About 7 Euro 1^{re}

Method

Watch the video "Method Without Subtitles" once. You can watch it a second time and if you feel lost, ask the teacher for the subtitled version
Did you need the subtitled version ? □ YES □ NO
What's the video about ?
Give another example which shows how the method described in the video works.
Proof Watch the video "Proof Without Subtitles" once.

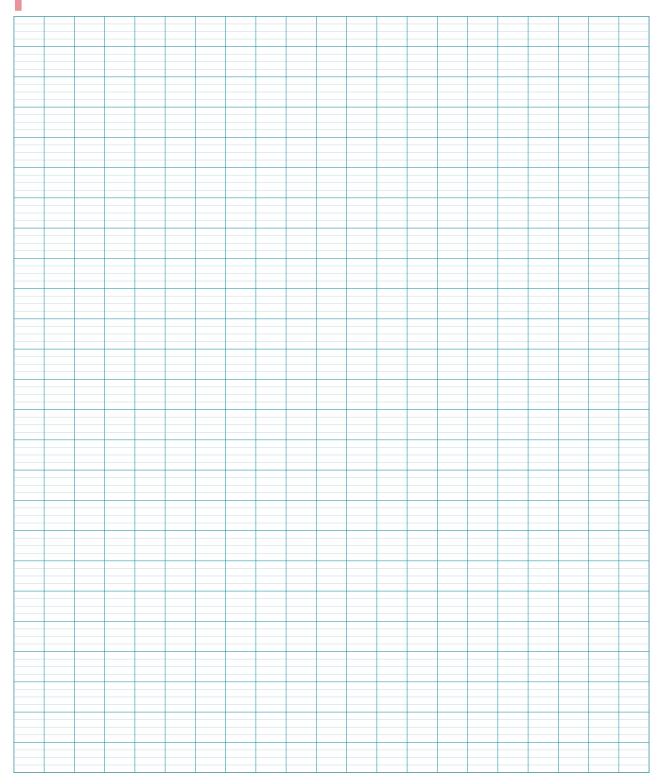
You can also watch this one a second time and/or ask the teacher for the subtitled version.

Did you need the subtitled version ? \qed YES \qed NO

Explain why this method is correct.

Beware

In the video, the man forgot an important detail: he actually proved that 5 times the original number is divisible by 7 if and only if x + 5y is divisible by 7...but that's it! We will admit that if 5n is a multiple of 7, then n is also a multiple of 7.



One step further

There is another method we can use to work out wether an integer is divisible by 7, here it is showed on an example: is 12345 divisible by 7?

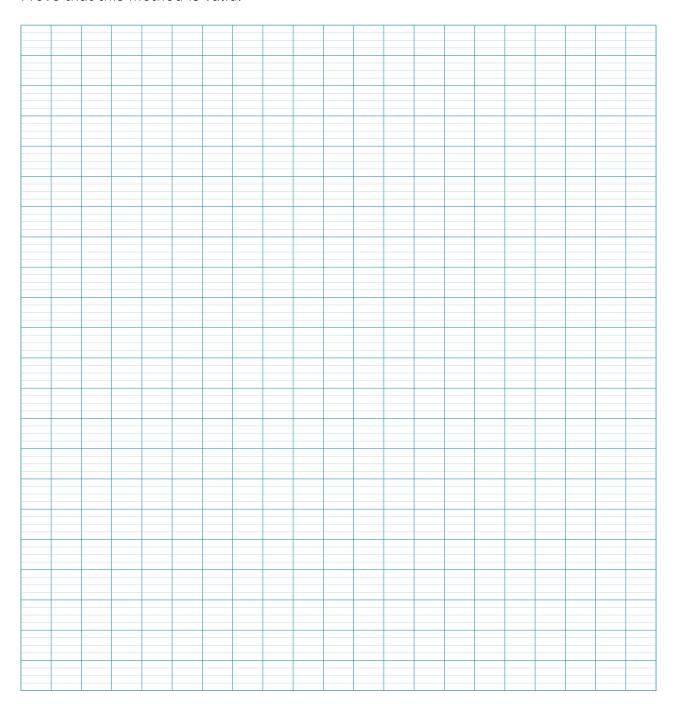
Let's take 12345, split it into 1234 and 5 and calculate $1234 - 2 \times 5$.

This gives us 1224, let's repeat the process:

$$122 - 2 \times 4 = 114$$

 $11-2\times8=-5$ so, as -5 is not divisible by 7, 12345 isn't either.

Prove that this method is valid.



APPENDIX: theorems used in the video (or not)

Theorem 1

Let a, b and c be three integers, with b and c not equal to zero.

If a is divisible by b then ca is divisible by b.

Example

4 is divisible by 2 so 4×123 is also divisible by 2.

Theorem 2

Let a, b and c be three integers such that a = b + cSuppose that c is divisible by k, then either a and b are also divisible by k, or none of them are.

Example

x is an integer.

- **1.** Suppose 30 = x + 5, then x is also divisible by 5.
- 2. Suppose 30 = x + 11, then x is not divisible by 11.

Theorem 3 (Gauss, to be used with a=7 and b=5)

Let *a*, *b* and *c* be three integers such that *a* divides *bc* and *a* and *b* have no common factor (except 1).

Then a divides c.

Example

If 7 divides 5*n*, since 7 and 5 have no common factors other than 1, we can conclude that 7 divides *n*.