Figures Calculus III

Felix Claeys, Brecht Verbeken, Simon Verbruggen

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1.2.3 Example evolute cycloid

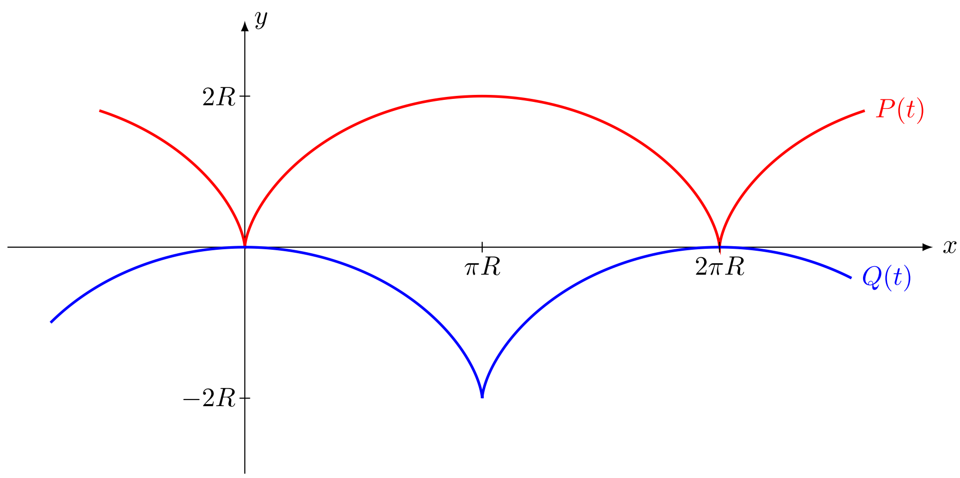

Figure 1: The cycloid $P(t) = [R(t - \sin t), R(1 - \cos t)]$ and its evolute $Q(t) = [R(t + \sin t), R(\cos t - 1)]$, which is a translation of $P(t)$.

1.2.4 Example evolute catenary

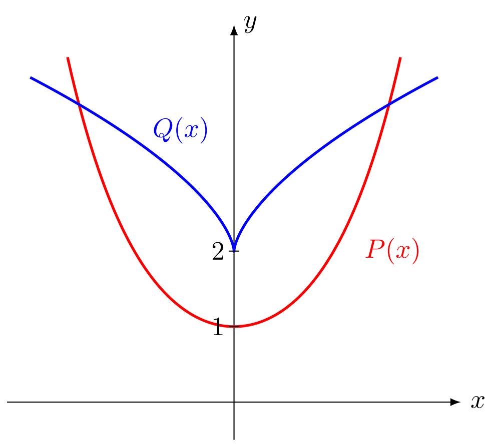

Figure 2: The catenary $P(x) = [x, a\cosh(x/a)]$ and its evolute $Q(x) = \left[x - \frac{a}{2}\sinh(2x/a), 2a\cosh(x/a)\right]$.

1.2.5 Example involute catenary (tractrix)

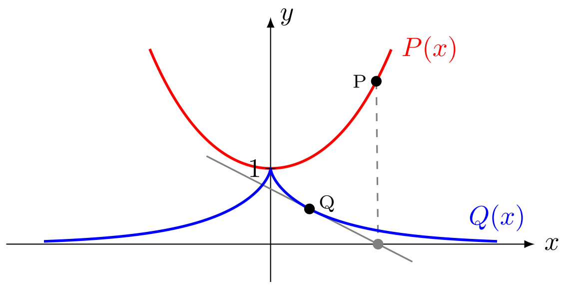

Figure 3: The catenary $P(x) = [x, a\cosh(x/a)]$ and its involute: the tractrix $Q(x) = \left[x - a\tanh(x/a), \frac{a}{\cosh(x/a)}\right]$. For a point Q on the tractrix, the intersection of the tangent to Q with the X-axis coincides with the orthogonal projection of the corresponding point on the catenary P.

1.2.8 Example envelope family of straight lines

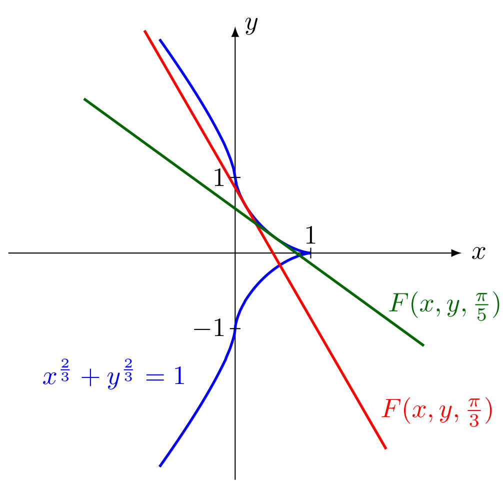

Figure 4: Some examples from the family of lines $F(x, y, a) = \frac{x}{\cos(a)} + \frac{y}{\sin(a)} = 1$, and the corresponding astroid: $x^{2/3} + y^{2/3} = 1$.

2.3 Gradient of scalar field

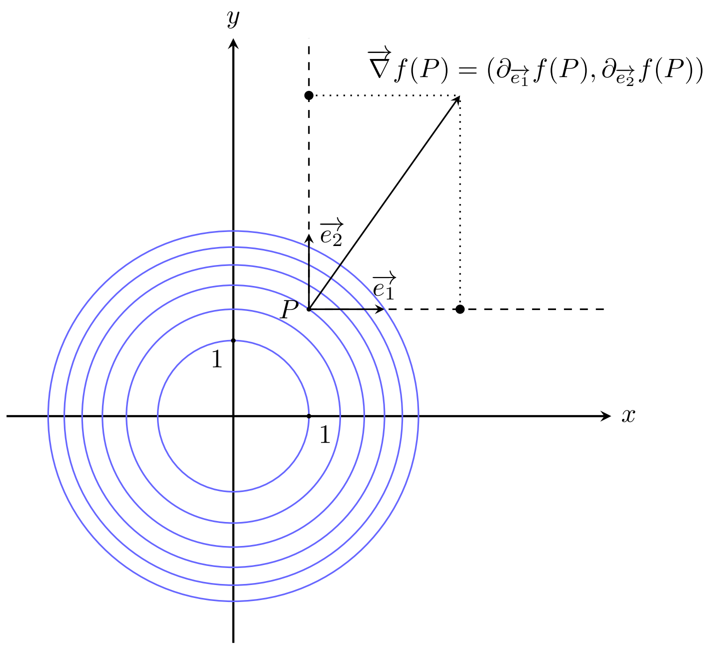

Figure 5:

3.1 Line integral of a scalar field

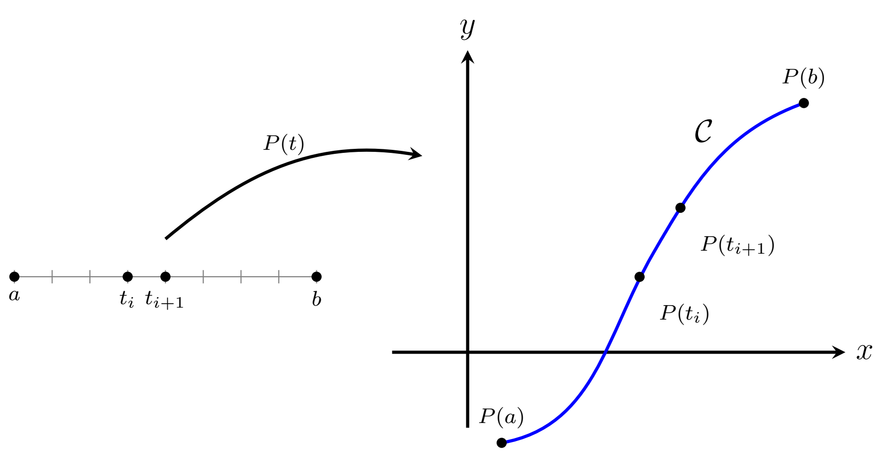

Figure 6:

3.2 Line integral of a vector field

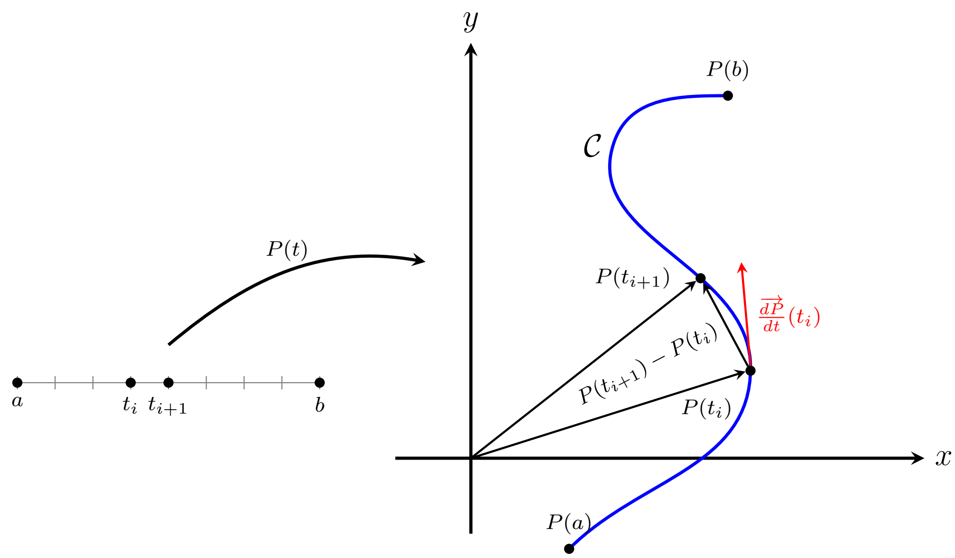

Figure 7:

3.4.2 Conservative field along a curve

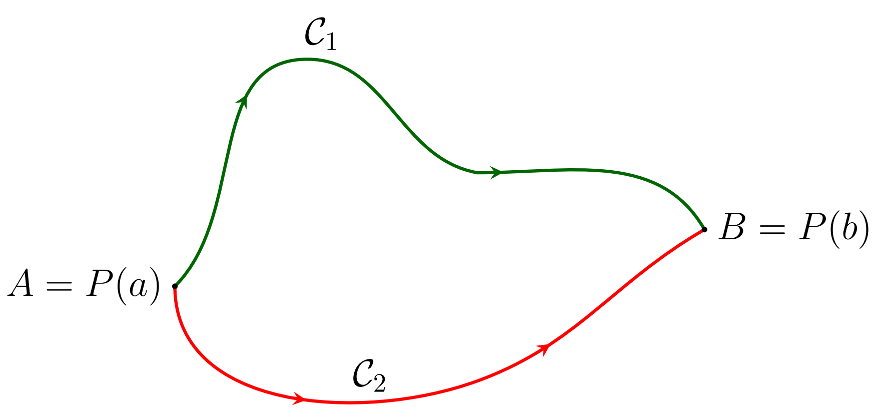

Figure 8:

3.4.3 Proof conservative field

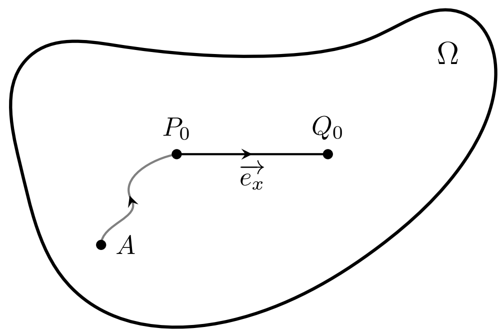

Figure 9:

3.5.1 Proof Greens theorem

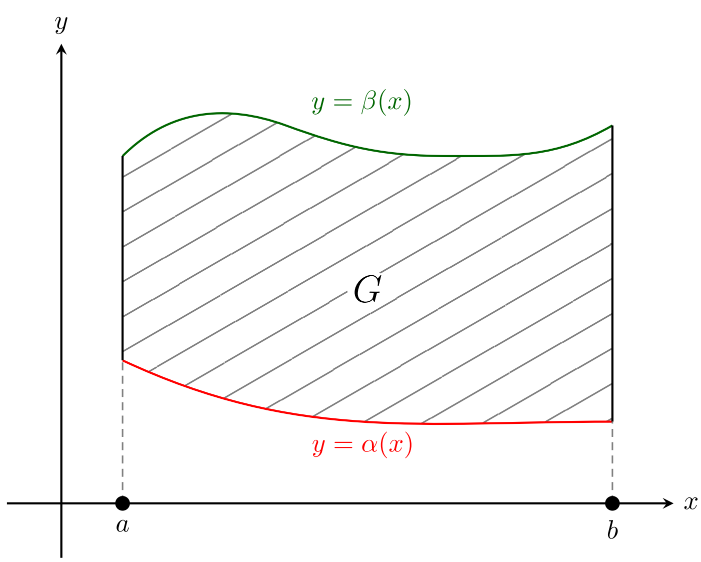

Figure 10:

3.5.2 Union of normal spaces

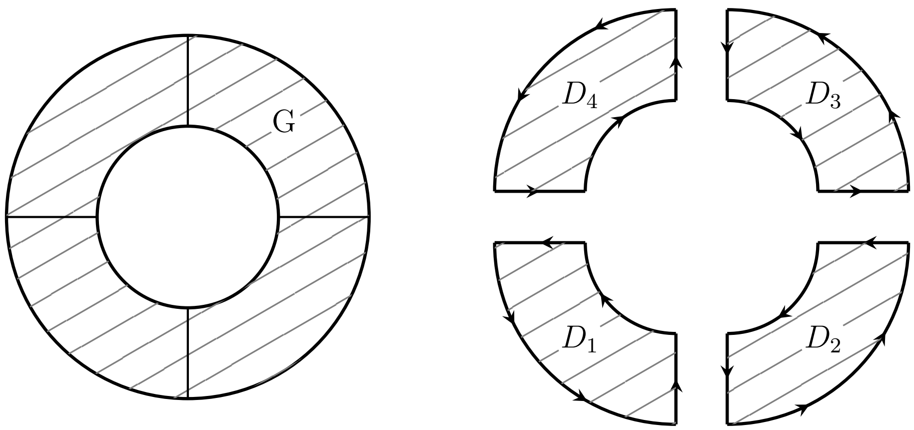

Figure 11:

3.5.4 Alternative formulation Greens theorem

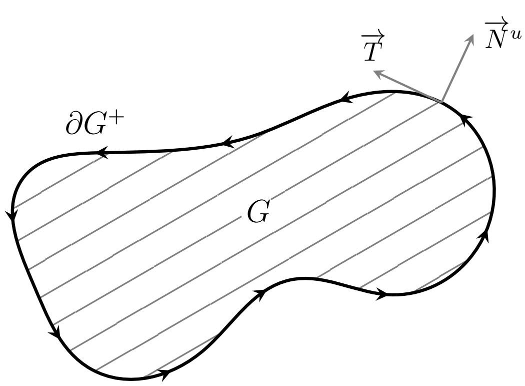

Figure 12:

4.1 Surface integral of a scalar field

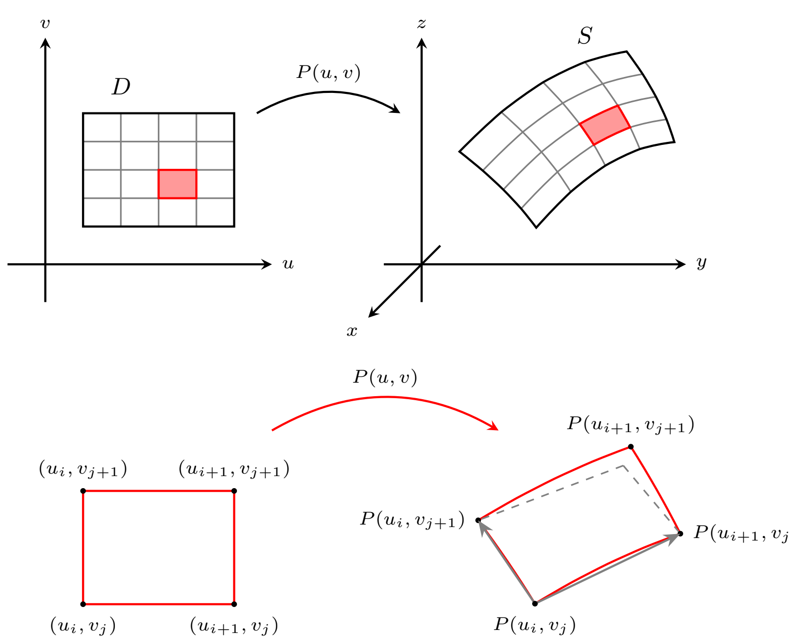

Figure 13:

4.4.1 The divergence theorem

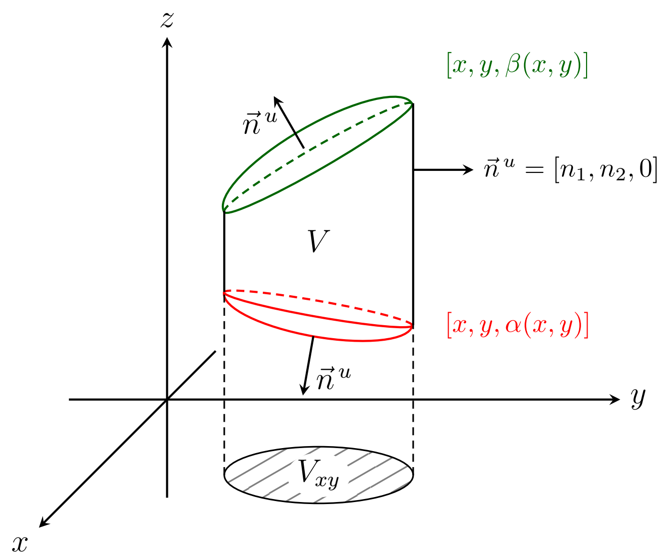

Figure 14:

4.6.0 The corkscrew rule

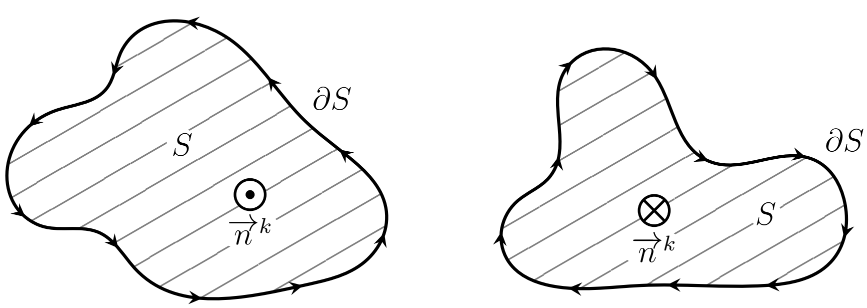

Figure 15:

4.6.1 Stokes theorem

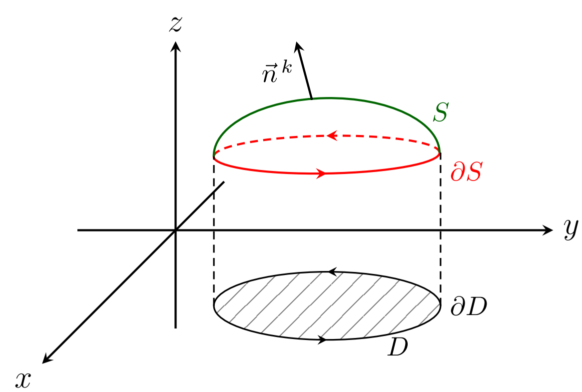

Figure 16:

5.1 Inverse function

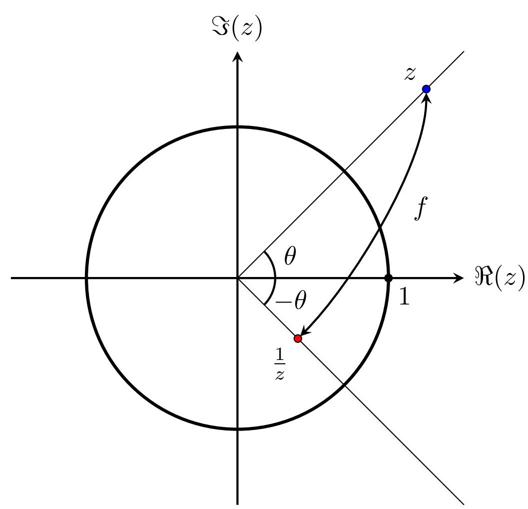

Figure 17:

5.1 Complex function

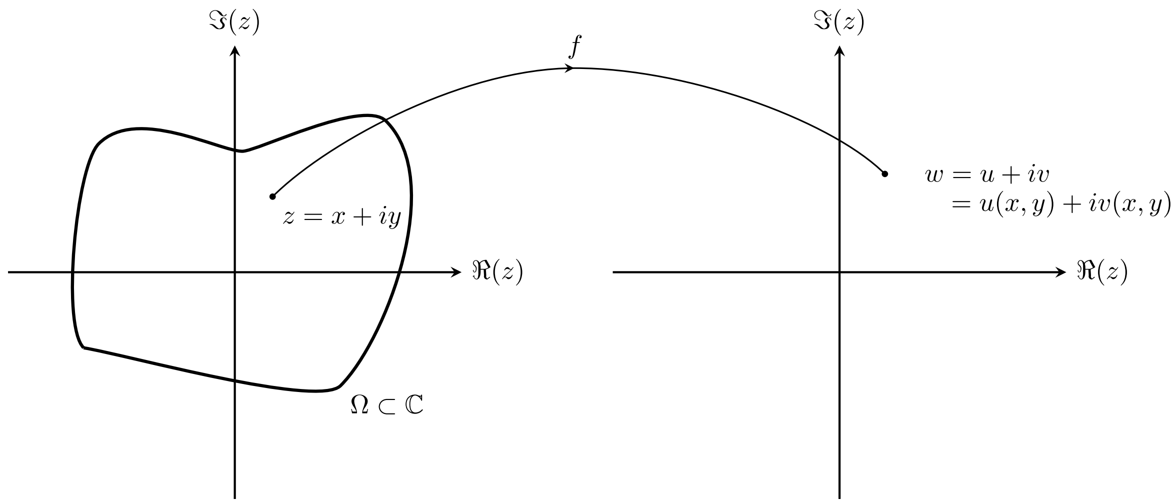

Figure 18:

5.2 Complex line integral

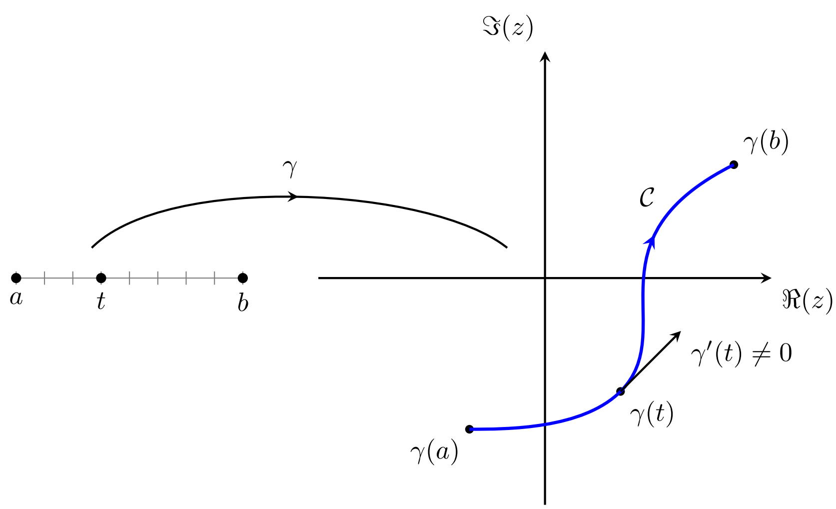

Figure 19:

6.2.1 Complex derivative

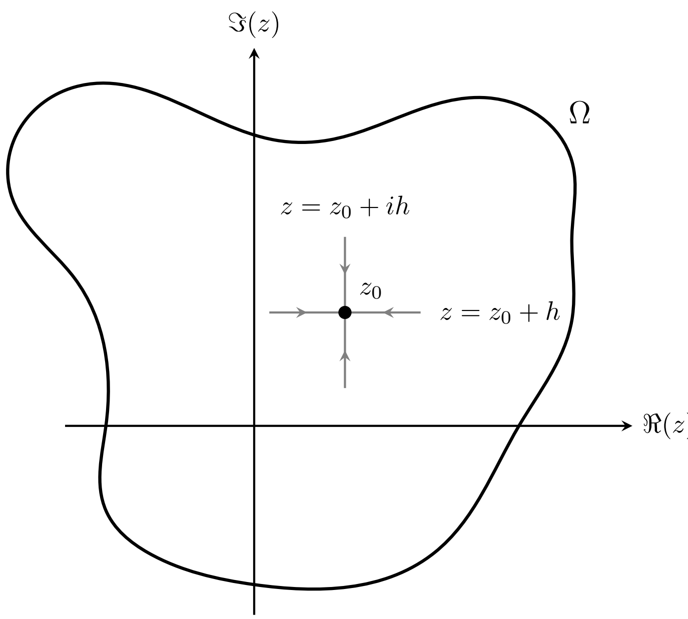

Figure 20:

6.3 Cauchy Goursat theorem for multiply connected domains

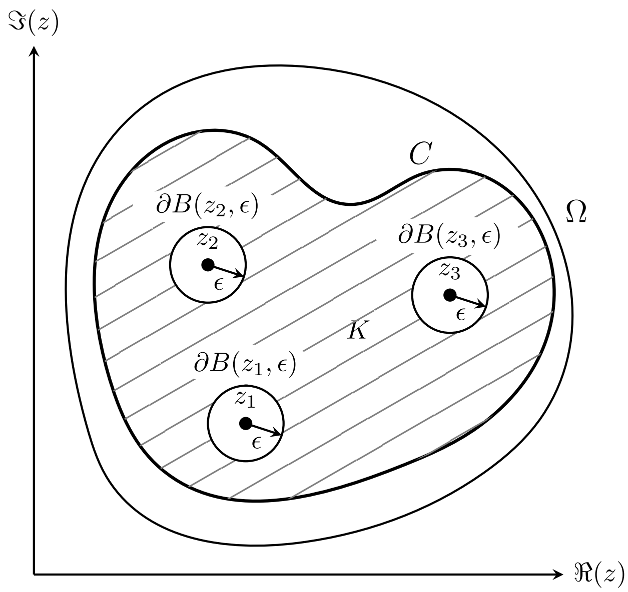

Figure 21:

6.3 Contour non simply connected

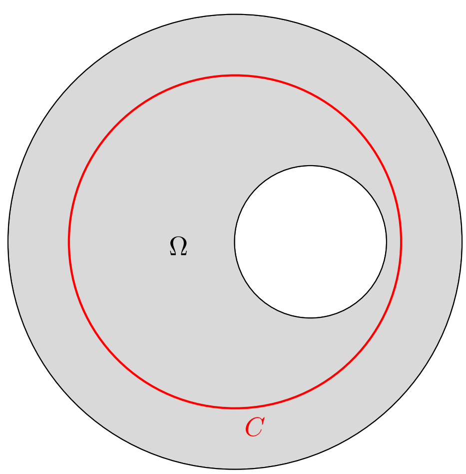

Figure 22:

6.3 Contour simply connected

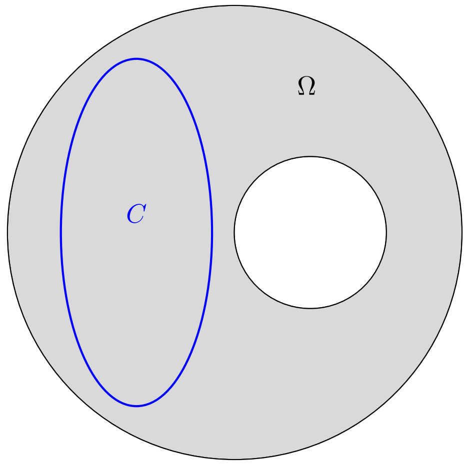

Figure 23:

6.3.3 Proof integral formula Cauchy

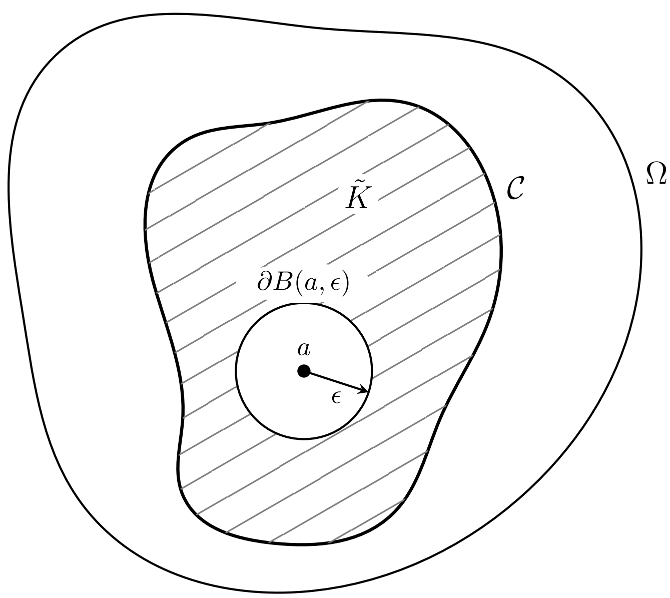

Figure 24:

7.2.4 Theorem convergence regions positive and negative power series

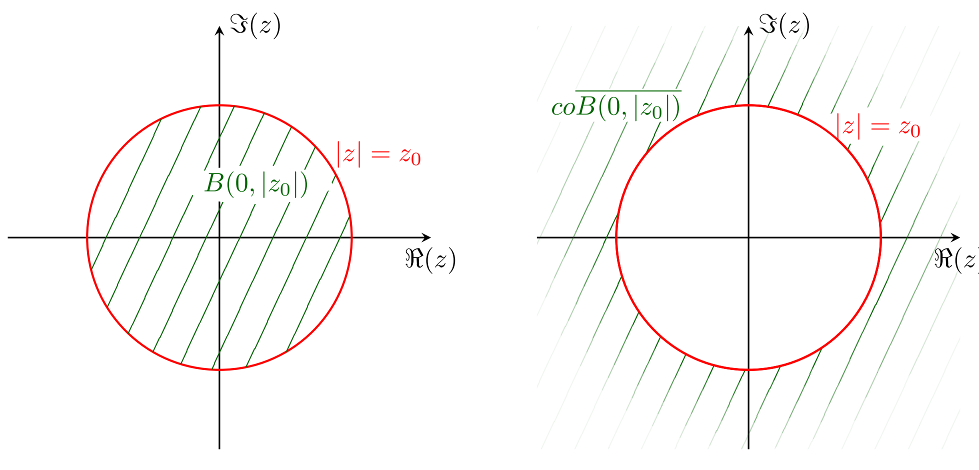

Figure 25: (Left) Region of convergence of a positive power series. (Right) Region of convergence of a negative power series.

8.2.1 Proof theorem Laurent series

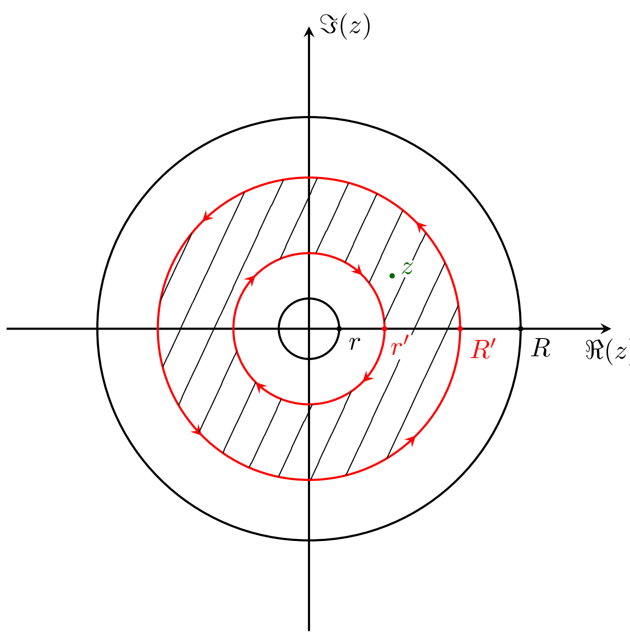

Figure 26: The region of convergence of the Laurent series is the annular region $B(0,R) \setminus \overline{B(0,r)}$. For every point z, it is possible to find R' and r' such that $0 < r < r' < |z| < R' < R$.

8.5.6 Residue theorem for region with multiple singularities

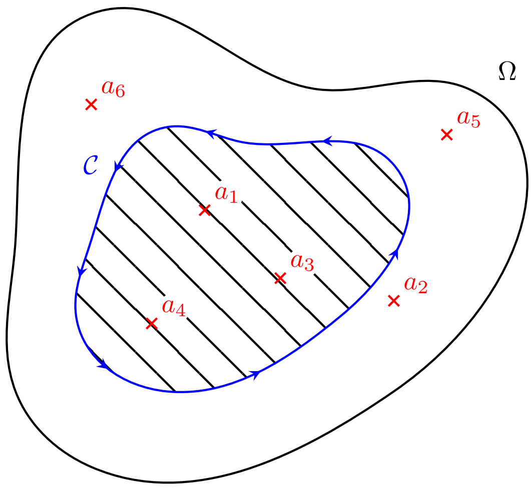

Figure 27: The contour $\mathcal{C}$ passes through none of the singularities a_i of the complex function f and encloses a compact set that lies entirely within Ω. $\mathcal{C}$ contains a finite number of singularities of f.

9.3 Estimation lemmas

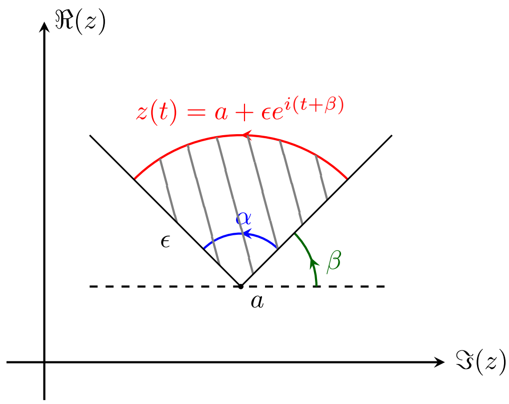

Figure 28: Small limit theorem: The circular arc C_ε^t with center a, radius $\varepsilon > 0$, and central angle α can be described by the parametric equation $z(t) = a + \varepsilon e^{i(t+\beta)}$, $t : 0 \to \alpha$.

9.5 Summation of series

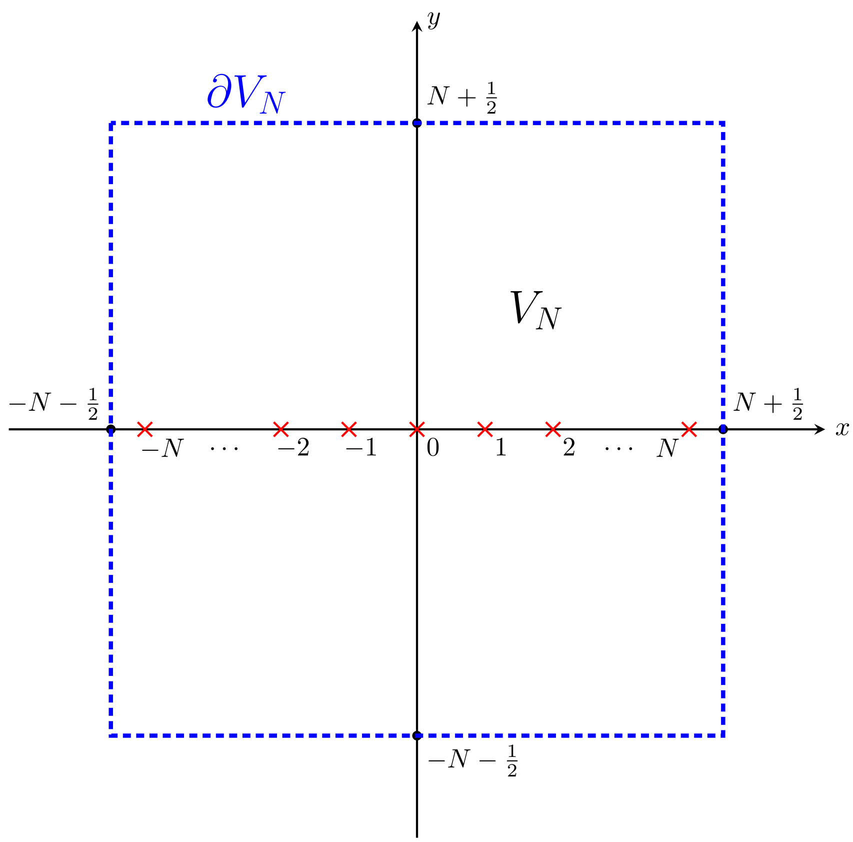

Figure 29: The function $g(z) = \cot(\pi z)$ has simple poles at $z = 0, \pm 1, \pm 2, \ldots$. If we consider the square V_N with center at the origin and side length $2N+1$, where $N \in \mathbb{N}$, then we can compute the contour integral over ∂V_N^+ using the residue theorem.