A painting

The width of a rectangular painting exceeds its height by 7 cm and its area is 288 cm².
Find the painting's dimensions.

Solution

Let x be the painting's height, then $x+7$ is its width, so we get

$$x(x+7) = 288$$

which is equivalent to

$$x^2 + 7x - 288 = 0$$

The discriminant is $49 + 4 \times 288 = 1201$ so we get two solutions, one is obviously negative and the other is

$$\frac{-7+\sqrt{1201}}{2} \simeq 13.8$$

So this is the height of the painting and it leads to a width of

$$\frac{7+\sqrt{1201}}{2} \simeq 20.8$$

Lengths have been rounded to the nearest millimeter.

A garden

The area of a rectangular garden measuring 16×24 m² will double when surrounded with a strip of x meters wide.

Find x.

Solution

When surrounded, the new dimensions of the garden are $16+2x$ and $24+2x$ so we can derive the following equation :

$$(16+2x)(24+2x) = 2 \times 16 \times 24$$

which immediately simplifies into

$$(8+x)(12+x) = 8 \times 24$$

and is equivalent to

$$x^2 + 20x - 96 = 0$$

Now this equation has a discriminant of 784, which is 28^2 and this give us two solutions, 4 and -24, but we only keep the positive one.
Thus the strip will be 4 meters wide.

A family of equations

Part I : example

1. Show that $3 + 2\sqrt{2} = \left(1 + \sqrt{2}\right)^2$.
2. Solve $x^2 + \left(3 + \sqrt{2}\right)x + 2 + \sqrt{2} = 0$ in $\mathbb{R}$.

Solution

1. You just have to expand $\left(1 + \sqrt{2}\right)^2$ with the special product rule to get $3 + 2\sqrt{2}$.
2. $$\begin{aligned}\Delta &= \left(3 + \sqrt{2}\right)^2 - 4 \times (2 + \sqrt{2}) \\ &= 9 + 6\sqrt{2} + 2 - 8 - 4\sqrt{2} \\ &= 3 + 2\sqrt{2} \\ &= \left(1 + \sqrt{2}\right)^2\end{aligned}$$

Part II : generalization

1. Show that given any real numbers a, b et c :
$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$
(one can start by expanding $((a + b) + c)^2$ or directly expand $(a + b + c)(a + b + c)$).
2. Let p be a real number such that $p > 1$, and (E) the following equation :
$$x^2 + (p + 1 + \sqrt{p})x + p + \sqrt{p} = 0$$
   a. Calculate the discriminant Δ of (E).
   b. Show that $(p - 1 + \sqrt{p})^2 = \Delta$.
   c. Consequently, deduce the solutions of (E) in $\mathbb{R}$.
3. Are the results found in question **2.a.** coherent with those found part I ?

Solution