

**B.E. I Sem. (CGPA) Civil Engineering
Exam.-2012-13****ENGINEERING MATHS - I****Paper : CE-101****Time Allowed : Three Hours****Maximum Marks : 60**

Note : Solve all questions.

- Q.1. 1) If y satisfies the equation $\frac{dy}{dx} = x^2 y - 1$ and $y = 1$ when $x = 0$. Use Taylor's expansion of y in powers of x to find y at $x < 0.03$.
- 2) Find the first four terms in the expansion of $\log_e \sin(x+h)$ in ascending powers of h . Hence find the value of $\log_e \sin 31^\circ$ to four places of decimals. Given $\log_e 2 = 0.69315$.

Or

- ✓ 1) Expand $\sin^{-1} \frac{2x}{1+x^2}$ in ascending powers of x , upto ~~x^6~~ by MacLaurin's theorem.
- ✓ 2) Expand $\sqrt{1 + \sin x}$ upto sixth power of x .

(2)

Q.2. 1)

Show that the length of the Portions of the tangent to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ intercepte between the coordinate axes is constant.

2) Find the Pedal equation to the curve $r = a(1+\cos\theta)$

Or

- 1) If the curve $x^{m+n} = a^{m+n} y^{2n}$, show that for any point (x, y) the m^{th} power of the sub tangent varies as the n^{th} power of the sub normal.
- 2) If the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at any point on it cuts the two axes at P and Q. Show that $OP + OQ = a$, O being the origin.

Q.3.

1) If $u = (1 - 2xy + y^2)^{-\frac{1}{2}}$ prove that

$$\frac{\partial}{\partial x} \left[(1 - x^2) \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left(y^2 \frac{\partial u}{\partial y} \right) = 0$$

2) Evaluate

$$\lim_{x \rightarrow \frac{\pi}{2}} (\cos x)^{\frac{\pi}{2}-x}$$

using L'Hospital's rule

Or

- 1) A given quantity of metal is to be moulded into a rod with rectangular base and square ends. Find the ratio of the length of the side of the square if the surface area of the rod is minimum.

- 2) If $u = x^y$ show that

$$(cm^4)^{\frac{1}{y}}$$

(IT)

(3)

$$\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$$

Q.4. (1) Show that $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$

2) Find the limits as $n \rightarrow \infty$ of the sum of the series

$$\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n}$$

Or

1) Find the whole perimeter of the cardioid $r = a(1+\cos\theta)$.

2) Find the surface of the solid generated by the revolution of the lemniscate $r^2 = a^2 \cos 2\theta$ about the line $\theta=0$.

Q.5. 1) Evaluate $\iint_A x^2 dx dy$ where A is the region in the first quadrant bounded by the hyperbola $xy = 16$.

2) Find the C.G. of the arc of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

Or

1) Evaluate $\int_0^1 x^4 (1 - \sqrt{x})^5 dx$

2) Evaluate $\int_{-1}^1 (1+x)^{p-1} (1-x)^{q-1} dx$

Q.6. 1) T or F $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta = \frac{1}{2} \left[\frac{1}{3} \right] \left[\frac{3}{4} \right]$

2) $\int_{-m}^m \sqrt{m + \frac{1}{2}} = \frac{\sqrt{\pi}}{2^{2m+1}} \sqrt{2m} (T \text{ or } F)$

3) Find the equation of tangent $y^2 = 3x^2 + 1$ at the point (1, 2)

4) T or F $S = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta} \right)^2} d\theta$

5) Evaluate $\int_0^{\pi/2} \log(\tan x + \cot x) dx$

