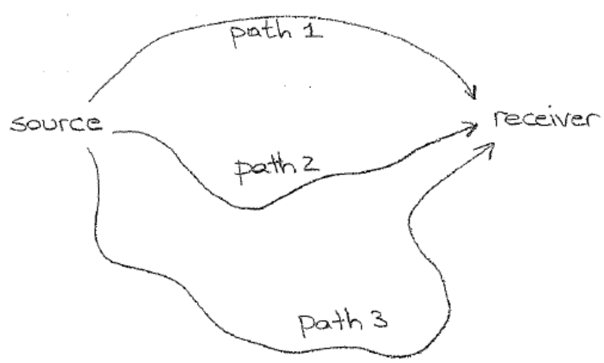
# EEE 391
## Basics of Signals and Systems
## Spring 2022–2023
## Homework 1
## due: 5 April 2023, Wednesday by 23:00 on Moodle

1) A signal source generates the signal $x(t) = 100\cos(3t)$. The signal is transmitted to a receiver over three different paths. Each path has a different length and delays the signal in time proportionately with its length: The first path delays by 1 sec, the second path delays by 2 sec, and the third path by 3 sec. The paths also attenuate the signal amplitude by 50%, 40%, and 80%, respectively.




The receiver combines the detected signals additively.

a) Find the signal $y(t)$ detected at the receiver.

b) Express $y(t)$ as a <u>single</u> sinusoid using the phasor addition technique to determine all three parameters of the sinusoid. Make a phasor diagram on the complex $z$ plane (approximately to scale) to verify your result graphically.

c) Find the Fourier series coefficients of the signal (without evaluating any integral to find the $a_k$'s) and make a plot of the frequency spectrum. Label all frequencies and complex amplitudes and the axes clearly.

2) For the following complex numbers:

a) $\frac{1-3j}{2+5j}$ b) $(\sqrt{2}+j4)(1+j)$ c) $j(j-2)e^{-j\frac{\pi}{2}}$ d) $\frac{e^{j\frac{\pi}{6}}+1}{1+j\sqrt{2}}$ e) $(\sqrt{6}+j^3)(1+e^{-j\pi})$

i) Express each of the complex numbers in polar form indicating its magnitude and angle (argument).

ii) Find its complex conjugate.

iii) Plot the number and its complex conjugate on the complex $z$ plane.

3) Determine whether the following signals are periodic or not. If periodic, find the fundamental frequency and the period.

a) $x(t) = 3\sin(3t)\cos(\frac{\pi}{5}t + \frac{\pi}{3}) + 2$

b) $x(t) = 6\sin(\sqrt{3}t + 5) + 2\cos(\pi t)$

c) $x(t) = \sin(12t) + \cos(16t)$

d) $x(t) = \sin^2(3t) + \cos^2(5t)$

4) A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period of $T = 8$ sec. The non-zero Fourier series coefficients of the signal are specified as follows:

$$a_o = -2 \qquad a_1 = a_{-1}^* = j \qquad a_5 = a_{-5} = 2 \qquad a_7 = a_{-7}^* = 2\, e^{j\frac{\pi}{3}}$$

Express $x(t)$ in the form:

$$x(t) = A_o + \sum_{k=1}^{\infty} A_k \cos(\omega_k t + \phi_k)$$

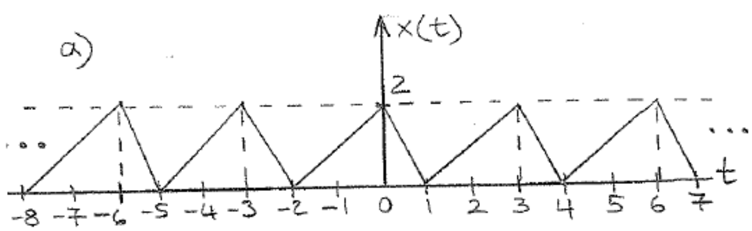
5) Find <u>all</u> the Fourier series coefficients of the following signal without evaluating any integrals:

$$x(t) = 2 + 5\cos(-t + \frac{\pi}{4}) - 2\sin(3t + 5) + 3\cos(5t + \frac{\pi}{2})\cos(4t) - e^{-jt}$$

a) What is the fundamental period of $x(t)$?
b) Is $x(t)$ a real signal?
c) Plot its frequency spectrum. Label all frequencies and complex amplitudes and the axes clearly.

6) If a continuous-time periodic signal has the Fourier series coefficients $a_k$, where $k = 0, \pm1, \pm2, \pm3, \ldots$, <u>derive</u> the Fourier series coefficients $b_k$ of the following signals in terms of $a_k$:
a) $x(-t)$ b) $x^*(t)$ c) $x(t - t_o)$ where $t_o$ is a constant d) $\frac{dx(t)}{dt}$ e) $\int_{-\infty}^{t} x(t)\, dt$
In part e), assume that the average value of $x(t)$ is zero.

7) Find all of the Fourier series coefficients of the following periodic signals:




b) $x(t) = -3|\sin(\omega_o t)| \ \ \forall t$

8) The sinusoid $x(t) = 3\sin(150\pi t - \frac{\pi}{6})$ is sampled with three different sampling periods:
a) 0.005 sec, b) 0.02 sec, c) 0.01 sec.
Analyze each case in detail by making a digital spectrum diagram with respect to $\hat{\omega}$. In each case, indicate whether the signal is undersampled, oversampled, or sampled at the Nyquist rate and whether folding occurs or not. For each part, find a closed-form expression for the reconstructed signal $y(t)$ and plot it on the same diagram as $x(t)$ using a different color or linestyle.