EEE 448/548: Reinforcement Learning & Dynamic Programming Fall 2023

Assignment 1

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Posted on Sep. 22 **Due Date: Oct. 6**

Disclaimer: *These assignments shall not be distributed outside this class.*

Content: Markov Chains

Problem 1. (40pt) I have four umbrellas, some at home, some in the office. It rains with probability p. I walk between home and office. I take an umbrella with me only if it rains. If it does not rain, I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in the other place and it rains, then I walk under the rain.

$a)$ **(20pt)** Model this problem as a Markov chain by identifying its state space and transition matrix. Draw the transition probability graph of the chain.

$b)$ **(20pt)** Identify a stationary distribution of the Markov chain. Is it unique? At steady state, what is the probability that I walk under the rain?

Problem 2. (40pt) A fair coin tossed repeatedly and independently until the pattern HTH appears.

$a)$ **(20pt)** Model this problem as a Markov chain by identifying its state space and transition matrix. Draw the transition probability graph of the chain.

$b)$ **(20pt)** Compute the expected number of tosses until the pattern HTH appears.

Problem 3. (20pt) Suppose that $w_0, w_1, \dots$ is a sequence of i.i.d. random variables taking integer values. Let x_0 be another random variable, independent of the sequence $\{w_k\}_{k=0}^{\infty}$, taking values in $X = \{1, \dots, N\}$. Let $f : X \times \mathbb{Z} \to X$ be a certain function. Define new random variables $x_1, x_2, \dots$ by

$$x_{k+1} = f(x_k, w_k) \quad \forall k = 0, 1, \dots.$$

Does the sequence $\{x_k\}_{k=0}^{\infty}$ form a Markov chain? Justify your answer.