

Midterm Exam 1, Ver.A, revision 3

March 31, 2016, Intro to Robotics

Name: _____

PeopleSoft ID: _____

Problem	Score	Possible
1		5
2		5
3		5
4		5
5		5
6		5
Totals		30

You may have on your desk:

- Your student ID card
- 1 handwritten 8.5"x11" double-sided crib sheet
- this exam (provided by Professor)

Grading: (problem difficulty) $\times \begin{cases} 2 & \text{for trying} \\ 3 & \text{for partial correct} \\ 5 & \text{for correct} \end{cases}$

Concepts: Covers chapters 1-4, 11.1--11.2

Rotations & transformations

- Composition of rotations about world or current frame
- Construct a homogenous transform

Kinematics

- Assign DH parameters
- Given DH parameters, construct A matrix
- Given two A matrices, construct T matrix

Inverse Kinematics

- Two-argument arc tangent function
- Solve inverse position kinematics for a 3-link arm

Jacobian

- Construct Jacobian given sketch and T matrices

Computer Vision

- Move from camera frame to world frame

Problem 1: ____/5

1. Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplications):
 - a. Rotate by Φ about the current y -axis
 - b. Rotate by θ about the world x -axis
 - c. Rotate by ψ about the current y -axis
 - d. Rotate by α about the world z -axis
 - e. Rotate by β about the current y -axis

2. Suppose the three coordinate frames

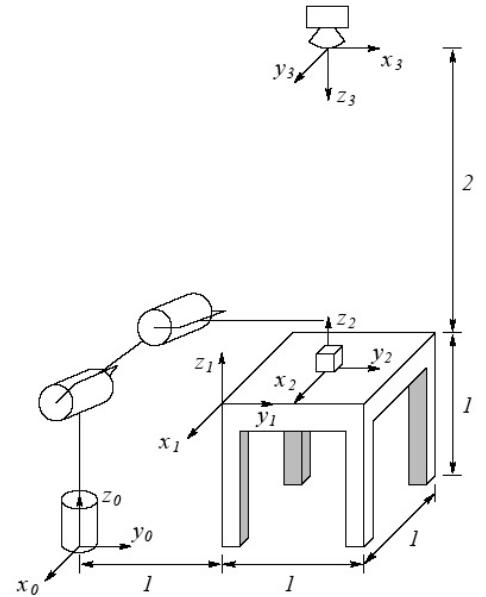
$o_1x_1y_1z_1$, $o_2x_2y_2z_2$, and $o_3x_3y_3z_3$ are given, and suppose

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the matrix $R_3^2 = \left[\quad \right]$

3. Consider the diagram at right. Robot is 1 meter from a table. The tabletop is 1 m high and 1 m square. A frame $o_1x_1y_1z_1$ is fixed to the side of the table as shown. A cube measuring 20 cm on a side is placed in the center of the table with frame $o_2x_2y_2z_2$ established at the center of the cube as shown. A camera is situated directly above the center of the block 2 meters above the table top with frame $o_3x_3y_3z_3$ attached as shown.

Find the **homogenous transform** relating the frame $o_1x_1y_1z_1$ to the camera frame.



Problem 2: ____/5 Rotation matrices

Axis/Angle Representation:

Define axis as unit vector in $o_0x_0y_0z_0$.

$$k = [k_x \ k_y \ k_z]^T$$

1.) Rotate world z-axis to align with vector k :

a. Rotate α about world z-axis.

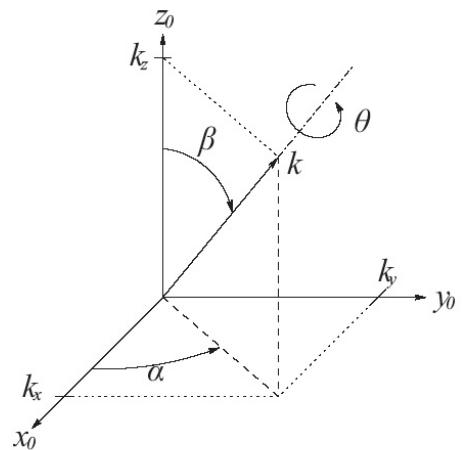
$$\sin(\alpha) = \text{_____}, \quad \cos(\alpha) = \text{_____}$$

$$\alpha = \text{_____}$$

b. then β about current y -axis

$$\sin(\beta) = \text{_____}, \quad \cos(\beta) = \text{_____}$$

$$\beta = \text{_____}$$



Matrix Identification. +1 for each correctly listed, -1 for each incorrectly listed, score is **max(0, sum points)**. A SO(k) matrix has determinant +1, but we will accept -1 as well.

a.) $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, b.) $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$, c.) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$, d.) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$,

e.) $\begin{bmatrix} 0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, f.) $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

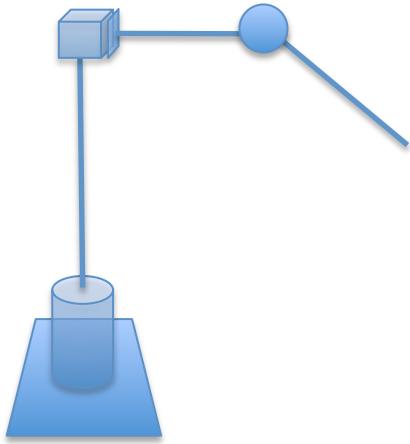
Which are valid so(k)? _____

Which are valid SE(3)? _____

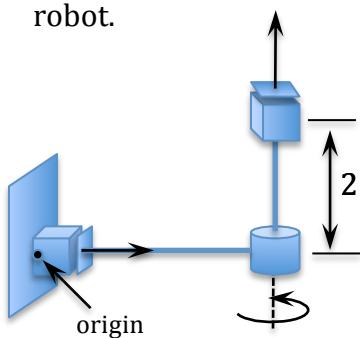
Which are a valid SO(3) _____

Problem 3: ____/5, Forward Kinematics

a.) For the 3-link robot below, draw the z and -axis according to the DH convention



b.) Give the DH parameters for this PRP robot.



* indicates variable

Link	a_i	α_i	d_i	θ_i
1				
2				
3				

Link	a_i	α_i	d_i	θ_i
1	0	0°	5	θ_1^*
2	0	90°	d_2^*	0°
3	10	0°	d_3^*	0°
4	3	0°	0	θ_4^*

c.) Compute the transformation matrix A_2 and A_4 using the DH parameters:

* indicates variable

$$A_2 = \left[\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \\ \text{ } \end{array} \right], A_4 = \left[\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \\ \text{ } \end{array} \right]$$

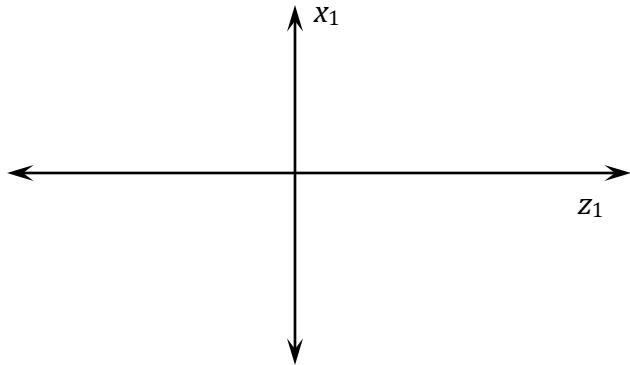
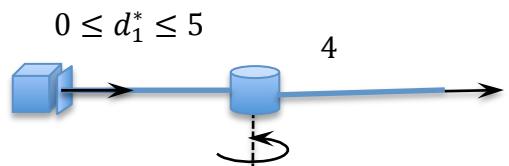
Problem 4: _____/5 Inverse kinematics

$$o = o_c^0 + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, o = [o_x, o_y, o_z]^T, o_c = [x_c, y_c, z_c]^T, \text{ solve for } o_c$$

$$-\pi/2 \leq \theta_2^* \leq \pi/2$$

Planar PR for inverse kinematics.

- a.) draw the workspace of the manipulator



- b.) given a point (x, y) in the workspace of the manipulator, give the d_1^* and θ_2^* values

Problem 5: ____/5 Jacobian

1. Calculate the manipulator Jacobian of the anthropomorphic manipulator at the position $o_3 = o_c$.
 - a. Write out the J matrix in terms of z_i and o_i .
 - b. Write out the z_i and o_i values.
 - c. Write out the J values. Calculate the cross products. You may use your previous calculations for the A and T matrices.

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 5 + q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} c_2 & 0 & -s_2 & -q_3 s_2 \\ s_2 & 0 & c_2 & q_3 c_2 \\ 0 & -1 & 0 & 5 + q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 6: ____/5 Computer Vision

- a. **Two frames** $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogenous transformation

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_{1(t)} = [2,4,5]^T$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

- b. For a camera with focal length $\lambda = 5$, find the image plane coordinates for the 3D points whose coordinates in the camera frame are given below. Indicate which points will not be visible to a physical camera.

ADD THIS TO YOUR NOTES – IT WILL NOT BE ON THE EXAM:

Transformation: $k \begin{bmatrix} x^c \\ y^c \\ z^c \end{bmatrix} = \begin{bmatrix} u \\ v \\ \lambda \end{bmatrix}$

- a. $(5,5,15)^c \rightarrow (u,v) =$
- b. $(-25,-25,50)^c \rightarrow (u,v) =$
- c. $(5,5,-5)^c \rightarrow (u,v) =$
- d. $(15,10,25)^c \rightarrow (u,v) =$
- c. The robot has two parallel laser beams located at $[-2,0,0]$, $[2,0,0]$, and pointing in the direction $[0,0,1]$. They are used to measure the distance to approaching cars. If the car on the image is of width 5, the camera has focal length $\lambda = 10$, and the laser beams in the image are located at $[-1,0]$, $[1,0]$.

How far is the car from the camera?

How big is the car?