ECE 374 B ✧ Spring 2023

Homework 7

- Groups of up to three people can submit joint solutions. Each problem should be submitted by exactly one person, and the beginning of the homework should clearly state the Gradescope names and email addresses of each group member. In addition, whoever submits the homework must tell Gradescope who their other group members are.
- **Submit your solutions electronically on the course Gradescope site as PDF files.** please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. You are given a list $D[n]$ of n words each of length k over an alphabet Σ in a language you don't know, although you are told that words are sorted in lexicographic order. Using $D[n]$, describe an algorithm to efficiently identify the order of the symbols in Σ. For example, given the alphabet $\Sigma = \{Q, X, Z\}$ and the list $D = \{QQZ, QZZ, XQZ, XQX, XXX\}$, your algorithm should return QZX. You may assume D always contains enough information to completely determine the order of the symbols. (Hint: use a graph structure, where each node represents one letter.)

2. Given a directed-acyclic-graph ($G = (V, E)$) with integer (positive or negative) edge weights:

   (a) Give an algorithm to find the **shortest** path from a node s to a node t.

   (b) Give an algorithm to find the **longest** path from a node s to a node t.

3. Your job is to arrange n ill-behaved children in a straight line, facing front. You are given a list of m statements of the form "i hates j." If i hates j, then you do not want to put i somewhere behind j, because then i is capable of throwing something at j.

   - Give an algorithm that orders the line (or says that it is not possible) in $O(m+n)$ time.
   - Suppose instead you want to arrange the children in rows such that if i hates j, then i must be in a lower numbered row than j. Give an efficient algorithm to find the minimum number of rows needed, if it is possible.

4. Can we solve the single-source longest-path problem by changing minimum to maximum in Dijkstra's algorithm? If so, then prove your algorithm correct. If not, then provide a counterexample. (Assume that the graph only has positive edge weights)

5. You are given a directed graph $G = (V, E)$ with possibly negative weighted edges:

   (a) Suppose you knew that the shortest path between any two vertices is guaranteed to have **at most** k edges. Give an algorithm that finds the shortest path between two vertices u and v in $O(k(n+m))$ time.

   (b) Suppose you knew the shortest path from u to v contains exactly k edges. Give an algorithm that finds the shortest path between two vertices u and v in $O(k(n+m))$ time. Note that the path need not be simple.