

پاسخ سوال صحیح و غلط:

$$\boxed{I} \quad h_1(n) = O(f(n)) \Rightarrow \exists d, n_1 > 0 : \forall n \geq n_1, h_1(n) \leq d f(n)$$

$$h_2(n) = O(g(n)) \Rightarrow \exists e, n_2 > 0 : \forall n \geq n_2, h_2(n) \leq e g(n)$$

$$\text{if } \text{Max}(n_1, n_2) = n_0 \quad , \quad \text{Max}(d, e) = c$$

$$, \quad h(n) = h_1(n) + h_2(n) \Rightarrow h(n) \leq c(f(n) + g(n))$$

$$\forall n \geq n_0 \Rightarrow h(n) = O(f(n) + g(n))$$

$$\Rightarrow O(f(n) + g(n)) = O(f(n)) + O(g(n))$$

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$$\boxed{II} \quad O(f(n)) \leq c' f(n)$$

$$O(g(n)) \leq c'' g(n)$$

$$\Rightarrow O(f(n)) \times O(g(n)) \leq \underbrace{c' c''}_c f(n) \times g(n) \quad \forall n \geq n_0$$

$$\Rightarrow O(f(n)) \times O(g(n)) = O(f(n) \times g(n))$$

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\boxed{III}

عبارت غلط می باشد. مثال نقض:

$$f(n) = g(n) = n^2 \Rightarrow O(f - g) = O(0)$$

$$n^2 + n = O(n^2) = O(f) \quad , \quad n^2 = O(n^2) = O(g)$$

$$O(f - g) = O(0) \neq O(f) - O(g) = n$$

$$\boxed{f} \quad f(n) = \Omega(g(n))$$

$$\Rightarrow \exists c, n_0 > 0, \forall n \geq n_0 : f(n) \geq c g(n)$$

$$\Rightarrow g(n) \leq \frac{1}{c} f(n)$$

$$\Rightarrow \exists d, n_0 > 0, \forall n \geq n_0 : g(n) \leq d f(n)$$

$$\Rightarrow g(n) = O(f(n))$$

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$$\boxed{a} \quad f(n) = O(g(n))$$

$$\Rightarrow \exists c, n_1 > 0, \forall n \geq n_1 : f(n) \leq c g(n)$$

$$f(n) = \Omega(g(n))$$

$$\Rightarrow \exists d, n_2 > 0, \forall n \geq n_2 : f(n) \geq d g(n)$$

$$\text{if } n_0 = \max(n_1, n_2)$$

$$\Rightarrow \exists c, d, n_0 > 0, \forall n \geq n_0 : d g(n) \leq f(n) \leq c g(n)$$

$$\Rightarrow f(n) = \Theta(g(n))$$

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$$\boxed{9} \quad \log(n^n) \geq \log(n!) \quad \forall n \geq 1$$

$$\Rightarrow \text{if } f(n) = \log(n!) \Rightarrow f(n) = O(\log(n^n))$$

$$\text{پس: } \log(n^n) = n \log(n) \Rightarrow f(n) = O(n \log(n))$$

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