ECE 374 B ✧ Spring 2024

Homework 9

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. A *strongly independent* set is a subset of vertices S in a graph G such that for any two vertices in S, there is no path of length less than or equal to two in G. Prove that *Strongly Independent Set* is NP-hard.

2. Are the following problems in P, NP, co-NP, NP-Hard, NP-complete? Either way, prove it.

   (a) A *kite* is a graph on an even number of vertices, say 2n, in which n of the vertices form a clique and the remaining n vertices are connected in a tail that consists of a path joined to one of the vertices of the clique. Given a graph and a goal g, the max kite problem asks for a sub-graph that is a kite and contains 2g nodes. What complexity classes does $kite$ belong in?

   (b) A *4kite* is exactly the same problem, but this time $g = 4$. What complexity classes does $4kite$ belong in?

3. Let $T = \{\langle M \rangle | M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.

4. Are the following problems decidable or undecidable? If the language is decidable, explain why, and if it's not, prove it.

   (a) $L_{5a} = \{\langle M \rangle \mid M \text{ is a TM that accepts string 1011 in } |1011|^6 \text{steps}\}$

   (b) $L_{5b} = \{\langle M, w \rangle \mid M \text{ is a TM that does not accept any strings in } |w|^6 \text{steps}\}$