with(*linalg*) :
A := *matrix*([[*a*, *b*, 0, 0], [−*b*, *a*, *b*, 0], [0, −*b*, *a*, *b*], [0, 0, −*b*, *a*]]);

$$\begin{bmatrix} a & b & 0 & 0 \\ -b & a & b & 0 \\ 0 & -b & a & b \\ 0 & 0 & -b & a \end{bmatrix}$$

(1)

B := *matrix*(4, 4, [1, 2, 1, 0, 1, 1, 1, 2, 3, 1/2, 2, 4, 4, 4, 5, 5]);

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 3 & \dfrac{1}{2} & 2 & 4 \\ 4 & 4 & 5 & 5 \end{bmatrix}$$

(2)

C := *matrix*(2, 4, [2, 3, 2, 3, 4, 6, 4, 6]);

$$\begin{bmatrix} 2 & 3 & 2 & 3 \\ 4 & 6 & 4 & 6 \end{bmatrix}$$

(3)

evalm(*A* + 3 * *B*);

evalm(*A*&*B*);

transpose(*C*);

rank(*C*); *rank*(*B*);

evalm(*A*&*transpose*(*C*) + (1/2) * *transpose*(*C*));

det(*B*);

det(*A*);

$$\begin{bmatrix} a+3 & b+6 & 3 & 0 \\ -b+3 & a+3 & b+3 & 6 \\ 9 & -b+\dfrac{3}{2} & a+6 & b+12 \\ 12 & 12 & -b+15 & a+15 \end{bmatrix}$$

$$\begin{bmatrix} a+b & 2a+b & a+b & 2b \\ 2b+a & -\dfrac{3}{2}b+a & a+b & 2a+4b \\ 3b+3a & 3b+\dfrac{1}{2}a & 2a+4b & 3b+4a \\ -3b+4a & -\dfrac{1}{2}b+4a & -2b+5a & -4b+5a \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$1$$

$$4$$

$$\begin{bmatrix} 2\,a+3\,b+1 & 4\,a+6\,b+2 \\ 3\,a+\dfrac{3}{2} & 6\,a+3 \\ 2\,a+1 & 4\,a+2 \\ -2\,b+3\,a+\dfrac{3}{2} & -4\,b+6\,a+3 \end{bmatrix}$$

$$-10$$

$$a^4+3\,a^2\,b^2+b^4 \tag{4}$$

inverse(B);
inverse(A);

$$\begin{bmatrix} \dfrac{3}{4} & -\dfrac{13}{20} & \dfrac{7}{10} & -\dfrac{3}{10} \\[6pt] \dfrac{1}{2} & \dfrac{9}{10} & -\dfrac{1}{5} & -\dfrac{1}{5} \\[6pt] -\dfrac{3}{4} & -\dfrac{23}{20} & -\dfrac{3}{10} & \dfrac{7}{10} \\[6pt] -\dfrac{1}{4} & \dfrac{19}{20} & -\dfrac{1}{10} & -\dfrac{1}{10} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{a\,(a^2+2\,b^2)}{a^4+3\,a^2\,b^2+b^4} & -\dfrac{b\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} & \dfrac{b^2\,a}{a^4+3\,a^2\,b^2+b^4} & -\dfrac{b^3}{a^4+3\,a^2\,b^2+b^4} \\[10pt] \dfrac{b\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} & \dfrac{a\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} & -\dfrac{a^2\,b}{a^4+3\,a^2\,b^2+b^4} & \dfrac{b^2\,a}{a^4+3\,a^2\,b^2+b^4} \\[10pt] \dfrac{b^2\,a}{a^4+3\,a^2\,b^2+b^4} & \dfrac{a^2\,b}{a^4+3\,a^2\,b^2+b^4} & \dfrac{a\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} & -\dfrac{b\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} \\[10pt] \dfrac{b^3}{a^4+3\,a^2\,b^2+b^4} & \dfrac{b^2\,a}{a^4+3\,a^2\,b^2+b^4} & \dfrac{b\,(a^2+b^2)}{a^4+3\,a^2\,b^2+b^4} & \dfrac{a\,(a^2+2\,b^2)}{a^4+3\,a^2\,b^2+b^4} \end{bmatrix} \tag{5}$$

evalm$(A\texttt{\^{}}2)$;

$$\begin{bmatrix} a^2 - b^2 & 2\,b\,a & b^2 & 0 \\ -2\,b\,a & a^2 - 2\,b^2 & 2\,b\,a & b^2 \\ b^2 & -2\,b\,a & a^2 - 2\,b^2 & 2\,b\,a \\ 0 & b^2 & -2\,b\,a & a^2 - b^2 \end{bmatrix} \tag{6}$$

$v := vector([1, 1, 0, 0]);$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

$evalm(B\&*v);$

$$\begin{bmatrix} 3 & 2 & \dfrac{7}{2} & 8 \end{bmatrix} \tag{8}$$

$evalm(v\&*B);$

$$\begin{bmatrix} 2 & 3 & 2 & 2 \end{bmatrix} \tag{9}$$

$C;$
$evalm(C);$

$$C$$

$$\begin{bmatrix} 2 & 3 & 2 & 3 \\ 4 & 6 & 4 & 6 \end{bmatrix} \tag{10}$$

$augment(A, B);$

$$\begin{bmatrix} a & b & 0 & 0 & 1 & 2 & 1 & 0 \\ -b & a & b & 0 & 1 & 1 & 1 & 2 \\ 0 & -b & a & b & 3 & \dfrac{1}{2} & 2 & 4 \\ 0 & 0 & -b & a & 4 & 4 & 5 & 5 \end{bmatrix} \tag{11}$$

$augment(A, v);$

$$\begin{bmatrix} a & b & 0 & 0 & 1 \\ -b & a & b & 0 & 1 \\ 0 & -b & a & b & 0 \\ 0 & 0 & -b & a & 0 \end{bmatrix} \tag{12}$$

$evalm(B);$
$p := vector([1, 2, 3, 4]);$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 3 & \dfrac{1}{2} & 2 & 4 \\ 4 & 4 & 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \tag{13}$$

sol := *linsolve*(*B*, *p*);

$$\left[\begin{array}{cccc} \dfrac{7}{20} & \dfrac{9}{10} & -\dfrac{23}{20} & \dfrac{19}{20} \end{array} \right]$$ **(14)**

Baup := *augment*(*B*, *p*); *rank*(*B*); *rank*(*Baup*);

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 3 & \dfrac{1}{2} & 2 & 4 & 3 \\ 4 & 4 & 5 & 5 & 4 \end{bmatrix}$$

$$4$$

$$4$$ **(15)**

evalm(*B*&*sol);

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \right]$$ **(16)**

evalm(*Baup*);
gausselim(*Baup*);

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 3 & \dfrac{1}{2} & 2 & 4 & 3 \\ 4 & 4 & 5 & 5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & -1 & 0 & 2 & 1 \\ 0 & 0 & -1 & -7 & -\dfrac{11}{2} \\ 0 & 0 & 0 & -10 & -\dfrac{19}{2} \end{bmatrix}$$ **(17)**

backsub(%);

$$\left[\begin{array}{cccc} \dfrac{7}{20} & \dfrac{9}{10} & -\dfrac{23}{20} & \dfrac{19}{20} \end{array} \right]$$ **(18)**

S := *matrix*([[0, *a*, 1, 0, *a*], [1, 0, 0, *a*, 0], [0, 1, *a*, 0, 1], [*a*, 0, 0, 1, 0], [0, *a*, 1, 0, *a*]]);
gausselim(*S*);

$$\begin{bmatrix} 0 & a & 1 & 0 & a \\ 1 & 0 & 0 & a & 0 \\ 0 & 1 & a & 0 & 1 \\ a & 0 & 0 & 1 & 0 \\ 0 & a & 1 & 0 & a \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a & 0 \\ 0 & a & 1 & 0 & a \\ 0 & 0 & \dfrac{a^2 - 1}{a} & 0 & 0 \\ 0 & 0 & 0 & -a^2 + 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

backsub(%)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \tag{20}$$