Final Exam, Version **ANSWERS**

May 10, 2016 Intro to Robotics

|  |  |  |
| --- | --- | --- |
| Problem | Score | Possible |
| 1 |  | 15 |
| 2 |  | 25 |
| 3 |  | 15 |
| 4 |  | 25 |
| 5 |  | 25 |
| 6 |  | 20 |
| Totals |  | 125 |

You may have on your desk:

* Your student ID card
* 2 handwritten 8.5”x11” double-sided crib sheets
* This exam (provided by Professor)

Grading: (problem difficulty)

Concepts: Covers chapters 1-4, 11.1--11.2

*Rotations & transformations*

* Composition of rotations about world or current frame
* Construct a homogenous transform

*Kinematics*

* Assign DH parameters
* Given DH parameters, construct A matrix
* Given two A matrices, construct T matrix

*Inverse Kinematics*

* Two-argument arc tangent function
* Solve inverse position kinematics for a 3-link arm

*Jacobian*

* Construct Jacobian given sketch and T matrices
* Solve Jacobian to find singularities

*Computer Vision*

* Move from camera frame to world frame

Problem 1: \_\_\_\_\_\_/15

1. (5 pt.s) Write the matrix product that will give the resulting rotation matrix

(*Do not perform the matrix multiplications*):

* 1. Rotate by *β* about the world *z*-axis
  2. Rotate by *α* about the world *x*-axis
  3. Rotate by *Φ* about the current *y*-axis
  4. Rotate by γ about the world *z*-axis
  5. Rotate by *ψ* about the current *y*-axis

1. (5 pts.) Suppose the three coordinate frames

, , and are given, and suppose

, and .

 Find the matrix

1. (5 pts.) Consider the diagram at right. Robot is 1 meter from a table. The tabletop is 1 m high and 1 m square. A frame is fixed to the side of the table as shown. A cube measuring 20 cm on a side is placed in the center of the table with frame established at the center of the cube as shown. A camera is situated directly above the center of the block 2 meters above the table top with frame attached as shown. Find the **homogenous transform** relating the frame to the camera frame, that is, .

Problem 2: \_\_\_\_\_\_/25 Jacobian Singularities

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link |  |  |  |  |
| 1 | 1 |  |  |  |
| 2 | 0 |  |  |  |
| 3 | 1 |  |  |  |

a.) a robot has the DH parameters and velocity Jacobian below.

(5 pts.) What are the singularities? (Give the determinant)

Expand the middle column, since it only has one term, which is negative

=

=

=

=

=

=

(5 pts.) What configuration variables result in singularities?

(

(5 pts.) Draw configurations for each type of singularity

left config. can’t generate velocity out of the page

or left to right

right config can’t generate velocity left to right

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link |  |  |  |  |
| 1 | 1 |  |  |  |
| 2 | 0 |  |  |  |
| 3 | 1 |  |  |  |

b.) A robot has the DH parameters and velocity Jacobian below.

(5 pts.)What are the singularities? (Give the determinant). Hint:

Expand the bottom row, since it only has one term, which is positive

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=

=

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(5 pts.) What configuration variables result in singularities?

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Problem 3: \_\_\_\_\_\_/15, Forward Kinematics

a.) (5 pts.) For the 3-link RPR robot below, draw the *z* and *x*-axis according to the DH convention.

Parallel-jaw

gripper

 origin

Note: there are several possible solutions for *x*3. The DH conventions are clear that z3 is parallel to z2

b.) (5 pts.) Give the DH parameters for this PRR robot.

variable

­

3

­

2

\* indicates variable

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link |  |  |  |  |
| 1 | 0 | 0 |  | 0 |
| 2 | 0 |  | 0 |  |
| 3 | 3 |  | 2 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Link |  |  |  |  |
| 1 | 3 |  |  |  |
| 2 | 4 |  |  |  |
| 3 | 5 |  |  |  |
| 4 | 3 |  |  |  |

c.) (5pt) Compute the transformation matrix *A2* using the DH parameters:

\* indicates variable

*A2* =, Equation 3.10, page 77

Problem 4: \_\_\_\_\_\_/25 Inverse kinematics

1

RRP robot for inverse kinematics.

*Note that in the drawing*

1. (5pt) Draw cross section of the manipulator’s *workspace* at

*x*0

*y*0

1

1

2

2

-3

-1

-2

-1

-2

*x*0

*z*0

1

2

2

-2

1

-1

1. (5pt) Draw cross section of the manipulator’s *workspace* at

What joint variables place the end-effector at point [*xc,yc,zc*] specified in the frame ? Assume the point is reachable. Let .

1. (5 pts.) =
2. (5 pts.)

to see this, let be the radius of the arm’s reach, and subtract off 1 for the fixed length of the arm. The height the arm must tilt is

1. (5 pts.) = This is the distance from to the point [*xc,yc,zc*] .

Problem 5: \_\_\_\_\_\_/25

Calculate the manipulator Jacobian of the 2-link RR arm at the position o2 = oc.

* 1. (10 pts.) Write out the *J* matrix in terms of *zi* and *oi*.
  2. (5 pts.) Write out the *zi* and *oi* values needed for part a. Don’t forget *o0*.
  3. (10 pts.) Write out the *J* values. Calculate the cross products.

Problem 6: \_\_\_\_\_\_/20 Computer Vision

1. (5 pts.) Two frames and are related by the homogenous transformation

A particle has position relative to frame .

What is the position of the particle in frame ?

1. (5 pts.) For a camera with focal length , find the image plane coordinates for the 3D points in the camera frame

(15,20,50)c 🡪 (*u,v*) = (*1.5,2*)

, ,

1. Astronomers are trying to measure the distance to an *object*. From a ground based observatory they take an image at June and another image at December, both times pointing their telescope orthogonally to the sun. The *object* appears to move against the field of much more distant stars and galaxies.

The telescope has focal length , the earth orbital radius is 150x109 m.

June (-0.7,-0.2)

December (0.3,-0.2)

(5 pts.) If both images are aligned, what is the homogenous transform from camera frame 1 to 2?

(5 pts.) If the image coordinates are (-0.7,-0.2) and then (0.3,-0.2), what is the distance to the object?[m]

SEE NEXT PAGE

This phenomenon is called ‘parallax’, and is how astronomers measure distances to stars in our galaxy. Usually we use telescopes mounted in satellites. Due to limited resolution of optics, this technique doesn’t work for stars that are far away.

Because this is the unit they employ for measuring distance, astromers favor this term over ‘light year’

A **parsec** (symbol: pc) is a unit of length used to measure large distances to objects outside the Solar System. One **parsec** is the distance at which one astronomical unit subtends an angle of one arcsecond. A **parsec** is equal to about 3.26 light-years (31 trillion kilometres or 19 trillion miles) in length.

For Star wars fans, this is what Han Solo meant by ‘Kessel run in 12 parsecs’…