

# ECE 374 B ✧ Spring 2024

## 🌀 Homework 9 🌀

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- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the  $\text{\LaTeX}$  solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).
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### 👉 **Some important course policies** 👉

- **You may use any source at your disposal**—paper, electronic, or human—but you *must* cite *every* source that you use, and you *must* write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
  - **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an **automatic zero**, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
    - Always give complete solutions, not just examples.
    - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
    - Never use weak induction.
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### **See the course web site for more information.**

If you have any questions about these policies,  
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. A *strongly independent* set is a subset of vertices  $S$  in a graph  $G$  such that for any two vertices in  $S$ , there is no path of length less than or equal to two in  $G$ . Prove that *Strongly Independent Set* is NP-hard.
  
2. Are the following problems in P, NP, co-NP, NP-Hard, NP-complete? Either way, prove it.
  - (a) A *kite* is a graph on an even number of vertices, say  $2n$ , in which  $n$  of the vertices form a clique and the remaining  $n$  vertices are connected in a tail that consists of a path joined to one of the vertices of the clique. Given a graph and a goal  $g$ , the max kite problem asks for a sub-graph that is a kite and contains  $2g$  nodes. What complexity classes does *kite* belong in?
  - (b) A *4kite* is exactly the same problem, but this time  $g = 4$ . What complexity classes does *4kite* belong in?
  
3. Let  $T = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$ . Show that  $T$  is undecidable.
  
  
  
  
  
  
  
  
  
  
4. Are the following problems decidable or undecidable? If the language is decidable, explain why, and if it's not, prove it.
  - (a)  $L_{5a} = \{ \langle M \rangle \mid M \text{ TM accepts string } 1011 \text{ in } |1011|^6 \text{ steps} \}$
  - (b)  $L_{5b} = \{ \langle M \rangle \mid M \text{ TM does not accept any strings in } |w|^6 \text{ steps} \}$