OpenHVSR User Manual (Ver. 1.0)

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About OpenHVSR

Credits

OpenHVSR is an algorithm developed in Matlab® (Release 2010a) by Ph.D. Samuel Bignardi. If you use this code for research purposes, please cite:

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- Herak, M, 2008. ModelHVSR—A Matlab tool to model horizontal-to-vertical spectral ratio of ambient noise. Computers & Geosciences vol. 34, pp 1514-1526.

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Aknowledgements

OpenHVSR includes some publicly available routines:

- To evaluate the confidence on the results, we included the "CONF_LIM" routine, available in the program ModelHVSR [4]. ModelHVSR is available for download from multiple sites (for e.g. https://sites.google.com/site/prinstessa/software-freeware) or by contactinh the author.
- he modeling routine implements the same modeling strategy present in ModelHVSR [4], which, in turn, is based based on [6, 7]. While in ModelHVSR a library compiled for 32 bit computers was provided along with the corresponding mex-fortran source code; here that code was ported to Matlab so that users on 64 bit machines can avoid dealing with mex files compilation and to ensure compatibility with any future release of Matlab.

• Routines for smoothing 2D data were written by Carlos Adrián Vargas Aguilera, Universidad de Guadalajara, México. They were downloaded from the Matlab Central, http://www.mathworks.com/matlabcentral/fileexchange/.

What is OpenHVSR

OpenHVSR is a computer program developed in the Matlab environment, designed for the simultaneous modeling and/or inversion of large, Horizontal-to-Vertical Spectral Ratio (HVSR or H/V) datasets and with the purpose of constructing 2D/3D subsurface models (topography included). The program is designed to provide a high level of interactive experience to the user and still to be of intuitive use

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0.0.1 To start the program

Place the program at any location in your computer, say for example it is placed in /my/folder/OpenHVSR/, then

- 1. Start Matlab (R2010a or higher).
- 2. Navigate to the /my/folder/OpenHVSR/ folder
- 3. open the file "START_OpenHVSR.m"
- 4. The program is set to 2D mode by default. If you want enable the 3D mode, just uncomment the line as follows:

```
mode = '2D'; mode 2D enabled
%mode = '3D';
```

or

```
%mode = '2D';
mode = '3D'; mode 3D enabled
```

5. run "START_OpenHVSR.m"

0.0.2 Load an example

We provide three examples which can be found in "OpenHVSR/EXAMPLES" folder. Examples 1 and 2 correspond to the same examples published in [3]. To load the examples,

- 1. Go to the menu "files" and select "Load HVI Project"
- 2. Navigate to the example you want to open and select the "project.m" file, then click "Open"

Example 1 _ Simulated _ data _ Figures _ 6 and 7 (2D mode): This is a simulated data example, i.e. HVSR curves presented here were simulated.

In this example nine different HVSR curves are assumed to be obtained along a linear profile and spaced at 100 m intervals.

The project-file "project.m" instructs OpenHVSR to load nine data files and nine files describing the subsurface.

In this case the provided subsurface model is ready for starting the inversion.

Example2_real_data_Figures_9and10 (2D mode): In this example eight different HVSR curves obtained along a linear profile and spaced at 1 km intervals are provided.

The project-file "project.m" instructs OpenHVSR to load the eight data files and eight files describing the subsurface.

In this case the provided subsurface model is the best model we found optimizing for the frequency band $0.4\text{-}6.0~\mathrm{Hz}.$

Example3 Simulated data for 3D (3D mode): In this example the same HVSR curves of example 1 are used, but the survey geometry (and consequently the project-file), was changed to illustrate a 3D subsurface with topography.

0.0.3 Create your own project

0.0.3.1 The project-file

Input to OpenHVSR is providede by a project-file which requires a basic understanding of Matlab language.

Such project-file specifies the locations of measurements, data filenames and an initial subsurface model under the measurement points. All the examples in the "OpenHVSR/EXAMPLES/" folder contain project-files which should be self explanatory, however, a minimal project-file structure is reported in table 1, which describes a survey where three HVSR-curves (hvsr_1.txt, hvsr_2.txt,hvsr_3.txt), obtained from recordings performed along a line and 100 m spaced are located in the "DATA" folder (lines 1 to 9). The initial subsurface assumed under hvsr_1.txt and hvsr_2.txt is described in the file "subsurface_A.txt" (lines 12,13), while the initial subsurface assumed under hvsr_3.txt is described in the file "subsurface_B.txt" (lines 15,16).

```
1
    % Data Section:
    SURVEYS\{1,1\} = [ 0 \ 0 \ 0];
                                        \% Location [x,y,z].
   SURVEYS {1,2} = 'DATA/hvsr 1.txt'; % HVSR curve.
 3
4
    SURVEYS\{2,1\} = [100 \ 0 \ 5];
5
    SURVEYS\{2,2\} = 'DATA/hvsr 2.txt';
6
    SURVEYS{3,1} = [200 \ 0 \ 5];
8
    SURVEYS{3,2} = 'DATA/hvsr 3.txt';
9
10
   % Initial Subsurface Section
11
   MODELS{1,1} = 'MODELS/subsurface A.txt';
12
   MODELS\{1,2\} = [1 \ 2];
13
14
   MODELS{2,1} = 'MODELS/subsurface B.txt';
15
   MODELS\{2,2\} = [3];
16
17
18
   % optional section
19
   % datafile columns
                        = [2 \ 3 \ 6];
20
   % datafile separator = 'data begins here';
```

Table 1: Example of a minimal input project-file

A project-file is a regular ASCII file, except that in order to be used as Matlab script it must have the ".m" file extension.

Two main variables must be defined here:

- SURVEYS
- MODELS

These two variables use the "cell" feature of Matlab which creates a container that has the same structure of a matrix but it can contain different sort of data, for example,

 $SURVEYS{3,2}$

is the entry of the SURVEYS cell at line 3 and column 2.

The "SURVEYS" variable in the project-file contains a description of one measurement for each row; column 1 contains the coordinates (x,y,z) of a measurement location (organized as a 1x3 vector and enclosed in squared brackets) and column 2 contains the path to the corresponding hysr file. For example

```
2 SURVEYS\{1,1\} = [0\ 0\ 0]; % Location [x,y,z]. 3 SURVEYS\{1,2\} = DATA/hvsr\ 1.txt'; % HVSR curve.
```

means that the curve contained in file "DATA/hvsr $_1$.txt", was obtained from the recordings performed at location (0,0,0).

Following the same strategy, the initial subsurface model is inputted through the "MODELS" variable. Every row of this variable describes a locally 1D subsurface model. The column 1 contains the path to the file describing the subsurface, while column 2 contains a list of IDs (organized as a row vector) which correspond to the rows of the SURVEYS variable that are associated with the subsurface model. For example

```
12 MODELS{1,1} = 'MODELS/subsurface_A.txt';
13 MODELS{1,2} = [1 2];
```

means that the subsurface model contained in file "MODELS/subsurface_A.txt" must be associated to the measurements described at rows 1 and 2 of the variable "SURVEYS". In other words, the curves in "DATA/hvsr_1.txt" and "DATA/hvsr_2.txt" files, corresponding to locations (0,0,0) and (100 0 5) respectively, are both associated to the same initial subsurface model described in file "MODELS/subsurface A.txt".

Finally, variables

- datafile columns
- datafile separator

are optional and can be used to load different data-file structures, provided that data-files are in ASCII format. By default, the program reads two-columns data files, with frequency values and corresponding HVSR curve amplitudes stored in the first and second column respectively. If the input files include an header (footer is not allowed), followed by a table of data, these can be read by specifing the structure of the file, either from the interface (through the "Files/HVSR files format" menu) or by uncommenting the lines 19,20 in the project-file. The datafile separator variable can be any string. The user must insert this string in

every data-file, between the header and the data sections, in order to indicate to the program where data begin. Finally, the datafile_columns variable specifies the column ID's corresponding to the frequecy scale, to the HVSR curve and to the standard deviation. For example

```
19 datafile_columns = [2 3 6];
20 datafile_separator = 'data_begins_here';
```

means that OpenHVSR will look for the string "data_begins_here" in order to find the data-section of the file. Further, It will read the data section as a matrix and it will use the data in the column 2 as frequency scale, column 3 as the HVSR curve amplitude and finally, colum 6 as standad deviation.

0.0.3.2 Data-files

Almost any data file structure is loadable provided it is an ASCII file. Files containing a header can be loaded with minimum effort by placing any string between the header and the data section of the file, for example in the following data-file

```
Site ID:
                     DPC INGV2014 MAE,
Instrument:
                     EXT-
No. of channels:
                     3
Sampling rate:
                     125~\mathrm{Hz}
Start recording:
                     25/07/14 04:00:00
End recording:
                    25/07/14 04:30:00
                          0h50'00'
Trace length:
Channel labels:
                  NORTH-SOUTH
                                  EAST-WEST UP-DOWN
freq. H/V Std. Dev. 95%conf. int.
                                         Vel. Ampl. Spec.
data begins here
0.01525 \ 0.000
                  0.000
                           0.000
                                    0.000
0.03051 \quad 0.000
                  0.000
                           0.000
                                    0.000
```

the string "data_begins_here" was chosen and inserted as separator. The user must then tell OpenHVSR that the data-files include a header, either from the interface (thrugh the menu Files/HVSR files format) or by inserting the line "datafile_separator = 'data_begins_here'" in the project-file.

OpenHVSR assumes that the frequency scale, the H/V curve and the standard deviation occupy the columns 1, 2 and 3 respectively, however, the user can customize the column order by modifying the "datafile_columns" variable as explained in section 0.0.3.1.

0.0.3.3 Subsurface-files

The subsurface (see for e.g. table 2) is inputted using one or more ASCII files where subsurface visco-elastic parameters are organized in a table. Each row corresponds to a layer and the last one corresponds to the half-space. Columns 1 to 6, correspond to:

- Vp (compressive "P" waves velocity)
- Vs (shear "S" waves velocity)
- Rho (density)
- H (thickness)
- Qp (P-waves daming)
- Qs (P-waves daming)

Note that the values of H, Qp, Qs must be set to "999." for the Half-space.

Table 2: Example of local 1D subsurface model

0.1 Interface description and use

The algorithm is composed of several routines integrated into a main graphical user interface (GUI), which is organized in tabs.

0.1.1 Tab 1. Main parameter settings and 2D profile optimization

Tab 1 (fig. 1), is dedicated to the general settings for the simultaneous modeling and/or inversion of a set of HVSR curves. In case of inversion, the input data consist of HVSR curves which are obtained after the elaboration of the corresponding field data by using any third party software (e.g. Grilla, Geopsy, Jsesame, etc.).

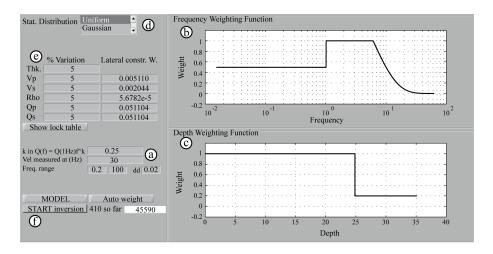


Figure 1: View of tab 1: modeling and inversion setup.

Setup of the forward modeling routine

The portion of the left panel (a), contains the control parameters necessary to setup of the forward model routine (FWD). For each location of the survey, the FWD calculates the theoretical transfer function of a layered subsurface model based on Tsai's approach and enhanced to manage frequency-dependent attenuation and body-wave dispersion [6, 7, 4]. The modeling algorithm represents the subsurface as a stack of viscoelastic homogeneous layers over a half space. Each layer is described by its thickness, H, and its mechanical behavior defined by the density (Rho), the compressive and shear wave velocities (Vp , Vs) and the corresponding attenuation factors (Qp, Qs) which are frequency dependent and follows

$$Q = Q_0 f^k \tag{1}$$

where Q_0 is the attenuation factor at 1 Hz and k is a constant which is assumed to be fixed for every location. Finally body waves dispersion, is considered trough the logarithmic low [1]

$$V(f) = V(f_{ref}) \left[1 + (\pi Q_0)^{-1} \ln \left(\frac{f}{f_{ref}} \right) \right]$$
 (2)

Text-boxes at (a) allow to set the constant k, the reference velocity $V(f_{ref})$, and the frequency band and frequency step to simulate the HVSR curves. The stand-alone modeling can be started by simply pressing the "MODEL" button (f).

Setup of the inversion

The remaining controls of this tab are used to setup the inversion. The inversion strategy is based on the guided Monte Carlo method, where at every iteration a randomly perturbed version of the best fitting model (i.e. the model which best reproduces the data) is produced and used to compute a set of simulated curves to be compared with the experimental HVSR curves. The generation of many trial models allows exploring the parameters space while looking for a new and better fitting model. The controls placed on this tab allow the user to:

- set the maximum percentage of change to be associated to each family of parameters (e)
- set the statistical distribution to be used to generate the perturbation; either uniform or normal (d)
- set the weights to be associated to different portions of the HVSR curve (i.e the weights on data) in case the user wants to focus on a particular portion of the HVSR curve (panel b). Right-clicking on this panel open a context menu which allow to modify the "Frequency Weighting Function".
- set the perturbation of the parameters as depth-dependent. This is done by modifying the "Depth Weighting Function" of panel (c). For each depth z the effective amount of perturbation will be the value of this function at z multiplied by the maximum percentage of change. Right-clicking on this panel open a context menu which allow to modify the "Depth Weighting Function".
- Specify which degree of freedom are locked (i.e. kept fixed) during inversion. The "Show Lock Table" button opens the "Lock Parameters" dialog window in figure 2. Here the locked parameters are shown on the right top table as unchecked. The right bottom table on the other hand can be used to relax the constrain on the maximum ratio V_p/V_s , so enabling higher V_p values in the selected layers in order to account for the water table.

Finally, since this inversion is focused on retrieving a 2D or 3D subsurface, we need a criterion to decide which set of simulated curves best reproduces the experimental data. This is achieved by calculating the value of an objective function (eq. 3) which is a positive, real value function of the subsurface parameters Vp, Vs, Ro, Qp, Qs, and H (called collectively **m**), and seeking its global minimum. Indeed, the objective function can be expressed as

$$E(\mathbf{m}) = aM(\mathbf{m}) + bS(\mathbf{m}) + \sum_{j=1}^{5} \alpha_j R_j(m_j)$$
(3)

where the first term on the r.h.s. represents the misfit between the data and the simulated curve. The second term introduces into the objective function a

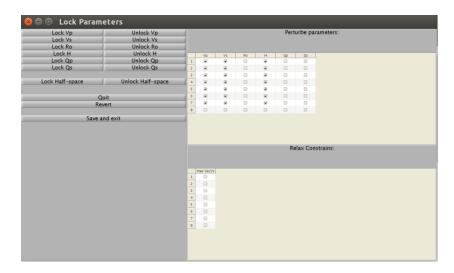


Figure 2: Lock Table

forcing condition on the slope of the curves, and the third term implements a smooth lateral variation constraint on a particular family of parameters $m_j = V_p$, V_s , Rho, Q_p , Q_s , for j=1 to 5 respectively. For details on equation 3, please refer to [3].

Parameters a and b can be set using the menu "Settings/Objective function" (see section 0.2.2) while the parameters α_j , which highlight the importance of the regularizing term for the different subsets of parameters are usually chosen by the user, even though an automatic selection strategy is available by pressing the "Automatic weighting" button. When pressed, a dialog window opens which asks the user to set the amount of lateral variation expected for each family of parameters.

As a final note, it is important to remember that the settings of tab 1 will also affect all other tabs.

0.1.2 Tab 2. Local 1D model optimization

Tab 2 (fig. 3), is devoted to the independent optimization of the local 1D subsurface at the site of the specific HVSR curve at hand.

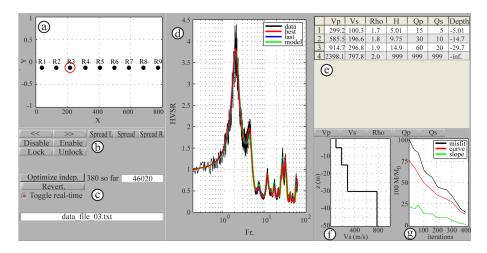


Figure 3: View of tab 2: Independent optimization of a specific (local) site.

Tab 2 shows:

- an aerial view (a) of all measurement sites (i.e. the location associated to each HVSR curve). The selected location is highlighted with a red circle. Further the user can navigate through the sites by using the buttons with the left/right arrows (b). Sites are normally indicated in blue but the user is allowed to lock any site in order to prevent further modifications or in order to introduce an abrupt discontinuity when it is known to exist. To lock/unlock a location use the "Lock" and "Unlock" buttons (b); the locked sites will turn red. Finally, a curve can be discarded (i.e. not considered) by using the "Disable" and "Enable" buttons. As a final note, when using the program in "3D mode" (see 0.0.1), the user is allowed to define a customized linear profile by using the "define profile" option of the context menu which is opened by right-clicking on panel (a)
- a set of controls (b and c) along with the name of the visualized data-file
- the central panel (d) shows the data space. Here the experimental curve (black) is shown along with its experimental uncertainty curves (gray, if available). On the same plot, the stand-alone modeled curve (green) which is obtained by pressing the "MODEL" button of Tab 1 and the simulated curves corresponding to the best fitting model (red curve) and to the last model generated during the Monte Carlo inversion (blue curve) are shown
- the table describing the current subsurface (e)
- the 1D profile of one media property (f), which can be selected by pressing the buttons on top.
- the misfit-vs-number of generated Monte Carlo trials plot (panel g), where the vaue of the misfit (i.e. aM + bS, see eq. 3), expressed as percentage

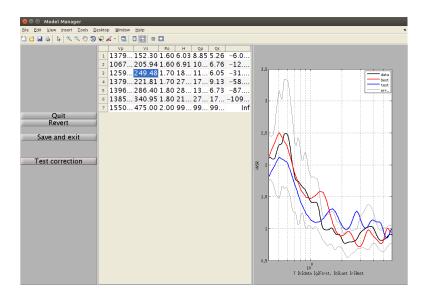


Figure 4: Model Manager

of the initial misfit, is plotted against the number of trial model generated (black line). The red and green lines, plot the behavior of terms aM and bS respectively.

To run the inversion algorithm press the "Optimize independently" button (c). When inversion is running, all tables and graphs can be set to either updated every time a new best model is found or dynamically, using the "toggle real-time" radio button (c). The inversion can be paused at any time and once stopped, the user is allowed to: inspect the 1D graph window (f), where the N most fitting models (N is settable) are plotted along with the best match. Further when a satisfactory fit is reached for a given location the user is allowed to extend the local subsurface model to the previous/sucessive location (using the spread left/right buttons) or even to the entire profile (spread button). Furthermore, the experienced user is also allowed to manually correct the subsurface model both on the basis of their own knowledge about the local geology, or alternatively after using the tools of tabs 4 and 5 (discussed later).

To open the "Model Manager" window (fig. 4), right-click on the model table (e) and select "modify". Once open, the user is allowed to manually edit the model table and test the effect of the correction prior to deciding whether to keep or discard the introduced changes.

Finally, if for any reason the user is not satisfied with the evolution of the inversion process, the "Revert" button will restore the initial subsurface model.

0.1.3 Tab 3. Profile view

Tab 3, (Fig. 5) is a 2D/3D view of the best fitting subsurface. For a 3D subsurface, the volume is fully sliceable and navigable.

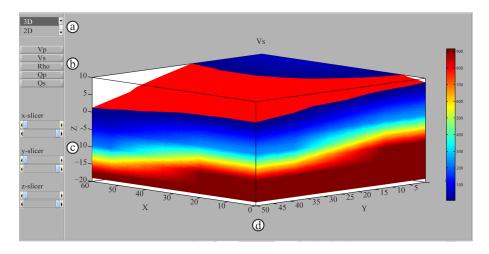


Figure 5: Tab 3: 2D/3D subsurface model view

0.1.4 Tab 4. Confidence

Tab 4 (Fig. 6) allows for plotting the confidence of the results.

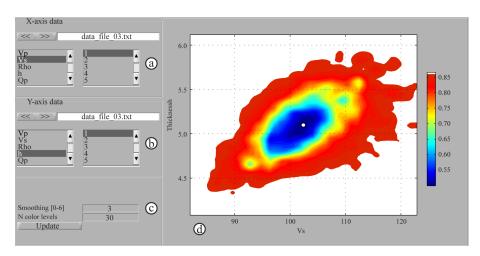


Figure 6: View of tab 4. This shows the confidence of the best model as a function of two selected parameters using the function CONF_LIM [4, 2, 5]. When the two parameters pertain to different measurement locations, this tool can be used to gain confidence on lateral variations.

This plot (shown in panel d) is based on the CONF_LIM routine [4] which, in turn, is based on the F-distribution computation [5, 2]. Since the parameter space is a multidimensional space, the confidence plot allows the user to inspect the distribution of misfit values with respect to two chosen degrees of freedom (d.o.f). The first d.o.f is selected using controls on panel (a). The buttons with the arrows allow to select which site to investigate, while the two drop-down lists allow to select the visco-elastic parameter and the layer under investigation. The second d.o.f is selected in the same way using panel (b), while graphical properties of the plot are set using the controls of panel (c). When the misfit is plotted as function of two d.o.f. pertaining to the same site (i.e. to the same HVSR curve and the same local 1D subsurface), the result of the figure is exactly the same of Herak's. The consistent improvement demonstrated by this method is that now the confidence plot can be expressed as a function of d.o.f. pertaining to different sites thus allowing the evaluation of the confidence regarding any found lateral heterogeneity.

0.1.5 Tab 5. Single parameter sensitivity

Finally, Tab 5 (Fig. 7) implements a novel strategy to investigate the confidence on the result. It allows for plotting the variation of misfit (first two term on r.h.s of eq. 3) with respect to a chosen parameter m, retaining the dependency on the frequency. This allows to judge how each portion of the HVSR curve is affected by a particular parameter (plot on panel c). Further, positive and negative variations, plotted in red and blue respectively, are associated to an increase and decrease of the frequency dependent contributions to the misfit.

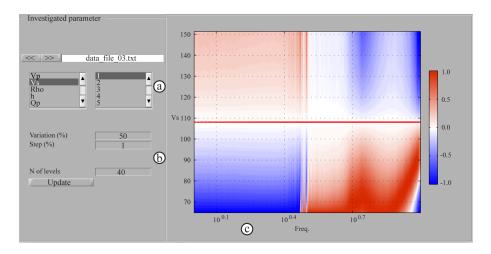


Figure 7: View of Tab 5. The frequency dependent variation of the misfit with respect to a single parameter (d.o.f) variation is shown (c). We refer to this as "single parameter sensitivity" and it can be used to understand the correct variation to be introduced into a model when improbable parameter values are present

The d.o.f. under investigation is set using the controls (a) while the amount of variation of the parameter is set using the controls (b).

0.2 Menus

Main aspects of the program menus:

0.2.1 Menu Files

- \bullet Load HVI $\mathit{Project}$: load a project-file from any point of the computer
- HVSR files format: allow to specify a string (that the user must insert beforehand into the HVSR data-files) to be used as header/data separator and allow to specify the columns where frequency scale and HVSR curve are stored. This potentially allows the program to read any file format
- Save as new project: allows to save the current subsurface and data as a completely new project that can be used independently. This is particularly useful when adding layers to the subsurface in order to refine the result
- Save Elaboration: saves the status of the current elaboration into a .mat (Matlab) file format for later use
- Resume Elaboration: resume an elaboration stored into a .mat file



Figure 8: Setup Manager

0.2.2 Settings

- Setup: open the "Setup Manager" window (fig. 8) to set the physical constraints of the subsurface visco-elastic parameters and allows to specify the target Earthquake
- Objective function: allows to setup parameters a and b of equation 3.

0.2.3 view

- HVSR: allows to switch between different visualizations of the HVSR curve
- profile: set graphical options
- coormap: allows to switch between colormaps

0.3 Suggestions

• We suggest to perform Inversions using multiple steps and using the "Save as new project" from one step to another. Start with a subsurface model with few layers and limit the curve inversion to low frequencies. Successively, when a good model is found, save as new project and edit the subsurface files splitting the deeper layer in two or three sublayers. Use the new project with thicknesses kept fixed but widening the frequency band to higher frequencies. Then, continue with the next, shallower layer, and so on. Usually this strategy reveal to be more effective for curves with the main peak at frequencies higher than 2 Hz.

• Changing set inversion parameters (such as weighting functions, frequency step or frequency band), during inversion is highly discouraged. However, this not affect the final result if the change results with a lower value of the misfit function. We encourage to save your work as a new project before changing any inversion parameter.

• Look at the subsurface model in table (Tab 2, e). When a certain portion of the HVSR curve is inverted by the Monte Carlo method, the elastic parameters of each layer are randomly perturbed. When a good match between the data and the simulated curves is found, the subsurface model that produced the synthetics is accepted. Unfortunately, if some parameter has a poor influence on the shape of the simulated curve and in turn, on the misfit (i.e. has a low sensitivity), a good match can be found even if an improbable value of such parameter is present. The tools of tabs 4 and 5 allow the user to judge if an improbable parameter value should be changed in order to obtain a more plausible model.

Bibliography

- [1] K. Aki and P. G. Richards. *Quantitative seismology, Theory and methods*. University Sciences Books, Susalito, California, second edition, 2002.
- [2] F. Bianco, E. Del Pezzo, M. Castellano, J. Ibanez, and F. Di Luccio. Separation of intrinsic and scattering seismic attenuation in the southern Apennine zone, Italy. *Geophysical Journal International*, 150:10–22, 2002.
- [3] S. Bignardi, A. Mantovani, and N. Abu Zeid. OpenHVSR: imaging the subsurface 2D/3D elastic properties through multiple hvsr modeling and inversion. *Computers & Geosciences*, 1:1–10, 2016.
- [4] M. Herak. ModelHVSR-A Matlab tool to model horizontal-to-vertical spectral ratio of ambient noise. *Computers & Geosciences*, 34:1514–1526, 2008.
- [5] K. Mayeda, S. Koyanagi, M. Hoshiba, K. Aki, and Y. Zeng. A comparative study of scattering, intrinsic, and coda q²1 for hawaii, long valley and central california between 1.5 and 15 Hz. Bulletin of the Seismological Society of America, 97:6643-6659, 1992.
- [6] N.C. Tsai. A note on the steady-state response of an elastic half-space. Bulletin of the Seismological Society of America, 60:795–808, 1970.
- [7] N.C. Tsai and G.W. Housne. Calculation of surface motions of a layered half-space. *Bulletin of the Seismological Society of America*, 60:1625–1651, 1970.