Remarks on: Swanson's Hamiltonian [J.Math.Phys 45(2);585(2004)

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Biswanath Rath
Maharaja Sriram Chandra Bhanja Deo University, Takatpur ,Baripada ,Mayurbhanj ,Odisha ,India

Remarks on: Swanson's Hamiltonian [J.Math.Phys **45**(**2**);585(2004)]

Biswanath Rath; Pravanjan Mallick and Prachiprava Mohapatra

Department of Physics, North Orissa University, Takatpur, Baripada -757003, Odisha, INDIA

(e.mail:biswanathrath10@gmail.com)

We find that the Hamiltonian $H = wa^{\dagger}a + \alpha a^2 + \beta(a^{\dagger})^2$ **satisfying the the condition** $w^2 - 4\alpha\beta \gg 0$ **can admit negative spectra , which is in contradiction to all real positive eigenvalues (as stated in the abstract).**

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Key words-Negative eigenvalue , matrix diagonalisation method .

I.Introduction

Nearly 14 years back M.S.Swanson [1] proposed a non-hermitiam quadratic Hamiltonian

$$H = wa^{\dagger} + \alpha a^2 + \beta(a^{\dagger})^2 \tag{1}$$

with $\alpha \neq \beta$ and suggested that the spectra becomes positive and real provided

$$w^2 - 4\alpha\beta \gg 0 \tag{2}$$

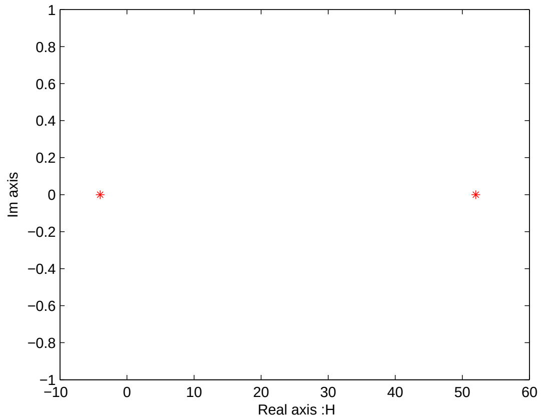

Figure 1: $H = 64a^\dagger a + 12a^2 + 20(a^\dagger)^2$

We feel that the above Hamiltonian is a diffent manifestation of Harmonic oscillator . As such the construction deals with non-hermitian quantum mechanics with real non-diagonal terms . By curiosity we performed matrix diagonalisation calculation to check the validity of pure real positive spectra satisfing the above condition. To our surprise the spectra can admit real negative energy in addition to positive energy.

II.Method and Results

Here we use simple matrix diagonalisation method [2] to find eigenvalues . For some values of w;α and β we summarise the value of negative energy as given below.

$E_0 = -0.5 \to w = 8; \alpha = 1.5; \beta = 2.5$

$E_0 = -1 \to w = 16; \alpha = 3; \beta = 5$

$E_0 = -2 \to w = 32; \alpha = 6; \beta = 10$

$E_0 = -4 \to w = 64; \alpha = 12; \beta = 20$

For the interest of the reader we plot the first two eigenvalues corresponding to $w = 64, \alpha = 12, \beta = 20$ in fig-1.

III. Conclusion

Our findings contradict the conclusion presented earlier[1] .One can verify our numerical findings using analytical method [3]. We believe that the non-hermitian Hamiltonian can admit negative energy in addition to positive energy . Probably a slight modified form of Swanson's Hamiltonian can also reflect all pure real negative eigenvalues [4] satisfying the condition in Eq(2).

References

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[4] B.Rath and P.Mallick,Open.Phys.**14**,492(2016).