

restart : with(Physics) :
 Setup(mathematicalnotation = true)
 [mathematicalnotation = true] (1)

Setup(coordinates = X)
 * Partial match of 'coordinates' against keyword 'coordinatesystems'
 Default differentiation variables for d_, D_ and dAlembertian are: {X = (x1, x2, x3, x4)}
 Systems of spacetime Coordinates are: {X = (x1, x2, x3, x4)}
 [coordinatesystems = {X}] (2)

g_[]

$$g_{\mu, \nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 (3)

Geodesics(tensornotation)

$$\frac{d^2}{d\tau^2} X^\mu(\tau)$$
 (4)

Geodesics()

$$\left[\frac{d^2}{d\tau^2} x4(\tau) = 0, \frac{d^2}{d\tau^2} x3(\tau) = 0, \frac{d^2}{d\tau^2} x2(\tau) = 0, \frac{d^2}{d\tau^2} x1(\tau) = 0 \right]$$
 (5)

Geodesics(output = solutions)

$$\{x1(\tau) = _C1 \tau + _C2, x2(\tau) = _C3 \tau + _C4, x3(\tau) = _C5 \tau + _C6, x4(\tau) = _C7 \tau + _C8\}$$
 (6)

g_[sc]
 Systems of spacetime Coordinates are: {X = (r, θ, φ, t)}
 Default differentiation variables for d_, D_ and dAlembertian are: {X = (r, θ, φ, t)}
 The Schwarzschild metric in coordinates [r, θ, φ, t]
 Parameters: [m]

$$g_{\mu, \nu} = \begin{bmatrix} \frac{r}{-r + 2m} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2 \sin(\theta)^2 & 0 \\ 0 & 0 & 0 & \frac{r - 2m}{r} \end{bmatrix}$$
 (7)

Geodesics(tensornotation)

$$\frac{d^2}{d\tau^2} X^\mu(\tau) + \Gamma^\mu_{\alpha\beta} \left(\frac{d}{d\tau} X^\alpha(\tau) \right) \left(\frac{d}{d\tau} X^\beta(\tau) \right) \quad (8)$$

Geodesics()

$$\left[\frac{d^2}{d\tau^2} \phi(\tau) = - \frac{2 \left(\frac{d}{d\tau} \phi(\tau) \right) \left(r(\tau) \left(\frac{d}{d\tau} \theta(\tau) \right) \cos(\theta(\tau)) + \left(\frac{d}{d\tau} r(\tau) \right) \sin(\theta(\tau)) \right)}{r(\tau) \sin(\theta(\tau))}, \quad (9)$$

$$\frac{d^2}{d\tau^2} \theta(\tau) = \frac{\sin(\theta(\tau)) \cos(\theta(\tau)) \left(\frac{d}{d\tau} \phi(\tau) \right)^2 r(\tau) - 2 \left(\frac{d}{d\tau} r(\tau) \right) \left(\frac{d}{d\tau} \theta(\tau) \right)}{r(\tau)},$$

$$\frac{d^2}{d\tau^2} r(\tau) = \frac{1}{(-r(\tau) + 2m) r(\tau)^3} \left(4 \left(-\frac{r(\tau)}{2} + m \right)^2 r(\tau)^3 (\cos(\theta(\tau)) \right.$$

$$\left. - 1) (\cos(\theta(\tau)) + 1) \left(\frac{d}{d\tau} \phi(\tau) \right)^2 - 4 \left(-\frac{r(\tau)}{2} + m \right)^2 r(\tau)^3 \left(\frac{d}{d\tau} \theta(\tau) \right)^2 \right.$$

$$\left. + 4m \left(-\frac{r(\tau)}{2} + m \right)^2 \left(\frac{d}{d\tau} t(\tau) \right)^2 - m \left(\frac{d}{d\tau} r(\tau) \right)^2 r(\tau)^2 \right), \frac{d^2}{d\tau^2} t(\tau) =$$

$$- \frac{2m \left(\frac{d}{d\tau} r(\tau) \right) \left(\frac{d}{d\tau} t(\tau) \right)}{r(\tau) (r(\tau) - 2m)} \Bigg]$$

theta(tau) = *(Pi, \^(2,-1))

$$\theta(\tau) = \frac{\pi}{2} \quad (10)$$