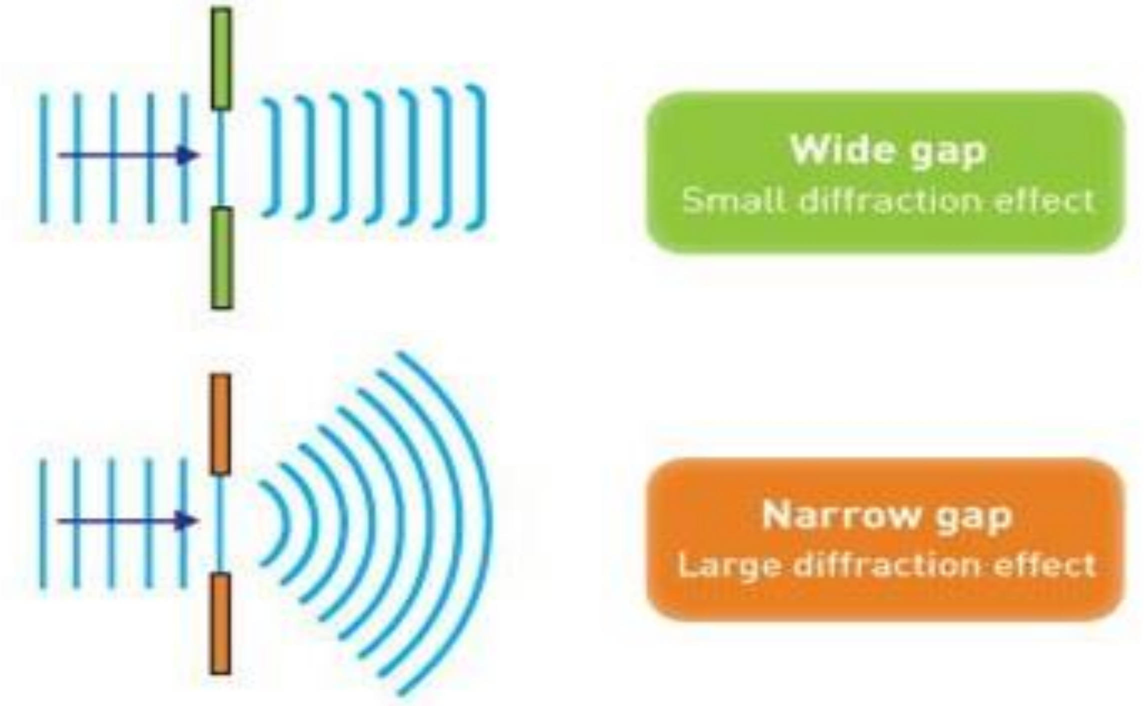
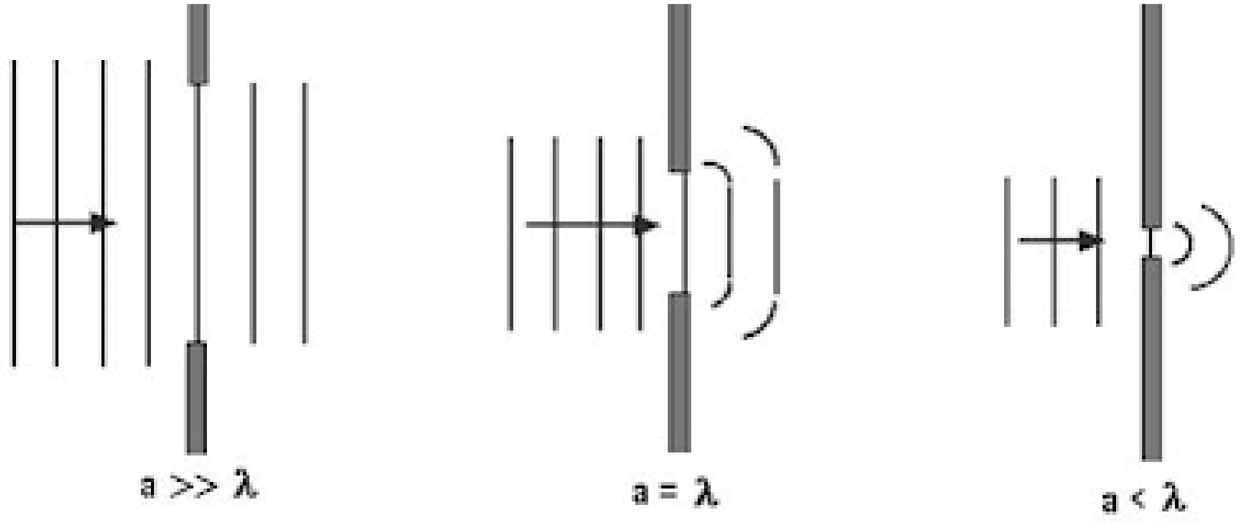
Module-1
Optics II (Diffraction)



What is Diffraction?

Diffraction of light is defined as the bending of light near the edge of the obstacle is called diffraction.



The size of obstacles should be equal to or smaller than the wavelength of light for diffraction.

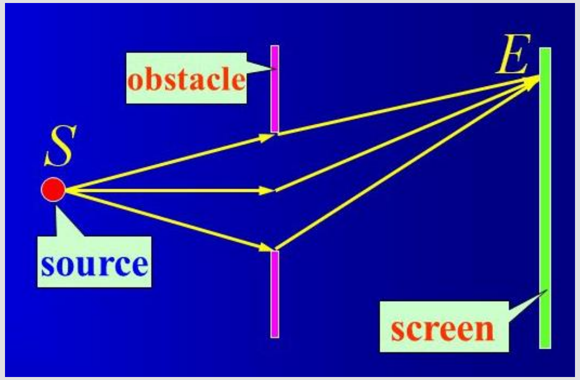
Types of Diffraction

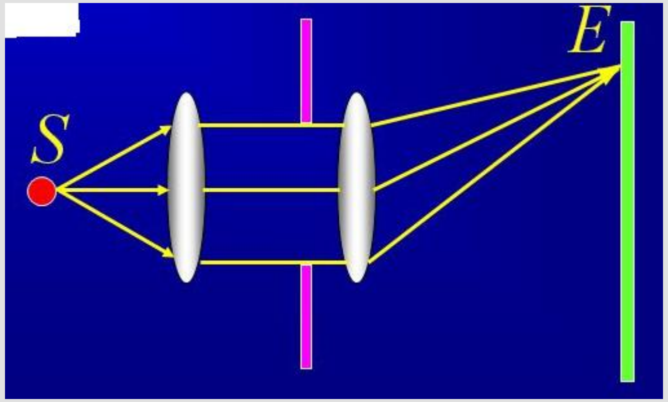
There are two main classes of diffraction, which are known as

Fraunhofer diffraction and

Fresnel diffraction.

Distinguish between Fraunhofer and Fresnel diffraction



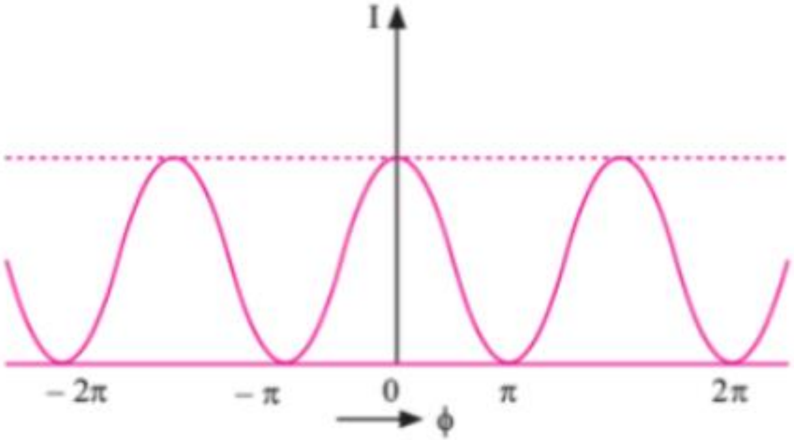


Fresnel diffraction	Fraunhofer diffraction
Distance of slit from source and screen is Finite.	Distance of slit from source and screen is Infinite.
Wavefront incident on the slit is spherical or cylindrical.	Wavefront incident on the slit is plane.
Wavefront incident on the screen is spherical or cylindrical.	Wavefront incident on the screen is plane.
Lenses are not required to observe Fresnel diffraction	Lenses are required to observe Fraunhofer diffraction

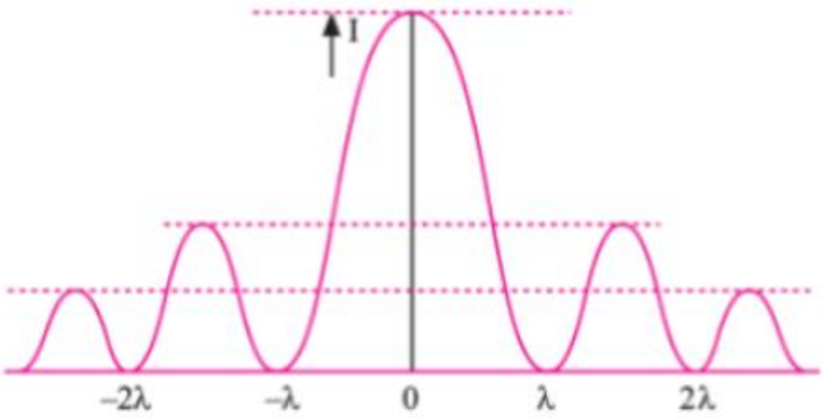
Interference pattern and diffraction pattern

Interference from two narrow slits

Diffraction from one (wider) slit

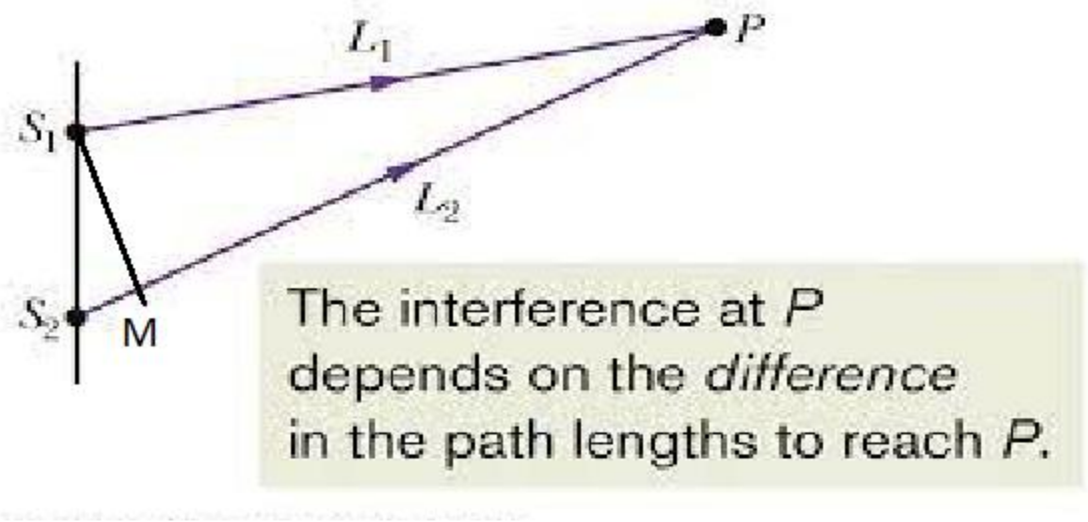


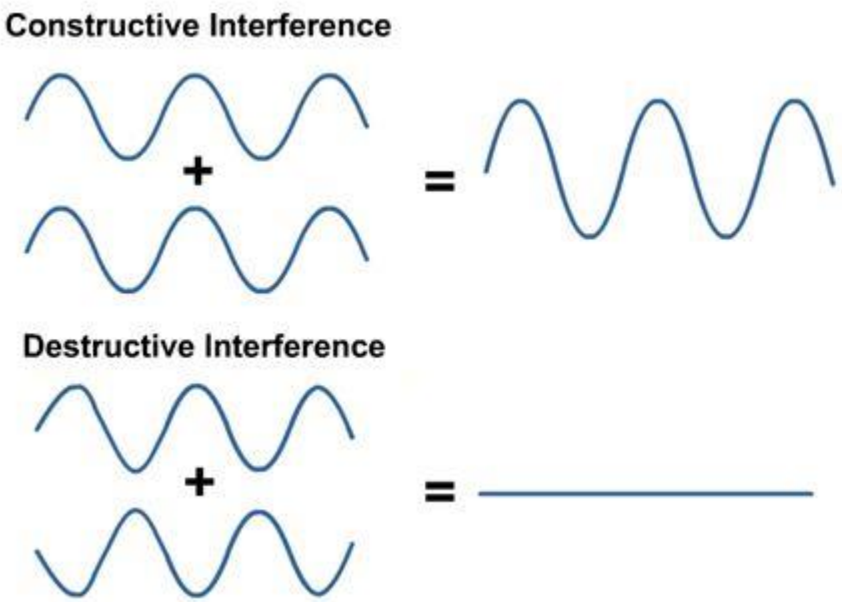
(a) Interference



(b) Diffraction

Path difference and Phase difference





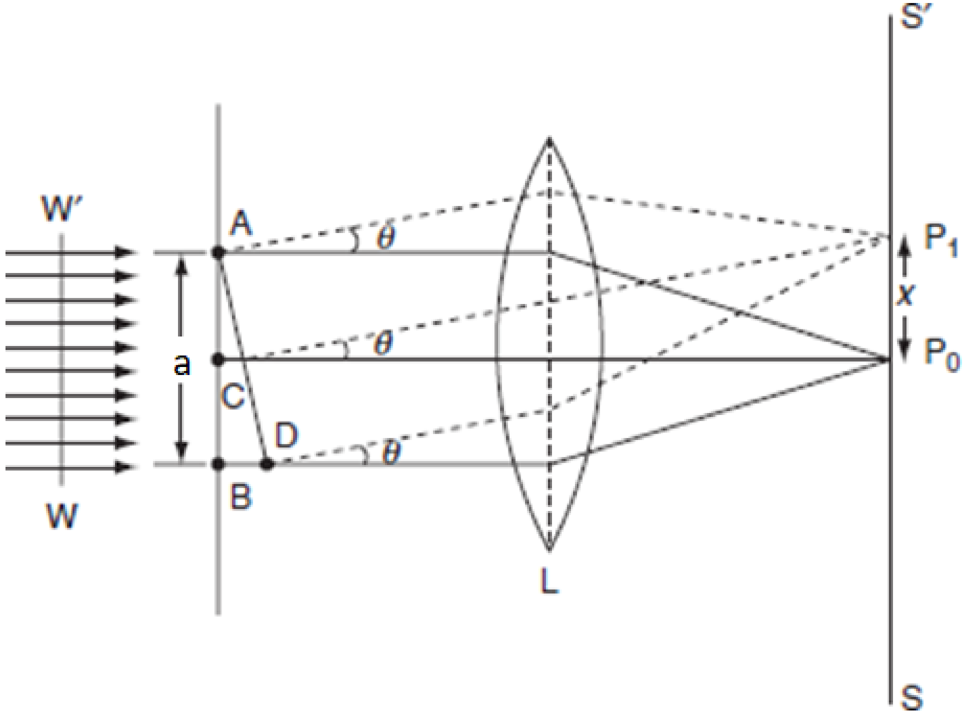
The path difference between two nearby waves is Δx

The phase difference between two nearby waves is

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

Intensity of light due to Fraunhofer Diffraction by single slit



But at point P1 all diffracted rays travels at angle θ. Point P1 may be **dark of bright** depends upon **path difference**.

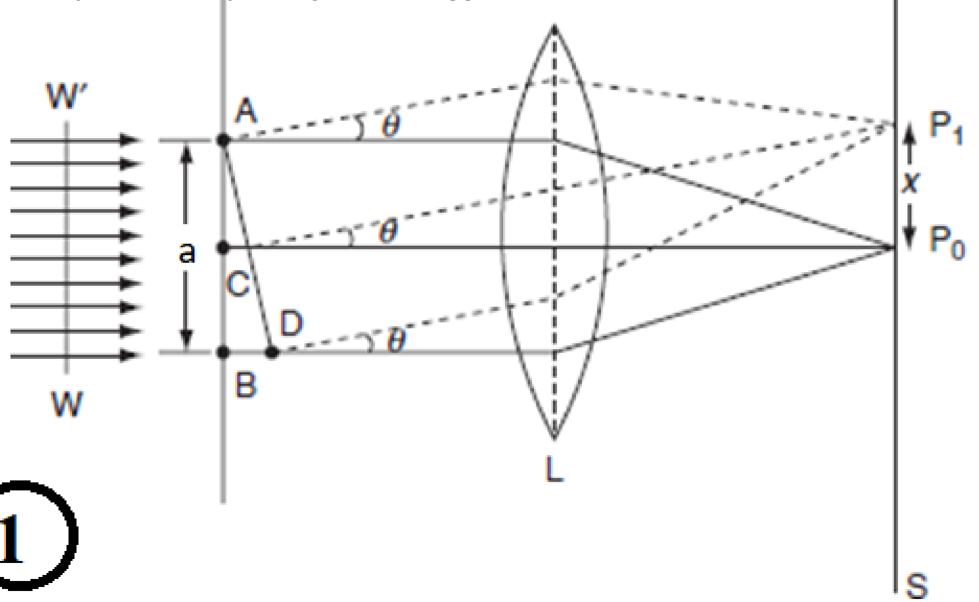
At centre, all rays travel in same path at point P_0 and produce central maxima.

The intensity of the wave is directly proportional to the square of its amplitude.

$$I \propto A^2$$

So let's first determine the amplitude at point P1 in order to calculate the intensity at point P1.

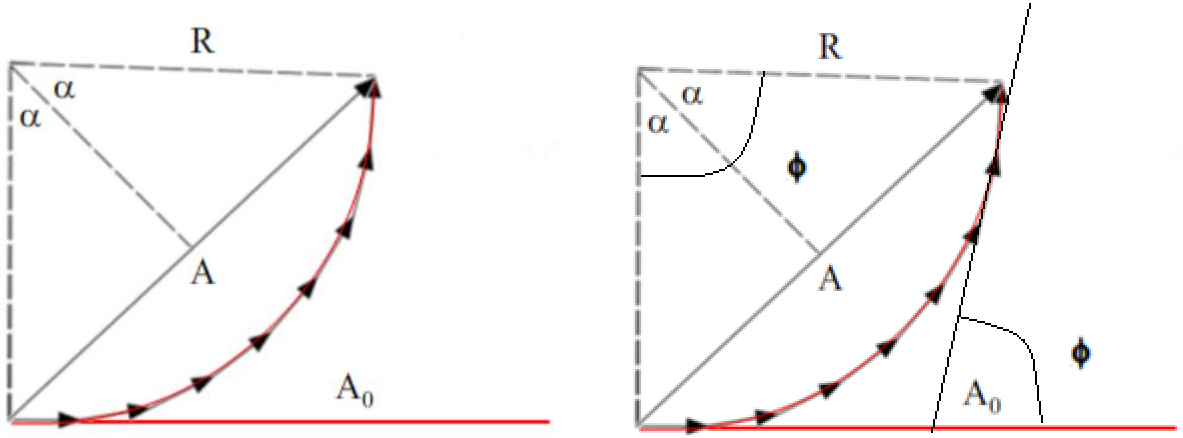
*at point P1 all diffracted rays travels at angle θ. Point P1 may be **dark of bright** depends upon **path difference**.*



1

2

The phasor diagram will be used to determine the amplitude



But the total path difference is $\Delta = \Delta x + \Delta x + \ldots = BD$

The path difference between two nearby waves is Δx

The phase difference between two nearby waves is

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\triangle\phi = \frac{2\pi}{\lambda} \times \triangle x$$

From ΔABD,

$$\sin\theta = \frac{BD}{AB}$$

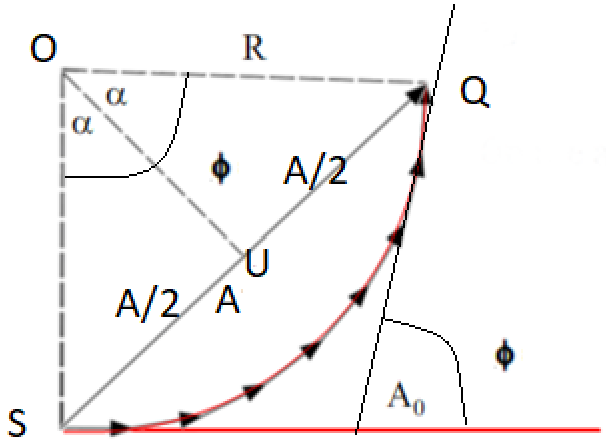
$$BD = AB \sin\theta$$

The path difference $BD = AB \sin\theta = a \sin\theta$

$$\text{Total phase difference} = \frac{2\pi}{\lambda} \times \text{Total path difference}$$

$$\phi = \frac{2\pi}{\lambda} \times BD$$

$$\phi = \frac{2\pi}{\lambda} \times a \sin\theta$$



1

Let us assume that

Let $\propto = \frac{\emptyset}{2}$ $\phi = \frac{2\pi}{\lambda} \times a \sin\theta$

$\alpha = \frac{\pi}{\lambda} a \sin\theta$

2

From ΔSUO

$\sin\propto = \frac{\frac{A}{2}}{R}$

$\frac{A}{2} = R\sin\propto$

$A = 2R\sin\propto$

3

From sector OSQ,

$\emptyset = \frac{length\ of\ arc}{radius}$

$\emptyset = \frac{A_0}{R}$

$R = \frac{A_0}{\emptyset}$

$R = \frac{A_0}{2\propto}$ (Because $\propto = \frac{\emptyset}{2}$)

$A = 2R\sin\propto$

$A = 2\frac{A_0}{2\propto}\sin\propto$

$A = \frac{A_0}{\propto}\sin\propto$

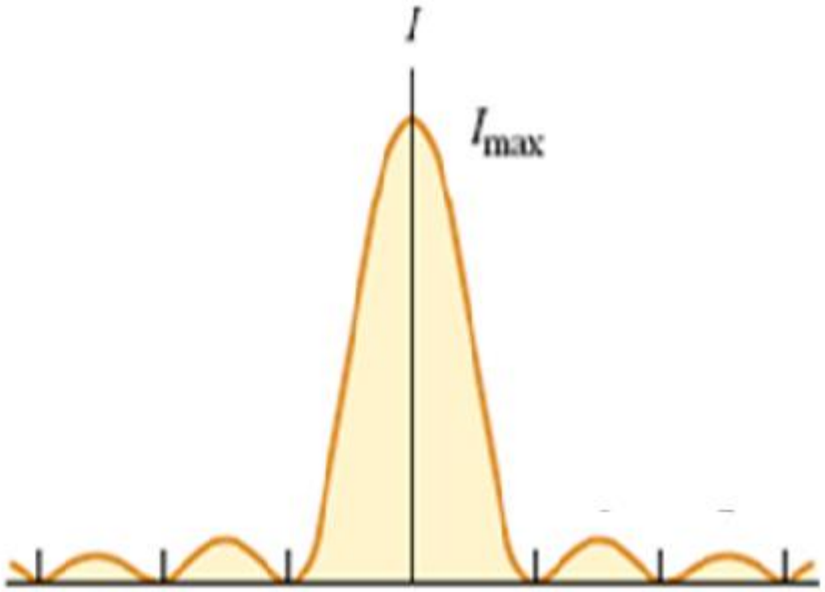
This is the resultant amplitude at an angle θ $A = A_0\frac{\sin\propto}{\propto}$ (where $\alpha = \frac{\pi}{\lambda} a\sin\theta$)

The intensity of light is square of amplitude $(A)^2 = (A_0\frac{\sin\propto}{\propto})^2$

$I_\theta = I_0\left(\frac{\sin\propto^2}{\propto^2}\right)$

This is the resultant intensity at an angle θ

$$I_\theta = I_0 \left(\frac{\sin\propto}{\propto}\right)^2$$



Condition for central maxima

$$I_\theta = I_0 \left(\frac{\sin\propto}{\propto}\right)^2$$

For central maxima $\left(\frac{\boldsymbol{sin} \propto}{\propto}\right)^2$ **=1**

The above condition true when

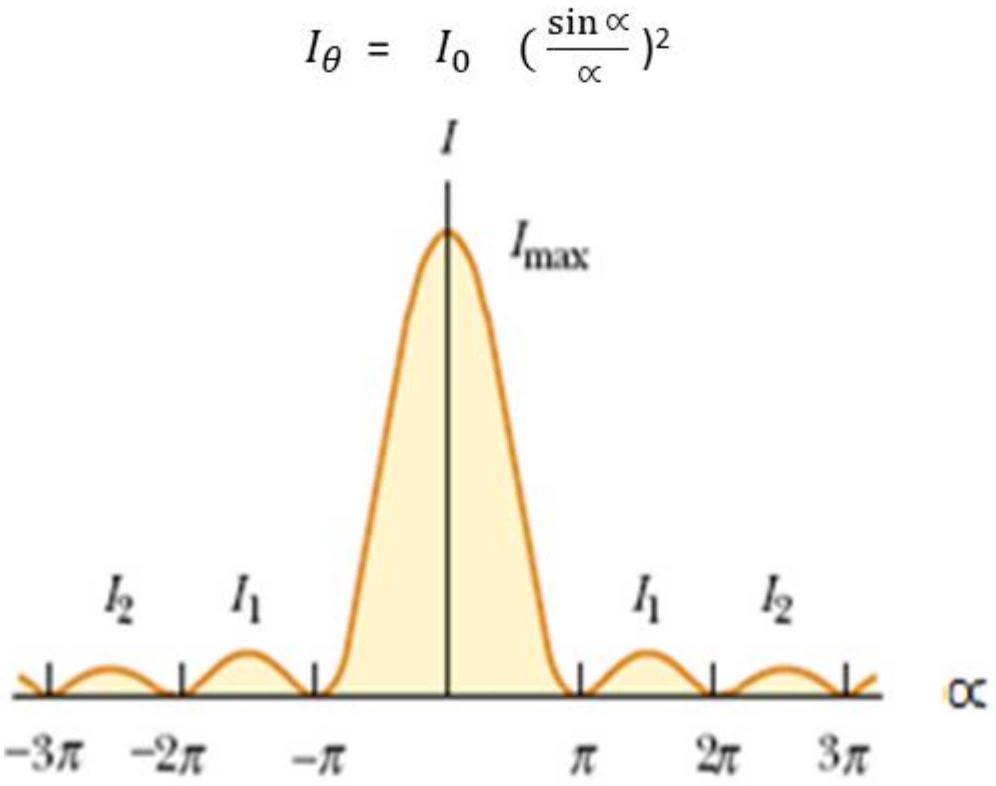
$$\frac{limit}{\alpha\to 0}\ \frac{\sin\ \propto}{\propto} = 1$$

It means α = 0

$$\boldsymbol{\alpha = \frac{\pi}{\lambda}\, a \sin \theta = 0}$$

a sin θ =0

θ =0



Condition for minimum intensity

For minima $\left(\frac{\sin\propto}{\propto}\right)^2 = 0$

Sin α = 0

for this $\boldsymbol{\alpha}$ has to be

= $\pm\boldsymbol{\pi}, \pm\boldsymbol{2\pi}, \pm\ \boldsymbol{3\pi}, \pm\boldsymbol{4\pi}$… = $\pm\boldsymbol{n\pi}$
(except zero 0)

From

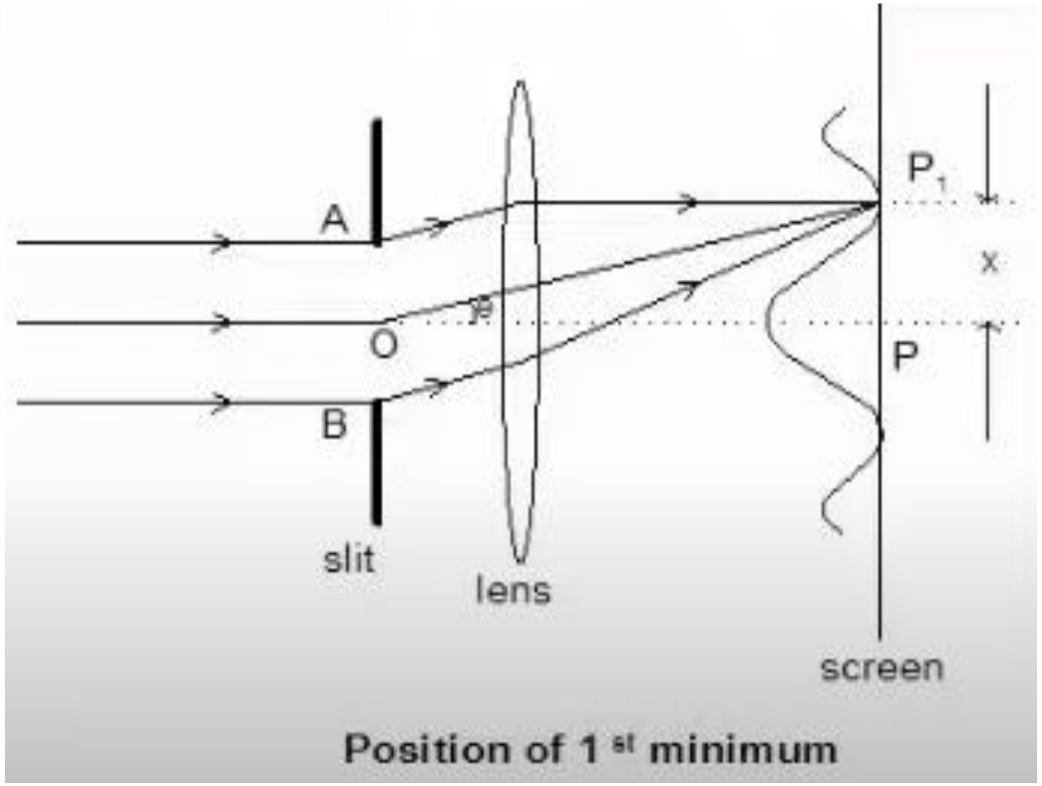
$\boldsymbol{\alpha} = \frac{\pi}{\lambda}$ **a sin** $\boldsymbol{\theta}$

$\pm\boldsymbol{n\pi} = \frac{\pi}{\lambda}$ a sin θ

nλ = a sin θ , where n = 1,2,3,…….

a sin θ= nλ

Position of first minima in single slit diffraction



In ΔOPP_1

$$\sin\theta = \frac{pp_1}{OP_1} = \frac{x}{f}$$

Condition for minima

$n\lambda = a \sin\theta$, where n = 1,2,3,…….

Condition for 1st minima (n=1)

$a \sin\theta = \lambda$

$$\sin\theta = \frac{\lambda}{a}$$

If lens is very close to the slit then, distance op is equal to the focal length of the lens.

From figure op =op$_1$ = f = focal length of the lens

$$\frac{\lambda}{a} = \frac{x}{f}$$

$X = \frac{f\lambda}{a}$ **(Position of first minima in single slit diffraction)**

The width of the central maximum can be calculated,

$$\mathbf{W = 2x = \frac{2f\lambda}{a}}$$

The most common questions asked in university exams

1. Derive an expression for intensity of diffracted light in single slit Fraunhofer diffraction.

2. For Fraunhofer diffraction by one slit, write the formula for intensity of light write the condition for central maxima and condition for minima

3. Derive the formula of the width of the central maxima in single slit Fraunhofer diffraction

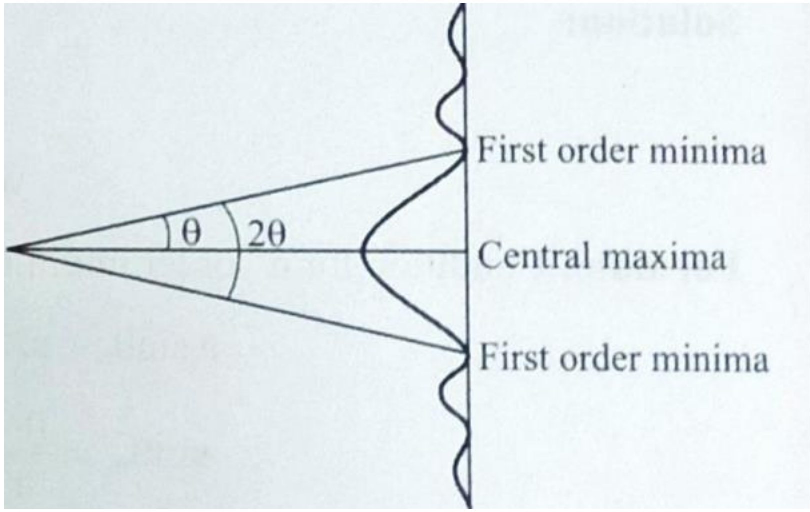
4. Distinguish between Fraunhofer and Fresnel diffraction

Find angular width of the central bright maximum in the Fraunhoffer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Given

Slit width (a) = 12×10^{-5} cm

Wavelength of the light (λ) = 6000 Å = 6000×10^{-8} cm



Formula: Condition for nth order minimum for single slit is given by,

$$a \sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

$$\sin\theta_1 = \frac{n\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{12 \times 10^{-5}} \qquad (n = 1)$$

$$\theta_1 = 30^0$$

Angular width of central maximum = $2\theta_1 = 60^0$

Example 2: Calculate angular separation between the first order minima on either side of central maximum when slit is 6×10^{-4} cm width. Given wavelength of light used $(\lambda) = 6000$ Å.

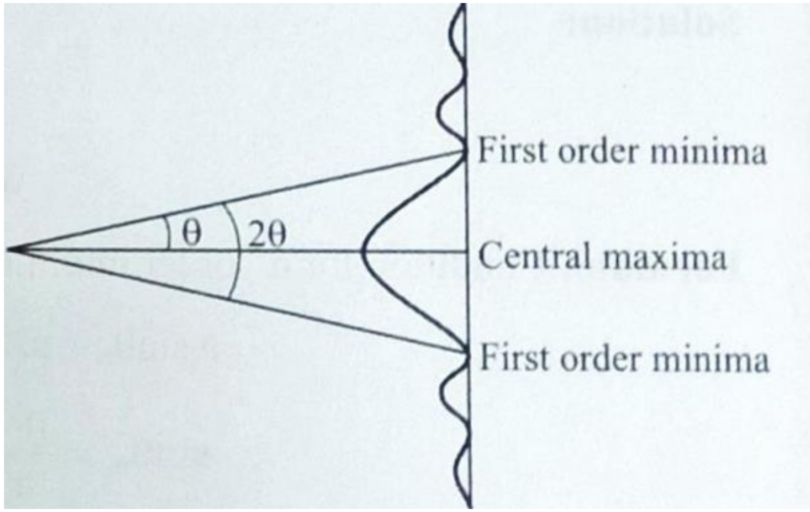
Given

Slit width (a) = 6×10^{-4} cm

Wavelength of the light $(\lambda) = 6000$ Å $= 6000 \times 10^{-8}$ cm

Order of diffraction (n) = 1

Formula: Condition for n^{th} order minimum for single slit is given by,



$$a \sin\theta_n = n\lambda$$

$$\sin\theta_1 = \frac{n\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{6 \times 10^{-4}}$$

$$\theta_1 = \sin^{-1}(0.1)$$

$$\theta_1 = 5^0\,44'$$

Angular separation = $(2\theta_1) = 11^0\,28'$

Example 3: Find the half angular width of the central bright maximum in the Fraunhoffer diffraction pattern of slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Given

Slit width (a) = 12×10^{-5} cm

Wavelength of the light (λ) = 6000 Å = 6000×10^{-8} cm

Formula: Condition for n^{th} order minimum for single slit is given by,

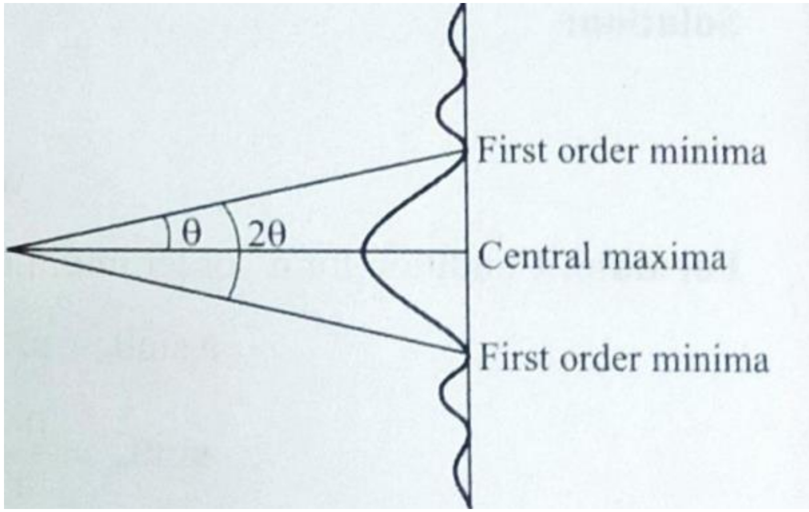
$$a \sin\theta_n = n\lambda$$

$$\sin\theta_n = \frac{n\lambda}{a}$$

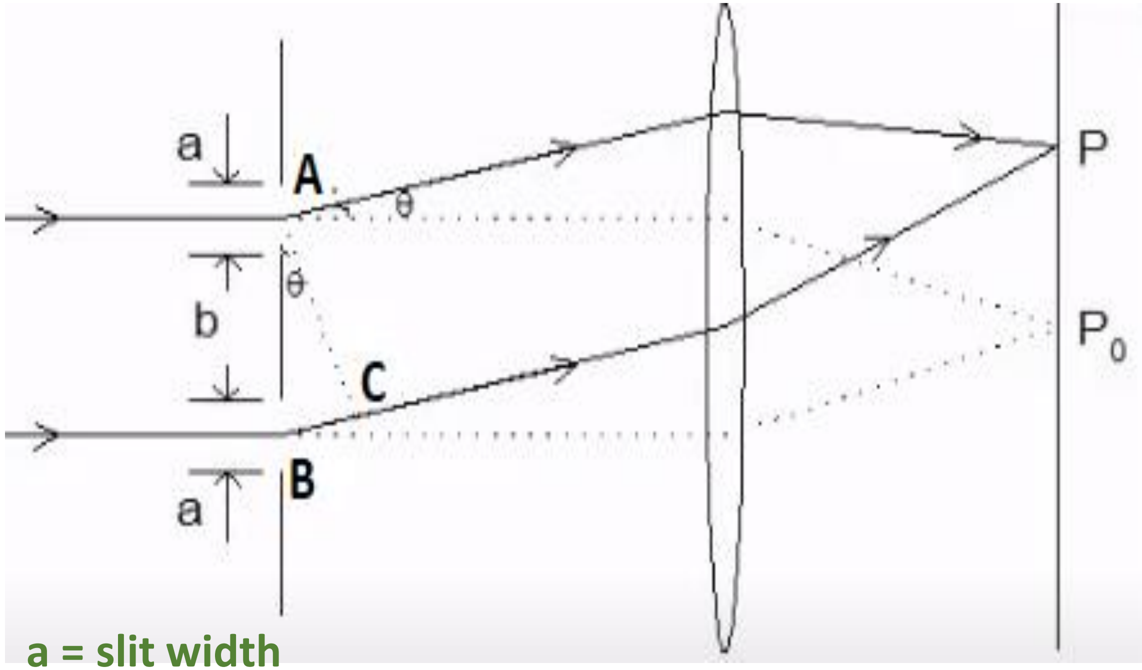
$$\sin\theta_1 = \frac{n\lambda}{a} = \frac{1 \times 6000 \times 10^{-8}}{12 \times 10^{-5}} \quad (n = 1)$$

$$\sin\theta_1 = 0.50$$

$$\theta_1 = 30°$$



Intensity of light due to Fraunhofer Diffraction by double slits



a = slit width

b = width of opaque

At point P all diffracted rays reach at angle θ.

The point will be dark or bright depends upon the path difference

At the centre of the screen all rays travels same distance and produce maximum intensity

The point P the path difference is

Path difference BC = (a/2 +b + a/2) sin θ

= (a + b) sin θ

The corresponding phase difference is

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\phi = \frac{2\pi}{\lambda}(a + b)\sin\theta$$

Assume $\boldsymbol{\beta = \frac{\pi}{\lambda}(a + b)\sin\theta}$

$$\phi = 2\,\beta$$

As a result of solving,

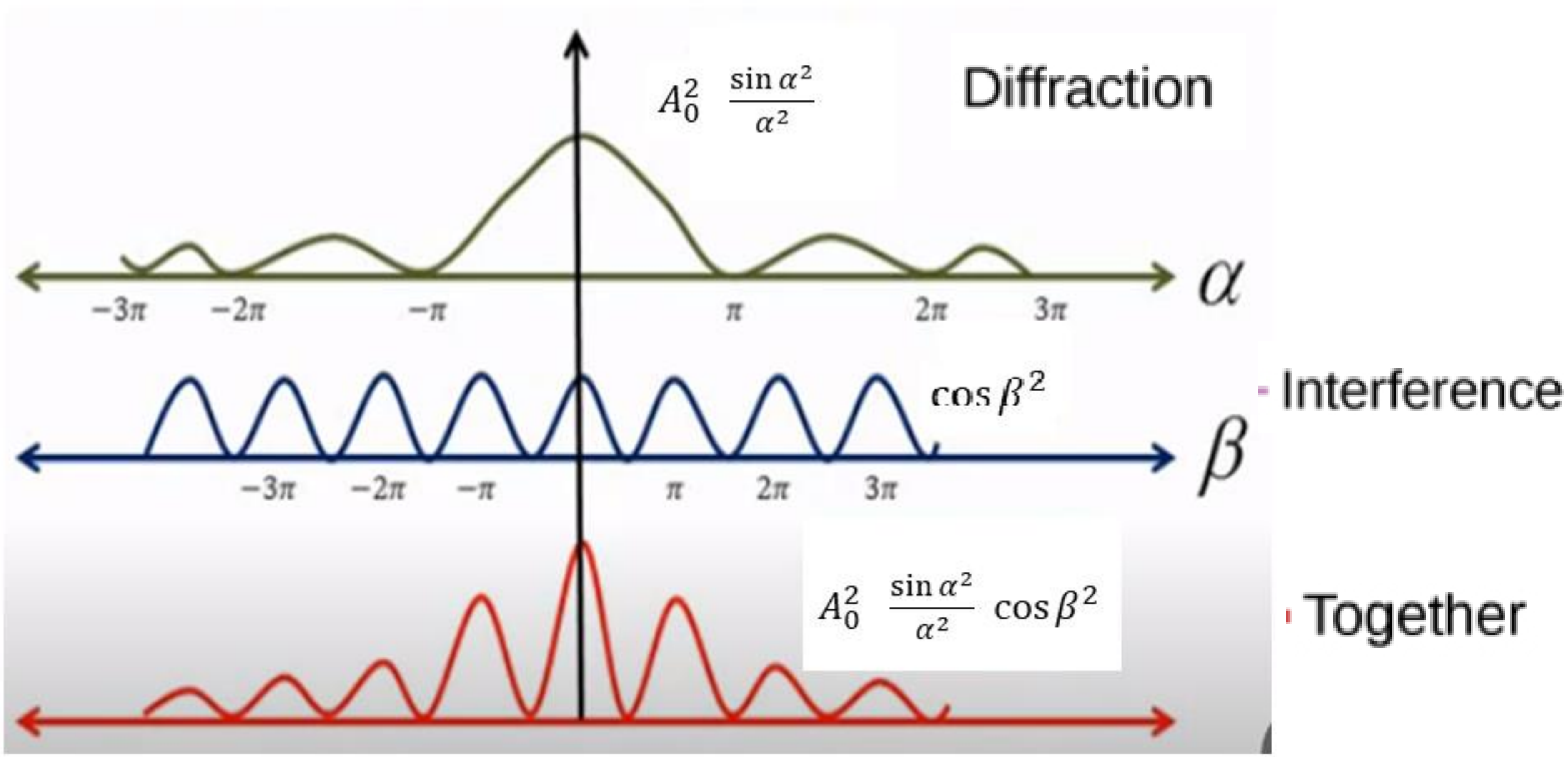
The resultant Intensity at angle θ due to double slits

$$A_\theta^2 = 4\ A_0^2\ \frac{\sin\alpha^2}{\alpha^2}(\cos\beta)^2$$

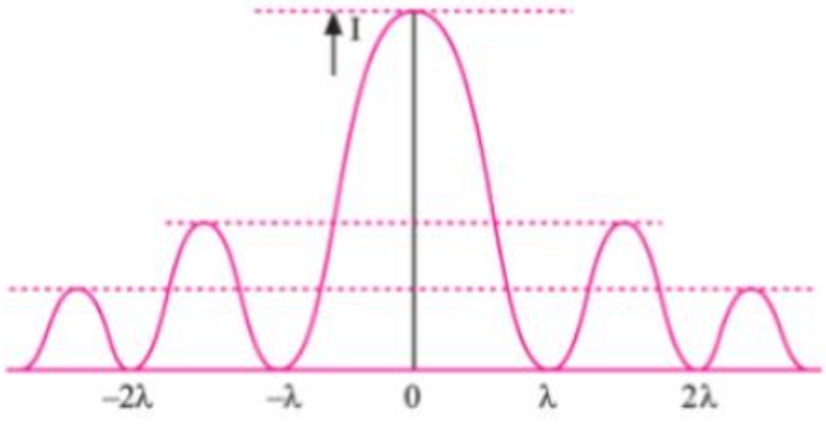
$\boldsymbol{\alpha = \frac{\pi}{\lambda}\, a \sin\theta}$ from single slit diffraction

$\boldsymbol{\beta = \frac{\pi}{\lambda}(a + b)\sin\theta}$ from Double slit diffraction

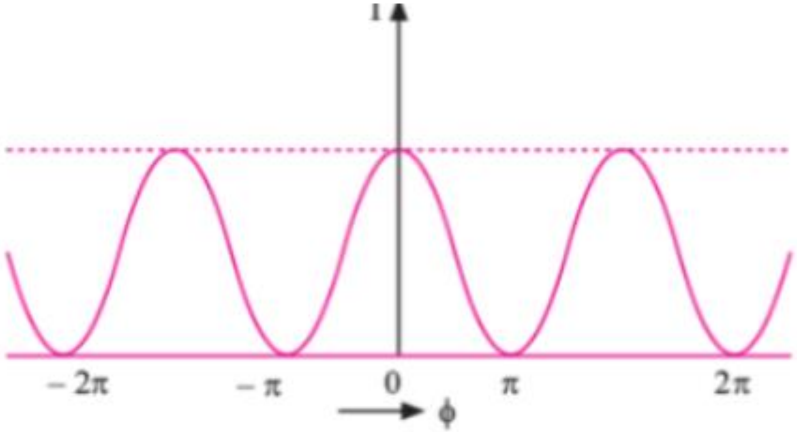
$$A_\theta^2 = 4\ A_0^2\ \frac{\sin\alpha^2}{\alpha^2}\cos\beta^2$$



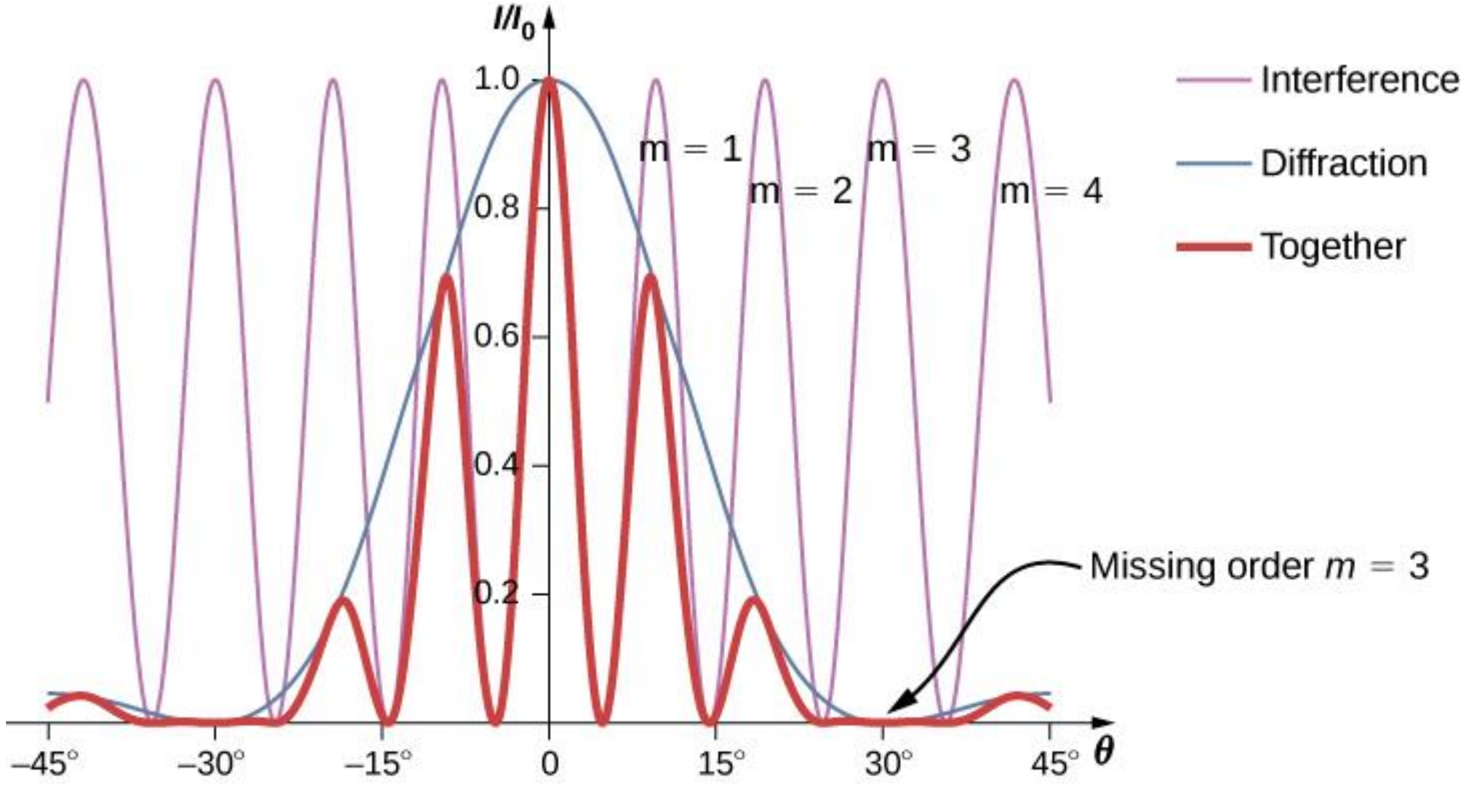
Missing order in double slit diffraction pattern

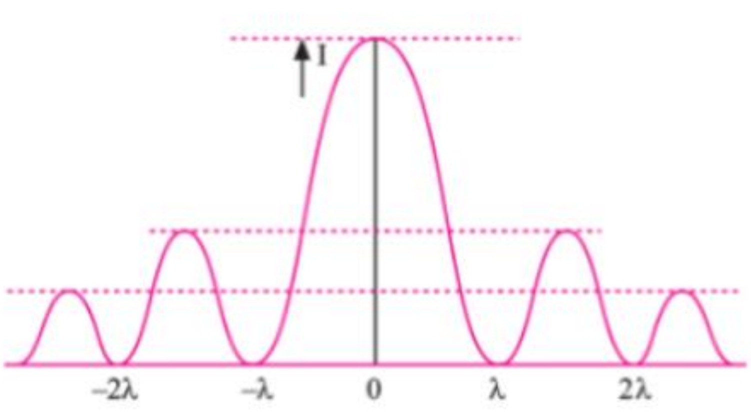


(b) Diffraction

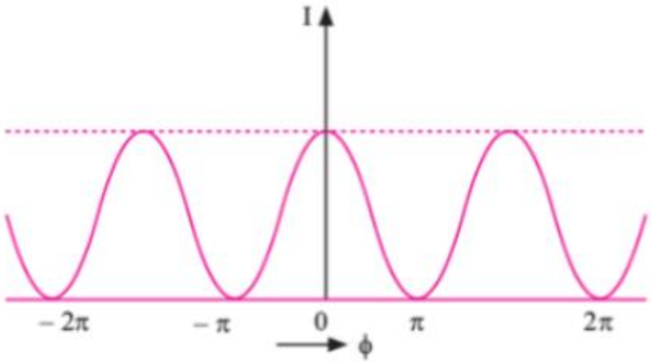


(a) Interference





(b) Diffraction



(a) Interference

Condition for minimum intensity due to single slit diffraction

$n\lambda$ = a sin θ , where n = 1,2,3,…….

a sin θ= mλ -------------------(1)

Condition for maxima intensity due to double slits interference

$n\lambda$ = (a + b) sin θ , where n = 1,2,3,……. **(12^{th} std)**

(a + b) sin θ = nλ ------------------ (2)

Divide equation (2) by (1)

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda}$$

$$\frac{(a+b)}{a} = \frac{n}{m}$$

Condition-1, then

if a= b

$$\frac{(a+b)}{a} = \frac{n}{m}$$

$$\frac{(a+a)}{a} = \frac{n}{m}$$

$$\frac{(2a)}{a} = \frac{n}{m}$$

$$2 = \frac{n}{m}$$

n = 2m if m= 1,2,3,….. then n= 2,4,6,…..

So, second, fourth, sixth orders of interference maxima are missing in diffraction pattern, because these maxima will coincide with 1st,2nd,3rd…. order diffraction minima due to single slit.

Condition-2, then

if 2a= b

$$\frac{(a+b)}{a} = \frac{n}{m}$$

$$\frac{(a+2a)}{a} = \frac{n}{m}$$

$$\frac{(3a)}{a} = \frac{n}{m}$$

$$3 = \frac{n}{m}$$

n = 3m if m= 1,2,3,….. then n= 3,6,9,…..

So, 3rd,6th,9th orders of interference maxima are missing in diffraction pattern, because these maxima will coincide with 1st,2nd,3rd…. order diffraction minima due to single slit.

Calculate the missing order for a plane diffraction grating, if the slit widths are 0.16 mm and they are 0.8 mm apart.

Given

Slit width (a) = 0.16 mm = 0.016 cm

Separation distance between slits (b) =0.8mm =0.08cm

Condition for n^{th} order plane diffraction grating maxima is given by,

$$(a + b) \sin\theta_n = n\lambda$$

Condition for p^{th} order minimum for single slit is given by,

$$a \sin\theta_n = p\lambda$$

$$\frac{(a+b)}{a} = \frac{n}{p}$$

$$\frac{0.016 + 0.08}{0.016} = \frac{n}{p}$$

$$\frac{n}{p} = 6$$

$$n = 6p$$

$$p = 1, 2, 3, \ldots\ldots \text{etc.}$$

$$\therefore \mathbf{n = 6, 12, 18, 24, \ldots\ldots}$$

Thus 6^{th}, 12^{th}, 18^{th}, 24^{th} order maxima will be missing in the diffraction patt

Let's revise the formula for amplitude before we go ahead

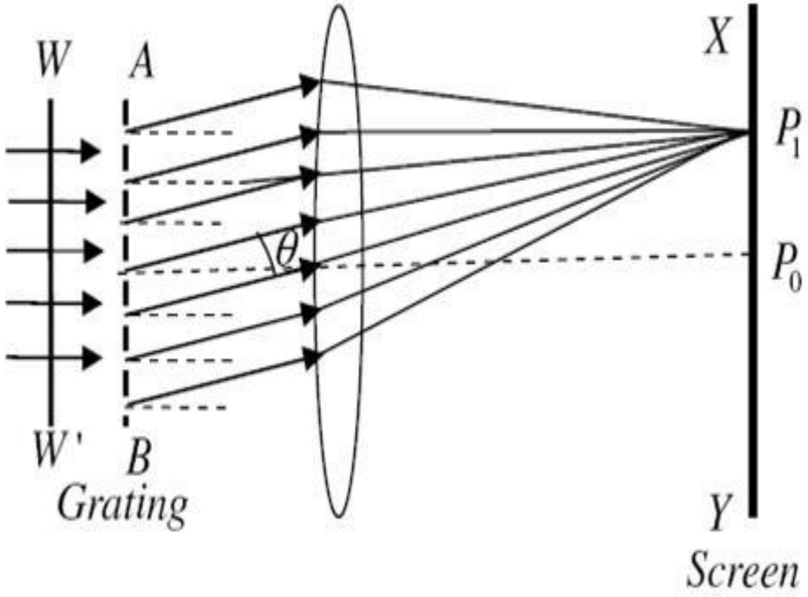
This is the resultant amplitude at an angle θ $\quad A = A_0 \dfrac{\sin \propto}{\propto}$ $\quad$ (where $\alpha = \dfrac{\pi}{\lambda}$ a sin θ)

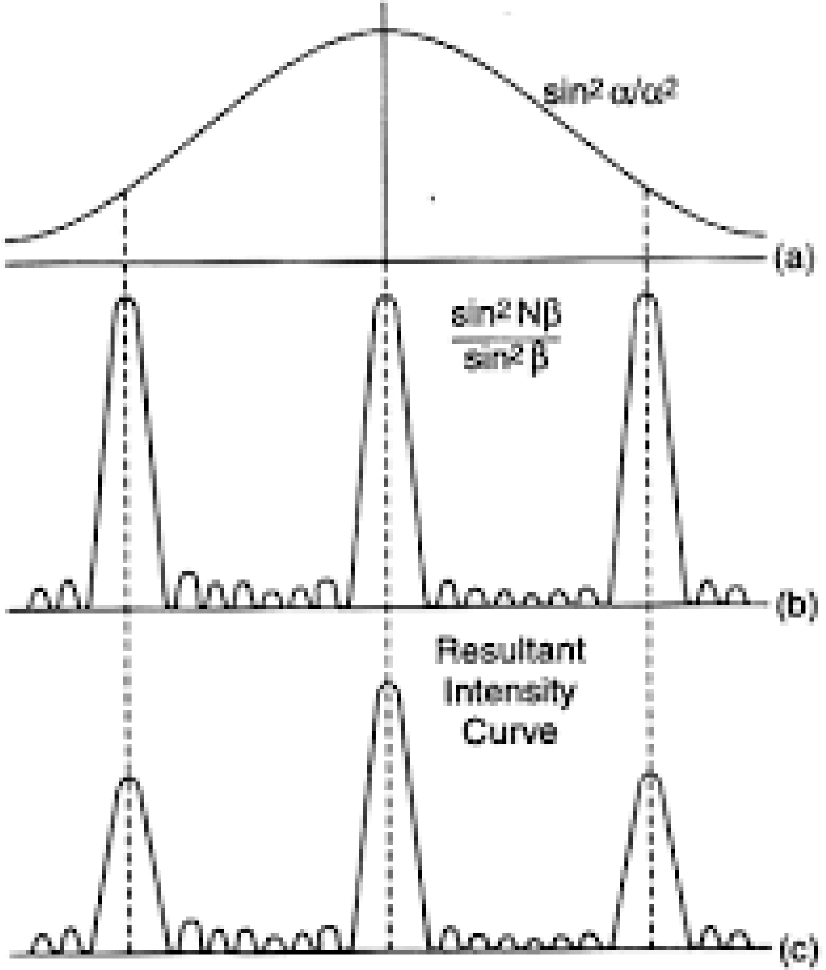
The resultant Intensity at angle θ due to double slits $\quad A_\theta = 4\, A_0 \dfrac{\sin \alpha}{\alpha} (\cos \beta)$

$\boldsymbol{\alpha = \frac{\pi}{\lambda}}$ **a sin** $\boldsymbol{\theta}$ $\quad$ from single slit diffraction

$\boldsymbol{\beta = \frac{\pi}{\lambda}}$ **(a + b) sin** $\boldsymbol{\theta}$ $\quad$ from Double slit diffraction

Fraunhofer Diffraction by N parallel slits.





The amplitude at angle θ is

$$A_\theta = A_0 \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\beta}$$

$$\alpha = \frac{\pi}{\lambda} a \sin\theta$$

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

The intensity if given by

$$I_\theta = A_\theta^2 = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

$A_0^2 \frac{\sin^2\alpha}{\alpha^2}$ This factor gives intensity distribution due to single slit

$\frac{\sin^2 N\beta}{\sin^2\beta}$ This factor gives intensity distribution due to N slits

For principal maxima (central maxima)

$$I_\theta = A_0^2 \frac{sin^2\alpha}{\alpha^2} \frac{sin^2 N\beta}{sin^2\beta}$$

$\frac{sin^2 N\beta}{sin^2\beta}$ This term should be maximum =1

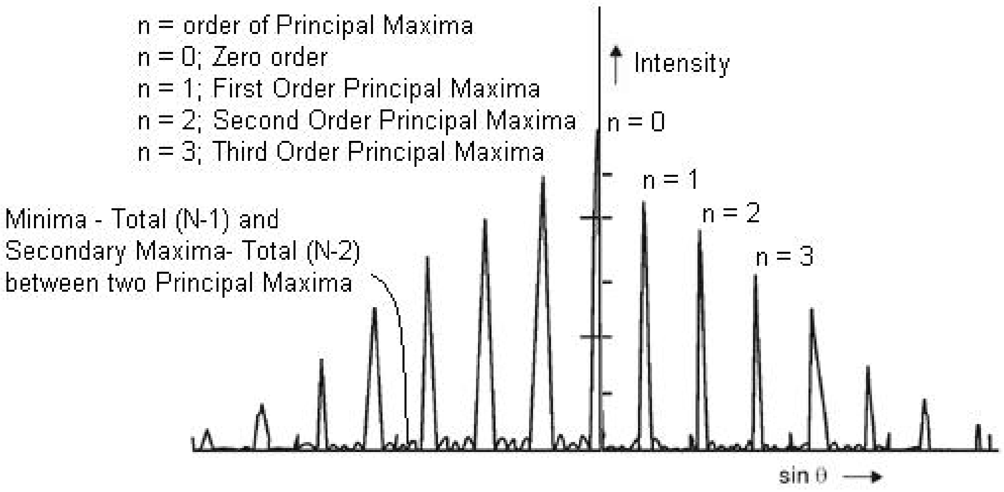
For maximum, Sin β = 0

This is possible when $\beta = \pm m\,\pi$ where m=0,1,2,3....

$$But, \;\; \beta = \frac{\pi}{\lambda}(a + b)\sin\theta$$

$$\beta = \frac{\pi}{\lambda}(a + b)\sin\theta = m\,\pi$$

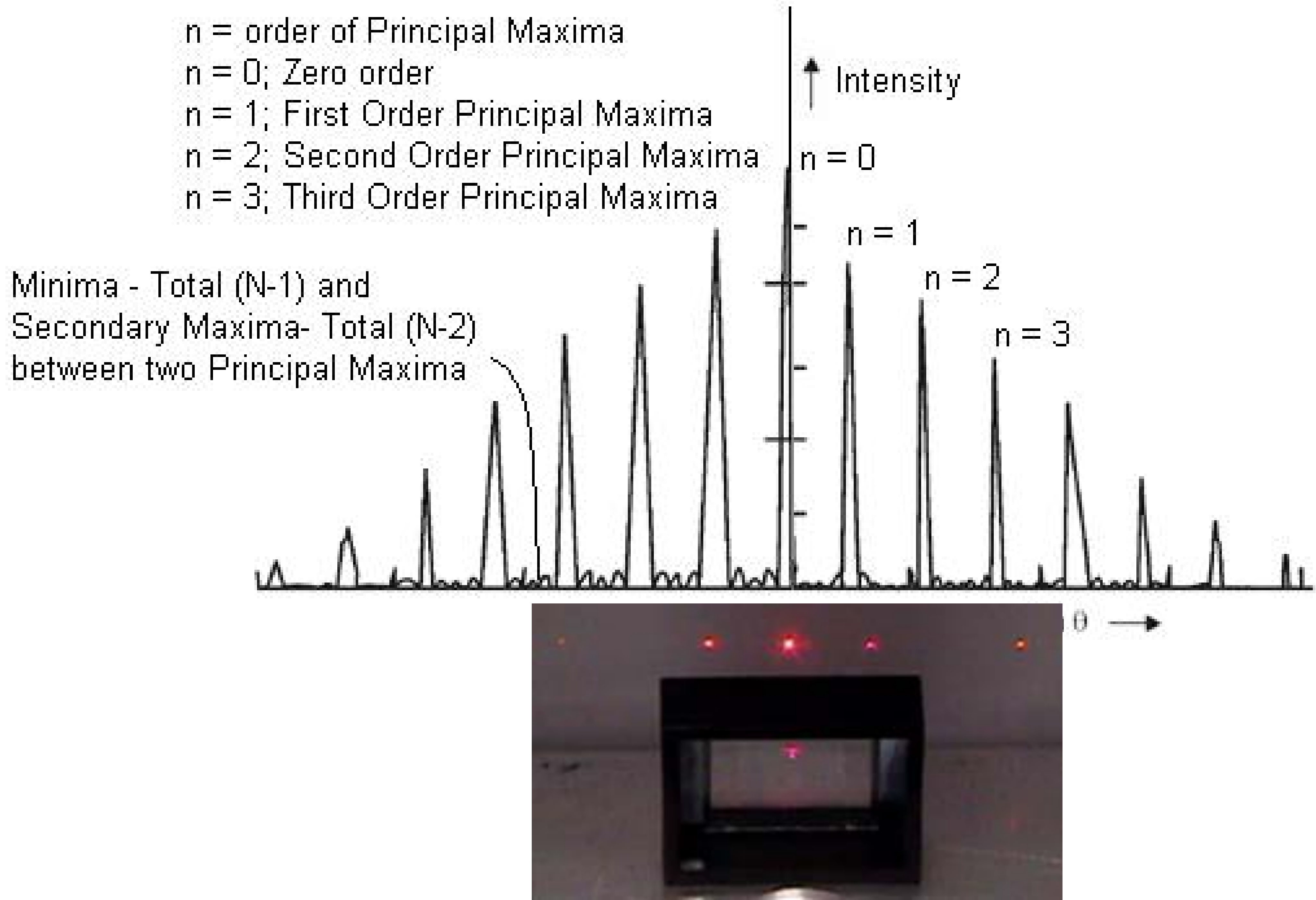
$(a + b)\sin\theta = m\,\lambda$ very imp formula for n slits or grating



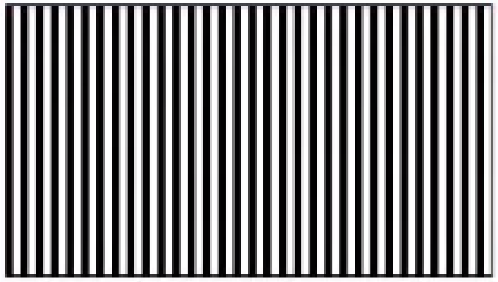
Where m= 0,1,2,3,4…….is called order of number.

m=0 corresponds to zero order maximum that is central maxima.

For m= 1,2,3… we get the first order, second order and so on principal maxima respectively.


n = order of Principal Maxima
n = 0; Zero order
n = 1; First Order Principal Maxima
n = 2; Second Order Principal Maxima
n = 3; Third Order Principal Maxima
Intensity
n = 0
n = 1
n = 2
n = 3
Minima - Total (N-1) and
Secondary Maxima- Total (N-2)
between two Principal Maxima
θ

Fraunhofer diffraction at N parallel slits (Diffraction grating)



Diffraction grating is defined as arrangement of large number of parallel slits placed closed to each other and separated by equal opaque spaces between them.

If a is width of the slit (i.e. space between the lines) and **b is the width of the line (i.e. width of opaque space),** the (a + b) is called as grating element.

If there are N no of lines per cm

Na + Nb = 1cm

N (a + b) = 1 cm

Grating element (a + b) is = $\frac{1}{N}$ cm

If there are N no of lines per inch

As we know 1 inch = 2.54 cm

Na + Nb = 2.54 cm

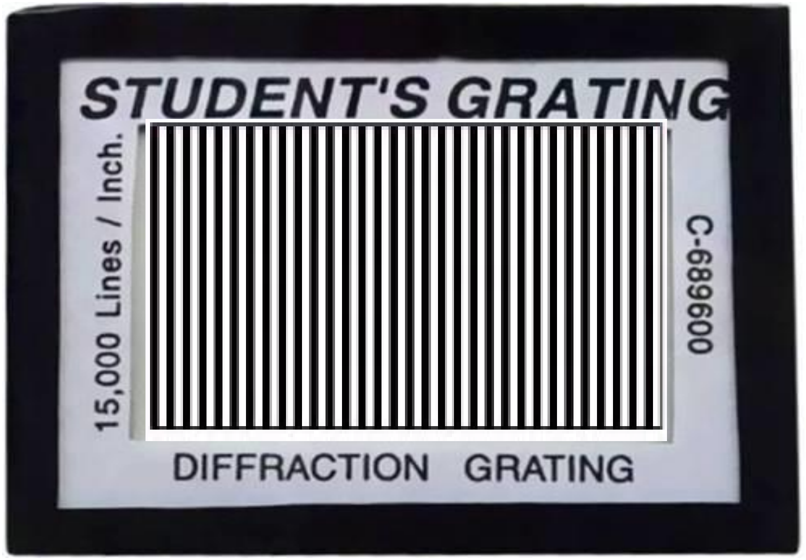
N (a + b) = 2.54 cm

Grating element (a + b) is = $\frac{\mathbf{2.54}}{\boldsymbol{N}}$ **cm**

There are two types of grating

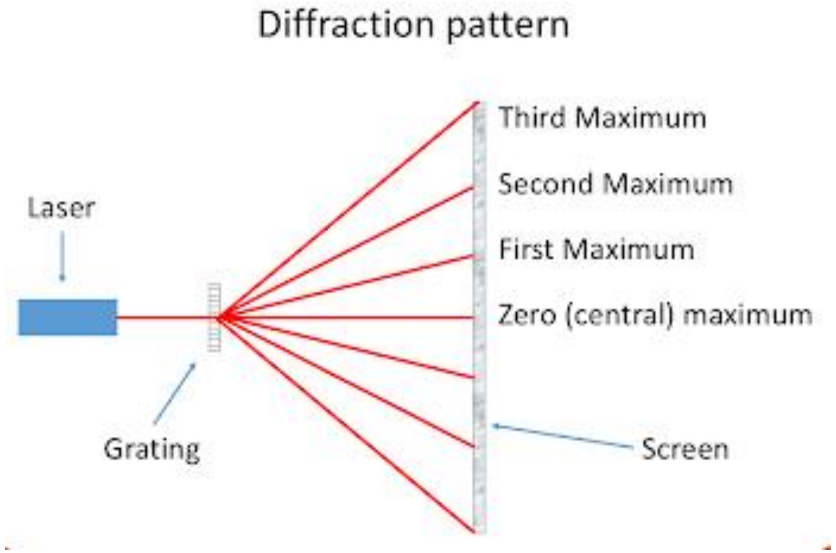
1. Transmission grating (light get transmitted) e.g. laboratory grating

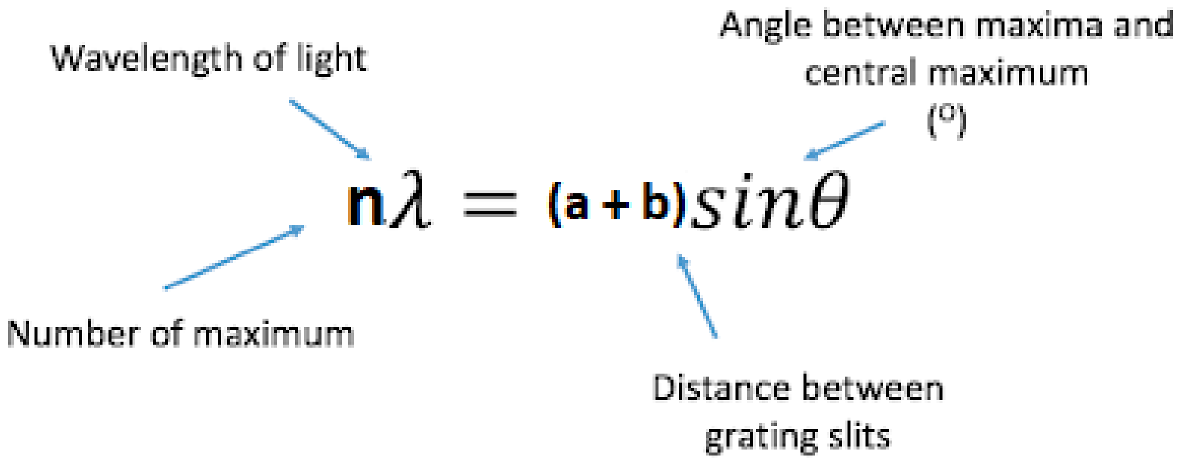
2. Reflection grating (light get reflected) e.g. Compact Disk (C.D)



Application of diffraction grating

Determination of wavelength of light by grating





Therefor the above equation is called as grating equation.

$$(a + b) \sin \theta = n \lambda$$

$$\lambda = \frac{(a + b) \sin \theta}{n}$$

$$(a + b) = \frac{1}{N}$$

$$\lambda = \frac{\boldsymbol{\sin \theta}}{\boldsymbol{nN}}$$

Calculate the highest order spectrum that can be obtained by a monochromatic light of wavelength 6000 Å by a grating of 6000 lines per centimeter.

Given

Wavelength of the light (λ) = 6000 Å = 6000×10^{-8} cm

Number of lines of grating (N) = 6000 lines /cm

Formula: Condition for n^{th} order plane diffraction grating maxima is given by,

$$(a + b) \sin\theta_n = n\lambda$$

The maximum value of sinθ gives highest order (n_{max}) of spectrum

Above condition can be written as

$$(a + b) \sin\theta_{max} = n_{max} \lambda$$

$$n_{max} = \frac{(a + b) \sin\theta_{max}}{\lambda}$$

$$(a + b) = \frac{1}{N} \quad \text{and} \quad \sin\theta_{max} = 1$$

$$n_{max} = \frac{1}{N \lambda} = \frac{1}{6000 \times 6000 \times 10^{-8}}$$

The highest order spectrum (n_{max}) = 2.7 ≈ 2

A parallel beam of monochromatic light is incident on a plane transmission grating having 3000 lines/cm. A third order diffraction is observed at 30°. Calculate the wavelength of monochromatic light.

Given

Number of lines on grating (N) = 3000 lines/ cm

Order of diffraction (n) = 3

Angle of diffraction (θ) = 30°

Formula: Condition for nth order plane diffraction grating maxima is given by,

$$(a + b) \sin\theta_n = n\lambda$$

$$\therefore \lambda = \frac{(a + b)\sin\theta_n}{n} \qquad \left(\because a + b = \frac{1}{N}\right)$$

$$= \frac{1 \times \sin\theta_n}{Nn} = \frac{1 \times \sin 30^0}{3000 \times 3}$$

$$\lambda = 5.555 \times 10^{-5} \text{ cm} = 5555 \times 10^{-8} \text{ cm}$$

The wavelength of monochromatic light (λ) = 5555 Å

A parallel beam of light of wavelength 5460 Å is incident at an angle of 30^0 on a plane transmission grating which has 6000 lines per cm. Find the highest order of spectrum that can be observed.

Given

Wavelength of the light (λ) = 5460 Å = 5460×10^{-8} cm

Angle of diffraction (θ) = 30^0

Number of lines on grating (N) = 6000 lines/cm

The maximum value of sinθ gives highest order (n_{max}) of spectrum

$$(a + b)\sin\theta_{max} = n_{max}\,\lambda$$

$$n_{max} = \frac{(a+b)\sin\theta_{max}}{\lambda}$$

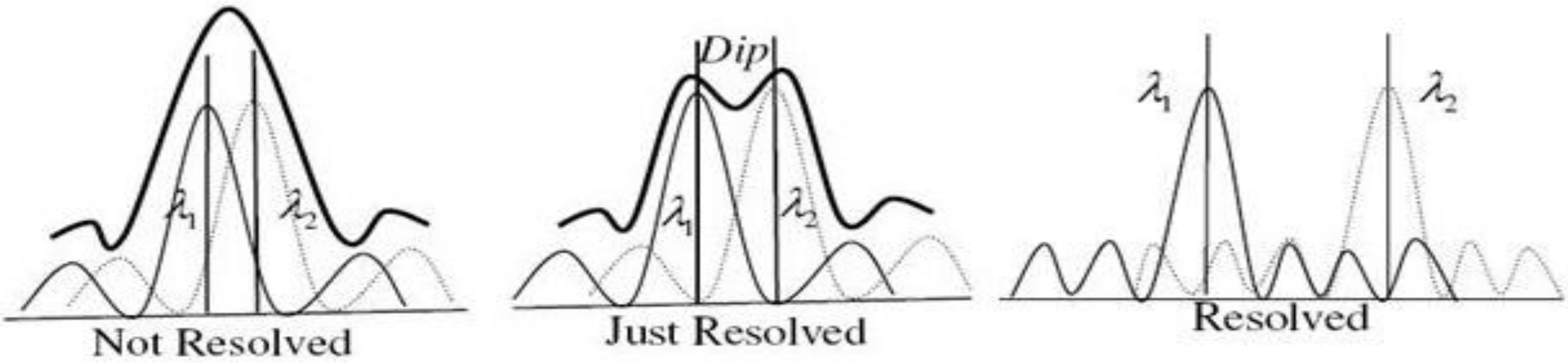
$$(a+b) = \frac{1}{N}, \ (\sin\theta_{max} = 1)$$

$$n_{max} = \frac{1}{N\lambda} = \frac{1}{6000 \times 5460 \times 10^{-8}}$$

Maximum numbers of observed orders (n_{max}) = 3.052 ≈ 3

Resolving power of grating

Rayleigh criterion



Unresolved

If distance between central maxima of two patter is less then the distance between maxima and minima of any two pattern

Just resolved

When central maxima of one pattern falls on first minima of second pattern then images are said to be just resolved

Well resolved

If the distance between central maxima of two pattern is more than distance between maxima and minima of any two pattern.

Resolving Power of Grating

Resolving power

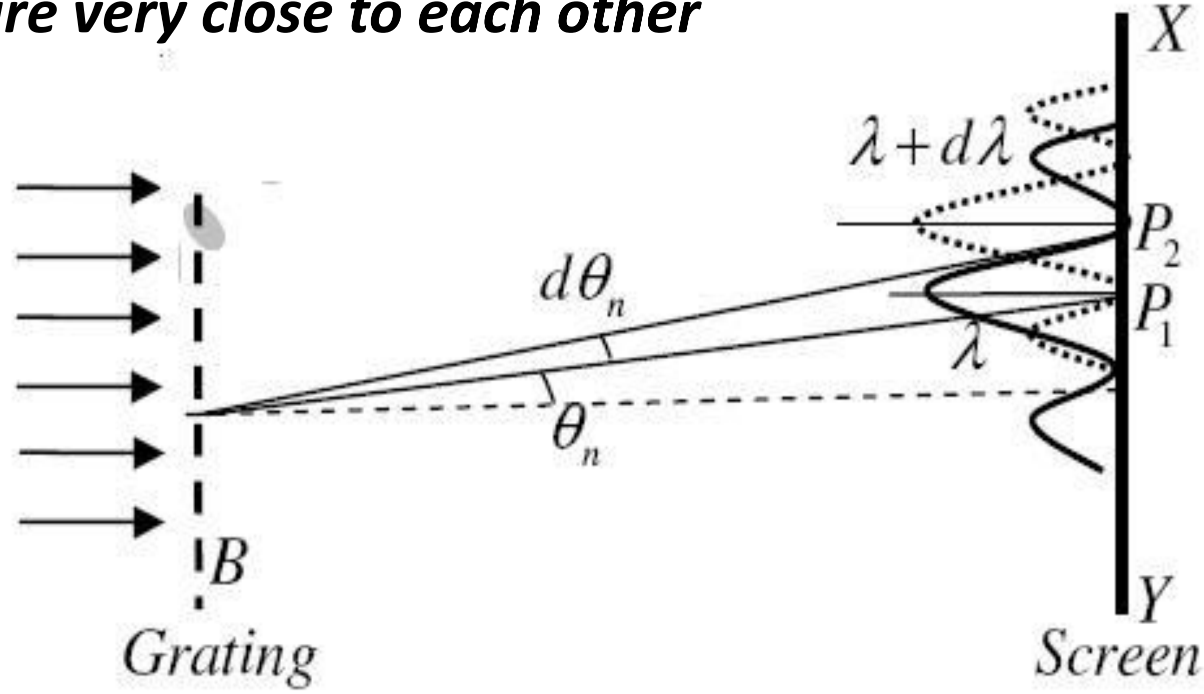
Ability of instrument to separate the images of two objects, which are very close to each other is called resolving power of instrument.

Resolving power of grating

It is ability of grating to separate maxima of two closed wavelengths which are very close to each other

or

*It is defined as **the capacity of a grating to form separate diffraction maxima of two wavelengths which are very close to each other***



Resolving power of grating is

$$R.P. = \frac{\lambda}{d\lambda} = mN$$

Where λ wavelength of any spectral line.

dλ is difference in wavelength

N is minimum no of lines required on the grating surface to just resolve the wavelengths λ and λ +dλ.

m is order of spectrum.

Thus the *resolving power is directly proportional to*

(i) The order of the spectrum *'m'*

(ii) The total number of lines on the grating *'N'*

Example 24: The light of wavelength 6000 Å is incident normally on a plane diffraction grating of 1000 lines per cm. Calculate (i) the difference between wavelengths that just appear separated in the first order and (ii) the resolving power in the third order spectrum.

Given

Wavelength $(\lambda) = 6000 \text{ Å} = 6000 \times 10^{-8}$ cm

Number of lines on grating (N) =1000 lines/cm

) Calculation of difference between two wavelengths.

$$\text{Resolving Power } = \frac{\lambda}{d\lambda} = nN$$

$$d\lambda = \frac{\lambda}{nN} = \frac{6000 \times 10^{-8}}{1 \times 1000} = 6 \times 10^{-8}$$

$$\mathbf{d\lambda = 6 \text{ Å}}$$

Calculation of resolving power in third order

$$\text{Resolving Power} = \frac{\lambda}{d\lambda} = nN$$

$$\frac{\lambda}{d\lambda} = nN = 3 \times 1000 = 3000$$

Resolving power in the third order spectrum $\left(\frac{\lambda}{d\lambda}\right) = \mathbf{3000}$

A grating has 620 rulings/mm and is 5.05 mm wide. What is the smallest wavelength interval that can be resolved in the third order at $\lambda = 481$ nm?

Given

$N = 5.05 \times 620 = 3131$ lines /mm

Wavelength $(\lambda) = 481\text{nm} = 481 \times 10^{-9}$ m

$$\text{Resolving power} = \frac{\lambda}{d\lambda} = nN$$

$$\therefore d\lambda = \frac{\lambda}{n \times N} = \frac{481 \times 10^{-9}}{3 \times 3131}$$

$$\mathbf{d\lambda = 0.051 \times 10^{-9} = 0.051\ nm.}$$