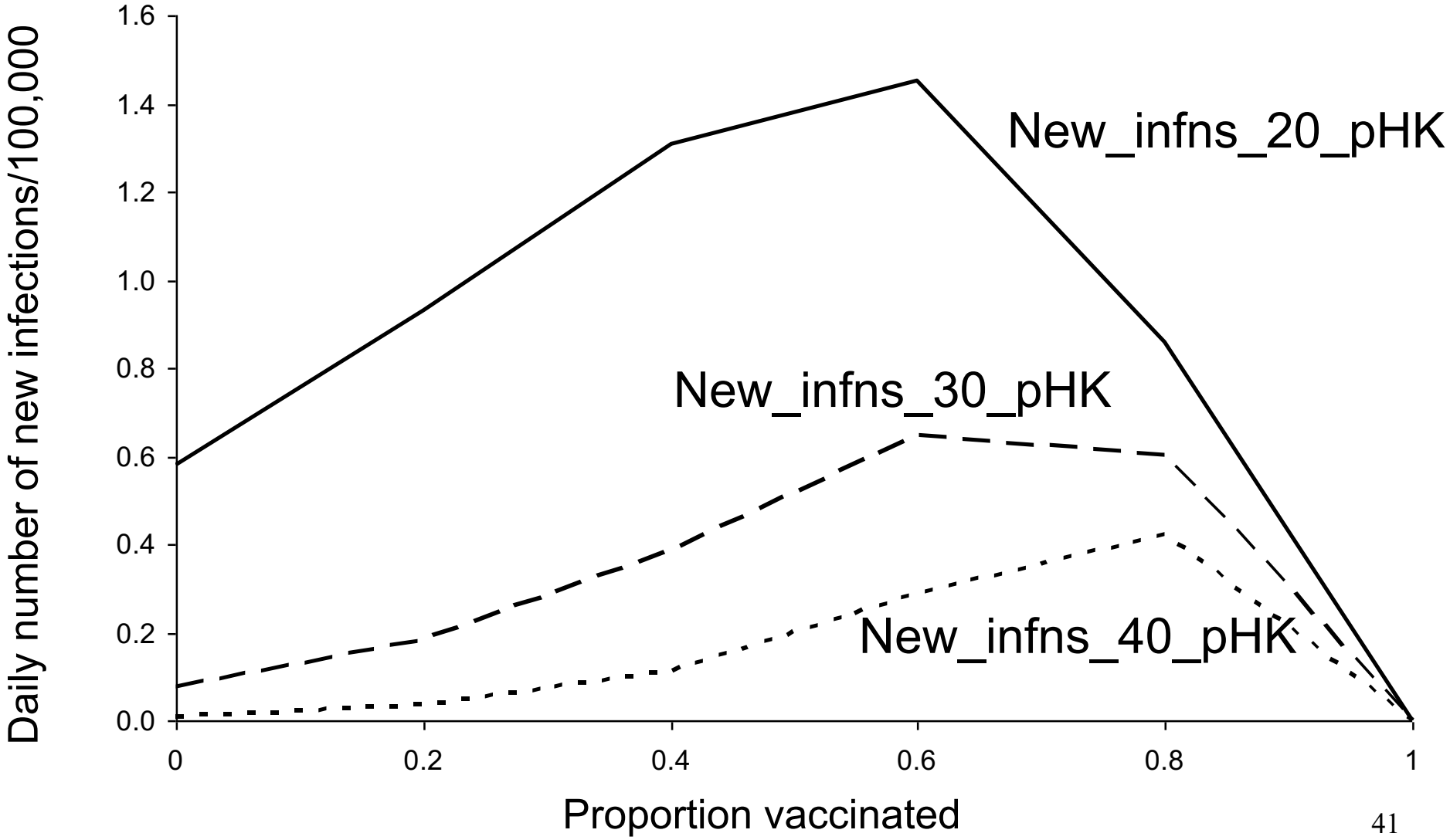
↓ ↑ ≥20 yr olds

a) Daily no. of new infections **decreases** for 5 year olds since both the force of infection and the proportion susceptible go down with the introduction of vaccination

b) Daily no. of new infections **increases** for 20 year olds since the increase in proportion susceptible following the introduction of vaccination outweighs the reduction in the force of infection.

The size of the change will depend on the vaccination coverage and the R_0 (λ) before the introduction of vaccination and is difficult to predict without a model...

Model predictions of the impact of different levels of effective vaccination coverage among newborns on the long-term daily number of new infections per 100,000 in the high transmission setting



Optional reading – producing the parameter plot in Berkeley Madonna

1. Select the “Parameter plot” option from the “Parameters” option in the menu.

2. Specify that you would like to run the model 6 times, with prop_vacc as the parameter that you’d like to vary, ranging between 0 and 1.00.

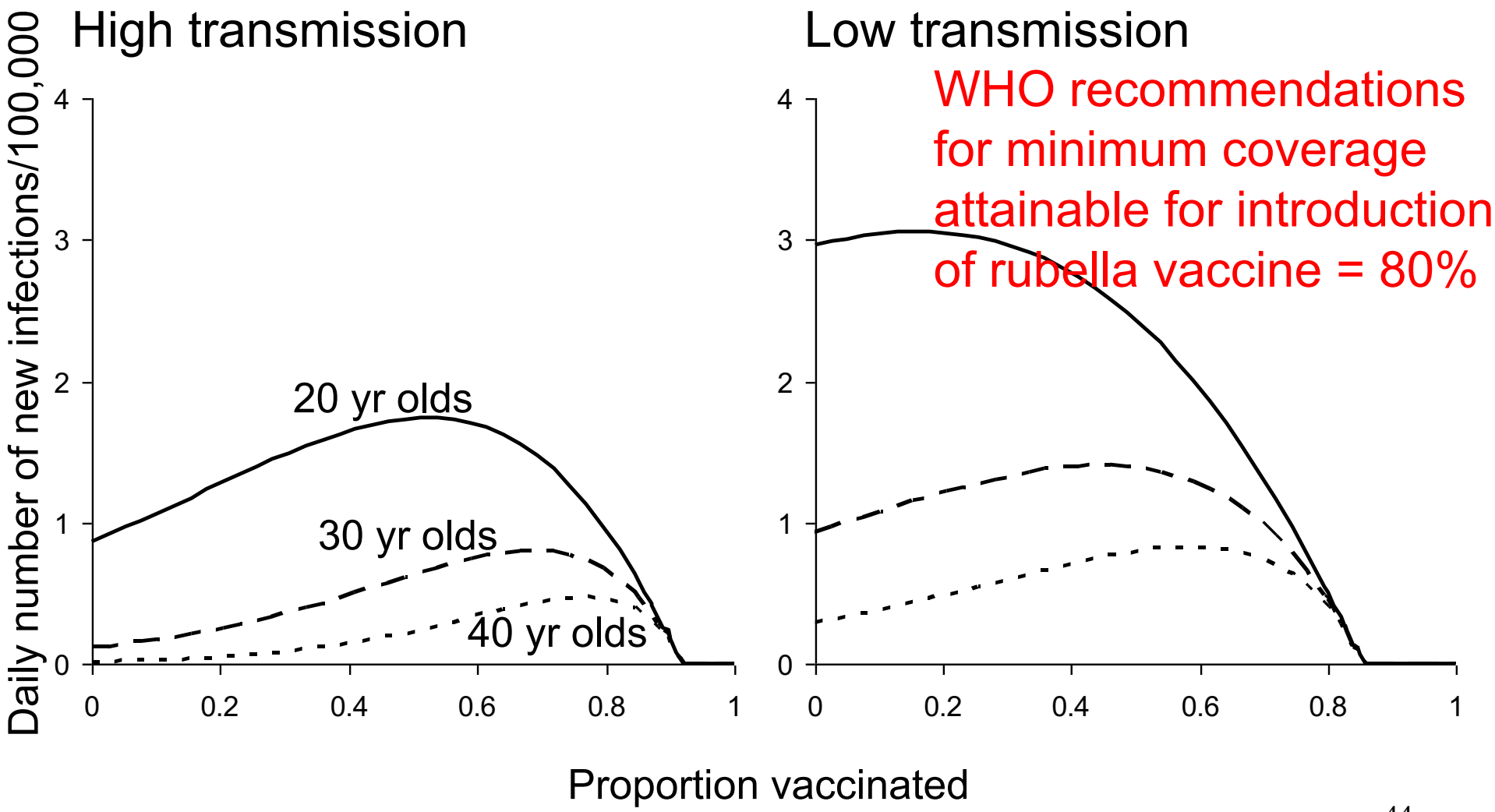
3. Specify that for each model run, you’d like to plot the final values of “new_infns_20”, “new_infns_30”, “new_infns_40” on the y-axis.

4. Click on the run button to see the plot.

Q1.6 What do you conclude about the most likely impact of the introduction of infant MMR vaccination on the daily number of rubella infections occurring among adults in the high transmission setting?

Based on the parameter plot, introducing infant MMR vaccination, with no other age groups vaccinated, could lead to an increase in the daily number of new infections per 100,000 among adults.

Model predictions of the impact of different levels of effective vaccination coverage among very young children on the infection incidence in the high and low transmission settings



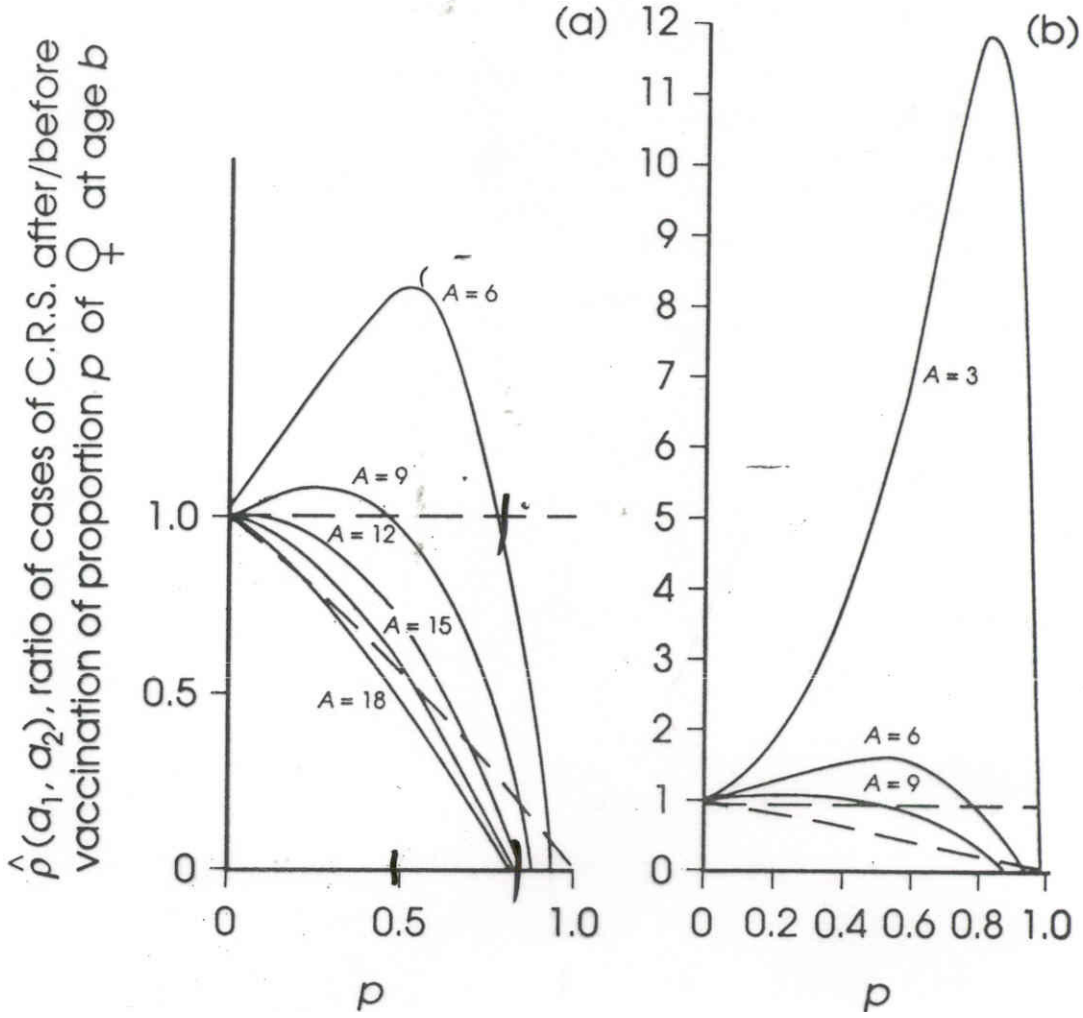
Q1.7 Should you be more cautious about introducing infant MMR vaccination in the high or low transmission setting? Why? How might you revise your vaccination strategy to limit the potential for adverse events to occur?

- For a given level of vaccination coverage among infants, the daily number of new infections per 100,000 in the low transmission setting is higher than that in the high transmission setting
- Therefore the CRS incidence will be higher in the low transmission setting that in the high transmission setting.

- But…the relative increase in the number of infections in adult age groups after introducing infant vaccination is introduced is higher in the high transmission setting than in the low transmission setting

- So…you might be more cautious about introducing infant vaccination in the high transmission setting than the low transmission setting

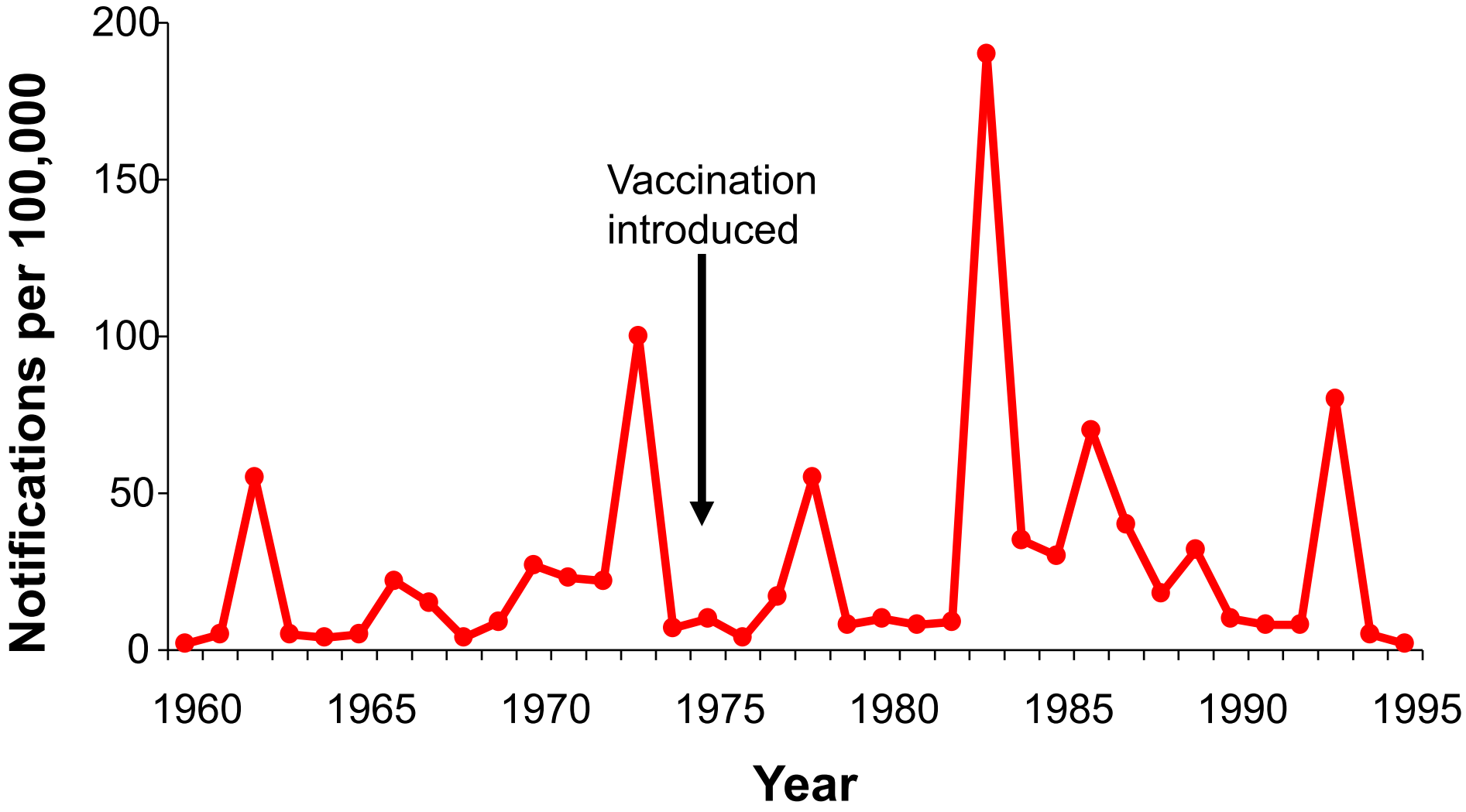
- A mixed vaccination strategy (e.g. vaccinating all women of child-bearing age, besides children) might be appropriate.



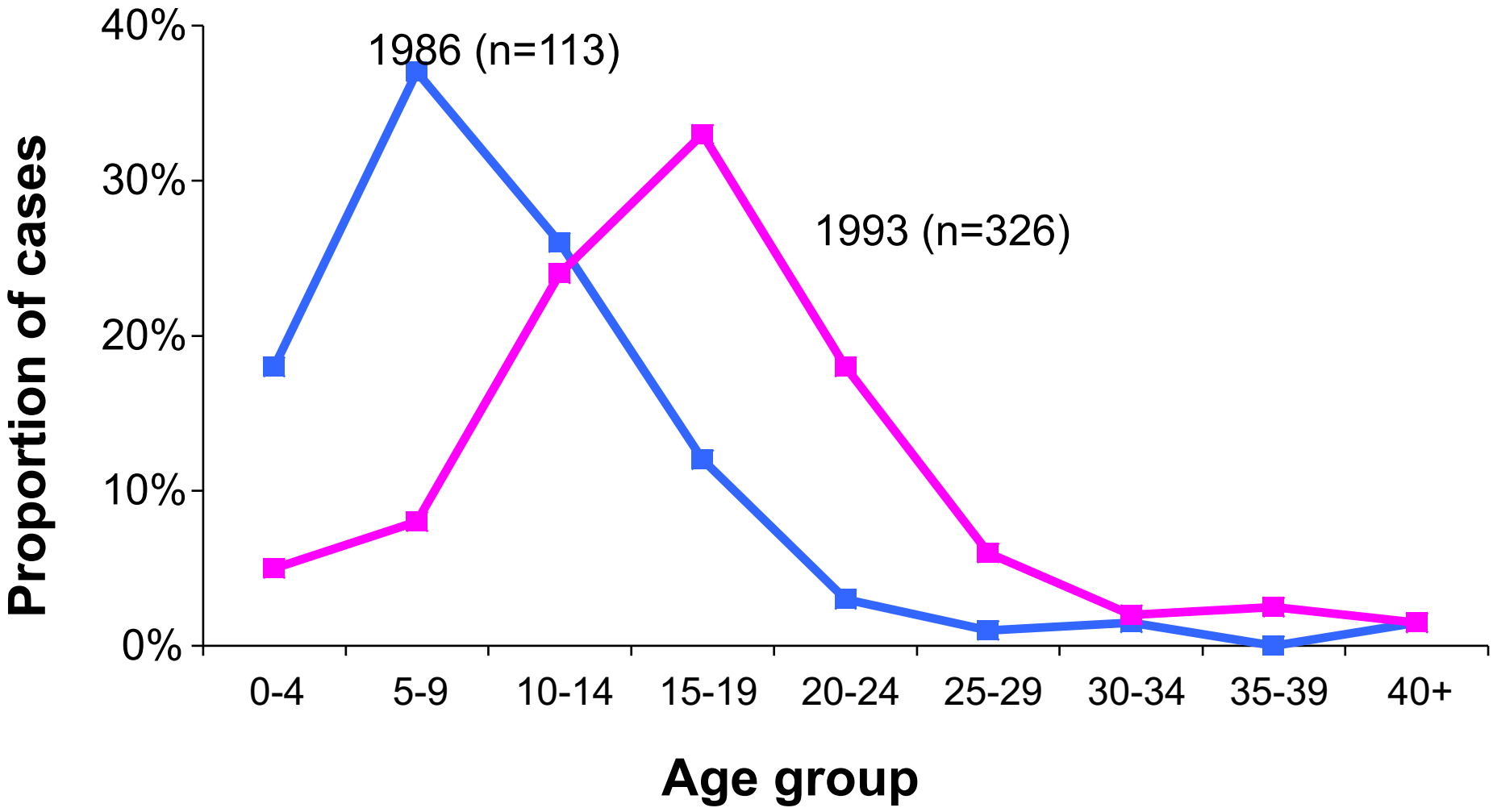
Vaccination coverage

Anderson and May (1991)

Notifications of Rubella in Greece

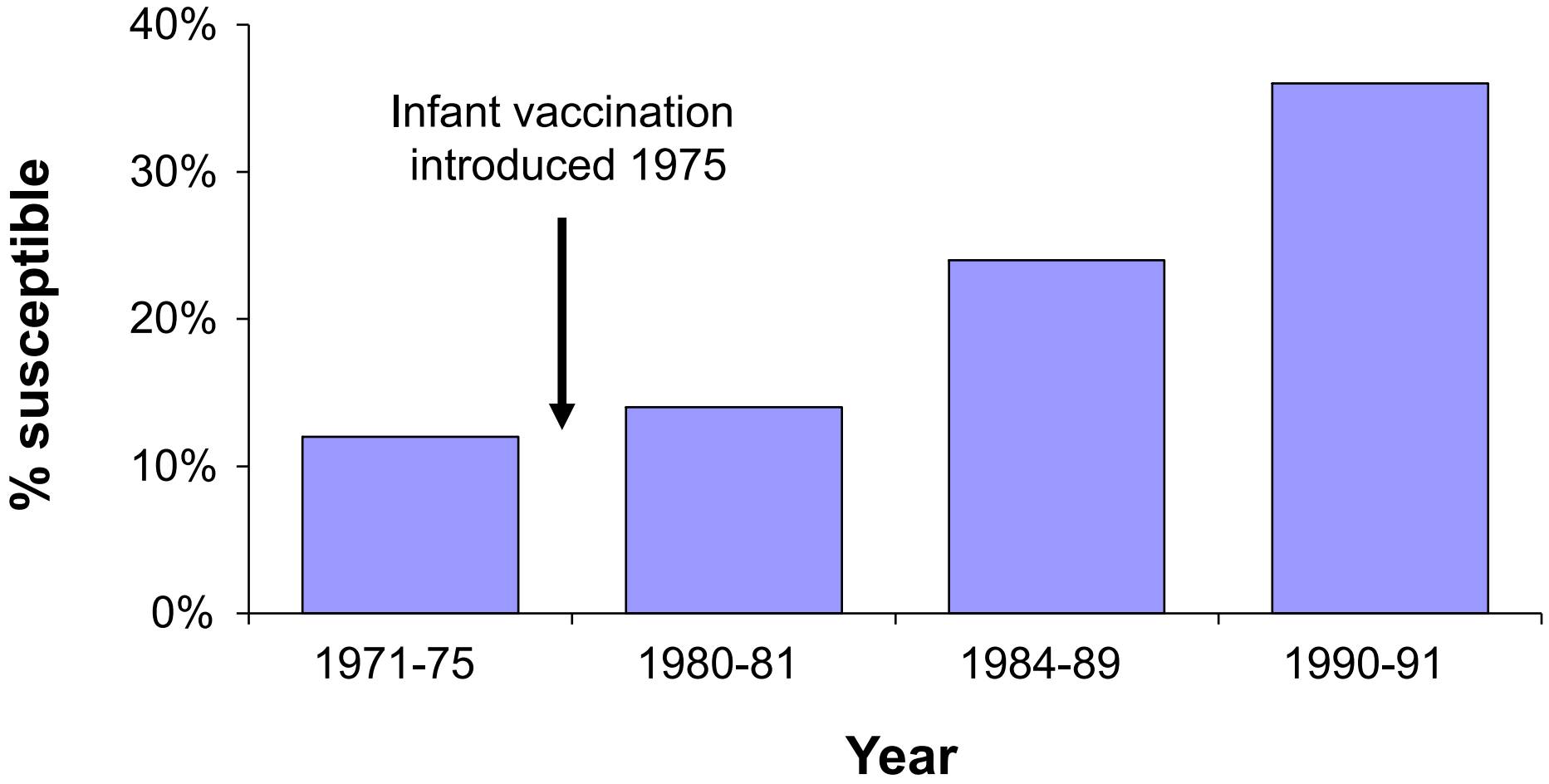


Age distribution of outpatient rubella cases, Athens



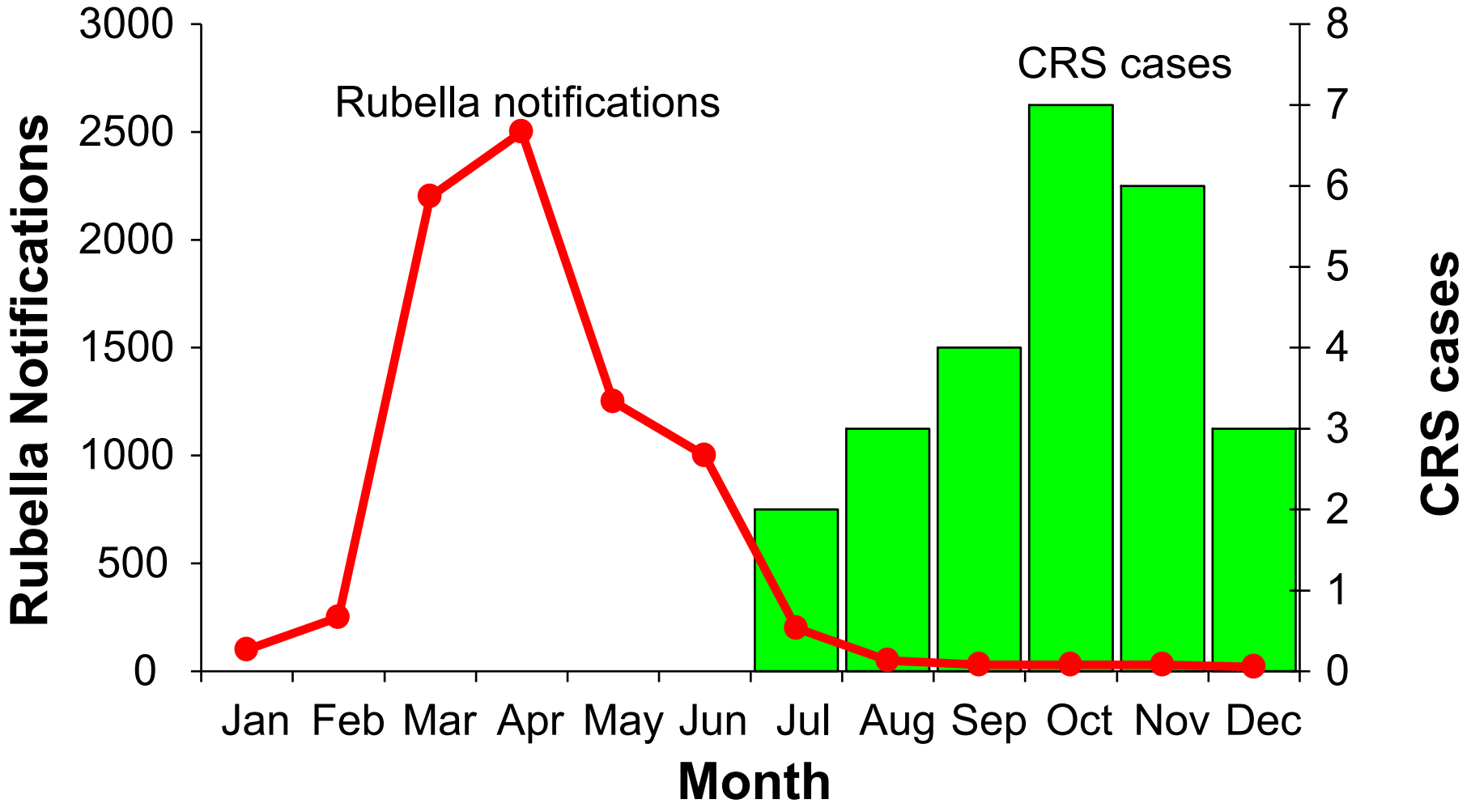
from Panagiotopoulos et al, BMJ, 1999

Proportion of pregnant women susceptible to rubella
Athens, 1975-91



data from Panagiotopoulos et al, BMJ, 1999

Rubella and CRS in Greece, 1993



Simulation exercises:

2 Exploring the effect of vaccinating children just before child-bearing age

Open Rubella model 2.mmd

You may need to open Berkeley Madonna first and then open the model from File -> open

Key features of the model

Everything is the same as for the previous model, except :

- R_0 reduced to about 7 to reflect the low transmission setting

- Model allows for annual vaccination among 13 year olds:
  - Difference equations have changed slightly to include vaccination
  - Para_vacc_13 reflects the proportion of 13 year olds that are vaccinated from year 100

```
prop_vacc_13 = 0.0
yr_start_vacc = 150
vacc[0..59] = 0.0
category aged 0-59 years
vacc[13] = IF (year>yr_start_vacc) THEN prop_vacc_13 ELSE 0
This equals zero before the introduction
```

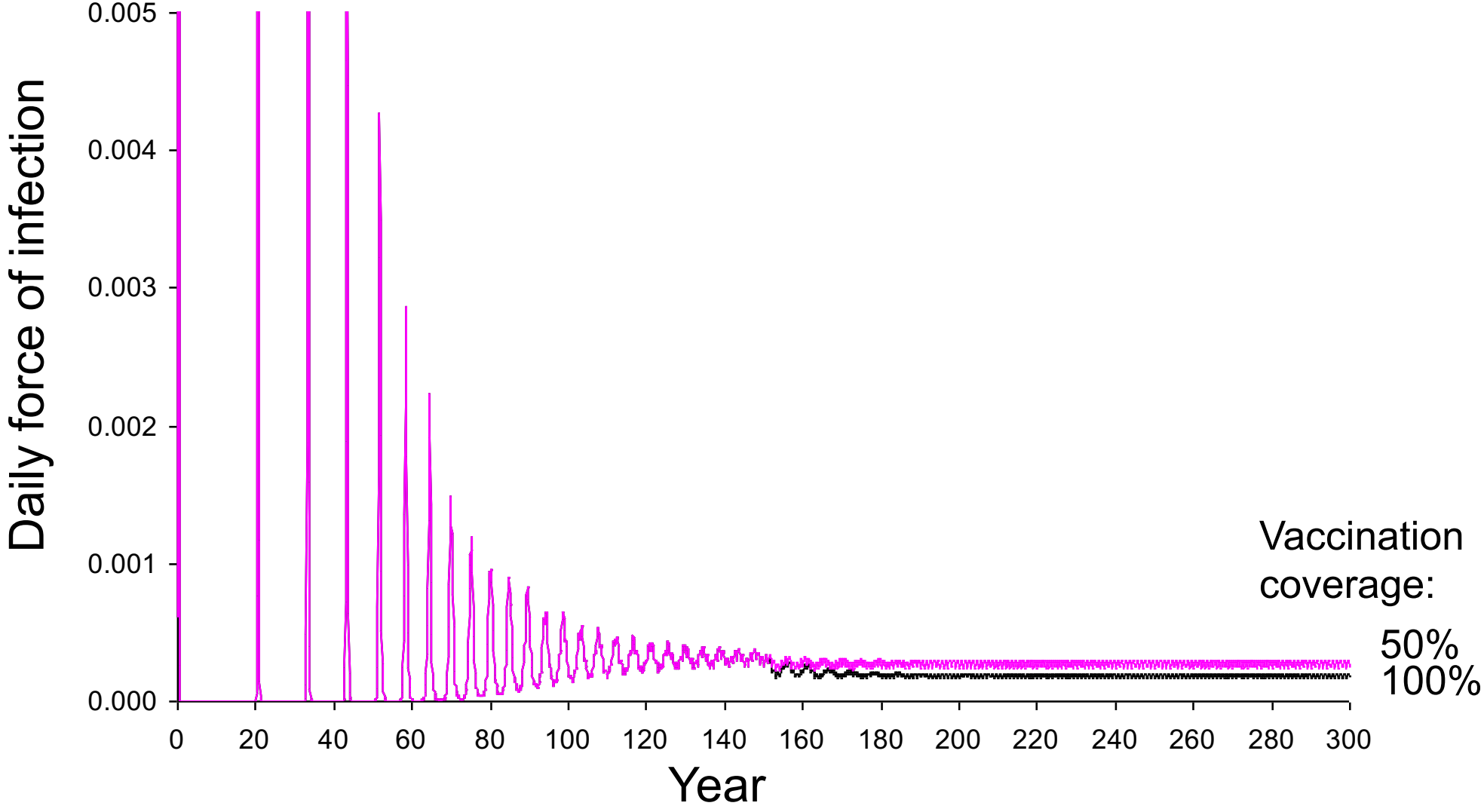
Q2.1 Run the model for values of prop_vacc_13 of 0.5 and 1.00.

How does annual vaccination among 13 year olds affect

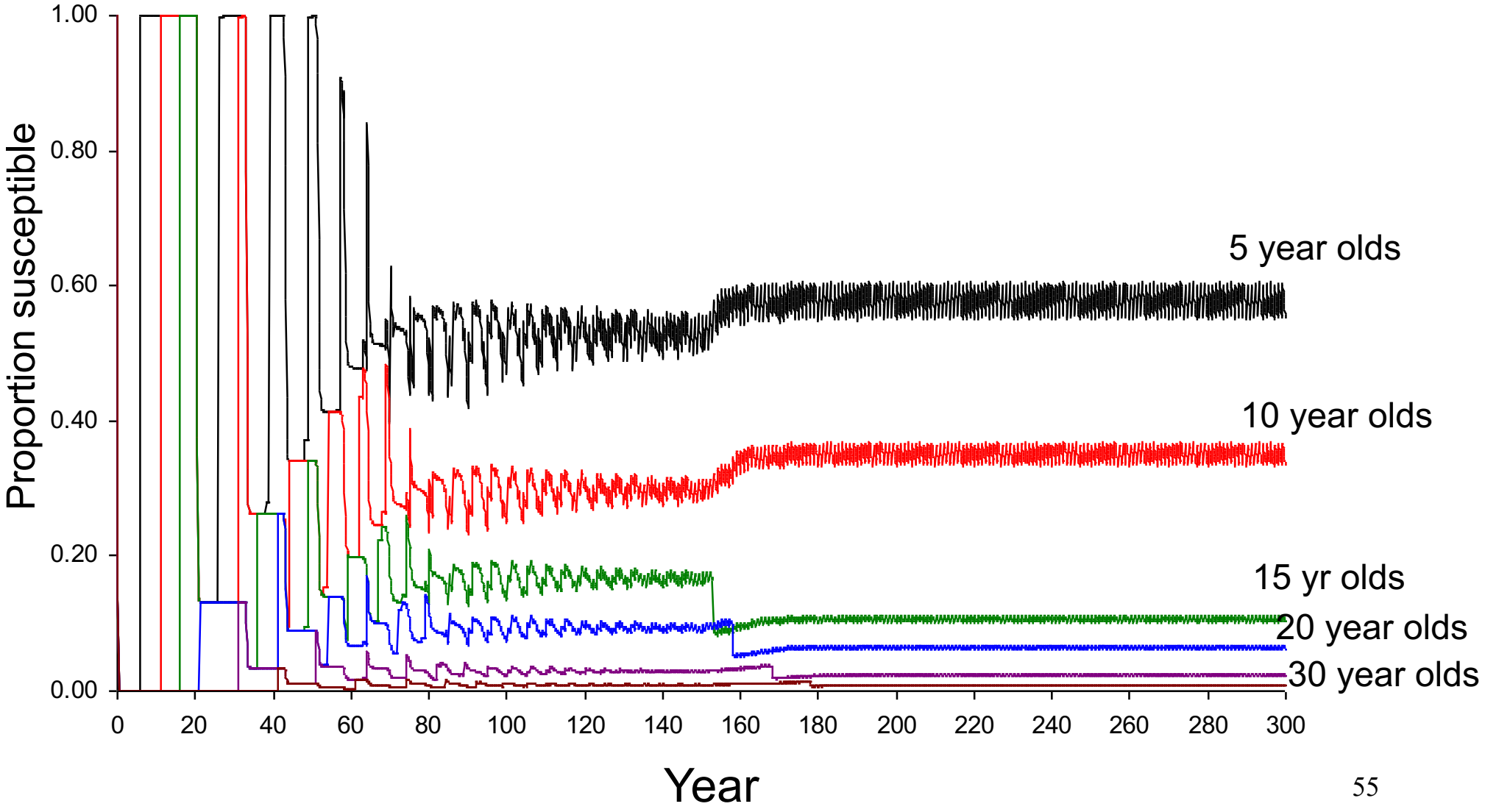
a) *the force of infection in the population?*
b) *the age-specific proportion of individuals who are susceptible?*
c) *the daily number of new infections per 100,000 in different age groups?*

Why does this occur?

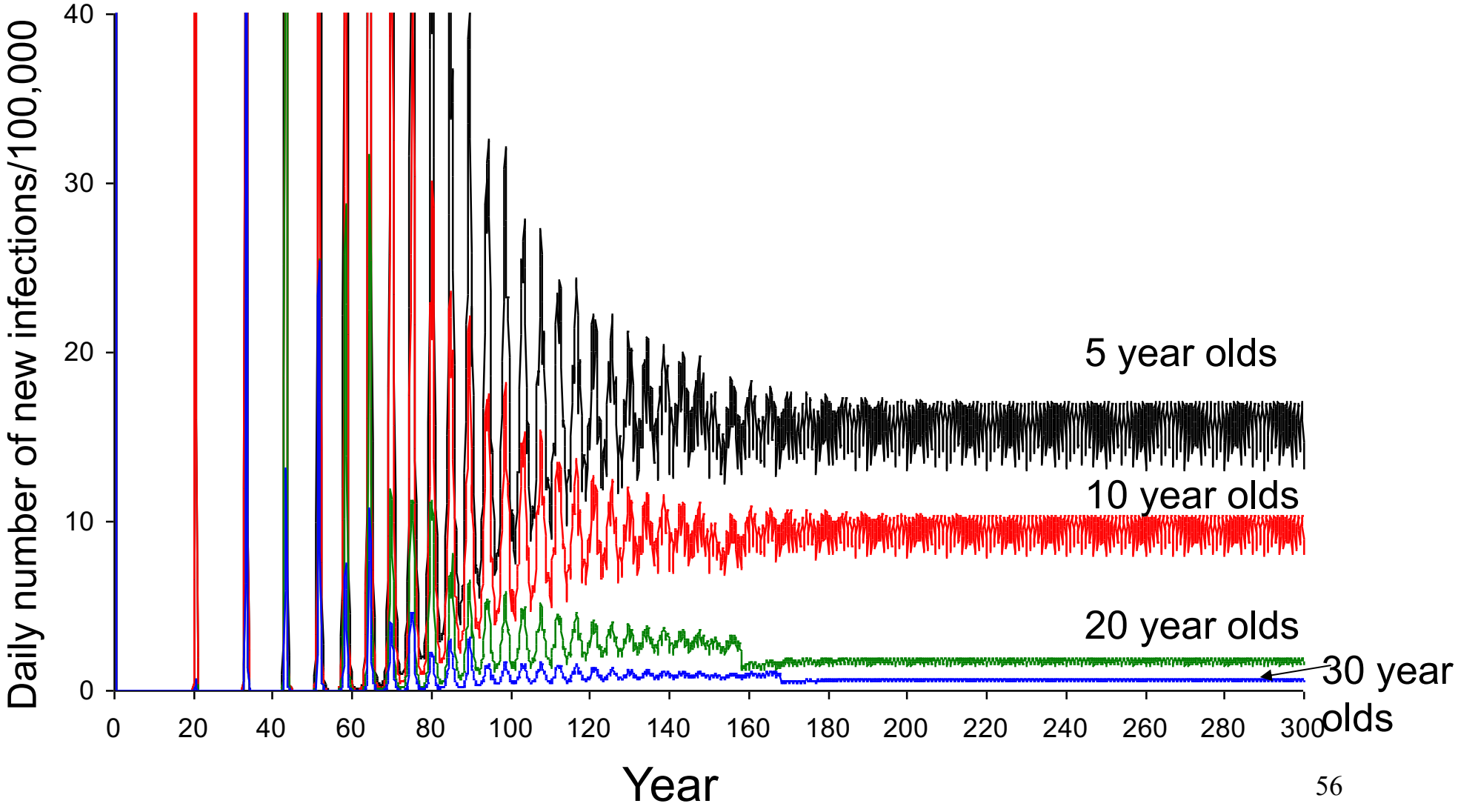
Q2.1, Q2.2 Model predictions of the impact of 50% or 100% effective vaccination coverage among 13 year olds on the force of infection in the low transmission setting



Q2.1b Model predictions of impact of 50% effective vaccination coverage among 13 yr olds on the proportion susceptible in the low transmission setting



Q2.1c Model predictions of the impact of 50% effective vaccination coverage among 13 year olds on the daily number of new infections per 100,000 in the UK



Q2.1 How does vaccination among 13 year olds affect

a) the force of infection in the population? b) the age-specific proportion of individuals who are susceptible? c) the daily number of new infections per 100,000 in different age groups?

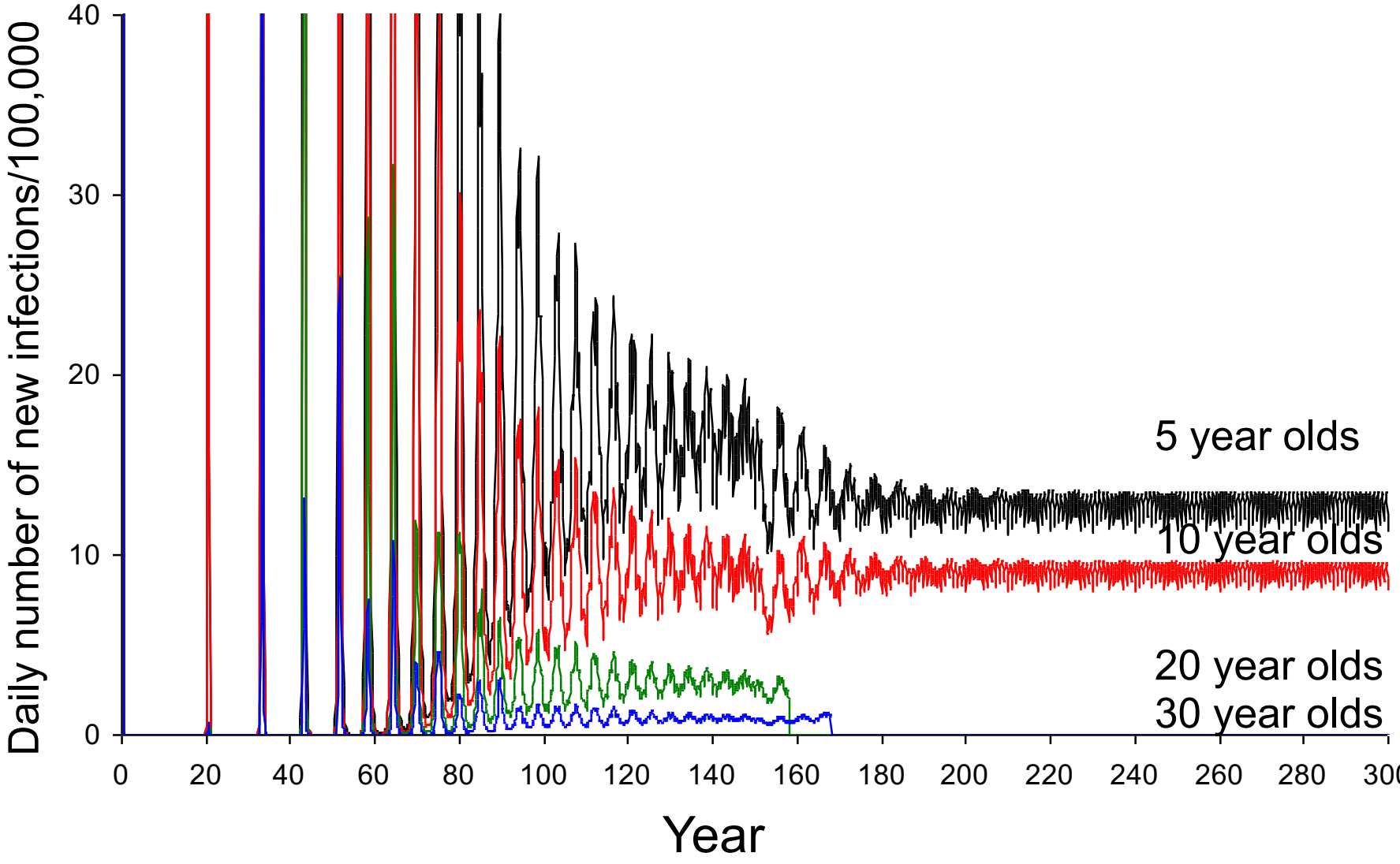
Why does this occur?

a) Force of infection goes down slightly

b) Proportion susceptible:
   - Big ↓ for 15+ year olds as they've been vaccinated
   - Slight ↑ for 5, 10 year olds due to less chance for infection (due to reduction in prevalence of infectious people)

c) Daily number of new infections per 100,000:
   - ↓ for 15+ year olds (as they've been vaccinated)
   - Little change for 5, 10 year olds due to small reduction in overall amount of transmission in population

Q2.2 Model predictions of the impact of 100% effective vaccination coverage among 13 year olds on the daily number of new infections per 100,000 in the low transmission setting



Q2.3 What are the relative benefits of a partial vaccination strategy as compared with the strategy of vaccinating all individuals in their first year of life?

- Vaccinating in the first year of life interrupts transmission for all
- Partial vaccination protects those most at risk and doesn't have a big effect on the amount of transmission in the whole population

Objectives

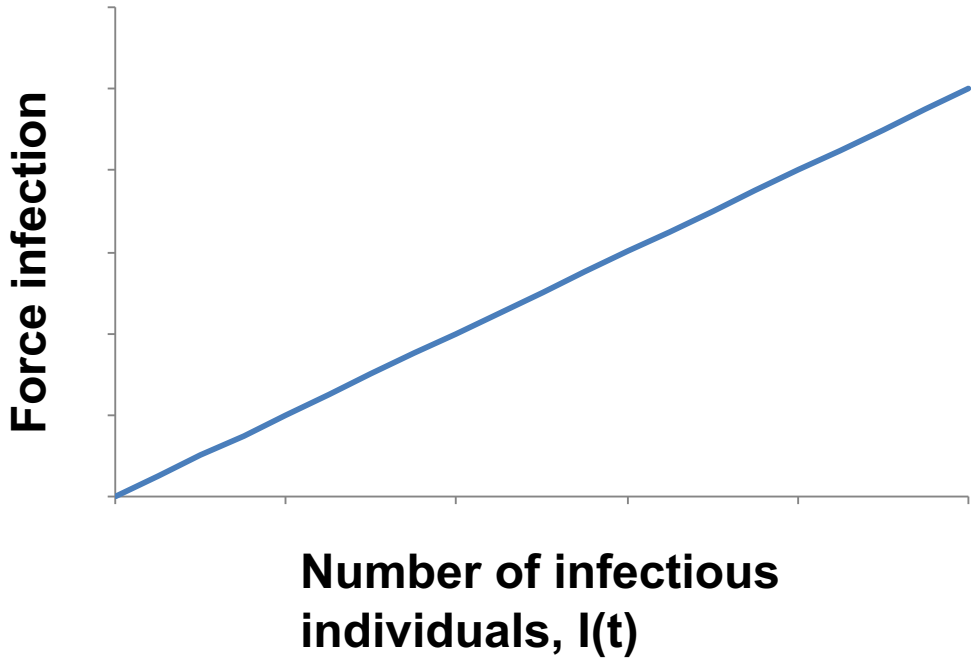
By the end of this session you should:

- Know why we might need to use age-structured models
- How age-structured models might be set up
- Know how different levels of vaccination coverage may affect:
    a) the age-specific proportion susceptible in the population
    b) the age-specific number of new infections per unit time
    c) the average age at infection.
- Be aware of the effect of mixing patterns on the impact of vaccination

The force (or risk) of infection

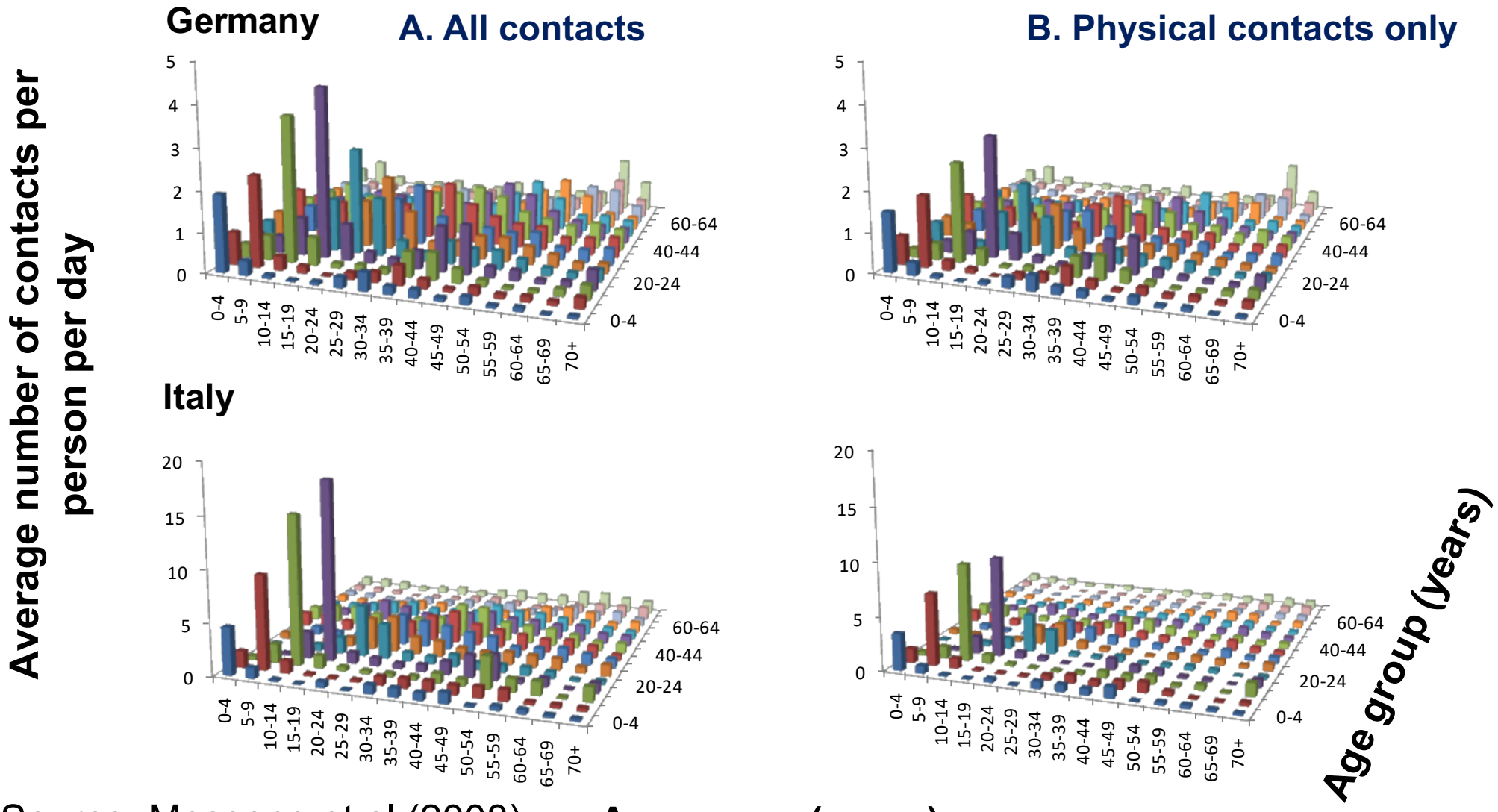
The simplest assumption: individuals mix randomly or the "mass action principle":

$$\lambda(t) = \beta I(t)$$



Is it reasonable to assume random mixing and does it matter?

Evidence for age-dependent transmission: Average numbers of individuals contacted per day, May 2005-Sept 2006 in Western populations (POLYMOD)



Source: Mossong et al (2008)

Ramamoorthy et al, Epidemics 2023 (Bologna)

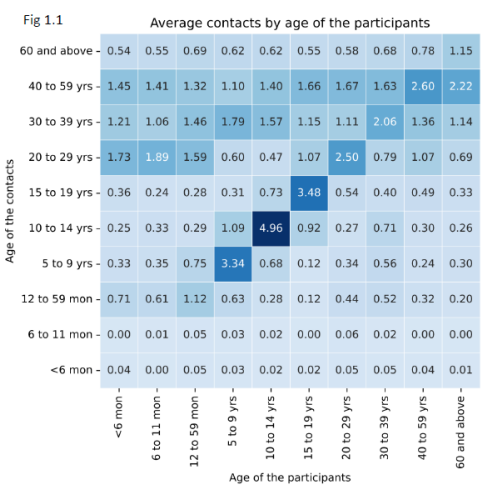
Insights into social mixing patterns in urban and rural population of southern India

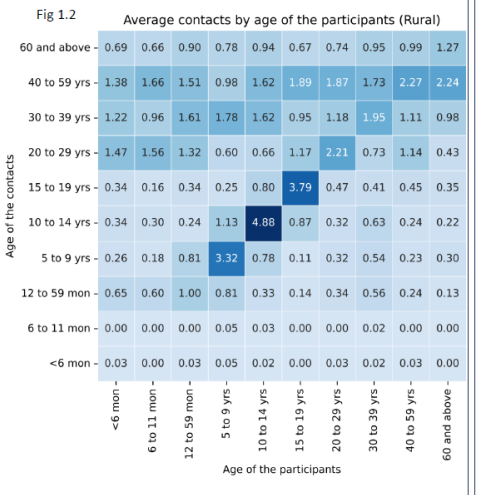
Agil Ramamoorthy, Rajan Srinivasan, Meenakshi. N, Gayathri. S, Moses Chapa Kiti, Obianuju Aguolu, Saad B. Omer, Benjamin Lopman, Venkata Raghava Mohan and all other GlobalMix study collaborators

1. Background

- Understanding social mixing patterns is vital for planning interventions during infectious disease outbreaks. Social interactions are intricately shaped by the unique cultural contexts and varying levels of community awareness.
- Studies that dwell into understanding the interaction pattern in high income countries has helped to identify at risk population and plan intervention to control disease transmission.
- This study aims to fill the gap in such research for middle-income countries, shedding light on high-risk populations with increased social contact.
- By identifying these patterns, the study informs targeted strategies like immunization and contact tracing to mitigate the impact of infectious disease efficiently.

2. Aims and Objectives





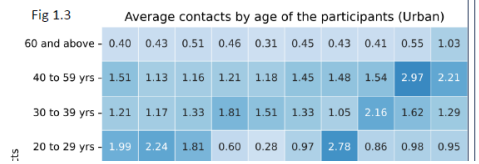


Figure-1
The average daily contacts among distinct
age cohorts demonstrates a uniformity

Ramamoorthy et al (Vellore)

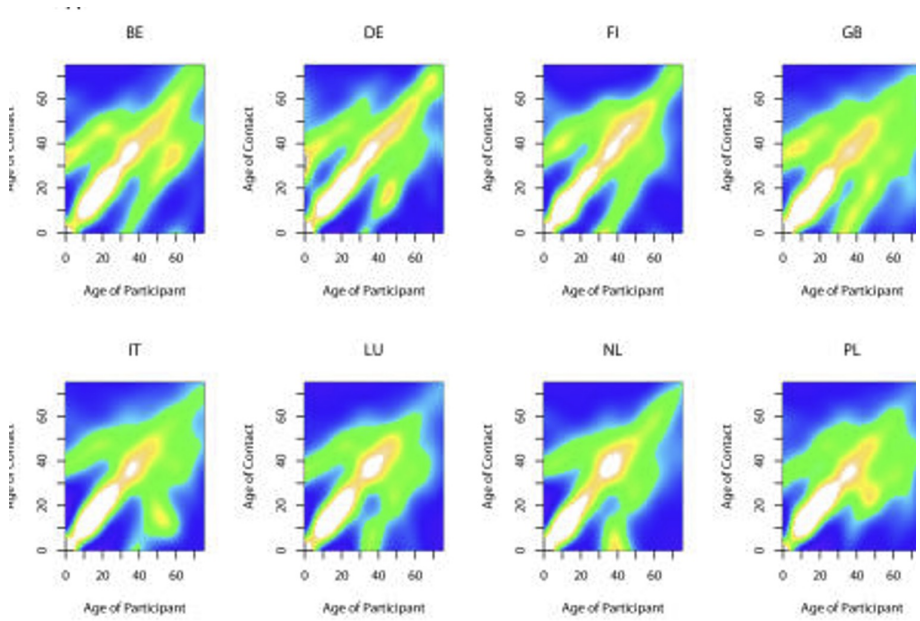
Fig 1.1

Average contacts by age of the participants

Age of the contacts	<6 mon	6 to 11 mon	12 to 59 mon	5 to 9 yrs	10 to 14 yrs	15 to 19 yrs	20 to 29 yrs	30 to 39 yrs	40 to 59 yrs	60 and above
60 and above	0.54	0.55	0.69	0.62	0.62	0.55	0.58	0.68	0.78	1.15
40 to 59 yrs	1.45	1.41	1.32	1.10	1.40	1.66	1.67	1.63	2.60	2.22
30 to 39 yrs	1.21	1.06	1.46	1.79	1.57	1.15	1.11	2.06	1.36	1.14
20 to 29 yrs	1.73	1.89	1.59	0.60	0.47	1.07	2.50	0.79	1.07	0.69
15 to 19 yrs	0.36	0.24	0.28	0.31	0.73	3.48	0.54	0.40	0.49	0.33
10 to 14 yrs	0.25	0.33	0.29	1.09	4.96	0.92	0.27	0.71	0.30	0.26
5 to 9 yrs	0.33	0.35	0.75	3.34	0.68	0.12	0.34	0.56	0.24	0.30
12 to 59 mon	0.71	0.61	1.12	0.63	0.28	0.12	0.44	0.52	0.32	0.20
6 to 11 mon	0.00	0.01	0.05	0.03	0.02	0.00	0.06	0.02	0.00	0.00
<6 mon	0.04	0.00	0.05	0.03	0.02	0.02	0.05	0.05	0.04	0.01

Age of the participants

POLYMOD

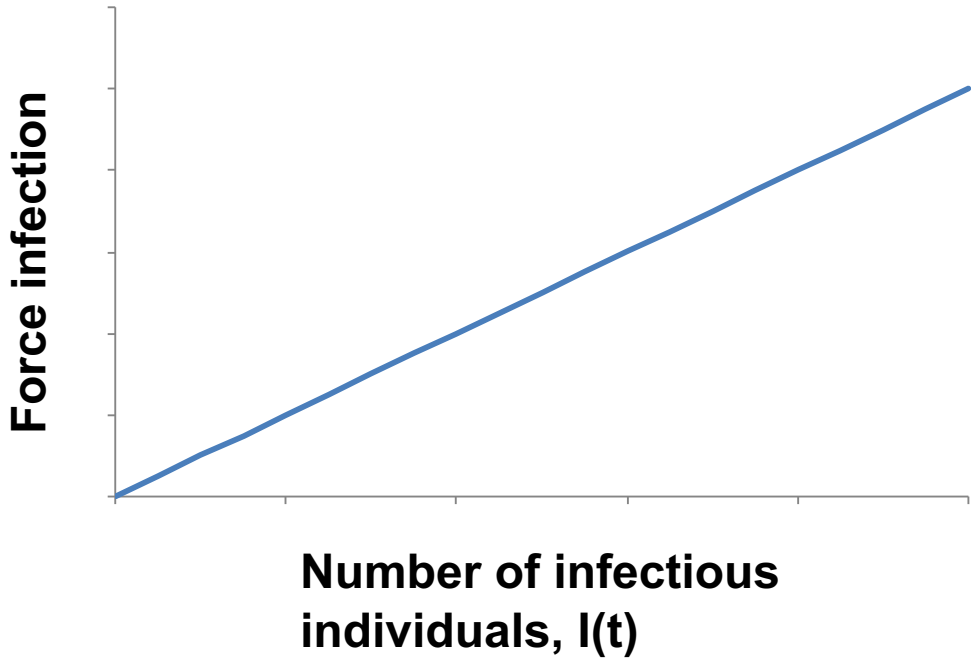


Who currently thinks it matters as to what we assume about contact patterns?

The force (or risk) of infection

The simplest assumption: individuals mix randomly or the "mass action principle":

$$\lambda(t) = \beta I(t)$$



Methods for incorporating non-random (heterogeneous) mixing into models

For heterogeneously mixing populations (eg in which contact patterns differ between the young and old):

$$\lambda_y(t) = \beta_{yy} I_y(t) + \beta_{yo} I_o(t)$$

$$\lambda_o(t) = \beta_{oy} I_y(t) + \beta_{oo} I_o(t)$$

β_{yy} = rate at which a specific young individual comes into effective contact with a specific young individual per unit time

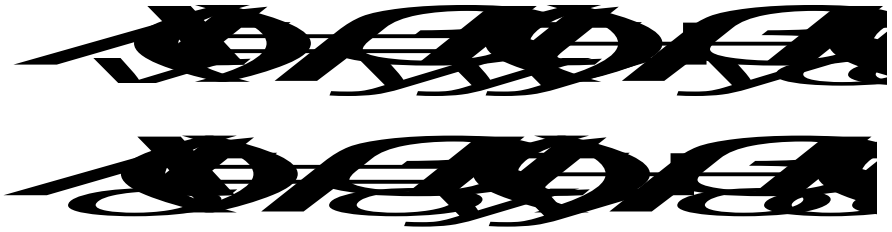
β_{yo} = rate at which a specific young (susceptible) individual comes into effective with a specific old (infectious) individual per unit time etc

$I_y(t)$ = number of infectious young individuals in the population

$I_o(t)$ = number of infectious old individuals in the population

Expressions for the force of infection using matrix notation

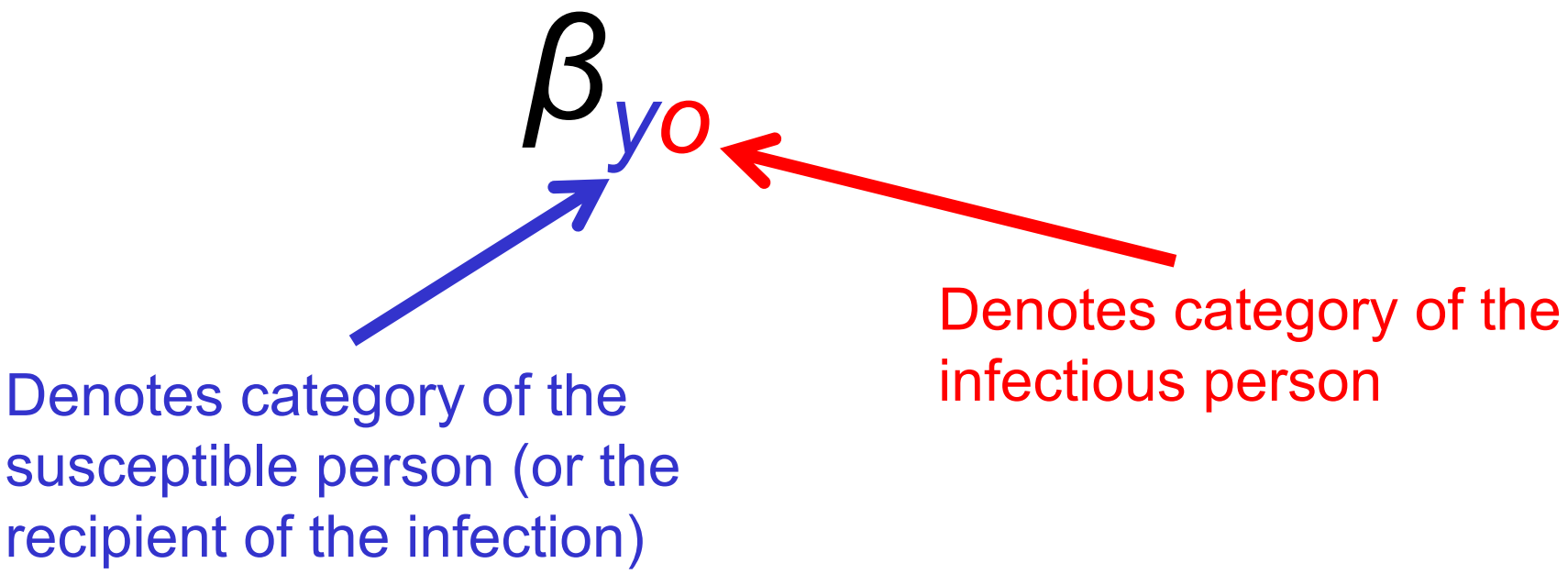
The equations:



can be summarized using the following matrix equation:

Matrix of "**Who Acquires Infection From Whom**" (WAIFW)

A note on notation



Therefore:

β_{yo} = rate at which a specific young (susceptible) individual comes into effective with a specific old (infectious) individual per unit time

Expressions for the force of infection using matrices

The equations:

$$\lambda_y(t) = \beta_{yy} I_y(t) + \beta_{yo} I_o(t)$$

$$\lambda_o(t) = \beta_{oy} I_y(t) + \beta_{oo} I_o(t)$$

can be summarized using the following matrix equations:

$$\begin{pmatrix} \lambda_y(t) \\ \lambda_o(t) \end{pmatrix} = \begin{pmatrix} \beta_{yy} & \beta_{yo} \\ \beta_{oy} & \beta_{oo} \end{pmatrix} \begin{pmatrix} I_y(t) \\ I_o(t) \end{pmatrix}$$

Matrix of "Who Acquires Infection From Whom" ("WAIFW")

β_{yy}, β_{yo} etc can be calculated given estimates of:

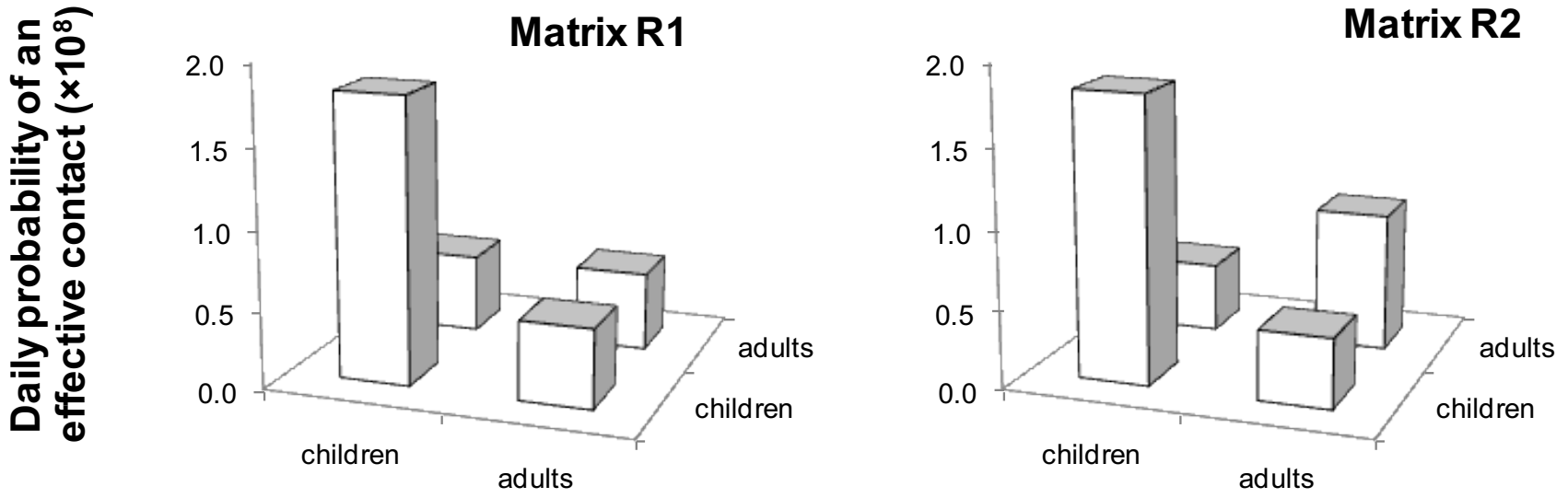
$$\lambda_y(t), \quad \lambda_o(t), I_y(t), I_o(t)$$

and assumptions about the WAIFW matrix structure

Example

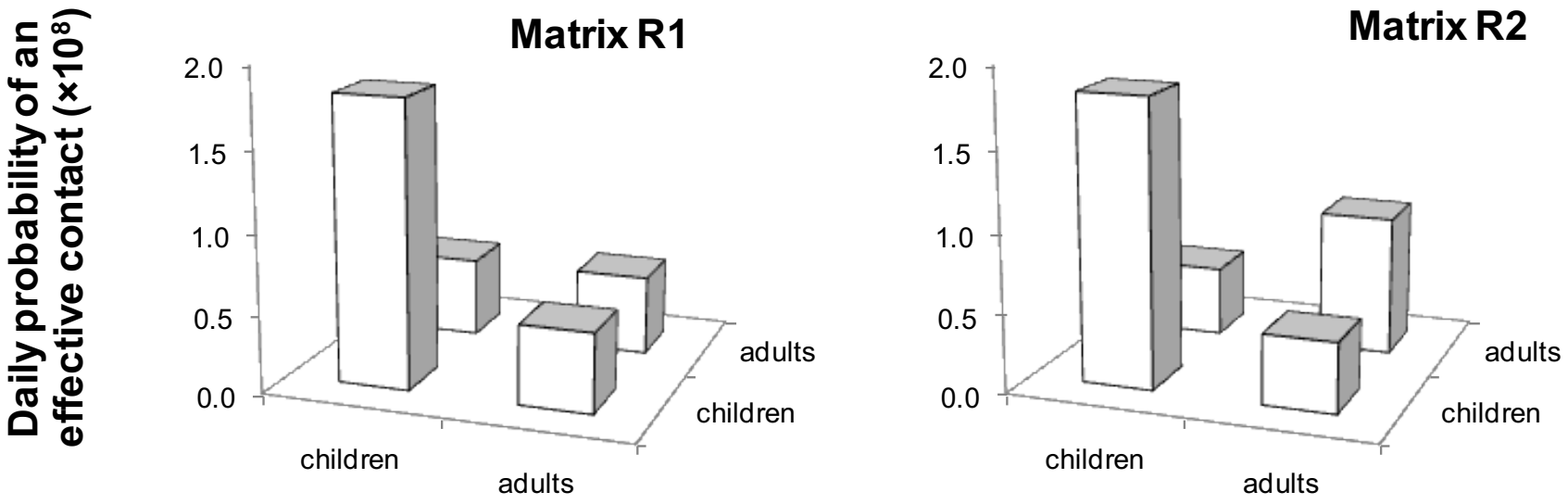
Suppose the force of infection differs between children and adults in a population

The following two contact matrices reproduce the same force of infection in children and adults before the introduction of vaccination among very young children



If we introduce vaccination among very young children, in which population will the impact of vaccination be greatest for a given level of coverage?

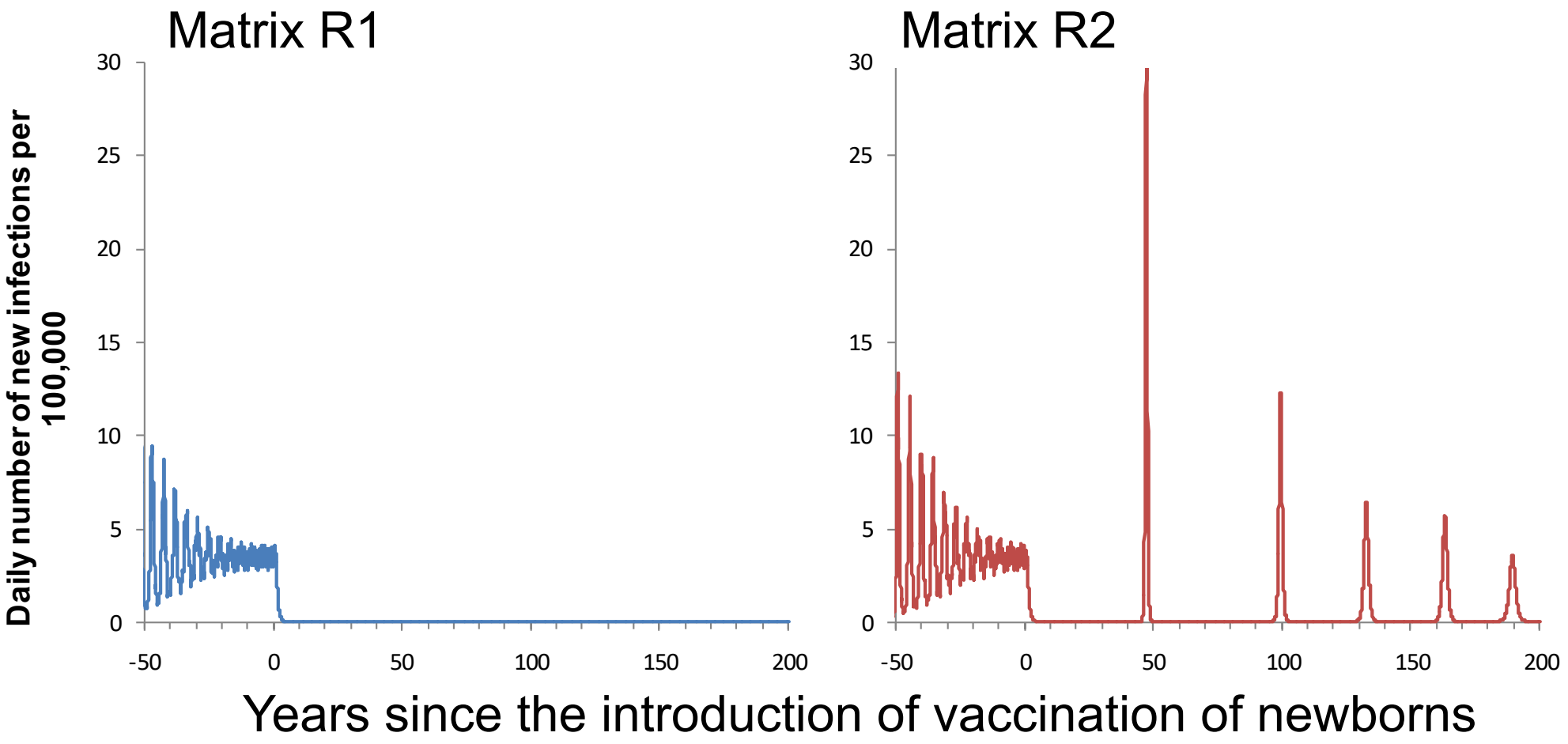
Example



If we introduce vaccination among very young children, in which population will the impact of vaccination be greatest for a given level of coverage?

Check your answer by running rubella_model3 and rubella_model4 for values of prop_vacc =0.72.

Model predictions of the daily number of new infections/100,000 for WAIFW matrices R1 and R2, 72% vaccination coverage among newborns



Optional reading about models rubella_model3 and rubella_model4

General equations similar to rubella_model1; goes up to age 75 years

Key difference relates to the force of infection:

force_of_infn[0..14] = beta_yy*Tot_infous_y + beta_yo*Tot_infous_o

force_of_infn[15..up_age] = beta_oy*Tot_infous_y + beta_oo*Tot_infous_o

Who now thinks it matters as to what we assume about contact patterns?

Conclusions of the effect of non-random mixing

- May see identical age-specific incidence in different settings with different contact patterns
- However, the impact of the intervention (and the herd immunity threshold) may differ because contact patterns differ
- R_0 will depend on the contact patterns in the population
- Transmission models need to include appropriate age-related contact assumptions when predicting the impact of an intervention
- Since contact patterns are not always well-understood, it is important to explore the effect of different contact patterns in sensitivity analyses

Objectives

By now, you should:

- Know why we might need to use age-structured models
- How age-structured models might be set up
- Know how different levels of vaccination coverage may affect:
    a) the age-specific proportion susceptible in the population
    b) the age-specific number of new infections per unit time
    c) the average age at infection.
- Be aware of the effect of mixing patterns on the impact of vaccination

Key steps in developing an age structured model

- Decide on the size of the age band that you want to model, e.g. single year age bands, 6 monthly, monthly?

- Set up separate compartments for each age band

- Decide how people move from one age band to the next
  - Constant ageing rate (=1/average duration of stay in the age band)
  - Only move people from one age group to the next at prescribed time points, e.g. at the end of the year, if using single year age bands (Schenzle or Realistic Age Structure (RAS) approach)

- Decide on whether people mix randomly or if there's age dependent mixing

- Ensure that infectious people in all age bands can infect others