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**ECE 5397/6397: Intro to Robotics**

**Class Worksheet – Lecture 4:** Coordinate transformation, Parameterizations of SO(3), Euler angles

Constraints on How many DOF?

1. column vectors in are unit vectors:

+=1, +=1, +=1

1. Columns in are mutually orthogonal:

,

Two-argument arctangent function is the unique angle such that

and

What is and and

for ,

for ,

**ZYZ Euler Angles:** specify the orientation of frame relative to frame

1. Rotate about *z*-axis by
2. Rotate about current y-axis by
3. Rotate about current z-axis by

For a given , how do we determine ? Why do we need to?

Solving for angles necessary for the inverse kinematics problem in Section 3.3

=

1. If and are not both zero, then , and
2. If >0, If <0
3. If , then and . Thus =0

=

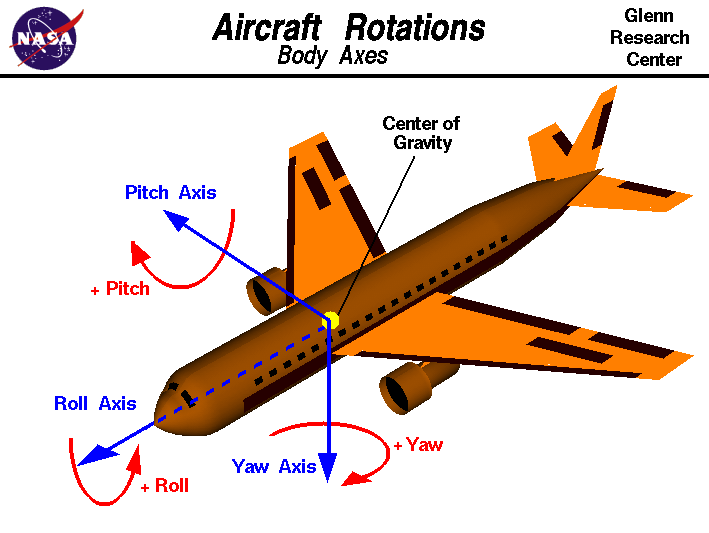
1. If , =1, ­­­­and =0 so 0

*remember the sine and cosine of sum identities:*

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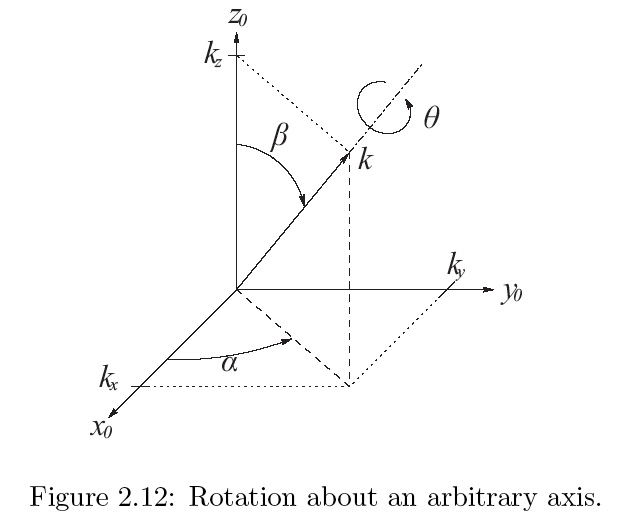
1. If , =, ­­­­and =0 so

==

**Roll,Pitch, Yaw Angles:** specify the orientation of frame relative to frame

1. Yaw about world *x*-axis by
2. Pitch about world *y*-axis by
3. Rotate about world z-axis by

(draw on image)



**Axis/Angle Representation:**

rotation about an arbitrary axis in space:

1. Define axis as unit vector in .

1. Rotate world *z*-axis to align with vector *k*:
   1. Rotate about world z-axis,
   2. then about current *y-*axis

, where

Solving:

What is the trace of ?

Then

How can you isolate

Thus

Problem: *If* , =,

**Rigid Motions**

A rigid motion is an ordered pair , where. It is a pure translation together with a pure rotation

The group of all rigid motions is known as the **Special Euclidean Group** and is denoted .

.

Let be the orientation of frame with respect to . Let be the vector from frame 0 to 1 and be the vector from frame 0 to 2. Suppose is rigidly attached to frame 2 with local coordinates

Draw , and :

Solve for