ECE 374 B - Summer 2024

Homework 2

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/su24/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com/su24/) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. **Solo Journeys: The Love Lives of 374 Students** A directed graph $G = (V, E)$ is *singly connected* if $u \rightsquigarrow v$ implies that G contains at most one simple path from u to v for all vertices $u, v \in V$. Give an efficient algorithm to determine whether a directed graph is singly connected.

2. **Premature Termination** Find diagram 8.7 in chapter 8 under additional resources of Lecture 16. Suppose we change the first line of diagram 8.7 to read "for the first |V|-1 vertices in topological order" instead. Will this algorithm still work? Why or why not? Justify your answer. (That way if your wrong I can still give you pity points.)

3. **When Dijkstra F*s Up** Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces an incorrect answer.

4. **Bitonic Befuddlement** A sequence is *bitonic* if it monotonically increases and then monotonically decreases, or if by a circular shift it monotonically increases and then monotonically decreases. For example the sequences $\langle 1, 4, 6, 8, 3, -2 \rangle$, $\langle 9, 2, -4, -10, -5 \rangle$, and $\langle 1, 2, 3, 4 \rangle$ are bitonic, but $\langle 1, 3, 12, 4, 2, 10 \rangle$ is not bitonic.

   Suppose that you are given a directed graph $G = (V, E)$ with weight function $w : E \to \mathbb{R}$, where all edge weights are unique, and you wish to find single-source shortest paths from a source vertex s. You are given one additional piece of information: for each vertex $v \in V$, the weights of the edges along any shortest path from s to v form a bitonic sequence.

5. **Ugh Yuck! A proof. Gross.** Prove the following statement. If G has a topological ordering, then G is a DAG. [Hint: contradiction is not your friend ;)]