# 1 Recursive Definitions

Give the recursive definition of the following languages. For both of these you should concisely explain why your solution is correct.

1. A language that contains all strings.
2. A language which holds all the strings containing the substring **000**.
3. A language $L_A$ that contains all palindrome strings using some arbitrary alphabet $\Sigma$.
4. A language $L_B$ that does not contain either three **0**'s or three **1**'s in a row. E.g., **001101** $\in L_B$ but **10001** is not in $L_B$.

# 2 Regular Expressions

Give regular expressions for each of the following languages over the alphabet {**0**, **1**}.

1. All strings containing the substring **000**.
2. All strings *not* containing the substring **000**.
3. All strings in which every run of **0**s has length at least 3.
4. All strings in which **1** does not appear after a substring **000**.
5. All strings containing at least three **0**s.
6. Every string except **000**. *[Hint: Don't try to be clever.]*
7. All strings $w$ such that *in every prefix of* $w$, the number of **0**s and **1**s differ by at most 1.

*8. All strings containing at least two **0**s and at least one **1**.

★9. All strings in which the substring **000** appears an even number of times.
(For example, **0001000** and **0000** are in this language, but **00000** is not.)