

with(*GroupTheory*)

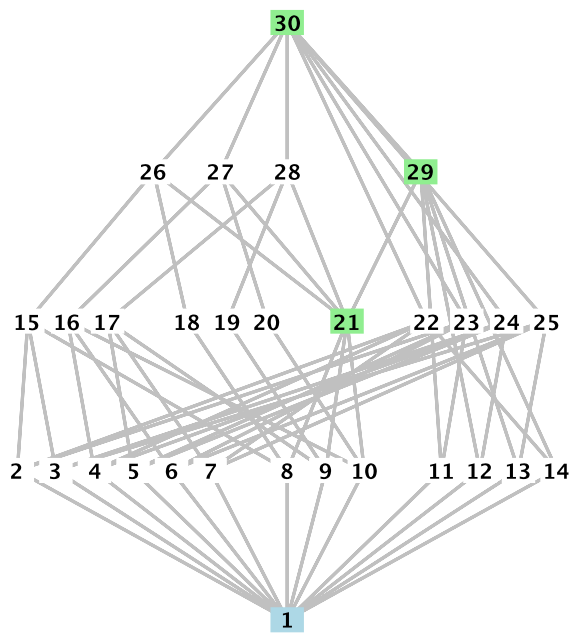
[ <|>, *AbelianInvariants*, *AllPerfectGroups*, *AllSmallGroups*, *AllTransitiveGroups*, *Alt*, *AlternatingGroup*, *AreConjugate*, *AreIsomorphic*, *BabyMonster*, *CayleyTable*, *CayleyTableGroup*, *Center*, *Centraliser*, *Centralizer*, *Centre*, *ConjugacyClass*, *ConjugacyClasses*, *Conjugator*, *ConwayGroup*, *Core*, *CustomGroup*, *CycleIndexPolynomial*, *CyclicGroup*, *Degree*, *DerivedLength*, *DerivedSeries*, *DerivedSubgroup*, *DicyclicGroup*, *DihedralGroup*, *DirectProduct*, *DrawCayleyTable*, *DrawSubgroupLattice*, *ElementaryGroup*, *Elements*, *Embedding*, *ExceptionalGroup*, *FPGroup*, *Factor*, *FischerGroup*, *FittingSubgroup*, *FrattiniSubgroup*, *GL*, *GaloisGroup*, *GeneralLinearGroup*, *GeneralOrthogonalGroup*, *GeneralUnitaryGroup*, *Generators*, *Group*, *GroupOrder*, *HaradaNortonGroup*, *HeldGroup*, *HigmanSimsGroup*, *Hypercenter*, *Hypercentre*, *IdentifySmallGroup*, *Index*, *Intersection*, *IsAbelian*, *IsAlternating*, *IsCommutative*, *IsElementary*, *IsFinite*, *IsNilpotent*, *IsNormal*, *IsPerfect*, *IsPrimitive*, *IsRegular*, *IsSimple*, *IsSoluble*, *IsSolvable*, *IsSubgroup*, *IsSymmetric*, *IsTransitive*, *JankoGroup*, *Labels*, *LeftCoset*, *LeftCosets*, *LowerCentralSeries*, *LyonsGroup*, *MathieuGroup*, *McLaughlinGroup*, *MetacyclicGroup*, *Monster*, *NilpotencyClass*, *NilpotentResidual*, *NonRedundantGenerators*, *NormalClosure*, *Normaliser*, *NormaliserSubgroup*, *NormalizerSubgroup*, *NumGroups*, *NumPerfectGroups*, *NumTransitiveGroups*, *ONanGroup*, *Operations*, *Orbit*, *Orbits*, *OrthogonalGroup*, *PCore*, *PGL*, *PGU*, *PSL*, *PSU*, *PSp*, *PerfectGroup*, *PermApply*, *PermCommutator*, *PermConjugate*, *PermCycleType*, *PermDegree*, *PermFixed*, *PermInverse*, *PermLeftQuotient*, *PermOrder*, *PermParity*, *PermPower*, *PermProduct*, *PermRightQuotient*, *PermSupport*, *PermutationGroup*, *PresentationComplexity*, *ProjectiveGeneralLinearGroup*, *ProjectiveGeneralUnitaryGroup*, *ProjectiveSpecialLinearGroup*, *ProjectiveSpecialUnitaryGroup*, *ProjectiveSymplecticGroup*, *QuaternionGroup*, *RandomElement*, *Relators*, *RightCoset*, *RightCosets*, *RubiksCubeGroup*, *RudvalisGroup*, *SL*, *Simplify*, *SmallGroup*, *SolubleResidual*, *SolvableResidual*, *SpecialLinearGroup*, *SpecialOrthogonalGroup*, *SpecialUnitaryGroup*, *Stabiliser*, *Stabilizer*, *Subgroup*, *SubgroupLattice*, *SubgroupMembership*, *Supergroup*, *SuzukiGroup*, *SylowSubgroup*, *Symm*, *SymmetricGroup*, *SymplecticGroup*, *ThompsonGroup*, *Tietze*, *TitsGroup*, *TransitiveGroup*, *TrivialGroup*, *UpperCentralSeries*]

$G := \text{SymmetricGroup}(4)$

$S_4$

(2)

*DrawSubgroupLattice*( $G$ )



$H := \text{DihedralGroup}(4)$

$D_4$

(3)

$\text{DrawCayleyTable}(H)$

	e	a	b	c	d	f	g	h
e	e	a	b	c	d	f	g	h
a	a	e	h	g	f	d	c	b
b	b	h	e	d	c	g	f	a
c	c	g	f	a	b	h	e	d
d	d	f	g	h	e	a	b	c
f	f	d	c	b	a	e	h	g
g	g	c	d	e	h	b	a	f
h	h	b	a	f	g	c	d	e

$$\text{GroupOrder}(G), \text{GroupOrder}(H) \quad 24, 8 \quad (4)$$

$$U := \text{DirectProduct}(G, H) \quad S_4 \times D_4 \quad (5)$$

$$V := \text{DirectProduct}(H, G) \quad D_4 \times S_4 \quad (6)$$

$$\text{AreIsomorphic}(U, V) \quad \text{true} \quad (7)$$

$$\text{GroupOrder}(U) = \text{GroupOrder}(G) \text{GroupOrder}(H) \quad 192 = 192 \quad (8)$$

$$id := \text{IdentifySmallGroup}(U) \quad 192, 1472 \quad (9)$$

$$W := \text{SmallGroup}(id) \quad \langle (1, 2)(3, 10)(4, 11)(5, 12)(6, 30)(7, 33)(8, 27)(9, 26)(13, 34)(14, 35)(15, 59)(16, \quad (10)$$

72)(17, 79)(18, 42)(19, 43)(20, 44)(21, 39)(22, 40)(23, 41)(24, 31)(25, 45)(28,  
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*AreIsomorphic(U, W)*

*true*

**(11)**