# EEE 391 Basics of Signals and Systems
# Spring 2022–2023 Homework 1
# due: 5 April 2023, Wednesday by 23:00

**Section: 2**
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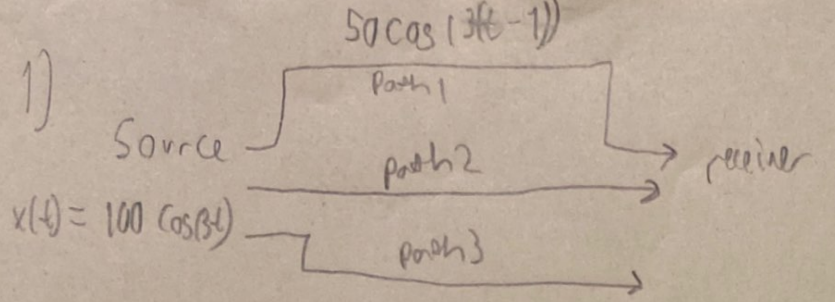
1)




Path1 ⇒ $50\cos(3(t-1))$
Path2 ⇒ $60\cos(3(t-2))$
Path3 ⇒ $20\cos(3(t-3))$

a) $y(t) = 50\cos(3(t-1)) + 60\cos(3(t-2)) + 20\cos(3(t-3))$

b) Since all the signals have the same frequency ⇒ we can use phasor addition

$$y(t) = \underbrace{50\cos(3t-3)}_{X_1 = 50e^{j(-3)}} + \underbrace{60\cos(3t-6)}_{X_2 = 60e^{j(-6)}} + \underbrace{20\cos(3t-9)}_{X_3 = 20e^{j(-9)}}$$

$$X_n = X_1 + X_2 + X_3 = 50(\cos(-3) + j\sin(-3)) + 60(\cos(-6) + j\sin(-6)) + 20(\cos(-9) + j\sin(-9))$$

$$= 50(\cos(3) - j\sin(3)) + 60(\cos(6) - j\sin(6)) + 20(\cos(9) - j\sin(9))$$

$$= \underbrace{\left(50\cos(3) + 60\cos(6) + 20\cos(9)\right)}_{U \approx -10.112 \quad -10.1} + j\underbrace{\left(-(50\sin(3) + 60\sin(6) + 20\sin(9))\right)}_{V \approx 1.47}$$

$$A = \sqrt{U^2+V^2} = \sqrt{(-10.1)^2 + (1.47)^2} \cong 10.2$$

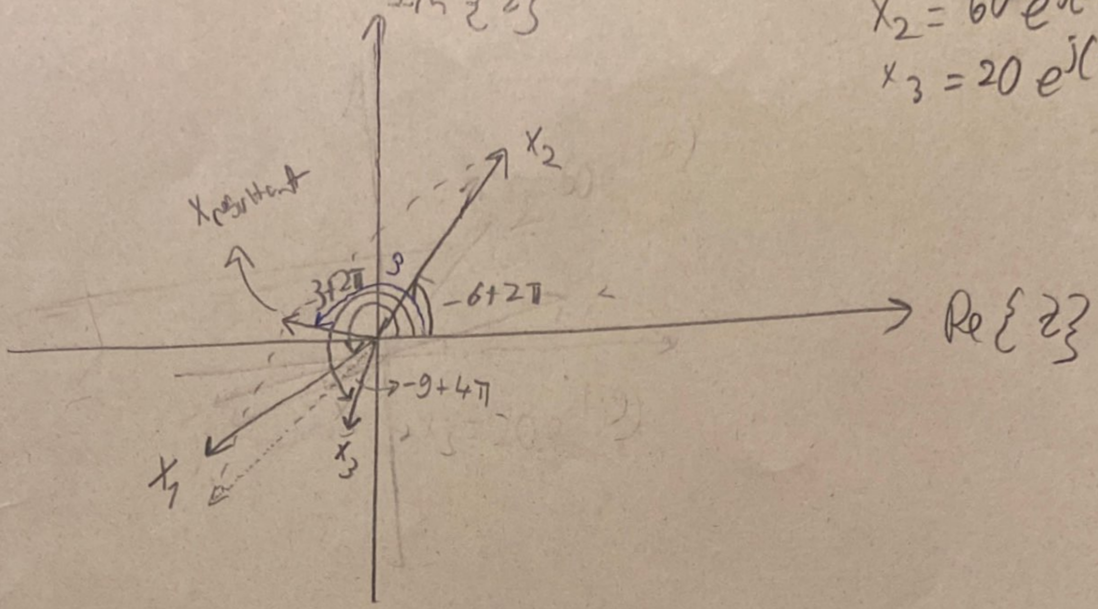
$$\phi = \operatorname{atan2}(V, U) = \operatorname{atan2}(1.47, -10.1) \cong 3.00$$

$$y(t) \cong \underbrace{10.2\cos(3t+3)}$$

$$\boxed{X_{resulted} = 10.2e^{3j}}$$

Phasor diagram

$X_1 = 50e^{j(-3)}$
$X_2 = 60e^{j(-6)}$
$X_3 = 20e^{j(-9)}$




c) $y(t) = 50\cos(3t-3) + 60\cos(3t-6) + 20\cos(3t-9)$

$$= 50\left(\frac{1}{2}\left(e^{j(3t-3)} + e^{-j(3t-3)}\right)\right) + 30\left(e^{j(3t-6)} + e^{-j(3t-6)}\right)$$

$$+ 10\left(e^{j(3t-9)} + e^{-j(3t-9)}\right)$$

$$= 25e^{-3j}e^{j3t} + 25e^{3j}e^{-j3t} + 30e^{-6j}e^{j3t} + 30e^{6j}e^{-j3t}$$

$$+ 10e^{-9j}e^{j3t} + 10e^{9j}e^{-j3t}$$

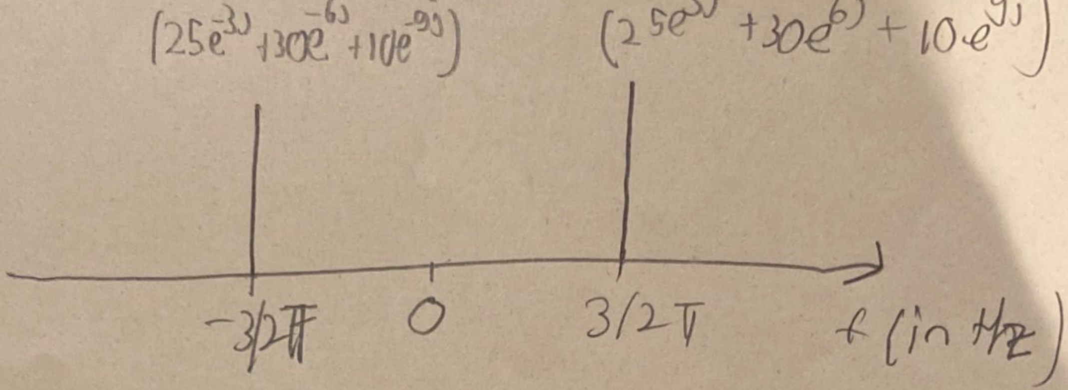
$$= \underbrace{\left(25e^{-3j} + 30e^{-6j} + 10e^{-9j}\right)}_{a_1}e^{j3t} + \underbrace{\left(25e^{3j} + 30e^{6j} + 10e^{9j}\right)}_{a_{-1}}e^{-j3t}$$

$w_0 = 2\pi f_0$

$$a_k = \begin{cases} 25e^{-3j} + 30e^{-6j} + 10e^{-9j} & \text{for } k=1 \\ 25e^{3j} + 30e^{6j} + 10e^{9j} & \text{for } k=-1 \\ 0 & \text{otherwise} \end{cases}$$

$w_0 = 3$ rad/s
$f_0 = \frac{3}{2\pi}$ Hz

Frequency Spectrum

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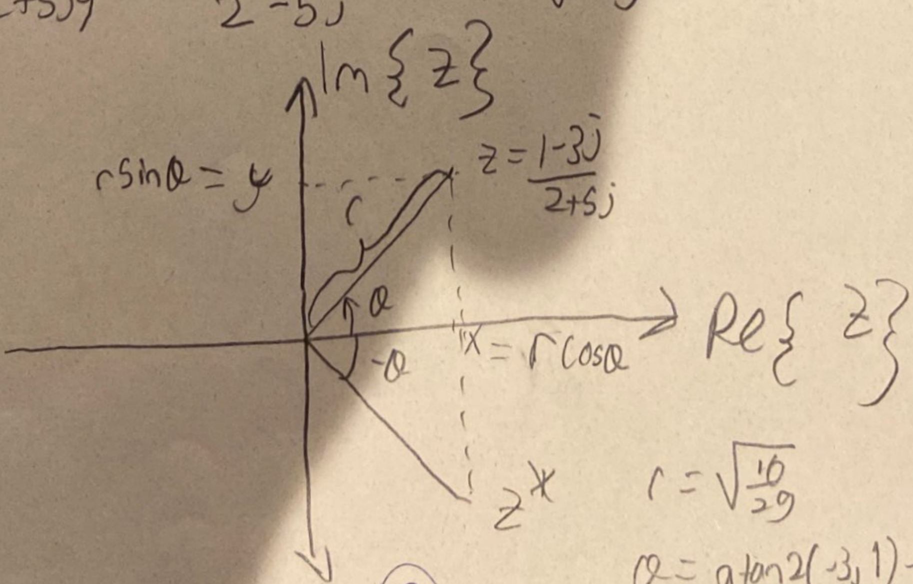
2) a) $\dfrac{1-3j}{2+5j} = \dfrac{r_1\angle\theta_1}{r_2\angle\theta_2} = \dfrac{r_1}{r_2}\angle\theta_1-\theta_2 = r\angle\theta$

i) $1-3j = \sqrt{1^2+(-3)^2}\angle \operatorname{atan2}(-3,1) = \sqrt{10}\angle \operatorname{atan2}(-3,1)$

$2+5j = \sqrt{2^2+5^2}\angle \operatorname{atan2}(5,2) = \sqrt{29}\angle \operatorname{atan2}(5,2)$

$\dfrac{1-3j}{2+5j} = \sqrt{\dfrac{10}{29}}\angle \operatorname{atan2}(-3,1) - \operatorname{atan2}(5,2)$

ii) $\left(\dfrac{1-3j}{2+5j}\right)^* = \dfrac{1+3j}{2-5j}$, also $\sqrt{\dfrac{10}{29}}\angle-(\operatorname{atan2}(-3,1)-\operatorname{atan2}(5,2))$




$r = \sqrt{\frac{10}{29}}$

$\theta = \operatorname{atan2}(-3,1) - \operatorname{atan2}(5,2)$

b) $\underbrace{(\sqrt{2}+j4)(1+j)}_{z} = z_1 \cdot z_2 = r_1\angle\theta_1 \, r_2\angle\theta_2 = r_1 r_2 \angle\theta_1+\theta_2$

i) $z_1 = \sqrt{2}+4j \Rightarrow r_1 = \sqrt{2+16} = \sqrt{18}, \; \theta_1 = \text{atan2}(4, \sqrt{2})$

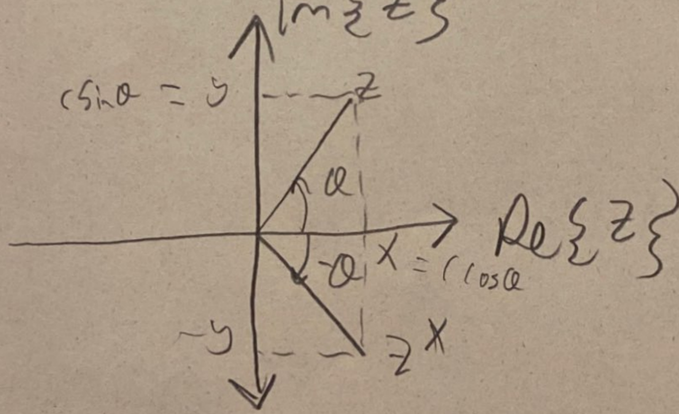
$z_2 = 1+j = r_2 = \sqrt{1^2+1^2} = \sqrt{2}, \; \theta_2 = \text{atan2}(1,1)$

$z = \sqrt{18}\cdot\sqrt{2} \angle (\text{atan2}(4,\sqrt{2}) + \text{atan2}(1,1))$

$= 6 \angle (\text{atan2}(4,\sqrt{2}) + \text{atan2}(1,1))$

ii) $z^* = (\sqrt{2}-4j)(1-j)$ or $6\angle -\text{atan2}(4,\sqrt{2}) - \text{atan2}(1,1)$

iii)




$r = 6$

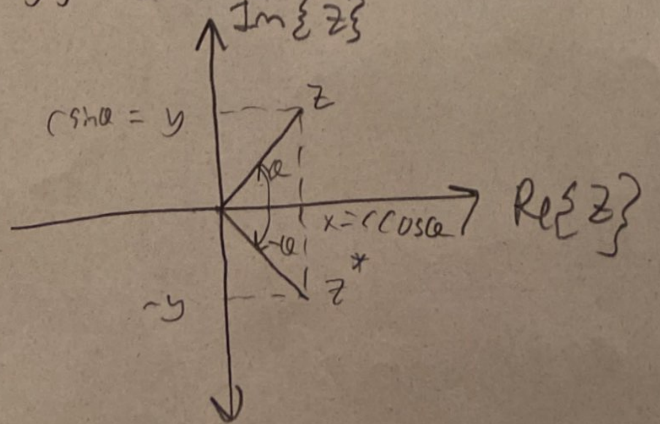
$\theta = \text{atan2}(4,\sqrt{2}) + \text{atan2}(1,1)$

c) $j(j-2)\underbrace{e^{-j\frac{\pi}{2}}}_{-j} = -j^2(j-2) = j-2$

i) $2-j = \sqrt{2^2+(-1)^2} \angle \text{atan2}(-1,2) = \sqrt{5} \angle \text{atan2}(-1,2)$

ii) $(2-j)^* = 2+j$ or $\sqrt{5}\angle -\text{atan2}(-1,2)$

iii)




$r = \sqrt{5}$

$\theta = \text{atan2}(-1,2)$

d) $$\frac{e^{j\frac{\pi}{6}}+1}{1+j\sqrt{2}} = z$$

$$z_1 = e^{j\frac{\pi}{6}}+1 = \underbrace{\cos\left(\frac{\pi}{6}\right)}_{\frac{\sqrt{3}}{2}} + j\underbrace{\sin\left(\frac{\pi}{6}\right)}_{\frac{1}{2}} + 1$$

$$= \frac{\sqrt{3}+2}{2} + \frac{1}{2}j$$

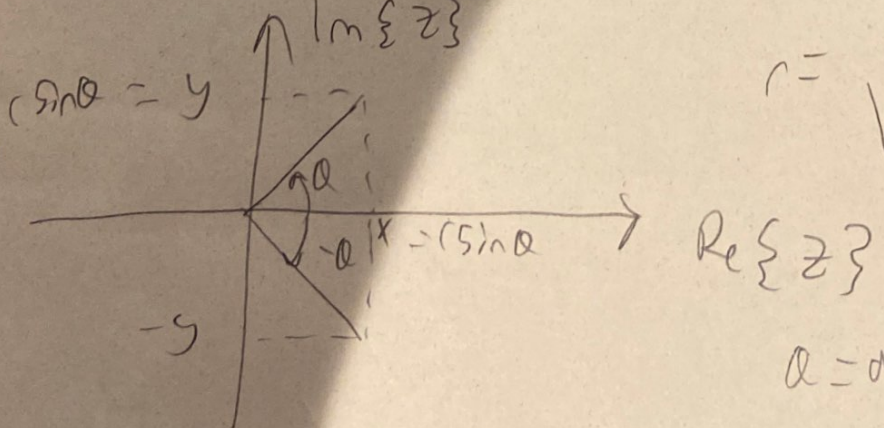
i)
$$z = \frac{e^{j\frac{\pi}{6}}+1}{1+j\sqrt{2}} = \frac{\sqrt{3}+2+j}{2+2\sqrt{2}j} = \frac{z_1}{z_2}$$

$$z_1 = \sqrt{(\sqrt{3}+2)^2+1^2}\ \angle\ \mathrm{atan2}(1, \sqrt{3}+2)$$

$$z_2 = \sqrt{2^2+(2\sqrt{2})^2}\ \angle\ \mathrm{atan2}(2\sqrt{2}, 2)$$

$$z = \sqrt{\frac{(\sqrt{3}+2)^2+1}{12}}\ \angle\left(\mathrm{atan2}(1, \sqrt{3}+2) - \mathrm{atan2}(2\sqrt{2}, 2)\right)$$

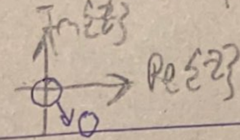
ii) $$z^* = \frac{\sqrt{3}+2-j}{2-2\sqrt{2}j}$$

iii)



$$r = \sqrt{\frac{(\sqrt{3}+2)^2+1}{12}}$$

$$\theta = \mathrm{atan2}(1, \sqrt{3}+2) - \mathrm{atan2}(2\sqrt{2}, 2)$$

e) $(\sqrt{6}+j^3)(1+e^{-j\pi}) = (\sqrt{6}-j)(1-1) = 0$

i) 0, ii) 0, iii) 


3) a) $x(t) = 3\sin(3t) + \cos\left(\frac{\pi}{5}t + \frac{\pi}{3}\right) + 2$

$$= 3\cos\left(\underbrace{3}_{w_1}t - \frac{\pi}{2}\right) + \cos\left(\underbrace{\frac{\pi}{5}}_{w_2}t + \frac{\pi}{3}\right) + 2$$

this signal is $\boxed{\text{not periodic}}$ since the frequencies $w_1$ and $w_2$ are not integer multiple of a fundamental freq, they are not harmonically related. It's easy to see, since $w_1$ is an integer and $w_2$ is an irrational number.

b) $x(t) = 6\sin(\sqrt{3}t + 5) + 2\cos(\pi t)$

$$= 6\cos\left(\underbrace{\sqrt{3}}_{w_1}t + 5 - \frac{\pi}{2}\right) + 2\cos(\underbrace{\pi}_{w_1}t)$$

$\sqrt{3}$ not harmonically related to $\pi$, $\boxed{\text{not periodic}}$

c) $x(t) = \sin(12t) + \cos(16t)$

$$= \cos\left(12t - \frac{\pi}{2}\right) + \cos(16t)$$

$\omega_0 = g.c.d(12, 16) = 4$, yes they are harmonically related, and $x(t)$ is $\boxed{\text{periodic}}$

$f_0 = \frac{4}{2\pi}$, $T_0 = \frac{1}{f_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

$= \frac{2}{\pi}$

d) $x(t) = \sin^2(3t) + \cos^2(5t)$

$$= \left(\frac{1}{2j}\left(e^{3t} - e^{-3t}\right)\right)^2 + \left(\frac{1}{2}\left(e^{5t} + e^{-5t}\right)\right)^2$$

$$= \frac{-1}{4}\left(e^{3t} - e^{-3t}\right)^2 + \frac{1}{4}\left(e^{5t} + e^{-5t}\right)^2$$

$$= \frac{-1}{4}\left(e^{6t} - 2e^{3t-3t} + e^{-6t}\right) + \frac{1}{4}\left(e^{10t} + 2e^{5t-5t} + e^{-10t}\right)$$

$$= \underbrace{-\frac{1}{4}e^{6t} - \frac{1}{4}e^{-6t}} + \frac{1}{2} + \frac{1}{4}e^{10t} + \frac{1}{4}e^{-10t} + \frac{1}{2}$$

$$= \frac{-1}{2}\left(\frac{1}{2}\left(e^{6t} + e^{-6t}\right)\right) + \frac{1}{2}\left(\frac{1}{2}\left(e^{10t} + e^{-10t}\right)\right) + 1$$

$$= \frac{-1}{2}\cos(6t) + \frac{1}{2}\cos(10t) + 1$$

$\omega_0 = g.c.d\{6, 10\} = 2$

$f_0 = \frac{2}{2\pi} = \frac{1}{\pi}\ Hz$ , yes periodic

4) $x(t)$, Periodic, continuous, real valued $\Rightarrow$ Complex conjugate Symmetry

$T_0 = 8$ sec, $a_0 = -2$, $a_1 = j$, $a_5 = 2$, $a_{-5} = 2$, $a_7 = 2e^{j\frac{\pi}{3}}$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\left(\frac{2\pi}{T_0}\right)kt} \Rightarrow A_0 + \sum_{k=1}^{\infty} A_k \cos(w_k t + \phi_k)$$

$$x(t) = a_{-1} e^{-j\left(\frac{2\pi}{8}\right)t} + a_1 e^{j\left(\frac{2\pi}{8}\right)t}$$
$$a_{-5} e^{-j\left(\frac{2\pi}{8}\right)5t} + a_5 e^{j\left(\frac{2\pi}{8}\right)5t}$$
$$a_{-7} e^{-j\left(\frac{2\pi}{8}\right)7t} + a_7 e^{j\left(\frac{2\pi}{8}\right)7t} + a_0$$

$$x(t) = -2 + \Big(\underbrace{-j}_{e^{-j\frac{\pi}{2}}} e^{-j\frac{\pi}{4}t} + \underbrace{j}_{e^{j\frac{\pi}{2}}} e^{j\frac{\pi}{4}t}\Big) + 2\left(e^{-j\frac{5\pi}{4}t} + e^{j\frac{5\pi}{4}t}\right)$$
$$+ \left(2e^{-j\frac{\pi}{3}} e^{-j\frac{7\pi}{4}t} + 2e^{j\frac{\pi}{3}} e^{j\frac{7\pi}{4}t}\right)$$

$$= -2 + \left(e^{-j\left(\frac{\pi}{4}t + \frac{\pi}{2}\right)} + e^{j\left(\frac{\pi}{4}t + \frac{\pi}{2}\right)}\right) + 4\left[\cos\left(\frac{5\pi}{4}t\right)\right]$$
$$+ 2\left(e^{-j\left(\frac{7\pi}{4}t + \frac{\pi}{3}\right)} + e^{j\left(\frac{7\pi}{4}t + \frac{\pi}{3}\right)}\right)$$

$$= \boxed{-2 + 2\cos\left(\frac{\pi}{4}t + \frac{\pi}{2}\right) + 4\cos\left(\frac{5\pi}{4}t\right) + 4\cos\left(\frac{7\pi}{4}t + \frac{\pi}{3}\right)}$$

5) $x(t) = 2 + 5\cos\left(-t + \frac{\pi}{4}\right) - 2\sin(3t+5) + 3\cos\left(5t + \frac{\pi}{2}\right)\cos(4t) \boxed{-e^{-jt}}$

$$5\cos\left(-t + \frac{\pi}{4}\right) = \frac{5}{2}\left(e^{-j\left(-t+\frac{\pi}{4}\right)} + e^{j\left(-t+\frac{\pi}{4}\right)}\right)$$

$$= \boxed{\frac{5}{2} e^{-j\frac{\pi}{4}} e^{jt} + \frac{5}{2} e^{j\frac{\pi}{4}} e^{-jt}}$$

$$-2\sin(3t+5) = \frac{\overbrace{-\cancel{2}}^{j = e^{j\frac{\pi}{2}}}}{\cancel{2}j}\left(e^{j(3t+5)} - e^{-j(3t+5)}\right)$$

$$= \boxed{e^{j\left(\frac{\pi}{2}+5\right)} e^{j3t} - e^{j\left(\frac{\pi}{2}-5\right)} e^{-j(3t)}}$$

$$3\cos\left(5t+\frac{\pi}{2}\right)\cos(4t) = \frac{3}{4}\left(e^{j\left(5t+\frac{\pi}{2}\right)} + e^{-j\left(5t+\frac{\pi}{2}\right)}\right) \cdot \left(e^{j4t} + e^{-j4t}\right)$$

$$= \frac{3}{4} j \left(e^{j5t} - e^{-j5t}\right)\left(e^{j4t} + e^{-j4t}\right)$$

$$= \frac{3}{4} j \left[e^{j9t} + e^{jt} - e^{-jt} - e^{-j9t}\right]$$

$$= \boxed{\frac{3}{4} e^{j\frac{\pi}{2}} e^{jt} - \frac{3}{4} e^{j\frac{\pi}{2}} e^{-jt} + \frac{3}{4} e^{j\frac{\pi}{2}} e^{j9t} - \frac{3}{4} e^{j\frac{\pi}{2}} e^{-j9t}}$$

$$x(t) = \left(\frac{5}{2} e^{j\frac{\pi}{4}} - \frac{3}{4} e^{j\frac{\pi}{2}} - 1\right) e^{-jt} + \left(\frac{5}{2} e^{-j\frac{\pi}{4}} + \frac{3}{4} e^{j\frac{\pi}{2}}\right) e^{jt}$$
$$+ \left(-e^{j\left(\frac{\pi}{2}-5\right)}\right) e^{-j3t} + \left(e^{j\left(\frac{\pi}{2}+5\right)}\right) e^{j3t} + \frac{3}{4} e^{j\frac{\pi}{2}} e^{j9t} - \frac{3}{4} e^{j\frac{\pi}{2}} e^{-j9t}$$

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$$x(t) = \left(\frac{5}{2} e^{j\frac{\pi}{4}} - \frac{3}{4} e^{j\frac{\pi}{2}} - 1\right) e^{-jt} + \left(\frac{5}{2} e^{-j\frac{\pi}{4}} + \frac{3}{4} e^{j\frac{\pi}{2}}\right) e^{jt}$$

$$- e^{j\left(\frac{\pi}{2} - 5\right)} e^{-j3t} + e^{j\left(\frac{\pi}{2}+5\right)} e^{j3t}$$

$$- \frac{3}{4} e^{j\frac{\pi}{2}} e^{-j9t} + \frac{3}{4} e^{j\frac{\pi}{2}} e^{j9t}$$

a) fundamental $w_0 = g.c.d \{1, 3, 9\} = 1$ $\quad w_0 = 2\pi f_0$

$$T_0 = \frac{1}{f_0} = \frac{1}{\frac{w_0}{2\pi}} = \frac{2\pi}{w_0} = 2\pi$$

$$a_k = \begin{cases} \frac{5}{2} e^{j\frac{\pi}{4}} - \frac{3}{4} e^{j\frac{\pi}{2}} - 1 & \text{for } k = -1 \\ \frac{5}{2} e^{-j\frac{\pi}{4}} + \frac{3}{4} e^{j\frac{\pi}{2}} & \text{''} \ k = 1 \\ -e^{j\left(\frac{\pi}{2}-5\right)} & \text{''} \ k = -3 \\ e^{j\left(\frac{\pi}{2}+5\right)} & \text{''} \ k = 3 \\ -\frac{3}{4} e^{j\frac{\pi}{2}} & \text{''} \ k = -9 \\ \frac{3}{4} e^{j\frac{\pi}{2}} & \text{''} \ k = 9 \\ 0 & \text{otherwise} \end{cases}$$

b) No, $x(t)$ is not a real signal, for the $k=1$ the signal does not represent complex conjugate symmetry

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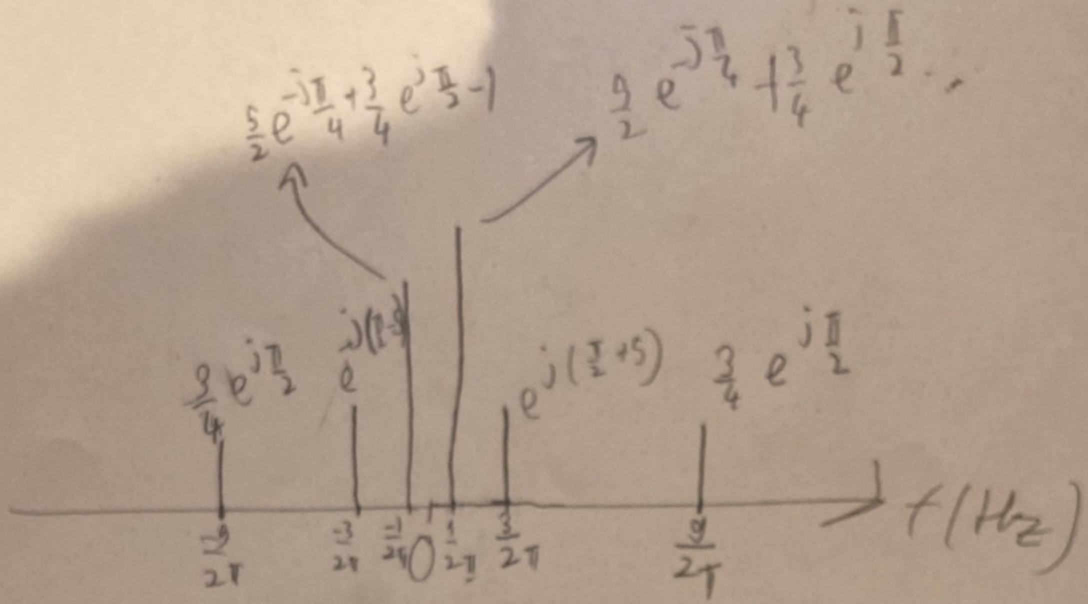
frequency spectrum




6) $a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jw_0kt}\, dt$ , $x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jw_0kt}$

a) $b_k = \frac{1}{T_0}\int_0^{T_0} x(-t)\, e^{-jw_0kt}\, dt$ , $-t = u$, $-dt = du$

$= \frac{-1}{T_0}\int_0^{-T_0} x(u)\, e^{jw_0ku}\, du = \frac{1}{T_0}\int_{-T_0}^{0} x(t)\, e^{jw_0kt}\, dt$ → periodic → $-T_0, 0$ / $0, T_0$ same

⇒ $= \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{jw_0kt}\, dt$ $a_{-k}$

⇒ $\boxed{b_k = a_{-k}}$

b) $x^*(t) \Rightarrow b_k = \left(\left(\frac{1}{T_0}\int_0^{T_0} x^*(t)\, e^{-jw_0kt}\, dt\right)^*\right)^*$

$$b_k = \underbrace{\left(\frac{1}{T_0}\int_0^{T_0} x(t)\, e^{jw_0kt}\, dt\right)}_{a_{-k}}^* = (a_{-k})^*$$

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c) $x(t-t_0) \to b_k = \frac{1}{T_0}\int_0^{T_0} x(t-t_0)\, e^{-jw_0kt}\, dt$ | $t-t_0=u$, $dt=du$

$$= \frac{1}{T_0}\int_{-t_0}^{T_0-t_0} x(u)\, e^{-jw_0k(u+t_0)}\, du$$

$$= \frac{e^{-jw_0kt_0}}{T_0}\int_{-t_0}^{T_0-t_0} x(u)\, e^{-jw_0ku}\, du$$

Since $x(u)$ periodic, $0 \to t_0 \Leftrightarrow -t_0 \to T_0 - t_0$

$$= e^{-jw_0kt_0}\underbrace{\frac{1}{T_0}\int_0^{T_0} x(u)\, e^{-jw_0ku}\, dv}_{a_k}$$

$$\Rightarrow b_k = a_k\, e^{-jw_0kt_0}$$

d) $\frac{dx(t)}{dt} \Rightarrow b_k = \frac{1}{T_0}\int_0^{T_0}\left(\frac{dx(t)}{dt}\right) e^{-jw_0kt}\,dt$

use integration by parts $\Rightarrow$ $u = e^{-jw_0kt}$ $\quad dv = dx(t)$

$du = -jw_0k\,e^{-jw_0kt}\,dt \quad v = x(t)$

$$b_k = \frac{1}{T_0}\left(\left[e^{-jw_0kt}\cdot x(t)\right]_0^{T_0} + jw_0k\underbrace{\int_0^{T_0} x(t)\,e^{-jw_0kt}\,dt}_{a_k \cdot T_0}\right)$$

$$= \frac{1}{\cancel{T_0}}\left(\overset{1}{e^{-j\frac{2\pi}{T_0}T_0\cdot t}}\cdot x(T_0) - \overset{1}{e^{0}}\cdot x(0) + jw_0k\,a_k\cancel{T_0}\right)$$

$x(T_0) - x(0) = 0$
since $x(t)$ periodic

$$= \boxed{a_k\, jw_0 k} \text{ or } \boxed{a_k\, j\left(\frac{2\pi}{T_0}\right)k}$$

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e) $\int_{-\infty}^{t} x(t)\,dt \Rightarrow b_k = \frac{1}{T_0}\int_0^{T_0}\left(\int_{-\infty}^{t} x(t)\,dt\right) e^{-jw_0kt}\,dt$

$$\Rightarrow \int_{-\infty}^{t} x(t)\,dt = \int_{-\infty}^{t}\left[\sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}\right]dt$$

$\Rightarrow$ rest at the next page

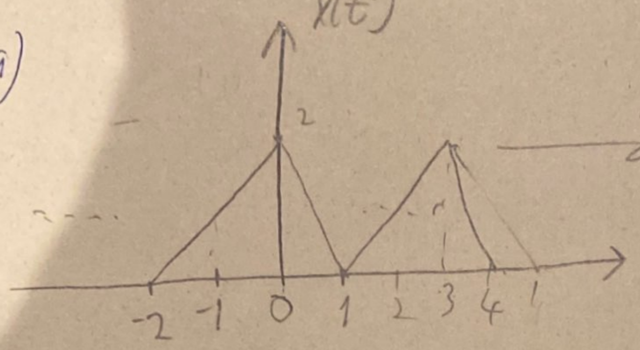
$$= \sum_{k=-\infty}^{\infty} a_k \int_{-\infty}^{t} e^{jk\omega_0 t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \left[ \frac{e^{jk\omega_0 t}}{jk\omega_0} \right]_{-\infty}^{t}$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{\left[ \frac{a_k}{jk\omega_0} \right]}_{b_k} e^{jk\omega_0 t}$$

$$\Rightarrow \boxed{b_k = \frac{a_k}{jk\omega_0}}$$

7) a)




$\rightarrow T_0 = 3\,sec$

$a_0 = 3$

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j\left(\frac{2\pi}{T_0}\right)kt} = \frac{1}{3}\int_0^{3} x(t)\, e^{-j\left(\frac{2\pi}{3}\right)kt}$$

$$x(t) = \begin{cases} -2t+2 & \text{for } 0 < t \le \frac{T_0}{3} \\ t+2 & \text{for } \frac{T_0}{3} < t \le T_0 \end{cases}$$

$$a_k = \frac{1}{3}\left( \int_0^1 \underbrace{(-2t+2)}_{U} \underbrace{e^{-j\left(\frac{2\pi}{3}\right)kt} dt}_{dV} + \int_1^3 \underbrace{(t+2)}_{V} \underbrace{e^{-j\frac{2\pi}{3}kt} dt}_{dV} \right)$$

$V_1 = -j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)kt}$

$$a_k = \frac{1}{3}\left( \left[(-2t+2)\left(-j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)kt}\right)\right]_0^1 + 2\int_0^1 -j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)kt} dt \right.$$
$$\left. + \left[(t+2)\left(-j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)kt}\right)\right]_1^3 - \int_1^3 -j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)kt} dt \right.$$

$$= \frac{1}{3}\left( j\frac{4\pi}{3}k - j\frac{4\pi}{3}k \int_0^1 e^{-j\left(\frac{2\pi}{3}\right)kt} dt + \right.$$
$$\left[ 5\left(-j\frac{2\pi}{3}k e^{-j2\pi k}\right) - 3\left(-j\frac{2\pi}{3}k e^{-j\left(\frac{2\pi}{3}\right)k}\right)\right]$$
$$\left. + j\frac{2\pi}{3}k \int_1^3 e^{-j\left(\frac{2\pi}{3}\right)kt} dt \right)$$

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$$= \frac{1}{3}\left(\frac{-6j\pi k - 9e^{-j\left(\frac{2\pi}{3}\right)k} + 9}{2\pi^2 k^2}\right.$$

$$+ \frac{e^{-2j\pi k}\left(30j\pi k + e^{\left(j\frac{4\pi}{3}k\right)}(-9-18j\pi k)+9\right)}{4\pi^2 k^2}$$

then

$$a_k = \frac{-2j\pi k - 3e^{-j\left(\frac{2\pi}{3}\right)k} + 3}{2\pi^2 k^2} + \frac{e^{-2j\pi k}\left(30j\pi k + e^{j\left(\frac{4\pi}{3}\right)k}(-9-18j\pi k)+9\right)}{12\pi^2 k^2}$$

b) $x(t) = -3|\sin(w_0 t)|$, $T_0 = 2\pi$

$$x(t) = \begin{cases} -3\sin(w_0 t) & 0<t<\pi \\ 3\sin(w_0 t) & \pi<t<2\pi \end{cases}$$

$$a_k = \frac{1}{2\pi}\left(\int_0^{\pi} -3\sin\left(\left(\frac{2\pi}{2\pi}\right)t\right)e^{-j\left(\frac{2\pi}{2\pi}\right)kt}\,dt\right.$$

$$+ \left.\int_{\pi}^{2\pi} 3\sin\left(\left(\frac{2\pi}{2\pi}\right)t\right)e^{-j\left(\frac{2\pi}{2\pi}\right)kt}\,dt\right)$$

by using wolfram alpha

$$\boxed{a_k = \frac{3e^{-2j\pi k}\left(1+e^{j\pi k}\right)}{2\pi(k^2-1)} - \frac{3\left(1+e^{-j\pi k}\right)}{2\pi(1-k^2)}}$$

8) $x(t) = 3\sin(150\pi t - \frac{\pi}{6})$

$w_0 = 150\pi$

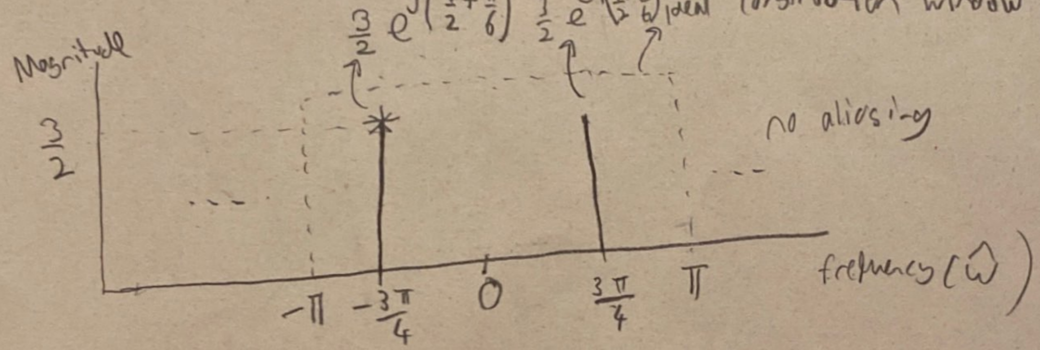
$f_0 = 75\,Hz$, $f_{Nyquist} = 150\,Hz$, $\frac{1}{150}$ sec

9) $0.005$ sec $\Rightarrow \frac{5}{1000} = \frac{1}{200}$ sec $\Rightarrow f_a = 200\,Hz$, ($f_a = f_s$ for part a)

Since $f_a > f_{Nyquist}$ this is a $\boxed{\text{oversampling}}$ case, $\boxed{\text{no aliasing}}$

$\hat{w} = w T_s = 150\pi \cdot \frac{1}{200} = \frac{3}{4}\pi \Rightarrow \hat{w} = \mp \frac{3\pi}{4}$ rad

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$



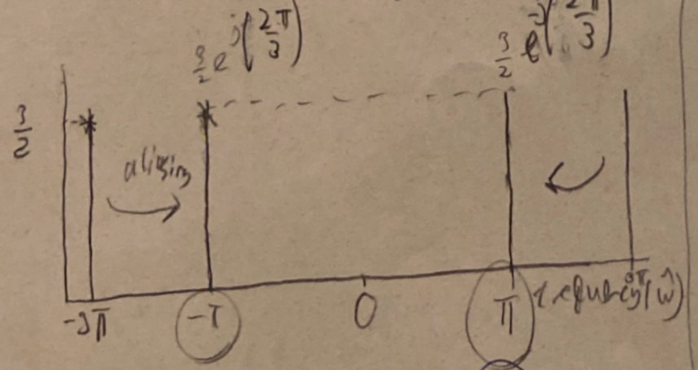

reconstruction:

$\hat{w} = \frac{w_0}{f_s} \Rightarrow w_0 = \mp \hat{w}_0 f_s = w_0 = \frac{3\pi}{4} \cdot 200 = 150\pi$ correct!

$y(t) = 3\sin(150\pi t - \frac{\pi}{6})$

b) $0.02$ sec $\Rightarrow \frac{2}{100} = \frac{1}{50}$ sec $\Rightarrow f_b = f_s = 50\,Hz$, $\boxed{\text{undersampling!}}$

$\hat{w}_0 = w_0 T_s = 150\pi \cdot \frac{1}{50} = 3\pi$




$\hat{w}_0 = \frac{w_0}{f_s} \Rightarrow w_0 = \mp \hat{w}_0 f_s = (3\pi - 2\pi)50$
$= 50\pi$

$y(t) = 3\sin(50\pi t - \frac{\pi}{6})$

$\boxed{\text{no folding}}$ since

$0 < f_s < f_0 \Rightarrow 0 < 50 < 75$

c) $0.01\ sec = \frac{1}{100} \Rightarrow f_c = f_s = 100\ Hz$

$\hat{\omega}_0 = \omega_0 \cdot T_s = 150\pi \cdot \frac{1}{100} = \frac{3}{2}\pi$

$\boxed{\text{Folding occurs}}$ since

$f_0 < f_s < 2f_0 \Rightarrow 75 < 100 < 150$

$\boxed{\text{undersampling}}$

reconstruction:

$w_0 = \mp \hat{w}_0 \cdot f_s$

$= \frac{\pi}{2} \cdot 100 = 50\pi$

$y(t) = 3\sin\left(50\pi t + \frac{\pi}{6}\right)$