2C Applied Mathematics 2019–20

COURSE *i*

Student Responsibilities

The information contained on this sheet may change! It is your responsibility as a student to check regularly for updates:

- this course's Moodle site;
- your student email.

Lectures

The first lecture is on Monday 13th January.

Lectures take place in Semester 2 of the academic year on Wednesdays and Fridays at 10am and 12pm in weeks 22–32 (except for the first lecture which is held on Monday in week 22). Please see the course Moodle page for details of locations, times and lecturers contact details.

Tutorials

You will have tutorials fortnightly on Mondays at either 10am or 12pm, starting in week 24. (Make sure you choose a tutorial section that meets at the same time as your lecture section.) You must enrol for both the course and your tutorial section via MyCampus. You should consult Moodle and/or MyCampus for your tutorial room.

At tutorials you will be expected to bring solutions to exercise sheets, together with any questions you have from your attempts, to discuss with tutors. You will also work on further exercises in the tutorials.

Assessment and Examinations

You will have six assessed feedback exercises. The best five of these make up 20% of your final grade. There will be a degree exam in summer exam period. The degree exam makes up 80% of your final grade.

Instructions for the submission of feedback execrises are available on the 2C Moodle page.

Course Aims

This course provides an introduction to Newtonian mechanics via problems related to the motion of point particles, and to the mathematics of population dynamics via a discussion of the growth of populations and competition between species. The course builds on knowledge of vector calculus, linear algebra and ordinary differential equations to construct mathematical descriptions of a variety of physical systems.

Course Prerequisites

Prerequisites for the course are 1RSXY and corequisites are Mathematics 2A (Multivariable calculus) and 2B (Linear Algebra). We will use results from the pre and corequisites extensively so it is very important that you are competent in differentiation, integration (of functions of one variable as well as several) vector alebra, matrices, eigenvalues and eigenvectors. In particular from 1SY you will require the knowledge of how to solve first and second order ordinary differential equations.

Lecture notes and books

You must take your own lecture notes. The material covered in lectures and tutorials defines the course. As an additional resource, some printed notes will be available on Moodle.

In view of the fact that this course consists of two quite different topics from Applied Mathematics, there is no textbook which is entirely suitable for the course, and hence no book is recommended for purchase. However, the following books may be consulted for background reading.

- W. Chester, *Mechanics*, George Allen & Unwin
- L. Edelstein-Keshet,*Mathematical Models in Biology*, SIAM Classics in Applied Maths **46**
- M. Lunn, *A first course in mechanics*, OUP

Contents

The main mathematical tools used in this course are vectors and the solution of ordinary differential equations. All relevant theory will be given at the appropriate stage in the course, but you are expected to be thoroughly competent in vector algebra, multivariable calculus and differential and integral calculus. In particular you should be comfortable with vector multiplication and the solution of separable ordinary differential equations and the use of integrating factors for first order ordinary differential equations. The courses 2A and 2B are co-requisites for this course and so we assume all background from 2A and 2B.

Modelling
Dimensional analysis. Single species population models, difference and differential equations. Exponential and logistic models. Phase portraits (in one dimensional this phase space is a line). Equilibria, stability, linearisation. Harvesting, bifurcation diagrams. Two species models, Jacobian and eigenvalues.

Mechanics
Vector functions of a single variable. Parametric curves. Tangent, arc-length, normal, binormal, curvature. Polar coordinates, cylindrical polar and spherical polars. Displacement, distance travelled, speed, velocity and acceleration in particle motion. Equations of motion of the form $\dot{\mathbf{x}} = f(t)\mathbf{x}(t) + \mathbf{g}(t)$ and others. Use of identities. Motion with constant velocity. Motion with constant acceleration. Linear momentum and Newton's laws. Impulse. Collisions, kinetic energy. Energy equation. Forces. Friction, springs, strings. Forces (continued). Coupled oscillators. Systems of interacting particles.

ILOs

Students should be able to formulate problems in particle mechanics in mathematical terms using vectors and/or differential equations as appropriate and solve the resulting equations to determine the motion of the particle. Students should be able to formulate models for the growth and decay of single populations, and to solve and interpret the equations that they derive.

Students should be able to:

a) Be able to analyse the dimensions of quantities and make predictions of parameter dependance based on dimensional analysis.

b) Be able to calculate the tangent vector and arc-length of a parametric curve and to sketch such curves.

c) Use plane, cylindrical and spherical polar coordinates to describe particle motion.

d) Explain the terms velocity, speed, acceleration, displacement and distance travelled in connection with the motion of a point particle and to connect these terms to properties of parametric curves.

e) Be able to formulate problems in particle mechanics in mathematical terms using vectors and/or differential equations as appropriate, and solve the resulting equations to determine the motion of the particle.

f) Define the terms linear momentum, impulse and force and use them in calculations.

g) Be able to construct and solve problems using: kinematics; Newton's Laws; concepts of energy, work and power; collisions and impulse; projectile motion and motion governed by Hooke's Law.

h) Be able to construct and solve problems in elementary mechanics using plane polar coordinates.

i) Be able to formulate problems in particle mechanics in mathematical terms using vectors and/or differential equations as appropriate, and solve the resulting equations to determine the motion of the particle.

j) Be able to derive and analyse single-population dynamical systems, including the exponential and logistic models with predation.

k) Be able to analyse the stability of equilibrium points in first order autonomous differential equations. and to construct simple bifurcation diagrams for single-population dynamics systems.

l) Learn and apply formulas which are taught in the course.