Figures Mathematical Techniques For Engineers Complex Analysis

Felix Claeys, Brecht Verbeken, Simon Verbruggen

October 12, 2025

2.1.1 Continuity Definition

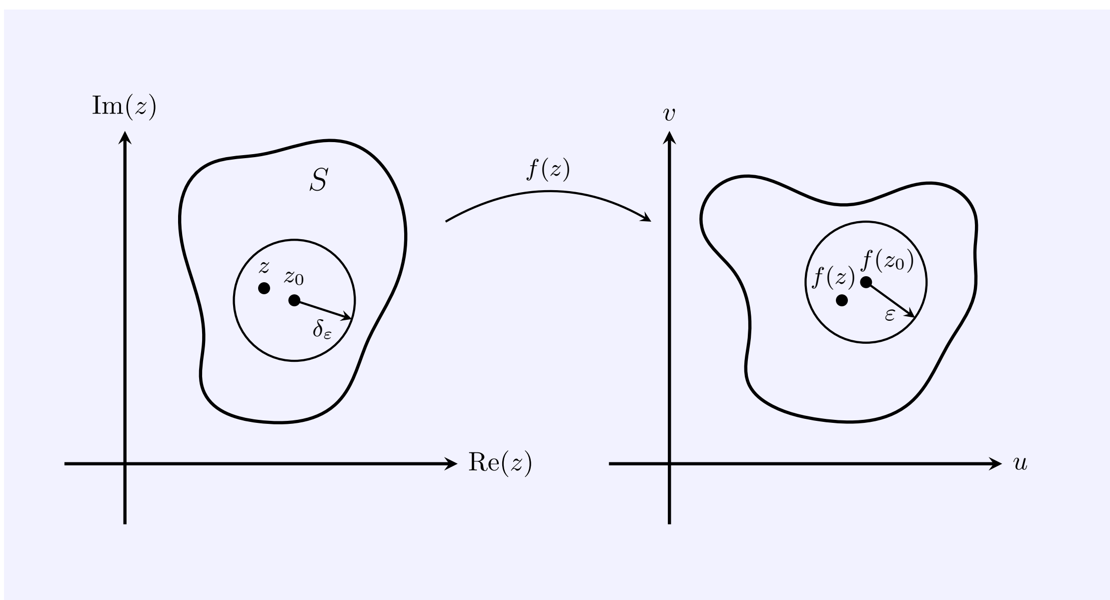

Figure 1: The function f is continuous in z_0 if $(\forall\varepsilon > 0)(\exists\delta_\varepsilon > 0)(\forall z \in S)(|z - z_0| < \delta_\varepsilon \Longrightarrow |f(z) - f(z_0)| < \varepsilon)$.

2.4 Geometrical Interpretation Of The Complex Derivative

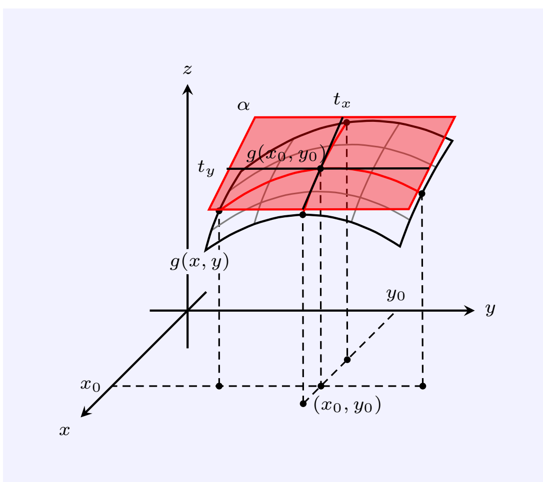

Figure 2: In every point $g(x_0, y_0)$ of a surface $g(x, y)$, a tangent plane can be drawn (red). The tangent lines t_x, t_y are oriented according to the x- and y-axis, respectively. They have a slope which corresponds to the partial derivatives $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$, respectively.

3.2.12 Symmetry of points

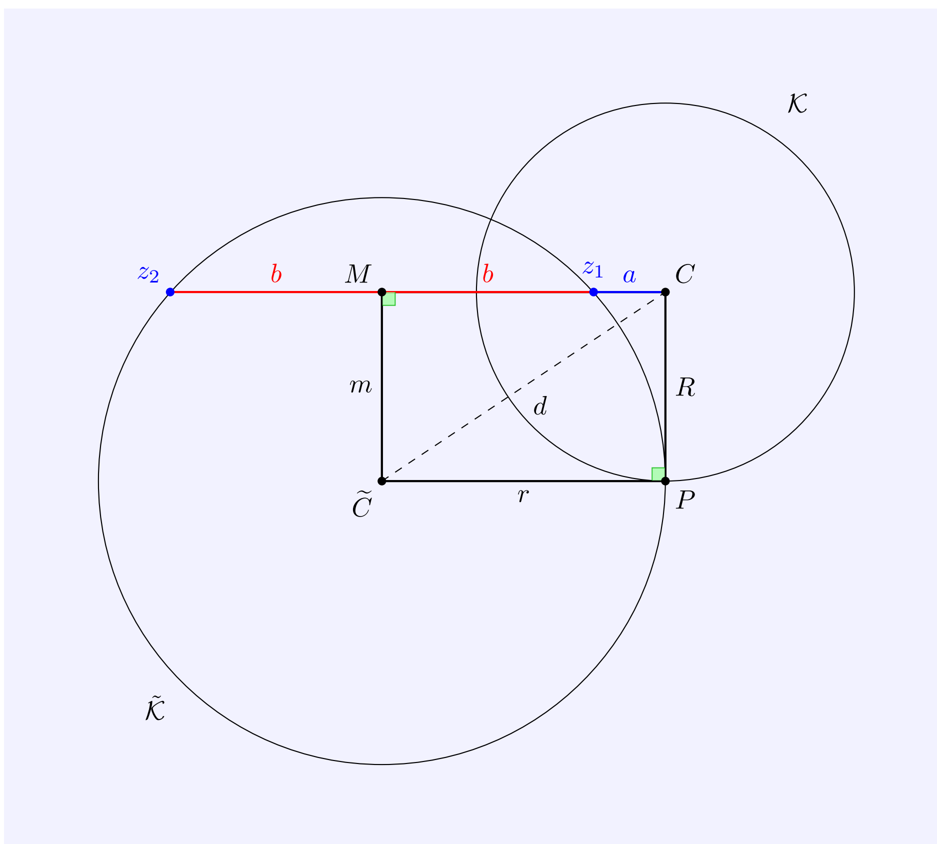

Figure 3: The points z_1 and z_2 are symmetrical with respect to the circle $\mathcal{K} = S(C, R)$.

3.5.1 Exponential function periodicity

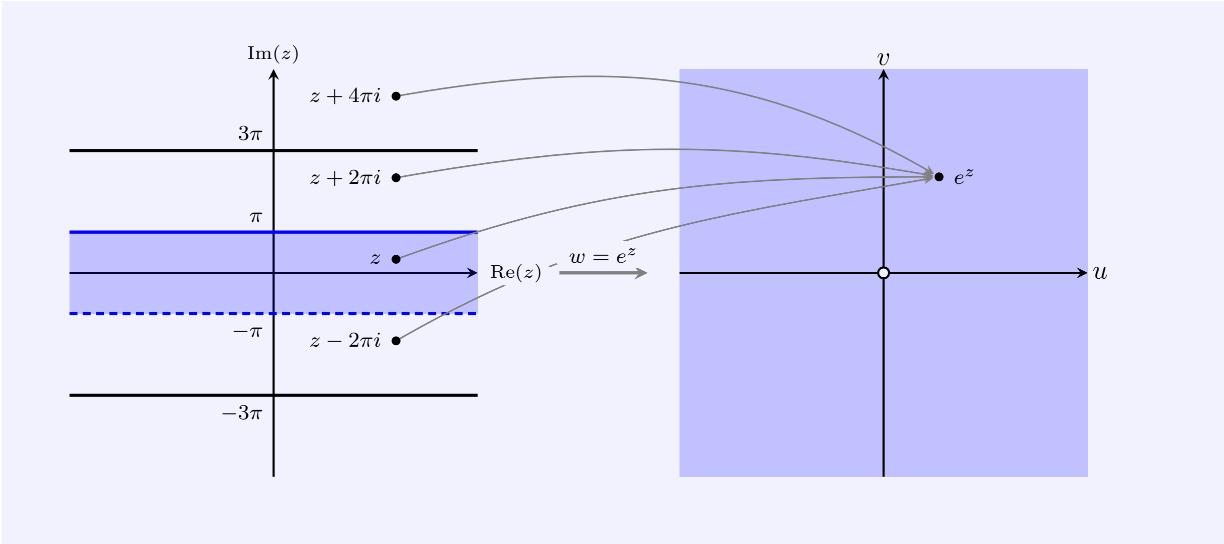

Figure 4: The exponential function $f(z) = e^z$ is periodic with period $2\pi i$.

3.5.2 Exponential function image vertical lines

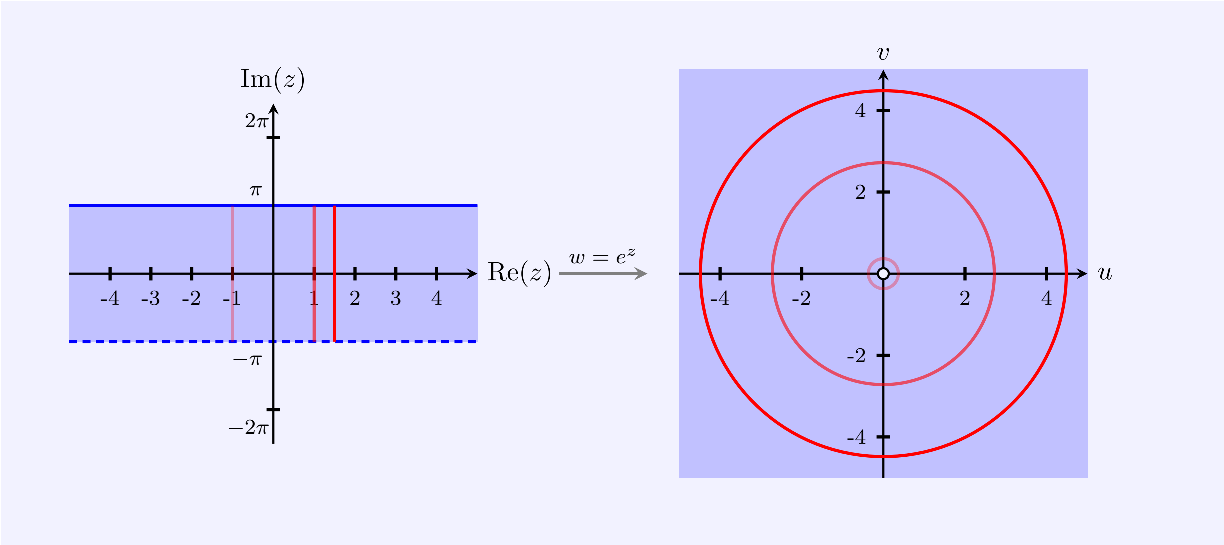

Figure 5: The exponential function $f(z) = e^z$ maps vertical lines onto circles centered at the origin.

3.5.3 Exponential function image horizontal lines

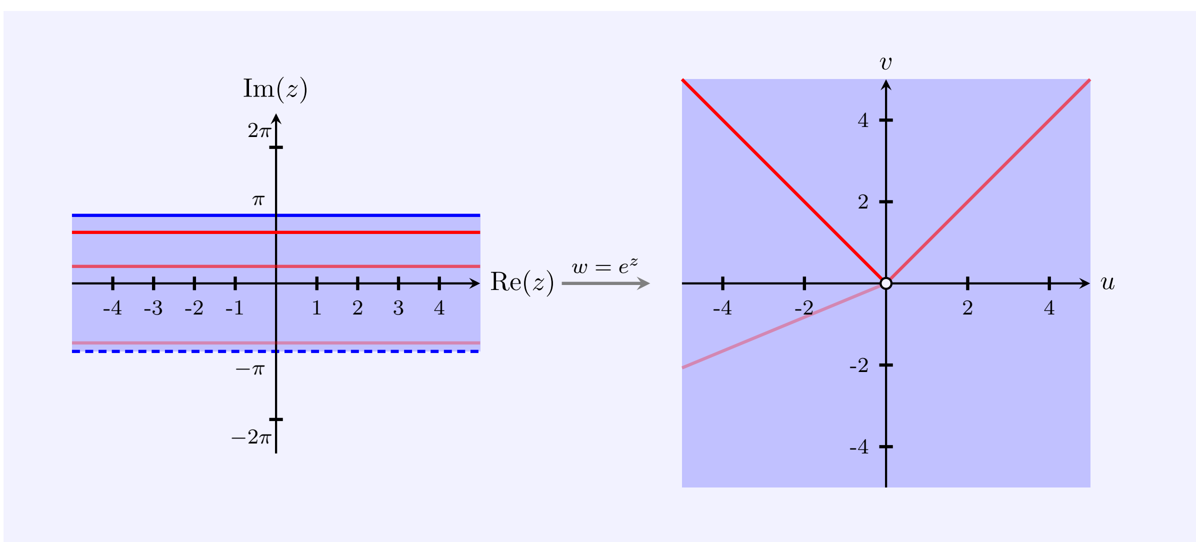

Figure 6: The exponential function $f(z) = e^z$ maps horizontal lines onto half-lines originating from the origin.

3.8.4 Exercise

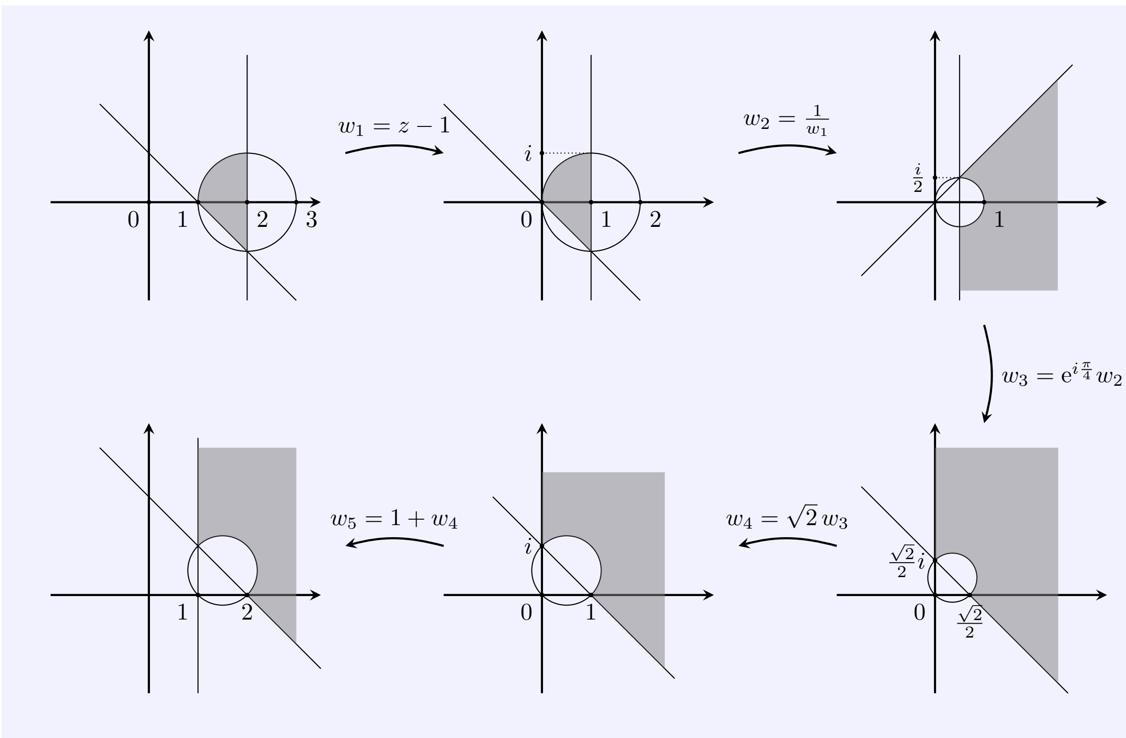

Figure 7: Image of domain $\mathcal{D} = \{z \in \mathbb{C} | \mathrm{Re}(z) + \mathrm{Im}(z) \geq 1, |z-2| \leq 1, \mathrm{Re}(z) \leq 2\}$ through the function $f(z) = \frac{z+i}{z-i}$.

5.1.1

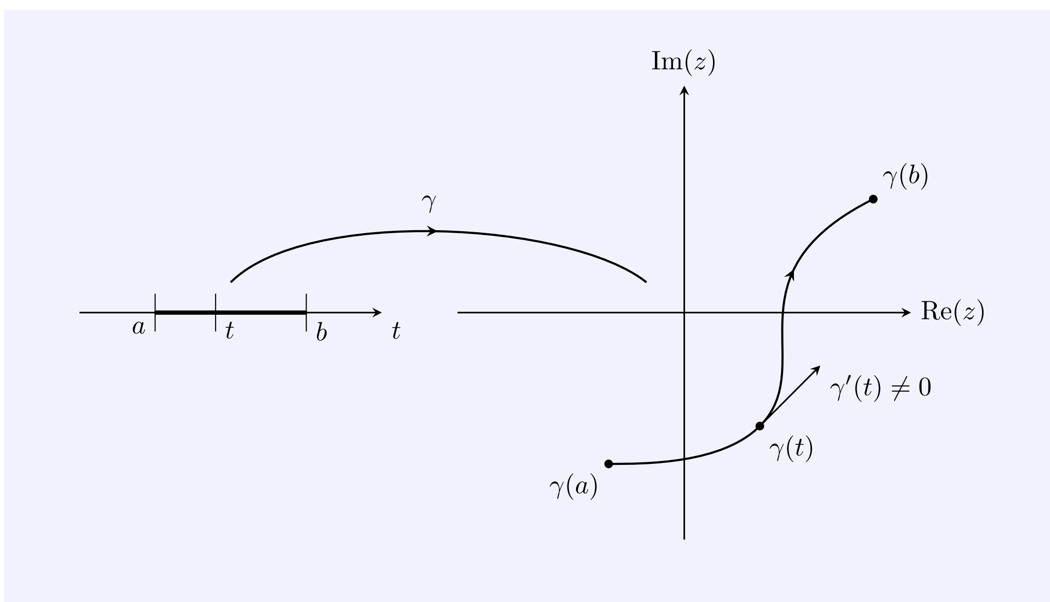

Figure 8: The complex line integral $\int_C f(z)\,dz$ for $z = \gamma(t)$, $t \in [a, b]$, with $\gamma(t)$ a smooth curve, can also be seen as an integral over t with the equation $\int_C f(z)\,dz = \int_a^b f(\gamma(t))\,\gamma'(t)\,dt$.

5.2.3

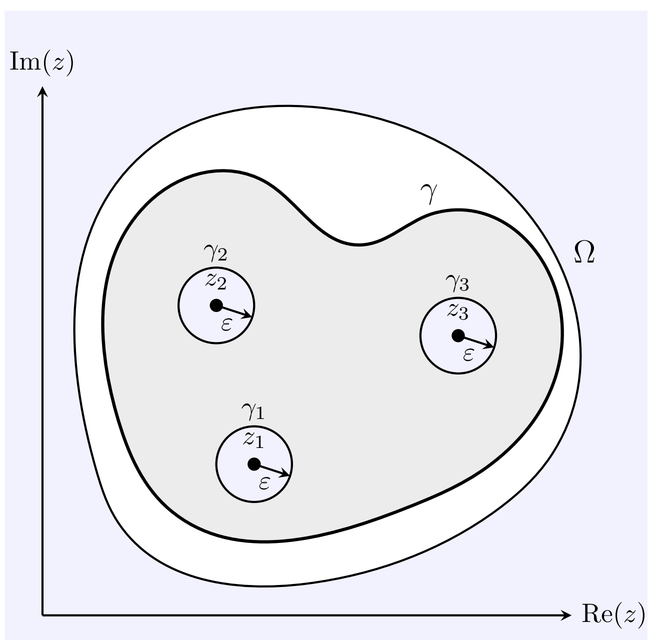

Figure 9: The integral over a contour C in Ω with an interior with a finite amount of singular poles is the sum of the integrals over the circles around these interior poles.

5.3 Cauchy integral formulas and consequences

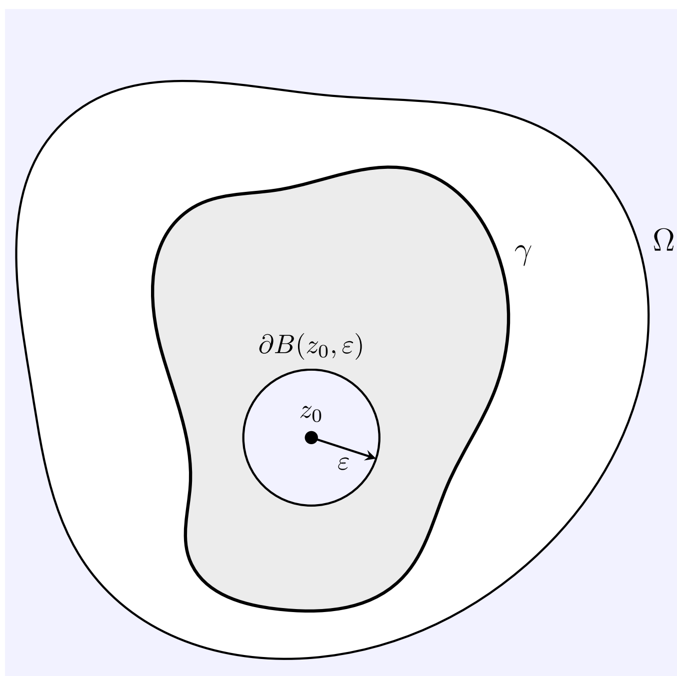

Figure 10: $\mathcal{C}$ is a bounded contour in Ω which encloses a compact set K lying completely within Ω. For a point $a \in \mathcal{C} \setminus K$, we parametrize the circle $\partial B(a, \epsilon)$, which lies entirely in the interior of $\mathcal{C}$.

5.4.4

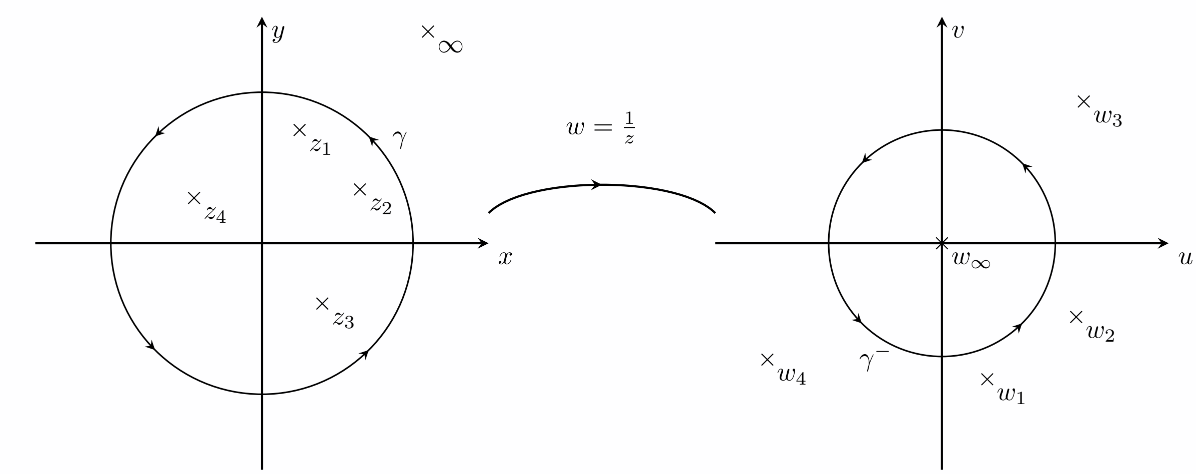

Figure 11: To define the residue of a isolated singular pole at $z = \infty$, we can evaluate the integral of a contour in which all other poles lie. By taking the reciprolal equation $w = \frac{1}{z}$ we get an integral around the origin that only contains the pole $w = 0$. Which proves $\operatorname{Res}(f(z), \infty) := -\frac{1}{2\pi i} \oint_\gamma f(z)\, dz$.

5.5.2 Uniqueness of holomorphic functions

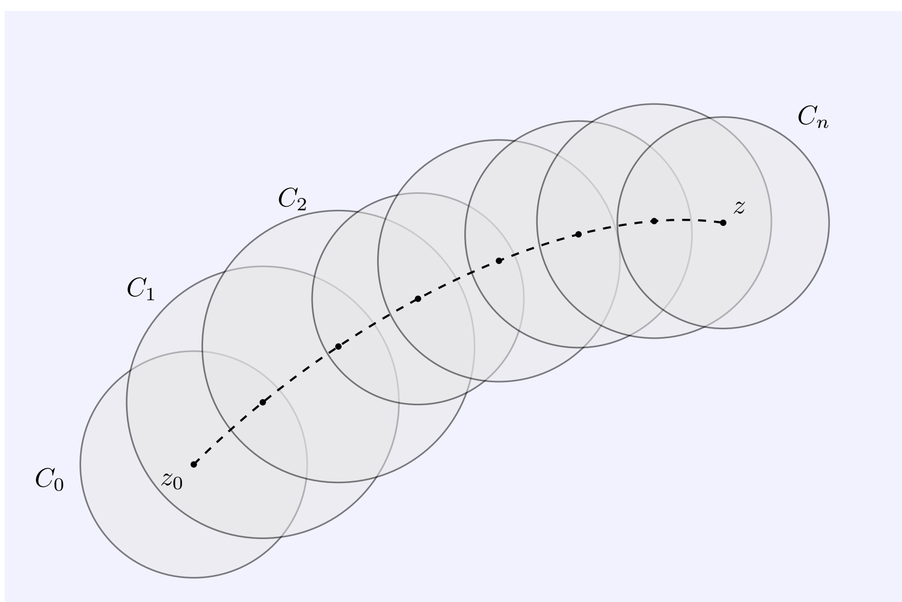

Figure 12: Let f be holomorphic in the space $\Omega \subseteq \mathbb{C}$. If $z_0 \in \Omega$ is an accumulation point of zeros of f, then $f \equiv 0$ over the entire space Ω.

5.6.2

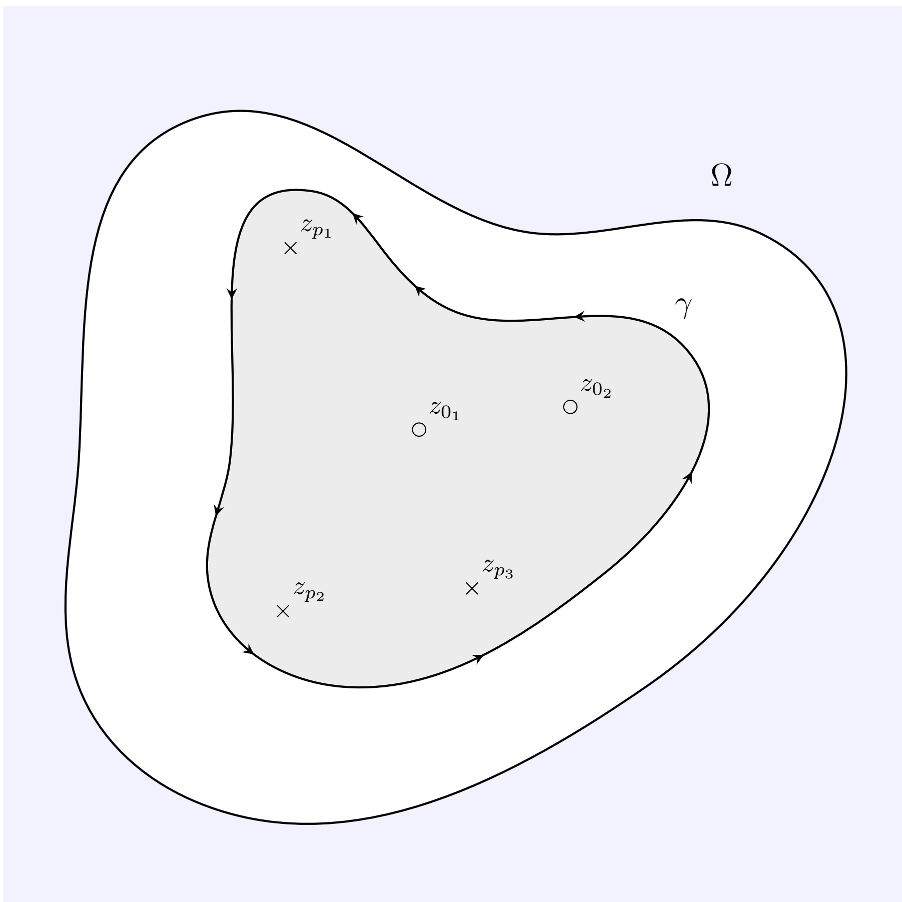

Figure 13: The argument principle gives for a closed, smooth Jordan curve γ, entirely within the domain $\Omega \subseteq \mathbb{C}$, and f a meromorphic function in Ω whose poles all lie inside γ, and such that $f(z) \neq 0$ for $z \in \gamma$. $\frac{1}{2\pi i} \oint_\gamma \frac{f'(z)}{f(z)}\, dz = N_\gamma(f) - P_\gamma(f)$,where $N_\gamma(f)$ and $P_\gamma(f)$ denote, respectively, the number of zeros and poles of f inside γ, each counted with multiplicity.

5.6.3 Argument Principle

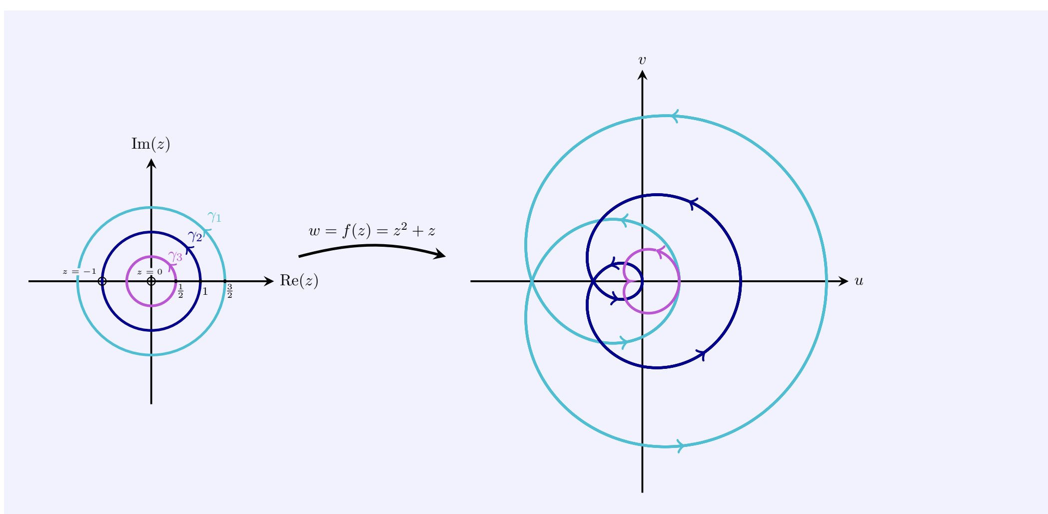

Figure 14: Illustration of the argument principle for $f(z) = z^2 + z$. The images of the circles $\gamma_1, \gamma_2, \gamma_3$ under f show how many times each curve winds around the origin, corresponding to the number of zeros of f inside each circle.

5.6.3 Rouches theorem

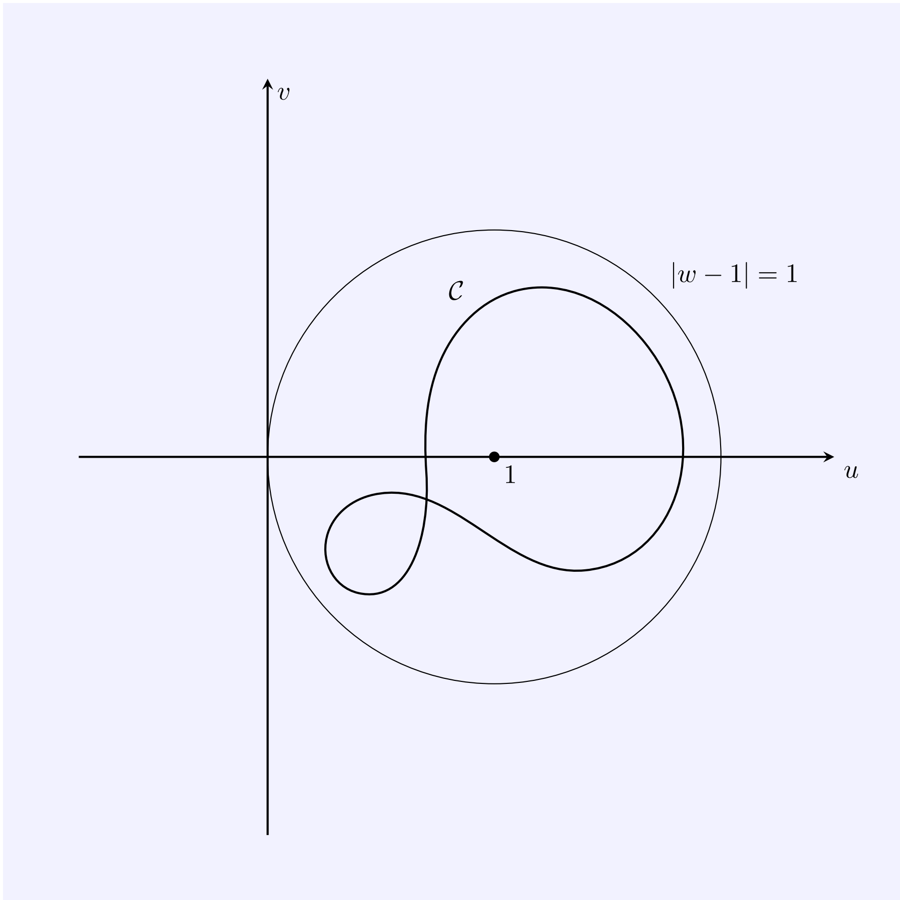

Figure 15: To prove the theorem of Rouché: For f and g, holomorphic functions on and inside a closed, smooth Jordan curve γ in $\mathbb{C}$. If $|g(z)| < |f(z)|$ for every $z \in \gamma$, then $N_\gamma(f) = N_\gamma(f+g)$. We use that $F(z) = \frac{f(z)+g(z)}{f(z)} = 1 + \frac{g(z)}{f(z)}$ and $|\frac{g(z)}{f(z)}| < 1$. We get a contour inside the unit circle around 1.