Midterm Exam 1, Ver.a
March 31, 2016, Intro to Robotics
Name: ______________________________

PeopleSoft ID: ______________________________

Problem	Score	Possible
1		5
2		5
3		5
4		5
5		5
6		5
Totals		30

You may have on your desk:

- Your student ID card
- 1 handwritten 8.5”x11” double-sided crib sheet
- this exam (provided by Professor)

$$\text{Grading: (problem difficulty)} \times \begin{cases} \text{2 for trying} \\ \text{3 for partial correct} \\ \text{5 for correct} \end{cases}$$

Concepts: Covers chapters 1-4, 11.1--11.2

Rotations & transformations

- Composition of rotations about world or current frame
- Construct a homogenous transform

Kinematics

- Assign DH parameters
- Given DH parameters, construct A matrix
- Given two A matrices, construct T matrix

Inverse Kinematics

- Two-argument arc tangent function
- Solve inverse position kinematics for a 3-link arm

Jacobian

- Construct Jacobian given sketch and T matrices

Computer Vision

- Move from camera frame to world frame

Problem 1: _____/5

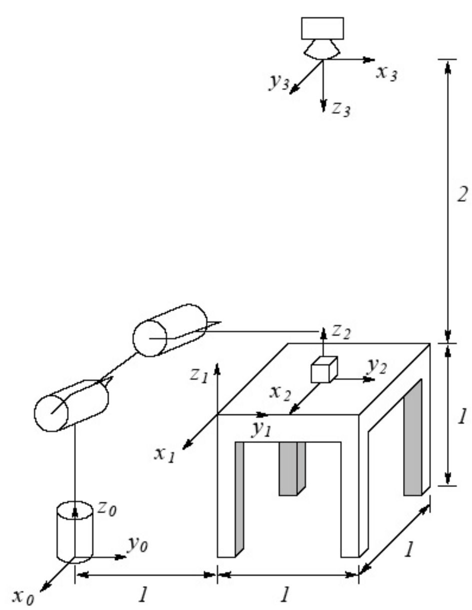
1. Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplications:
    a. Rotate by Φ about the current y-axis
    b. Rotate by θ about the world x-axis
    c. Rotate by ψ about the current y-axis
    d. Rotate by α about the world z-axis
    e. Rotate by β about the current y-axis

2. Suppose the three coordinate frames
$o_1x_1y_1z_1$, $o_2x_2y_2z_2$, and $o_3x_3y_3z_3$ are given, and suppose

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the matrix $R_3^2 = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix}$

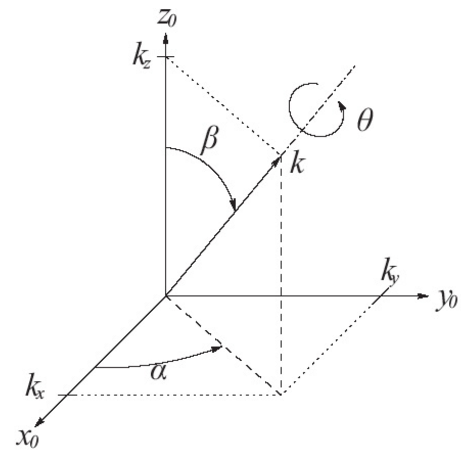
3. Consider the diagram at right. Robot is 1 meter from a table. The tabletop is 1 m high and 1 m square. A frame $o_1x_1y_1z_1$ is fixed to the side of the table as shown. A cube measuring 20 cm on a side is placed in the center of the table with frame $o_2x_2y_2z_2$established at the center of the cube as shown. A camera is situated directly above the center of the block 2 meters above the table top with frame $o_3x_3y_3z_3$ attached as shown.



Find the **homogenous transform** relating the frame $o_1x_1y_1z_1$ to the camera frame.

Problem 2: _____/5 Rotation matrices

Axis/Angle Representation:



Define axis as unit vector in $o_0x_0y_0z_0$.

$$k = [k_x \quad k_y \quad k_z]^T$$

1.) Rotate world z-axis to align with vector k:

a. Rotate α about world z-axis.

$$\sin(\alpha) = \frac{k_y}{\sqrt{k_x^2 + k_y^2}}, \qquad \cos(\alpha) = \frac{k_x}{\sqrt{k_x^2 + k_y^2}}$$

$$\alpha =$$

b. then β about current y-axis

$$\sin(\beta) = \sqrt{k_x^2 + k_y^2}, \qquad \cos(\beta) = k_z$$

$$\beta =$$

Matrix Identification. +1 for each correctly listed, -1 for each incorrectly listed, score is **max**(0, sum points)

a.) $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, b.) $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$, c.) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$, d.) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$,

e.) $\begin{bmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, e.) $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

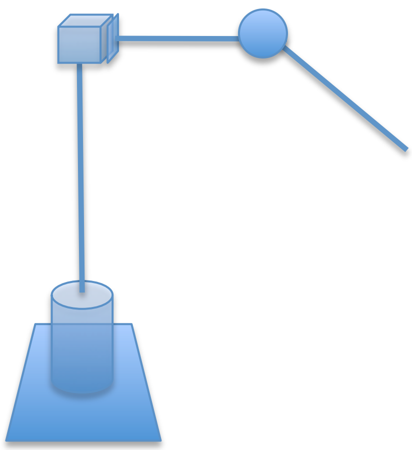
Which are valid so(k)? ____________

Which are valid SE(3)? _____________

Which are a valid SO(3) ______________

Problem 3: ______/5, Forward Kinematics

a.) For the 3-link robot below, draw the z and x-axis according to the DH convention



b.) Give the DH parameters for this PRP planar robot.

* indicates variable

Link	a_i	α_i	d_i	θ_i
1				
2				
3				

c.) Compute the transformation matrix A_2 and A_4 using the DH parameters:

Link	a_i	α_i	d_i	θ_i
1	0	0°	5	θ_1^*
2	0	90°	d_2^*	0°
3	10	0°	d_3^*	0°
4	3	0°	0	θ_1^*

* indicates variable

$A_2 = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$, $A_4 = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$

Problem 4: ______/5 Inverse kinematics

$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, o = \left[o_x, o_y, o_z\right]^T, o_c = [x_c, y_c, z\_c]^T,$ **solve for** o_c

Already did spherical robot and articulated manipulator
Do cylindrical robot

Planar PRP for inverse kinematics.

Problem 5: _____/5 Jacobian

1. Calculate the manipulator Jacobian of the anthropomorphic manipulator at the position $o_3 = o_c$.
   a. Write out the J matrix in terms of z_i and o_i.
   b. Write out the z_i and o_i values.
   c. Write out the J values. Calculate the cross products. You may use your previous calculations for the A and T matrices.

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 5 + q1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_2 & 0 & -s_2 & -q3s_2 \\ s_2 & 0 & c_2 & q3c_2 \\ 0 & -1 & 0 & 5 + q1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 6: _____/5 Computer Vision

a. **Two frames** $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogenous transformation

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_{1(t)} = [2,4,5]^T$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

b. For a camera with focal length $\lambda = 5$, find the image plane coordinates for the 3D points whole coordinates in the camera frame are given below. Indicate which points will not be visible to a physical camera.

Transformation: $k \begin{bmatrix} x^c \\ y^c \\ z^c \end{bmatrix} = \begin{bmatrix} 00 \\ 00 \\ 00 \end{bmatrix}$

a. $(5,5,15)^c$ → $(u,v) =$

b. $(-25,-25,50)^c$ → $(u,v) =$

c. $(5,5,-5)^c$ → $(u,v) =$

d. $(15,10,25)^c$ → $(u,v) =$

c. The robot has two parallel laser beams located at [-0.2,0,0], [0.2,0,0], and pointing in the direction [0,0,1]. They are used to measure the distance to approaching cars. If the car on the image is of width 5, the camera has focal length $\lambda = 10$, and the laser beams are located at [-1,0,0], [1,0,0].

How far is the car from the camera?

How big is the car?