Roll No. ....

Total No. of Questions : 5] [Total No. of Printed Pages : 4

EH-245

B.E. IInd Semester (CGPA) Inform. Tech.

Examination, 2019

Mathematics

Paper - IT - 201

Time : 3 Hours] **[Maximum Marks : 60**

Note :- Attempt all **five** questions. Each question has an internal choice.

1\. (a) Using Lagrange's mean value theorem, prove that

$$\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$$

where $0 < a < b$

(b) Expand $\log x$ in power of $(x-1)$ by Taylor's theorem and hence find the value of $\log_e 1.1$.

OR

(a) Expand $\log(1+e^x)$ in ascending powers of x as for as the term containing x^4.

(b) The power dissipated in a resistor is given by $p = E^2/R$. Find by using calculus the approximate percentage change in P when E is increased by 3% and R is decreased by 2%.

2. (a) Find the radius of curvature to the cycloid

$x = a(\theta + \sin\theta)$

$y = a(1 - \cos\theta)$ at the point θ.

(b) Find the centre of curvature of the curve

$x^3 + xy^2 - 6y^2 = 0$ at $(3, 3)$

OR

(a) Find all the asymptotes of the curve

$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

(b) Find the envelope of the family of straight lines given by the normal equation :

$x\cos c + y\sin c = p$

3. (a) The tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on $x^2y^2 = x^2 - y^2$.

(b) If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, prove that

$$x^2\frac{\partial^2 h}{\partial x^2} + y^2\frac{\partial^2 h}{\partial xy^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = \sin hx - \sin 2u$$

OR

(a) If $u = \log(x^3+y^3+z^3-3xyz)$, show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{9}{(x+y+z)^2}$$

(b) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$

4. (a) Find the limit when $n \to \infty$ of the series :

$$\frac{n}{n^2} + \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots.. + \frac{n}{n^2+(n-1)^2}$$

(b) Prove that $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}$

OR

(a) Evaluate $\displaystyle\int_a^b \sin x\, dx, 0 \le a,\ b \le \pi/2$ as the limit of a sum.

(b) Evaluate

$$\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)......\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$$

5. (a) Find the area enclosed by the curve $y^2(a-x)=x^3$ and its asymptotes.

(b) Find the perimeter of the cardioid $r=a(1-\cos\theta)$.

OR

(a) Show that the volume of the solid generated by the revolution of the curve $(a-x)y^2=a^2x$, about its asymptote is $\pi^2a^3/2$.

(b) Find the common area to the circles $r=a\sqrt{2}$ and $r=2a\cos\theta$.

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