Total No. of Questions : 6

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EH-181

B.E. II Semester (CGPA) Civil Engineering Exam. 2014

ENGINEERING MATHEMATICS - II

Paper : CE-201

Time Allowed : Three Hours

Maximum Marks : 60

Note : Attempt all the questions. Each questions carry equal marks.

Q.1. Objective / Short / True or False / Type question.

a) The differential equation

$(ay^2 + x + x^8)dx + (y^8 - y + bx)\,dy = 0$ is exact if

(i) $b = a$ (ii) $b = 2a$

(iii) $a = 1, b = 3$ (iv) $b \neq 2a$

b) The number of arbitrary constants in the general solution of a differential equation is equal to the ________ of the differential order.

c) The homogeneous linear differential equation whose auxiliary equation has roots 1, −1 is ________

d) Which of the basis of solutions are for the differential

equation $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 0$

(i) $x \ln x$ (ii) $\ln x, e^x$ (iii) $\dfrac{1}{x}, \dfrac{1}{x^2}$ (iv) $\dfrac{1}{x^2}e^x, x \ln x$

e) The expansion of $5x^3+x$ in terms of Legendre Polynomial is

f) $L^{-1}\left\{\dfrac{s}{(2s+3)^2}\right\}$ is ________

$\frac{dy}{dx} + Px = Q$

g) The condition under which the Laplace transform of $f(t)$ exists are ________

h) If x = c is a point of discontinuity then the Fourier series of f(x) at x = c gives f(x) =

i) The smallest period of the function $sin\left(\frac{2n\pi\, x}{K}\right)$ is ________

j) By eliminating a and b from z = a(x + y) + b the p.d.e. formed is ________

Q.2. a) Solve the differential equation 15

$$\left(x^3 - x\right)\frac{dy}{dx} - \left(3x^2 - 1\right)y = x^5 - 2x^3 + x$$

b) Solve : $x\, dx + y\, dy = \frac{a^2\left(x\, dy - y\, dx\right)}{x^2 + y^2}$

OR

a) Solve : $(2x^2y^2 + y)\, dx - (x^3y - 3x)\, dy = 0$

b) Solve : $(p - 1)e^{3x} + p^3\, e^{2y} = 0$

Q.3. a) Solve :

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + \left(p^2 + q^2\right)y = e^{cx} + pqx^2$$

b) Solve : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = (\log x)\sin(\log x)$

OR

a) The radial displacement in a rotating disc at a distance r-from the axis is given by

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$$r^2\frac{d^2y}{dr^2}+r\frac{dy}{dr}-u+Kr^3=0,$$ where K is a constant. Solve the equation under the conditions $u = 0$ when $r = 0$, $u = 0$ when $r = a$.

b) Solve : $\frac{dx}{dt}=2y, \frac{dy}{dt}=2z,$ and $\frac{dz}{dt}=2x$

Q.4. a) Show that $x^n f_n(x)$ is a solution of the equation $xy'' + (1-2n)\,y' + xy = 0$.

b) State and prove Rodrigue's formula.

OR

a) Prove that $\left(1-2xt+t^2\right)^{-1/2}=\sum_{n=0}^{\infty} t^n p_n(x)$

b) If $f(x) = 0, -1 < x \leq 0$
$= x, 0 < x < 1$

then show that $f(x)=\frac{1}{4}P_0(x)+\frac{1}{2}P_1(x)$
$+\frac{5}{16}P_2(x)-\frac{3}{32}P_4(x)+\cdots$

Q.5. a) Define unit step function. Express the function

$$f(t)=\begin{cases}\sin t, & t<\pi\\ t, & t\geq\pi\end{cases}$$, in terms of unit step function and hence find its Laplace transform.

b) i) Find the Laplace transform of $\sin 2t.\cos 3t + \cos(at+5)$

ii) Find $L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\}$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 = \frac{2}{\pi}\int_0^{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} \frac{2}{\pi}\times\frac{\pi^3}{3}$ (4)

$a_0 = \frac{2\pi^2}{3}$

OR

a_n

b_n

a) Find the Laplace transform of $\left(\sin\sqrt{t}\right)$ and hence obtain the value of $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.

b) Using Laplace transform method solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos 2t$ given that $x(0) = 1$ and $x\left(\pi/2\right) = -1$

Q.6. a) Prove that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in $-\pi < x < \pi$. Hence show that

i) $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$

ii) $\sum \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

iii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$

iv) $\sum \frac{1}{n^4} = \frac{\pi^4}{90}$

b) Solve by Charpits' method $(p^2 + q^2)\, y = qz$

OR

a) Obtain the first three coefficients in the Fourier series for y, where y is given by:

x:	0	1	2	3	4	5
y :	4	8	15	7	6	2

b) Solve $r + 2s + t = 2(y - x) + \sin(x - y)$

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